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A Robust Single-Valued Neutrosophic Soft Aggregation Operators in Multi-Criteria Decision Making

Chiranjibe Jana * and Madhumangal Pal

Department of Applied Mathematics with Oceanology and Computer Programming, Vidyasagar University, Midnapore 721102, India; mmpalvu@gmail.com

* Correspondence: jana.chiranjibe7@gmail.com; Tel.: +91-9647135650

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Abstract: Molodtsov originated soft set theory that was provided a general mathematical framework for handling with uncertainties in which we meet the data by affix parameterized factor during the information analysis as differentiated to fuzzy as well as neutrosophic set theory. The main object of this paper is to lay a foundation for providing a new approach of single-valued neutrosophic soft tool which is considering many problems that contain uncertainties. In present study, a new aggregation operators of single-valued neutrosophic soft numbers have so far not yet been applied for ranking of the alternatives in decision-making problems. To this propose work, single-valued neutrosophic soft weighted arithmetic averaging (SVNSWA) operator, single-valued neutrosophic soft weighted geometric averaging (SVNSWGA) operator have been used to compare two single-valued neutrosophic soft numbers (SVNSNs) for aggregating different single-valued neutrosophic soft input arguments in neutrosophic soft environment. Then, its related properties have been investigated. Finally, a practical example for Medical diagnosis problems provided to test the feasibility and applicability of the proposed work.

Keywords: single-valued neutrosophic soft number and its operations; SVN soft weighted arithmetic averaging operator; SVN soft weighted geometric averaging operator; decision-making

1. Introduction

Multi-criteria decision-making (MCDM) problems seek great attention in modern decision science. The method is addressed to select the best alternative among the finite set of alternatives as claimed by decision makers under the preference values of the alternatives. MCDM problems extensively applied with quantitative or qualitative attribute values and have a board application in medical diagnosis [1,2], ecology [3], sensor network [4] management science and engineering [5,6], economic [7], market prediction and engineering technology [8], transport service problem [9] etc. As our modern society move forward with the decision-making process, so it always faces imprecise, vague and uncertain facts to take a decision in solving decision-making problems. In order to solve imprecise and uncertain data, [10] initiated the idea of intuitionistic fuzzy set (IFS), a powerful extension of fuzzy set (FS) [11]. Even though (FS) and (IFS) are very powerful set to model decision problems containing uncertainties, in some cases these sets are not sufficient to overcome indeterminate and inconsistent statistics experience in real world problems. As SVNS [12] have strong acceptance for modeling of problems including the incomplete, indeterminate and inconsistent data. The aggregated information for the execution of the criteria for alternatives, weighted and order weighted aggregation operators [13–22] takes a significant role during the combination of the information process. The above aggregation operators based decision-making problems are not enough for the solution of real-world problems because they have insufficient of parameterizations. In real life problems involve different parameters. Most of MCDM problems, researchers can not consider parameterizations factor when they aggregate the information of the alternatives. Therefore, there are lack of information of the alternatives about the involves parameter. Motivated by the aforementioned limitations, this research developed a novel fuzzy-based MCDM approach for the evaluation of medical diagnosis problems. Initially, we introduced the SVSNs to quantify evaluation information on criteria and alternatives. We combined the SVNNs with the concept of soft set. Subsequently, this study proposed single-valued neutrosophic soft weighted averaging (SVNSWA) operator and single-valued neutrosophic soft weighted geometric (SVNSWG) operator to aggregate criteria by considering their parameterizations factors. We haved studied idempotency, boundedness, shift-invariance and Homogeneity property of these two kinds of soft weighted aggregation operators. The main advantage of these operators is that they are able to make smooth description of the real-world problems by the use of parameterizations factor. In order to rank the alternatives, aggregation operators lead to aggregate the over all information of the objects for the preferences of the decision maker into a collective one and hence find to a desirable according to its score values. To the best of our knowledge, the research developed on FSS and SVNSS is only about their basic theory and its applications, but, there have been no research done on single-valued neutrosophic soft aggregation numbers. So, it is a new issue and have a scope for future development in decision science. Therefore, decision-making problems in single-valued neutrosophic soft environment under proposed aggregation operators which makes us enough motivation to develop the propose problems. The main object of this article is to exhibit some aggregation operators under SVN data called as single-valued neutrosophic soft aggregation for collect the distinct priorities of the choices of this technique.

The remainder of this paper is organized as: In next Section, briefly survey some essential ideas of the FSS and SVNSS. In Section 3, we define some operational principles of single-valued neutrosophic soft numbers and then define single-valued neutrosophic soft weighted averaging (SVNSWA) operator, single-valued neutrosophic soft weighted geometric (SVNSWGA) operator and established its related properties. In next Section, we utilize those operators to create single-valued neutrosophic soft multi-criteria group decision-making problems. An interpretative case is specified for the selection of most illness patient in Section 5. In Section 6, a comparative analysis has been made between the existing works and the proposed study. Finally, in Section 7, follows a remark.

2. Literature Review

Neutrosophic set (NS) a tremendous branch of philosophy was proposed by Smarandache [23,24]. This proposed approach is characterized by three functions called (truth-, indeterminacy-,falsity)-membership functions. Therefore, (NS) has strong acceptance to develop models carrying indeterminate and inconsistent data. However, since codomain of membership functions of a (NS) is real standard or nonstandard subsets of $]^{-}0, 1^{+}[$, in some applications areas engineering and real scientific fields they have some difficulties in modeling of problems. To overcome difficulties in these areas, Wang et al. [12] defined the view of single-valued neutrosophic set (SVNs). As (SVNs) have strong acceptance for modeling of problems including the incomplete, indeterminate and inconsistent data. So, scholars have been investigating on how to find a proper one alternative and have obtained some achievements. Ye utilized [25] arithmetic and geometric aggregation functions under simplified neutrosophic numbers to develop MCDM problems. Garg and Nancy [26] have followed to the study of linguistic SVN prioritized aggregation function to propose a MADM problem. Wan et al. [27] introduced Frank Choquet Bonferroni mean operators and utilized this operator develop MCDM problems in single-valued bipolar neutrosophic environment. Shi and Ye [28] introduced Dombi aggregation operator to originate neutrosophic cubic Dombi (NCD) aggregation functions to study a decision-making problems. Wei and Zhang [29] utilized combination of power averaging and Bonferroni mean operator to developed SVN Bonferroni power aggregation operators to develop a MADM problem. Ulucay et al. [30] developed a decision-making problem using similarity measure method under bipolar neutrosophic environment. Abdel-Basset et al. [31] studied MCGDM based on neutrosophic hierarchy method. Abdel-Basset et al. [32] proposed strategic planning and decision-making based on neutrosophic AHP-SWOT analysis. Dalapati et al. [33] proposed cross entropy based MAGDM based on interval neutrosophic information. In [34], Bausys and Zavadskas provided VIKOR method based MCDM problems using interval neutrosophic numbers. Biswas et al. [35] utilized TOPSIS method for MCDM problems under SVN environment. Broumi et al. [36] introduced an algorithm to solve a neutrosophic shortest problems from source node to destination node. Sahin and Liu [37] derived correlation coefficient between two SVN hesitant fuzzy numbers. Jana et al. [38] studied trapezoidal neutrosophic aggregation functions and utilized these operators develop MADM problems. Recently, researchers have drawn attention to model interval rough sets with their application problems [39].

But the technique of the above papers are not enough for the solution of real-world problems because they have insufficient of parameterizations. In that context, soft set theory plays an important role to overcome such barrier and effectively applied to solve the conditions. Maji et al. [40,41] provided with the bridge connection between FS and IFS with soft sets theory [42]. Some hybrid models together with soft set theory have been develop in various uncertain environments such as on fuzzy soft set theory with parameterizations [43,44], fuzzy soft expert sets [45], generalized intuitionsitic fuzzy soft sets [46], IVIF soft sets [47,48] and its applications, bipolar intuitionistic fuzzy soft sets and decision-making [49], Hesitant fuzzy soft sets [50]. Jana and pal [51,52] have studied soft intersection BCK/BCI-algebras, and soft intersection group structure based on (α, β)-soft intersectional sets. Selvachandran and Peng [53] has found a modified TOPSIS method using vauge parameterized vegue soft set and gave its application in decision making. Recently, Arora and Garg [54] provided a new approach of aggregation operator using parameterized factor in intuitionistic fuzzy soft environment. In the same time, a tool combination of neutrosophic set and soft set have gave a momentum to the solution of real life problems in many directions. Karaaslan [55] used possibility theory to develop PNS-decision-making method using neutrosophic soft OR-product and AND-product. Broumi and Samarandache [56] proposed single-valued neutrosophic soft expert set and its application in decision-making. Ali et al. [57] gave an application of bipolar neutrosophic soft sets in decision making in the environment of bipolar neutrosophic set. Deli et al. [58] motivated to develop a decision-making method called ivnpivn-soft sets using neutrosophic information. In [59], Khalid and Abbas used soft set theory in distance measure.

In this study, multi-criteria decision-making approach is characterized by single-valued neutrosophic soft numbers (SVNSNs). The SVNSWA and SVNSWG aggregation operators are presented. Then, a medical diagnosis problems is solved by using these proposed operators.

3. Basic Concepts of FSS and SVNSS

In what follows, U, E and $\mathcal{P}(U)$ respectively denote universal set, parameter set and power set of U. Also, A $\subseteq E$.

Definition 1 ([11])**.** *Let X be a non-empty set. A fuzzy set μ of X is defined as a mapping $\mu : X \rightarrow [0,1]$, where $[0,1]$ is the usual interval of real numbers. We take $\mathbb{F}(X)$ as the set of all fuzzy subsets of X.*

Definition 2 ([42])**.** *A pair $(\mathcal{F}, E)$ is called a soft set over U if $\mathcal{F}$ is a mapping given by $\mathcal{F} : E \rightarrow \mathcal{P}(U)$. In other words, a soft over the universe U is a parameterized family of subsets of the universal set U. For $\varepsilon \in A$, $\mathcal{F}(\varepsilon)$ may be considered as the set of ε-elements of the soft $(\mathcal{F}, A)$, or as the set of ε-approximate elements of the soft set.*

The following example illustrate the above idea.

Example 1. *Let (X,τ) be a topological space, i.e. τ is a family of subsets of the set X called the open sets of X. Then, the family of open neighborhood $N(x)$ of point x, where $N(x) = \{V \in \tau | x \in V\}$, may be consider as the soft set $(N(x),\tau)$.*

Definition 3 ([40])**.** *Let U be the universe set and E be the set of parameters. Let $\mathcal{P}(U)$ be the power set of U and $A \subseteq E$, and $\mathcal{P}(U)$ is the collection of all fuzzy subsets of U, then $(\mathcal{F}, A)$ is called fuzzy soft set, where $\mathcal{F} : A \to \mathcal{P}(U)$.*

Example 2. *Let $U = \{M_1, M_2, M_3, M_4\}$ be the set of four mobiles under consideration and $E = \{beautiful(e_1),\ costly(e_2),\ batterybackup(e_3)$ and $apps(e_4)\}$ be a set of parameters then FSS for describing "attractiveness of the mobiles"*
is $(\mathcal{F}, A) = \{\mathcal{F}_{e_1}, \mathcal{F}_{e_2}, \mathcal{F}_{e_3}\}$, where $A = \{e_1, e_2, e_3\} \subseteq E$ and (F, A) can be defined as:
$\mathcal{F}_{e_1} = \{(M_1, 0.6),(M_2, 0.4),(M_3, 0.5),(M_4, 0.3)\}$,
$\mathcal{F}_{e_2} = \{(M_1, 0.7),(M_2, 0.6),(M_3, 0.5),(M_4, 0.4)\}$ *and*
$\mathcal{F}_{e_3} = \{(M_1, 0.9),(M_2, 0.5),(M_3, 0.3),(M_4, 0.6)\}$.

Definition 4 ([23])**.** *Let X be finite, with a generic element in X denoted by x. A NS $\tilde{c}$ in X is defined by*

$$\tilde{C} = \left\{ \langle T_C(x), I_C(x), F_C(x)\rangle | x \in X \right\},$$

where its truth-function T_C is presented by $T_C : X \to]0^-, 1^+[$, indeterminacy-function I_C presented $I_C : X \to]0^-, 1^+[$, and falsity- function $\hat{F}_C$ interpreted as $F_C : X \to]0^-, 1^+[$. Also, T_C, I_C and F_C are real standard or non-standard subsets of $]0^-, 1^+[$. There is no restriction on the sum of T_C, I_C and F_C, and so $0^- \leq T_C + I_C + F_C \leq 3^+$.

For real applications of NS, Wang et al. [12] introduced SVNs in the following definition.

Definition 5 ([12])**.** *Let X be a finite set, with a generic element in X denoted by x. A SVNS is defined as:*

$$\tilde{C} = \left\{ \langle T_C(x), I_C(x), F_C(x)\rangle | x \in X \right\},$$

where $T_C : X \to [0, 1]$ indicated the truth, $I_C : X \to [0, 1]$ is the indeterminacy and $F_C : X \to [0, 1]$ is the falsity function of x to C with the condition $0 \leq T_C + I_C + F_C \leq 3$.

Definition 6. *Let U be universal set and E be the parameter set. For $N \subset E$. Let $\mathcal{P}(U)$ called the subsets of single-valued neutrosophic sets of U. The term $(\mathcal{F}_C, C)$ is called single-valued neutrosophic soft sets of U, where $\mathcal{F}_C$ is a function follows as, $\mathcal{F}_C : N \to \mathcal{P}(U)$.*

Example 3. *Let $U = \{O_1, O_2, O_3, O_4\}$ be the set of four mobiles under consideration and*
$E = \{beautiful(e_1),\ costly(e_2),\ batterybackup(e_3)$ and $apps(e_4)\}$ be a set of parameters under SVNSS for describing "attractiveness of the mobiles" is $(C, N) = \{\mathcal{F}_{e_1}, \mathcal{F}_{e_2}, \mathcal{F}_{e_3}\}$, where $A = \{e_1, e_2, e_3\} \subseteq E$ and $(\tilde{C}, N)$ can be defined as:
$\tilde{C}_{e_1} = \{(O_1, 0.6, 0.4, 0.2),(O_2, 0.4, 0.5, 0.1),(O_3, 0.5, 0.2, 0.3),(O_4, 0.3, 0.6, 0.1)\}$,
$\tilde{C}_{e_2} = \{(O_1, 0.7, 0.1, 0.2),(O_2, 0.6, 0.3, 0.1),(O_3, 0.5, 0.3, 0.3),(O_4, 0.4, 0.4, 0.1)\}$ *and*
$\tilde{C}_{e_3} = \{(O_1, 0.9, 0.1, 0.3),(O_2, 0.5, 0.2, 0.2),(O_3, 0.3, 0.5, 0.1),(O_4, 0.6, 0.4, 0.4)\}$.

For the sake of simplicity, we denote the pair of $\tilde{C}_{e_t}(x_c) = \{\langle T_c(x), I_c(x), F_c(x)\rangle | x_s \in U\}$, i.e., $\tilde{C}_{e_{st}} = \langle T_{st}, I_{st}, F_{st}\rangle$ is called as single-valued neutrosophic soft (SVNSN) numbers. For the application purpose, it is necessary to define score function for ranking it. For this, a score function of $\tilde{C}_{e_{st}}$ is defined as

$$\Psi(\tilde{C}_{e_{st}}) = T_{st} - F_{st} \tag{1}$$

where, $\Psi(\tilde{C}_{e_{st}}) \in [0,1]$. By this definition, it is clear that the larger the $\Psi(\tilde{C}_{e_{st}})$, the larger is SVNSN $\tilde{C}_{e_{st}}$.

Example 4. *Let $\tilde{C}_{e_{11}} = \langle 0.6, 0.2, 0.2\rangle$ and $\tilde{C}_{e_{12}} = \langle 0.3, 0.5, 0.5\rangle$ be two SVNSNs, then by Equation (2), we get $\Psi(\tilde{C}_{e_{11}}) = 0.4$ and $\Psi(\tilde{C}_{e_{12}}) = -0.2$. Since $\Psi(\tilde{C}_{e_{11}}) > \Psi(\tilde{C}_{e_{12}})$ which imply $\tilde{C}_{e_{11}} > \tilde{C}_{e_{12}}$.*

However, there are some situation, where above function can not be used to compare SVNSNs. For example, let $\tilde{C}_{e_{11}} = \langle 0.6, 0.2, 0.2\rangle$ and $\tilde{C}_{e_{12}} = \langle 0.5, 0.1, 0.1\rangle$, then it is not possible to compare SVNSNs, which one of them is bigger as $\Psi(\tilde{C}_{e_{11}}) = \Psi(\tilde{C}_{e_{12}})$. To overcome this situation, we define accuracy function of $\tilde{C}_{e_{st}}$ as follows:

$$\mathcal{H}(\tilde{C}_{e_{st}}) = T_{st} + I_{st} + F_{st} \tag{2}$$

where, $\mathcal{H}(\tilde{C}_{e_{st}}) \in [0,1]$. Based on score function Ψ and accuracy function $\mathcal{H}$, defined order relation on two SVNSNs $\tilde{P}_{e_{st}}$ and $\tilde{Q}_{e_{st}}$ as follows:

(i) If $\Psi(\tilde{P}_{e_{st}}) < \Psi(\tilde{Q}_{e_{st}})$, then $\tilde{P}_{e_{st}} \prec \tilde{Q}_{e_{st}}$
(ii) If $\Psi(\tilde{P}_{e_{st}}) > \Psi(\tilde{Q}_{e_{st}})$, then $\tilde{P}_{e_{st}} \succ \tilde{Q}_{e_{st}}$
(iii) If $\Psi(\tilde{P}_{e_{st}}) = \Psi(\tilde{Q}_{e_{st}})$, then

  (1) If $\mathcal{H}(\tilde{P}_{e_{st}}) < \mathcal{H}(\tilde{Q}_{e_{st}})$, then $\tilde{P}_{e_{st}} \prec \tilde{Q}_{e_{st}}$.
  (2) If $\mathcal{H}(\tilde{P}_{e_{st}}) > \mathcal{H}(\tilde{Q}_{e_{st}})$, then $\tilde{P}_{e_{st}} \succ \tilde{Q}_{e_{st}}$.
  (3) If $\mathcal{H}(\tilde{P}_{e_{st}}) = \mathcal{H}(\tilde{Q}_{e_{st}})$, then $\tilde{P}_{e_{st}} \sim \tilde{Q}_{e_{st}}$.

4. Single-Valued Neutrosophic Soft Weighted Arithmetic Averaging (SVNSWAA) Operator

In this Section, an aggregation operators namely single-valued neutrosophic soft weighted averaging (SVNSWA) operator and single-valued neutrosophic soft weighted geometric averaging (BFSWGA) operator for neutrosophic soft numbers (SVNSNs) are proposed.

4.1. Operational Law for SVNSNs

Definition 7. *Let $\tilde{C}_e = \langle T, I, F\rangle$ and $\tilde{C}_{e_{11}} = \langle T_{11}, I_{11}, F_{11}\rangle)$ and $\tilde{C}_{e_{12}} = \langle T_{12}, I_{12}, F_{12}\rangle)$ be the three SVNSNs over the universe X, then following operations are defined as follows:*

(i) $\tilde{C}_{e_{11}} \oplus \tilde{C}_{e_{12}} = \big(\big\langle T_{11} + T_{12} - T_{11}T_{12}, I_{11}I_{12}, F_{11}F_{12}\big\rangle\big)$
(ii) $\tilde{C}_{e_{11}} \otimes \tilde{C}_{e_{12}} = \big(\big\langle T_{11}T_{12}, I_{11} + I_{12} - I_{11}I_{12}, F_{11} + F_{12} - F_{11}F_{12}\big\rangle\big)$
(iii) $\lambda\tilde{C}_e = \big(1-(1-T)^{\lambda}, I^{\lambda}, F^{\lambda}\big)$
(iv) $\tilde{C}_e^{\lambda} = \big(T^{\lambda}, 1-(1-I)^{\lambda}, 1-(1-F)^{\lambda}\big)$.

Definition 8. *Let $\tilde{C}_{e_{st}} = (T_{st}, I_{st}, F_{st})$ $(s = 1, 2, \ldots, m; t = 1, 2, \ldots, n)$ be a number of SVNSNs and ϕ_t, θ_s are the are weight vectors for the parameter e_t's and expert y_s's respectively, satisfying $\phi_t \geq 0$, $\theta_s \geq 0$ such that $\sum_{t=1}^{n} \phi_t = 1$ and $\sum_{s=1}^{m} \theta_s = 1$. Then single-valued neutrosophic soft weighted averaging (SVNSWA) operator is function $SVNSWA : \tilde{C}^n \to \tilde{C}$ such that*

$$SVNSWA(\tilde{C}_{e_{11}}, \tilde{C}_{e_{12}}, \ldots, \tilde{C}_{e_{mn}}) = \bigoplus_{t=1}^{n} \phi_t \Big(\bigoplus_{s=1}^{m} \theta_s \tilde{C}_{e_{st}}\Big). \tag{3}$$

We get the following theorem that follows on SVNSWA operator.

Theorem 1. *$\tilde{C}_{e_{st}} = (T_{st}, I_{st}, F_{st})$ $(s = 1, 2, \ldots, m; t = 1, 2, \ldots, n)$ be a number of (SVNSNs), then aggregated value of them using the SVNSWA operator is also a SVNSNs, and $SVNSWA(\tilde{C}_{e_{11}}, \tilde{C}_{e_{12}}, \ldots, \tilde{C}_{e_{mn}})$*

$$= \left\langle 1 - \prod_{t=1}^{n} \left(\prod_{s=1}^{m} (1 - T_{st})^{\theta_s} \right)^{\phi_t}, \prod_{t=1}^{n} \left(\prod_{s=1}^{m} \left(I_{st} \right)^{\theta_s} \right)^{\phi_t}, \prod_{t=1}^{n} \left(\prod_{s=1}^{m} \left(F_{st} \right)^{\theta_s} \right)^{\phi_t} \right\rangle. \quad (4)$$

Theorem 1 can be proved by the method of mathematical induction as follows:

Proof. For $m = 1$, we get $\theta_1 = 1$. Then by Definition 7 of operational law,

$$\begin{aligned} SVNSWA(\tilde{C}_{e_{11}}, \tilde{C}_{e_{12}}, \ldots, \tilde{C}_{e_{mn}}) &= \bigoplus_{t=1}^{n} \phi_t \left(\tilde{C}_{e_{1t}} \right) \\ &= \left\langle 1 - \prod_{t=1}^{n} \left(1 - T_{1t} \right)^{\phi_t}, \prod_{t=1}^{n} (I_{1t})^{\phi_t}, \prod_{t=1}^{n} (F_{1t})^{\phi_t} \right\rangle \\ &= \left\langle 1 - \prod_{t=1}^{n} \left(\prod_{s=1}^{1} (1 - F_{st})^{\theta_s} \right)^{\phi_t}, \prod_{t=1}^{n} \left(\prod_{s=1}^{1} (I_{st})^{\theta_s} \right)^{\phi_t}, \prod_{t=1}^{n} \left(\prod_{s=1}^{1} (F_{st})^{\theta_s} \right)^{\phi_t} \right\rangle. \end{aligned}$$

Again, for $n = 1$ and $\phi_1 = 1$ and hence,

$$\begin{aligned} SVNSWA(\tilde{C}_{e_{11}}, \tilde{C}_{e_{12}}, \ldots, \tilde{C}_{e_{mn}}) &= \left(\bigoplus_{s=1}^{m} \theta_{s1} \tilde{C}_{e_{1t}} \right) \\ &= \left\langle 1 - \prod_{t=1}^{m} \left(1 - T_{s1} \right)^{\theta_s}, \prod_{s=1}^{m} (I_{s1})^{\theta_s}, \prod_{s=1}^{m} (F_{s1})^{\theta_s} \right\rangle \\ &= \left\langle 1 - \prod_{t=1}^{1} \left(\prod_{s=1}^{m} (1 - T_{st})^{\theta_s} \right)^{\phi_t}, \prod_{t=1}^{1} \left(\prod_{s=1}^{m} (I_{st})^{\theta_s} \right)^{\phi_t}, \prod_{t=1}^{1} \left(\prod_{s=1}^{m} (F_{st})^{\theta_s} \right)^{\phi_t} \right\rangle. \end{aligned}$$

Thus, (5) is true for $m = 1$ and $n = 1$. Assume that (5) is true for $n = p_1 + 1$, $m = p_2$ and $n = p_1$, $m = p_2 + 1$, then it follows that

$$\bigoplus_{t=1}^{p_1+1} \phi_t \left(\bigoplus_{s=1}^{p_2} \theta_s \tilde{C}_{e_{st}} \right) = \left\langle 1 - \prod_{t=1}^{p_1+1} \left(\prod_{s=1}^{p_2} (1 - T_{st})^{\theta_s} \right)^{\phi_t}, \prod_{t=1}^{p_1+1} \left(\prod_{s=1}^{p_2} (I_{st})^{\theta_s} \right)^{\phi_t}, \prod_{t=1}^{p_1+1} \left(\prod_{s=1}^{p_2} (F_{st})^{\theta_s} \right)^{\phi_t} \right\rangle.$$

Also,

$$\bigoplus_{t=1}^{p_1} \phi_t \left(\bigoplus_{s=1}^{p_2+1} \theta_s \tilde{C}_{e_{st}} \right) = \left\langle 1 - \prod_{t=1}^{p_1} \left(\prod_{s=1}^{p_2+1} (1 - T_{st})^{\theta_s} \right)^{\phi_t}, \prod_{t=1}^{p_1} \left(\prod_{s=1}^{p_2+1} (I_{st})^{\theta_s} \right)^{\phi_t}, \prod_{t=1}^{p_1} \left(\prod_{s=1}^{p_2+1} (F_{st})^{\theta_s} \right)^{\phi_t} \right\rangle.$$

Now for $n = p_1 + 1$ and $m = p_2 + 1$, we obtained

$$\begin{aligned} \bigoplus_{t=1}^{p_1+1} \phi_t \left(\bigoplus_{s=1}^{p_2+1} \theta_s \tilde{C}_{e_{st}} \right) &= \bigoplus_{t=1}^{p_1+1} \phi_t \left(\bigoplus_{s=1}^{p_2} \theta_s \tilde{C}_{e_{st}} \oplus \theta_{p_2+1} \tilde{C}_{e_{(p_2+1)t}} \right) \\ &= \bigoplus_{t=1}^{p_1+1} \bigoplus_{s=1}^{p_2} \phi_t \theta_s \tilde{C}_{e_{st}} \bigoplus_{t=1}^{p_1+1} \phi_t \theta_{p_2+1} \tilde{C}_{e_{(p_2+1)t}} \Big) \end{aligned}$$

$$= \left\langle 1 - \prod_{t=1}^{p_1+1} \left(\prod_{s=1}^{p_2} (1 - T_{st})^{\theta_s} \right)^{\phi_t} \oplus 1 - \prod_{t=1}^{p_1+1} \left((1 - T_{(p_2+1)t})^{\theta_{p_2+1}} \right)^{\phi_t}, \right.$$

$$\left. \prod_{t=1}^{p_1+1} \left(\prod_{s=1}^{p_2} (I_{st})^{\theta_s} \right)^{\phi_t} \oplus \prod_{t=1}^{p_1+1} \left((I_{(p_2+1)t})^{\theta_{p_2+1}} \right)^{\phi_t}, \prod_{t=1}^{p_1+1} \left(\prod_{s=1}^{p_2} (F_{st})^{\theta_s} \right)^{\phi_t} \oplus \prod_{t=1}^{p_1+1} \left((F_{(p_2+1)t})^{\theta_{p_2+1}} \right)^{\phi_t} \right\rangle$$

$$= \left\langle 1 - \prod_{t=1}^{p_1+1} \left(\prod_{s=1}^{p_2+1} (1 - T_{st})^{\theta_s} \right)^{\phi_t}, \prod_{t=1}^{p_1+1} \left(\prod_{s=1}^{p_2+1} (I_{st})^{\theta_s} \right)^{\phi_t}, \prod_{t=1}^{p_1+1} \left(\prod_{s=1}^{p_2+1} (F_{st})^{\theta_s} \right)^{\phi_t} \right\rangle.$$

Thus, (5) is true for $n = p_1 + 1$, $m = p_2 + 1$, therefore by induction the results is hold for all $m, n \geq 1$.

Since, $0 \leq T_{st} \leq 1 \Leftrightarrow 0 \leq \prod_{s=1}^{m}(1-T_{st})^{\theta_s} \leq 1$ and hence, $0 \leq 1 - \prod_{t=1}^{n}(\prod_{s=1}^{m}(1-T_{st})^{\theta_s})^{\phi_t} \leq 1$. Also, $0 \leq I_{st} \leq 1 \Leftrightarrow 0 \leq \prod_{s=1}^{m}(I_{st})^{\theta_s} \leq 1 \Leftrightarrow 0 \leq \prod_{t=1}^{n}(\prod_{s=1}^{m}(I_{st})^{\theta_s})^{\phi_t} \leq 1$, and $0 \leq F_{st} \leq 1 \Leftrightarrow 0 \leq \prod_{s=1}^{m}(F_{st})^{\theta_s} \leq 1 \Leftrightarrow 0 \leq \prod_{t=1}^{n}(\prod_{s=1}^{m}(F_{st})^{\theta_s})^{\phi_t} \leq 1$. Thus, $0 \leq 1 - \prod_{t=1}^{n}(\prod_{s=1}^{m}(1-T_{st})^{\theta_s})^{\phi_t} + \prod_{t=1}^{n}(\prod_{s=1}^{m}(I_{st})^{\theta_s})^{\phi_t} + \prod_{t=1}^{n}(\prod_{s=1}^{m}(F_{st})^{\theta_s})^{\phi_t} \leq 3$. Hence, aggregated value obtained by SVNSWA is again a SVNSN. □

Corollary 1 ([60])**.** *For only one parameter e_1, i.e., $n = 1$, then SVNSWA operator reduces to SVNWA.*

$$SVNSWA(\tilde{C}_{e_{11}}, \tilde{C}_{e_{21}}, \ldots, \tilde{C}_{e_{m1}}) = \left\langle 1 - \prod_{s=1}^{m}(1-T_s)^{\theta_s}, \prod_{s=1}^{m}(I_s)^{\theta_s}, \prod_{s=1}^{m}(F_s)^{\theta_s} \right\rangle. \tag{5}$$

Therefore, it is justified that aggregation operator defined under SVNS environment is taken as a special case of the proposed operator.

Example 5. *Let $Y = \{y_1, y_2, y_3, y_4\}$ be the set of experts which are going to narrate the "attractiveness of two-wheeler bikes" under the set of parameters $E = \{e_1 = stylish,\ e_2 = weight,\ e_3 = milage,\ e_4 = price\}$. The rating value of the experts is assumed to be given in the form of SVNSNs$(C, E) = (T_{st}, I_{st}, F_{st})_{4\times 3}$ for each parameters which are given in the following table (Table 1).*

Table 1. Neutrosophic soft numbers.

Experts	e_1	e_2	e_3
y_1	$\langle 0.6, 0.2, 0.3\rangle$	$\langle 0.5, 0.5, 0.2\rangle$	$\langle 0.6, 0.3, 0.5\rangle$
y_2	$\langle 0.5, 0.4, 0.4\rangle$	$\langle 0.3, 0.2, 0.1\rangle$	$\langle 0.5, 0.4, 0.5\rangle$
y_3	$\langle 0.7, 0.1, 0.4\rangle$	$\langle 0.4, 0.3, 0.6\rangle$	$\langle 0.3, 0.1, 0.6\rangle$
y_4	$\langle 0.4, 0.5, 0.2\rangle$	$\langle 0.7, 0.2, 0.1\rangle$	$\langle 0.2, 0.6, 0.3\rangle$

Let $\phi = (0.3, 0.2, 0.5)^T$ and $\theta = (0.2, 0.1, 0.3, 0.4)^T$ be the weight vectors for the parameters and experts respectively. Then, we get by using Theorem 1 as:

$SVNSWA(\tilde{B}_{e_{11}}, \tilde{B}_{e_{12}}, \ldots, \tilde{B}_{e_{43}})$

$$= \left\langle 1 - \prod_{t=1}^{3}\left(\prod_{s=1}^{4}(1-T_{st})^{\theta_s}\right)^{\phi_t}, \prod_{t=1}^{3}\left(\prod_{s=1}^{4}(I_{st})^{\theta_s}\right)^{\phi_t}, \prod_{t=1}^{3}\left(\prod_{s=1}^{4}(I_{st})^{\theta_s}\right)^{\phi_t} \right\rangle$$

$\Big\langle 1 - \Big(\Big\{(1-0.6)^{0.2}(1-0.5)^{0.1}(1-0.7)^{0.3}(1-0.4)^{0.4}\Big\}^{0.3} \Big\{(1-0.5)^{0.2}(1-0.3)^{0.1}(1-0.4)^{0.3}(1-0.7)^{0.4}\Big\}^{0.2} \Big\{(1-0.6)^{0.2}(1-0.5)^{0.1}(1-0.3)^{0.3}(1-0.2)^{0.4}\Big\}^{0.5}\Big), \Big\{(0.2)^{0.2}\ (0.4)^{0.1}\ (0.1)^{0.3}\ (0.5)^{0.4}\Big\}^{0.3}$

$\Big\{(0.5)^{0.2}\ (0.2)^{0.1}\ (0.3)^{0.3}\ (0.2)^{0.4}\Big\}^{0.2}$

$\Big\{(0.3)^{0.2}\ (0.4)^{0.1}\ (0.1)^{0.3}\ (0.6)^{0.4}\Big\}^{0.5}, \Big\{(0.3)^{0.2}\ (0.4)^{0.1}\ (0.4)^{0.3}\ (0.2)^{0.4}\Big\}^{0.3}$

$\Big\{(0.2)^{0.2}\ (0.1)^{0.1}\ (0.6)^{0.3}\ (0.1)^{0.4}\Big\}^{0.2}$

$\Big\{(0.5)^{0.2}\ (0.5)^{0.1}\ (0.6)^{0.3}\ (0.3)^{0.4}\Big\}^{0.5}\Big\rangle$

$= \Big\langle (0.5317, 0.2755, 0.3256) \Big\rangle.$

We prove easily the following properties by using the operator SVNSWA.

Theorem 2 (Idempotency Property). *Let $\tilde{C}_{e_{st}} = (T_{st}, I_{st}, F_{st})$ $(s = 1, 2, \ldots, m; t = 1, 2, \ldots, n)$ be a number of SVNSNs are all equal, i.e., $\tilde{C}_{e_s}t = \tilde{C}_e$ for all s, t, then*

$$SVNSWA(\tilde{C}_{e_{11}}, \tilde{C}_{e_{12}}, \ldots, \tilde{C}_{e_{mn}}) = \tilde{C}_e. \tag{6}$$

Proof. Since $\tilde{C}_{e_{st}} = \tilde{C}_e = \langle T, I, F \rangle$ for all s, t. Then,

$$\begin{aligned} SVNSWA(\tilde{C}_{e_{11}}, \tilde{C}_{e_{12}}, \ldots, \tilde{C}_{e_{mn}}) &= \left\langle 1 - \prod_{t=1}^{n} \Big(\prod_{s=1}^{m} (1-T)^{\theta_s} \Big)^{\phi_t}, \prod_{t=1}^{n} \Big(\prod_{s=1}^{m} (I)^{\theta_s} \Big)^{\phi_t} \prod_{t=1}^{n} \Big(\prod_{s=1}^{m} (F)^{\theta_s} \Big)^{\phi_t} \right\rangle \\ &= \left\langle 1 - \Big((1-T)^{\sum_{s=1}^{m} \theta_s} \Big)^{\sum_{t=1}^{n} \phi_t}, \Big((I)^{\sum_{s=1}^{m} \theta_s} \Big)^{\sum_{t=1}^{n} \phi_t} \Big((F)^{\sum_{s=1}^{m} \theta_s} \Big)^{\sum_{t=1}^{n} \phi_t} \right\rangle \\ &= \Big\langle 1 - (1-T), I, F \Big\rangle \\ &= \Big\langle T, I, F \Big\rangle. \end{aligned}$$

The proof is completed. □

Theorem 3 (Boundedness Property). *Let $\tilde{C}_{e_{st}} = (T_{st}, I_{st}, F_{st})$ $(s = 1, 2, \ldots, m; t = 1, 2, \ldots, n)$ be a collection of SVNSNs. Let $\tilde{C}^{-}_{e_{st}} = \langle \min_t \min_s \{T_{st}\}, \max_t \max_s \{I_{st}\}, \max_t \max_s \{F_{st}\} \rangle$ and $\tilde{C}^{+}_{e_{st}} = \langle \max_t \max_s \{T_{st}\}, \min_t \min_s \{I_{st}\}, \min_t \min_s \{F_{st}\} \rangle$. Then,*

$$\tilde{C}^{-}_{e_{st}} \leq SVNSWA(\tilde{C}_{e_{11}}, \tilde{C}_{e_{12}}, \ldots, \tilde{C}_{e_{mn}}) \leq \tilde{C}^{+}_{e_{st}}.$$

Proof. Since, $\tilde{C}_{e_{st}} = (T_{st}, I_{st}, F_{st})$ be a SVNSNs then $\min_t \min_s \{T_{st}\} \leq T_{st} \leq \max_t \max_s \{T_{st}\}$ which implies that $1 - \max_t \max_s \{T_{st}\} \leq 1 - T_{st} \leq 1 - \min_t \min_s \{T_{st}\} \Leftrightarrow (1 - \max_t \max_s \{T_{st}\})^{\theta_s} \leq (1 - T_{st} \leq (1 - \min_t \min_s \{T_{st}\})^{\theta_s} \Leftrightarrow 1 - \max_t \max_s \{T_{st}\} \leq \prod_{s=1}^{m} (1 - T_{st})^{\theta_s} \leq 1 - \min_t \min_s \{T_{st}\} \Leftrightarrow (1 - \max_t \max_s \{T_{st}\})^{\sum_{t=1}^{n} \phi_t} \leq \prod_{t=1}^{n} (\prod_{s=1}^{m} (1 - T_{st})^{\theta_s})^{\sum_{t=1}^{n} \phi_t} \leq (1 - \min_t \min_s \{T_{st}\})^{\sum_{t=1}^{n} \phi_t} \Leftrightarrow 1 - \max_t \max_s \{T_{st}\} \leq \prod_{t=1}^{n} (\prod_{s=1}^{m} (1 - T_{st})^{\theta_s})^{\sum_{t=1}^{n} \phi_t} \leq 1 - \min_t \min_s \{T_{st}\}$. Therefore,

$$\max_t \max_s \{T_{st}\} \leq 1 - \prod_{t=1}^{n} \Big(\prod_{s=1}^{m} (1 - T_{st})^{\theta_s} \Big)^{\sum_{t=1}^{n} \phi_t} \leq \min_t \min_s \{T_{st}\}. \tag{7}$$

Again,

$$\min_t \min_s \{I_{st}\} \leq I_{st} \leq \max_t \max_s \{I_{st}\}$$

which finds $(\min_t \min_s \{I_{st}\})^{\sum_{s=1}^{m} \theta_s} \leq \prod_{s=1}^{m} (I_{st})^{\theta_s} \leq (\max_t \max_s \{I_{st}\})^{\sum_{s=1}^{m} \theta_s} \Leftrightarrow \min_t \min_s \{I_{st}\} \leq \prod_{s=1}^{m} (I_{st})^{\theta_s} \leq \max_t \max_s \{I_{st}\} \Leftrightarrow (\min_t \min_s \{I_{st}\})^{\phi_t} \leq (\prod_{s=1}^{m} (I_{st})^{\theta_s})^{\phi_t} \leq (\max_t \max_s \{I_{st}\})^{\phi_t} \Leftrightarrow (\min_t \min_s \{I_{st}\})^{\sum_{t=1}^{n} \phi_t} \leq \prod_{t=1}^{n} (\prod_{s=1}^{m} (I_{st})^{\theta_s})^{\phi_t} \leq (\max_t \max_s \{I_{st}\})^{\sum_{t=1}^{n} \phi_t}$, hence we get,

$$\min_t \min_s \{I_{st}\} \leq \prod_{t=1}^{n} (\prod_{s=1}^{m} (I_{st})^{\theta_s})^{\phi_t} \leq \max_t \max_s \{I_{st}\}. \tag{8}$$

and,

$$\min_t \min_s \{F_{st}\} \leq F_{st} \leq \max_t \max_s \{F_{st}\}$$

which follows $(\min_t \min_s \{F_{st}\})^{\sum_{s=1}^{m} \theta_s} \leq \prod_{s=1}^{m} (F_{st})^{\theta_s} \leq (\max_t \max_s \{F_{st}\})^{\sum_{s=1}^{m} \theta_s} \Leftrightarrow \min_t \min_s \{F_{st}\} \leq \prod_{s=1}^{m} (F_{st})^{\theta_s} \leq \max_t \max_s \{F_{st}\} \Leftrightarrow (\min_t \min_s \{F_{st}\})^{\phi_t} \leq (\prod_{s=1}^{m} (F_{st})^{\theta_s})^{\phi_t} \leq (\max_t \max_s \{F_{st}\})^{\phi_t} \Leftrightarrow (\min_t \min_s \{F_{st}\})^{\sum_{t=1}^{n} \phi_t} \leq \prod_{t=1}^{n} (\prod_{s=1}^{m} (F_{st})^{\theta_s})^{\phi_t} \leq (\max_t \max_s \{F_{st}\})^{\sum_{t=1}^{n} \phi_t}$, hence we get,

$$\min_t \min_s \{F_{st}\} \leq \prod_{t=1}^{n} (\prod_{s=1}^{m} (F_{st})^{\theta_s})^{\phi_t} \leq \max_t \max_s \{F_{st}\}. \tag{9}$$

Let $\beta \equiv SVNSWA(\tilde{C}_{e_{11}}, \tilde{C}_{e_{12}}, \ldots, \tilde{C}_{e_{mn}}) = \langle T_\beta, I_\beta, F_\beta \rangle$, then from Equations (7)–(9), $\min_t \min_s \{T_{st}\} \leq T_\beta \leq \max_t \max_s \{T_{st}\}$ and $\min_t \min_s \{I_{st}\} \leq I_\beta \leq \max_t \max_s \{I_{st}\}$, and $\min_t \min_s \{F_{st}\} \leq F_\beta \leq \max_t \max_s \{F_{st}\}$. Then by definition of score function

$$\Psi(\beta) = T_\beta - F_\beta \leq \max_t \max_s \{T_{st}\} - \min_t \min_s \{F_{st}\} = \Psi(\tilde{C}^{+}_{e_{st}})$$

$$\Psi(\beta) = T_\beta - F_\beta \geq \min_t \min_s \{T_{st}\} - \max_t \max_s \{F_{st}\} = \Psi(\tilde{C}^{-}_{e_{st}}).$$

Now, there are three cases arises:

Case 1. If $\Psi(\tilde{C}_{e_{st}}) < \Psi(\tilde{C}^{+}_{e_{st}})$ and $\Psi(\tilde{C}_{e_{st}}) > \Psi(\tilde{C}^{-}_{e_{st}})$, then by comparison of two SVNSNs, we have

$$\tilde{C}^{-}_{e_{st}} \leq SVNSWA(\tilde{C}_{e_{11}}, \tilde{C}_{e_{12}}, \ldots, \tilde{C}_{e_{mn}}) \leq \tilde{C}^{+}_{e_{st}}.$$

Case 2. If $\Psi(\tilde{C}_{e_{st}}) = \Psi(\tilde{C}^{+}_{e_{st}})$, i.e., $T_\beta + F_\beta + F_{st} = \max_t \max_s \{T_{st}\} + \min_t \min_s \{I_{st}\} + \min_t \min_s \{F_{st}\}$, then by above inequalities $T_\beta = \max_t \max_s \{T_{st}\}$ and $I_\beta = \min_t \min_s \{I_{st}\}$, and $F_\beta = \min_t \min_s \{F_{st}\}$

Therefore,

$$\mathcal{H} = T_\beta + I_\beta + F_\beta = \max_t \max_s \{T_{st}\} + \min_t \min_s \{I_{st}\} + \min_t \min_s \{F_{st}\} = \mathcal{H}(\tilde{C}^{+}_{e_{st}}),$$

then by comparison of two SVNSNs, we have

$$SVNSWA(\tilde{C}_{e_{11}}, \tilde{C}_{e_{12}}, \ldots, \tilde{C}_{e_{mn}}) = C^{+}_{e_{st}}.$$

Case 3. If $\Psi(\tilde{C}_{e_{st}}) = \Psi(\tilde{C}^{-}_{e_{st}})$, i.e., $T_\beta + I_\beta + F_\beta = \min_t \min_s \{T_{st}\} + \max_t \max_s \{I_{st}\} + \max_t \max_s \{F_{st}\}$, then by above inequalities $T_\beta = \min_t \min_s \{T_{st}\}$, $I_\beta = \max_t \max_s \{I_{st}\}$, and $F_\beta = \max_t \max_s \{F_{st}\}$.

Hence,

$$\mathcal{H} = T_\beta + I_\beta + F_{st} = \min_t \min_s \{T_{st}\} + \max_t \max_s \{I_{st}\} + \max_t \max_s \{F_{st}\} = \mathcal{H}(\tilde{C}^{-}_{e_{st}}),$$

then by comparison of two SVNSNs, we have

$$SVNSWA(\tilde{C}_{e_{11}}, \tilde{C}_{e_{12}}, \ldots, \tilde{C}_{e_{mn}}) = C^{-}_{e_{st}}.$$

Thus, proof is completed. □

Theorem 4 (Shift-invariance property). *If $\tilde{C}_e = \langle T, I, F \rangle$ be another SVNSN, then*

$$SVNSWA(\tilde{C}_{e_{11}} \oplus \tilde{C}_e, \tilde{C}_{e_{12}} \oplus \tilde{C}_e, \ldots, \tilde{C}_{e_{mn}} \oplus \tilde{C}_e) = SVNSWA(\tilde{C}_{e_{11}}, \tilde{C}_{e_{12}}, \ldots, \tilde{C}_{e_{mn}}) \oplus \tilde{C}_e$$

Proof. Since $\tilde{C}_e$ and $\tilde{C}_{e_{st}}$ are SVNSNs. Then, we have $\tilde{C}_e \oplus \tilde{C}_{e_{st}} = \Big\langle 1-(1-T)(1-T_{st}), II_{st}, FF_{st} \Big\rangle$. Hence, $SVNSWA(\tilde{C}_{e_{11}} \oplus \tilde{C}_e, \tilde{C}_{e_{12}} \oplus \tilde{C}_e, \ldots, \tilde{C}_{e_{mn}} \oplus \tilde{C}_e)$

$$\begin{aligned}
&= \bigoplus_{t=1}^{n} \phi_t \Big(\bigoplus_{s=1}^{m} \theta_t (\tilde{C}_{e_{st}} \oplus \tilde{C}_e) \Big) \\
&= \Big\langle 1 - \prod_{t=1}^{n} \Big(\prod_{s=1}^{m} (1-T_{st})^{\theta_s} (1-T)^{\theta_s} \Big)^{\phi_t}, \prod_{t=1}^{n} \Big(\prod_{s=1}^{m} (I_{st})^{\theta_s} (I)^{\theta_s} \Big)^{\phi_t}, \prod_{t=1}^{n} \Big(\prod_{s=1}^{m} (F_{st})^{\theta_s} (F)^{\theta_s} \Big)^{\phi_t} \Big\rangle \\
&= \Big\langle 1 - (1-I) \prod_{t=1}^{n} \Big(\prod_{s=1}^{m} (1-T_{st})^{\theta_s} \Big)^{\phi_t}, I \prod_{t=1}^{n} \Big(\prod_{s=1}^{m} (I_{st})^{\theta_s} \Big)^{\phi_t}, F \prod_{t=1}^{n} \Big(\prod_{s=1}^{m} (F_{st})^{\theta_s} \Big)^{\phi_t} \Big\rangle \\
&= \Big\langle 1 - \prod_{t=1}^{n} \Big(\prod_{s=1}^{m} (1-T_{st})^{\theta_s} \Big)^{\phi_t}, \prod_{t=1}^{n} \Big(\prod_{s=1}^{m} (I_{st})^{\theta_s} \Big)^{\phi_t}, \prod_{t=1}^{n} \Big(\prod_{s=1}^{m} (F_{st})^{\theta_s} \Big)^{\phi_t} \Big\rangle \oplus \Big\langle T, I, F \Big\rangle \\
&= SVNSWA(\tilde{C}_{e_{11}}, \tilde{C}_{e_{12}}, \ldots, \tilde{C}_{e_{mn}}) \oplus \tilde{C}_e.
\end{aligned}$$

Hence the result. □

Theorem 5 (Homogeneity property)**.** *For any real number $\lambda > 0$, we have*

$$SVNSWA(\lambda\tilde{C}_{e_{11}}, \lambda\tilde{C}_{e_{12}}, \ldots, \lambda\tilde{C}_{e_{mn}}) = \lambda SVNSWA(\tilde{C}_{e_{11}}, \tilde{C}_{e_{12}}, \ldots, \tilde{C}_{e_{mn}}).$$

Proof. Let $\tilde{C}_{e_{st}} = (T_{st}, I_{st}, F_{st})$ $(s = 1, 2, \ldots, m; t = 1, 2, \ldots, n)$ be a number of SVNSNs and $\lambda > 0$ be any real number. Then, $\lambda\tilde{C}_{e_{st}} = \Big\langle 1-(1-T_{st})^{\lambda}, (I_{st})^{\lambda}, (F_{st})^{\lambda} \rangle$. Thus,

$$\begin{aligned}
SVNSWA(\lambda\tilde{C}_{e_{11}}, \lambda\tilde{C}_{e_{12}}, \ldots, \lambda\tilde{C}_{e_{mn}}) &= \Big\langle 1 - \prod_{t=1}^{n} \Big(\prod_{s=1}^{m} (1-T_{st})^{\lambda\theta_s} \Big)^{\phi_t}, \prod_{t=1}^{n} \Big(\prod_{s=1}^{m} (I_{st})^{\lambda\theta_s} \Big)^{\phi_t}, \prod_{t=1}^{n} \Big(\prod_{s=1}^{m} (F_{st})^{\lambda\theta_s} \Big)^{\phi_t} \Big\rangle \\
&= \Big\langle 1 - \Big(\prod_{t=1}^{n} \Big(\prod_{s=1}^{m} (1-T_{st})^{\theta_s} \Big)^{\phi_t} \Big)^{\lambda}, \Big(\prod_{t=1}^{n} \Big(\prod_{s=1}^{m} (I_{st})^{\theta_s} \Big)^{\phi_t} \Big)^{\lambda}, \\
&\quad \Big(\prod_{t=1}^{n} \Big(\prod_{s=1}^{m} (F_{st})^{\theta_s} \Big)^{\phi_t} \Big)^{\lambda} \Big\rangle \\
&= \lambda\, SVNSWA(\tilde{C}_{e_{11}}, \tilde{C}_{e_{12}}, \ldots, \tilde{C}_{e_{mn}}).
\end{aligned}$$

Hence the proof is completed. □

4.2. Single-Valued Neutrosophic Soft Weighted Geometric Averaging (SVNSWGA) Operator

In this Section, we defined single-valued neutrosophic soft weighted geometric averaging (SVNSWGA) operator and studied

Definition 9. *Let $\tilde{C}_{e_{st}} = (T_{st}, I_{st}, F_{st})$ $(s = 1, 2, \ldots, m; t = 1, 2, \ldots, n)$ be a number of SVNSNs and ϕ_t, θ_s are the are weight vectors for the parameter e_t's and expert y_s's respectively, satisfying $\phi_t \geq 0$, $\theta_s \geq 0$ such that $\sum_{t=1}^{m} \phi_t = 1$ and $\sum_{s=1}^{n} \theta_s = 1$. Then single-valued neutrosophic soft weighted geometric (SVNSWGA) operator is a function $SVNSWGA : \tilde{C}^n \rightarrow \tilde{C}$ such that*

$$SVNSWGA(\tilde{C}_{e_{11}}, \tilde{C}_{e_{12}}, \ldots, \tilde{C}_{e_{mn}}) = \bigotimes_{t=1}^{n} \Big(\bigotimes_{s=1}^{m} \tilde{C}_{e_{st}}^{\theta_s} \Big)^{\phi_t}.$$

Theorem 6. *Then single-valued neutrosophic soft weighted geometric (SVNSWGA) operator is a function $SVNSWGA : \tilde{C}^n \rightarrow \tilde{C}$ such that*

$$SVNSWA(\tilde{C}_{e_{11}}, \tilde{C}_{e_{12}}, \ldots, \tilde{C}_{e_{1n}}) = \Big\langle \prod_{t=1}^{n} \Big(\prod_{s=1}^{m} (T_{st})^{\theta_s} \Big)^{\phi_t}, 1 - \prod_{t=1}^{n} \Big(\prod_{s=1}^{m} (1-I_{st})^{\theta_s} \Big)^{\phi_t}, 1 - \prod_{t=1}^{n} \Big(\prod_{s=1}^{m} (1-F_{st})^{\theta_s} \Big)^{\phi_t} \Big\rangle. \quad (10)$$

Proof. For $m = 1$ and $\theta_1 = 1$ then by Definition 7, we have

$$\begin{aligned} SVNSWGA(\tilde{C}_{e_{11}}, \tilde{C}_{e_{12}}, \ldots, \tilde{C}_{e_{1n}}) &= \bigotimes_{t=1}^{n} \tilde{C}_{e_{st}}^{\phi_t} \\ &= \Big\langle \prod_{t=1}^{n} (T_{1t})^{\phi_t}, 1 - \prod_{t=1}^{n} (1 - I_{1t})^{\phi_t}, 1 - \prod_{t=1}^{n} (1 - F_{1t})^{\phi_t} \Big\rangle \\ &= \Big\langle \prod_{t=1}^{n} \Big(\prod_{s=1}^{1} (T_{st})^{\theta_s} \Big)^{\phi_t}, 1 - \prod_{t=1}^{n} \Big(\prod_{s=1}^{1} (1 - I_{st})^{\theta_s} \Big)^{\phi_t}, 1 - \prod_{t=1}^{n} \Big(\prod_{s=1}^{1} (1 - F_{st})^{\theta_s} \Big)^{\phi_t} \Big\rangle. \end{aligned}$$

For $n = 1$ and $\phi_1 = 1$, then by Definition 9, we get

$$\begin{aligned} SVNSWGA(\tilde{C}_{e_{11}}, \tilde{C}_{e_{21}}, \ldots, \tilde{C}_{e_{m1}}) &= \bigotimes_{t=1}^{n} \tilde{C}_{e_{st}}^{\theta_s} \\ &= \Big\langle \prod_{s=1}^{m} (T_{s1})^{\theta_s}, 1 - \prod_{s=1}^{m} (1 - I_{s1})^{\theta_s}, 1 - \prod_{s=1}^{m} (1 - F_{s1})^{\theta_s} \Big\rangle \\ &= \Big\langle \prod_{t=1}^{1} \Big(\prod_{s=1}^{m} (T_{st})^{\theta_s} \Big)^{\phi_t}, 1 - \prod_{t=1}^{1} \Big(\prod_{s=1}^{m} (1 - I_{st})^{\theta_s} \Big)^{\phi_t}, 1 - \prod_{t=1}^{1} \Big(\prod_{s=1}^{m} (1 - F_{st})^{\theta_s} \Big)^{\phi_t} \Big\rangle. \end{aligned}$$

Assume that (10) is true for $n = p_1 + 1$, $m = p_2$ and $n = p_1$, $m = p_2 + 1$, then it follows that

$$\bigotimes_{t=1}^{p_1+1} \Big(\bigotimes_{s=1}^{p_2} \tilde{C}_{e_{st}}^{\theta_s} \Big)^{\phi_t} = \Big\langle \prod_{t=1}^{p_1+1} \Big(\prod_{s=1}^{p_2} (T_{st})^{\theta_s} \Big)^{\phi_t}, 1 - \prod_{t=1}^{p_1+1} \Big(\prod_{s=1}^{p_2} (1 - I_{st})^{\theta_s} \Big)^{\phi_t}, 1 - \prod_{t=1}^{p_1+1} \Big(\prod_{s=1}^{p_2} (1 - F_{st})^{\theta_s} \Big)^{\phi_t} \Big\rangle.$$

Also,

$$\bigotimes_{t=1}^{p_1} \Big(\bigotimes_{s=1}^{p_2+1} \tilde{C}_{e_{st}}^{\theta_s} \Big)^{\phi_t} = \Big\langle \prod_{t=1}^{p_1} \Big(\prod_{s=1}^{p_2+1} (T_{st})^{\theta_s} \Big)^{\phi_t}, 1 - \prod_{t=1}^{p_1} \Big(\prod_{s=1}^{p_2+1} (1 - I_{st})^{\theta_s} \Big)^{\phi_t}, 1 - \prod_{t=1}^{p_1} \Big(\prod_{s=1}^{p_2+1} (1 - F_{st})^{\theta_s} \Big)^{\phi_t} \Big\rangle.$$

Now for $n = p_1 + 1$ and $m = p_2 + 1$, we obtained

$$\begin{aligned} \bigotimes_{t=1}^{p_1+1} \Big(\bigotimes_{s=1}^{p_2+1} \tilde{C}_{e_{st}}^{\theta_s} \Big)^{\phi_t} &= \bigotimes_{t=1}^{p_1+1} \Big(\bigotimes_{s=1}^{p_2} \tilde{C}_{e_{st}}^{\theta_s} \otimes \tilde{C}_{e_{(p_2+1)t}}^{\theta_{p_2+1}} \Big)^{\phi_t} \\ &= \bigotimes_{t=1}^{p_1+1} \Big(\bigotimes_{s=1}^{p_2} \tilde{C}_{e_{st}}^{\theta_s} \Big)^{\phi_t} \bigotimes_{t=1}^{p_1+1} \Big(\Big(\tilde{C}_{e_{(p_2+1)t}}^{\theta_{p_2+1}} \Big)^{\phi_t} \Big) \\ &= \Big\langle \prod_{t=1}^{p_1+1} \Big(\prod_{s=1}^{p_2} (T_{st})^{\theta_s} \Big)^{\phi_t} \otimes \prod_{t=1}^{p_1+1} \Big((T_{(p_2+1)t})^{\theta_{p_2+1}} \Big)^{\phi_t}, \\ &\quad 1 - \prod_{t=1}^{p_1+1} \Big(\prod_{s=1}^{p_2} (1 - I_{st})^{\theta_s} \Big)^{\phi_t} \otimes 1 - \prod_{t=1}^{p_1+1} \Big((1 - I_{(p_2+1)t})^{\theta_{p_2+1}} \Big)^{\phi_t}, \\ &\quad 1 - \prod_{t=1}^{p_1+1} \Big(\prod_{s=1}^{p_2} (1 - F_{st})^{\theta_s} \Big)^{\phi_t} \otimes 1 - \prod_{t=1}^{p_1+1} \Big((1 - F_{(p_2+1)t})^{\theta_{p_2+1}} \Big)^{\phi_t} \Big\rangle \\ &= \Big\langle \prod_{t=1}^{p_1+1} \Big(\prod_{s=1}^{p_2+1} (T_{st})^{\theta_s} \Big)^{\phi_t}, 1 - \prod_{t=1}^{p_1+1} \Big(\prod_{s=1}^{p_2+1} (1 - I_{st})^{\theta_s} \Big)^{\phi_t}, \\ &\quad 1 - \prod_{t=1}^{p_1+1} \Big(\prod_{s=1}^{p_2+1} (1 - F_{st})^{\theta_s} \Big)^{\phi_t} \Big\rangle. \end{aligned}$$

Thus, (10) is true for $n = p_1 + 1$, $m = p_2 + 1$, therefore by induction the results is hold for all $m, n \geq 1$.

Since, $0 \leq I_{st} \leq 1 \Leftrightarrow 0 \leq \prod_{s=1}^{m} (1 - I_{st})^{\theta_s} \leq 1 \Leftrightarrow 0 \leq \prod_{t=1}^{n} \Big(\prod_{s=1}^{m} (1 - I_{st})^{\theta_s} \Big)^{\phi_t} \leq 1 \Leftrightarrow 0 \leq 1 - \prod_{t=1}^{n} \Big(\prod_{s=1}^{m} (1 - I_{st})^{\theta_s} \Big)^{\phi_t} \leq 1$, and $0 \leq F_{st} \leq 1 \Leftrightarrow 0 \leq \prod_{s=1}^{m} (1 - F_{st})^{\theta_s} \leq$

$1 \Leftrightarrow 0 \leq \prod_{t=1}^{n}\left(\prod_{s=1}^{m}(1-F_{st})^{\theta_s}\right)^{\phi_t} \leq 1 \Leftrightarrow 0 \leq 1-\prod_{t=1}^{n}\left(\prod_{s=1}^{m}(1-F_{st})^{\theta_s}\right)^{\phi_t} \leq 1$ On the other hand, $0 \leq T_{st} \leq 1 \Leftrightarrow 0 \leq \prod_{s=1}^{m}(T_{st})^{\theta_s} \leq 1 \Leftrightarrow 0 \leq \prod_{t=1}^{n}\left(\prod_{s=1}^{m}(T_{st})^{\theta_s}\right)^{\phi_t} \leq 1$. Therefore, $0 \leq 1-\prod_{t=1}^{n}\left(\prod_{s=1}^{m}(1-I_{st})^{\theta_s}\right)^{\phi_t}+1-\prod_{t=1}^{n}\left(\prod_{s=1}^{m}(1-F_{st})^{\theta_s}\right)^{\phi_t}+\prod_{t=1}^{n}\left(\prod_{s=1}^{m}(T_{st})^{\theta_s}\right)^{\phi_t} \leq 3.$

Thus, aggregated value obtained by SVNSWG operator is again a SVNSN. □

Example 6. *Let $Y = \{y_1, y_2, y_3, y_4\}$ be the set of experts which are going to narrate the "attractiveness of two-wheeler bikes" under the set of parameters $E = \{e_1 = stylish,\ e_2 = weight,\ e_3 = milage,\ e_4 = price\}$. The rating value of the experts is assumed to be given in the form of SVNSNs$(B, E) = (T_{st}, I_{st}, F_{st})_{4\times 3}$ for each parameters which are given in the following table (Table 2).*

Table 2. Neutrosophic soft numbers.

Experts	e_1	e_2	e_3
y_1	$\langle 0.6, 0.3, 0.3\rangle$	$\langle 0.7, 0.2, 0.5\rangle$	$\langle 0.4, 0.2, 0.2\rangle$
y_2	$\langle 0.5, 0.4, 0.2\rangle$	$\langle 0.5, 0.3, 0.2\rangle$	$\langle 0.8, 0.1, 0.1\rangle$
y_3	$\langle 0.4, 0.1, 0.2\rangle$	$\langle 0.7, 0.1, 0.5\rangle$	$\langle 0.5, 0.2, 0.3\rangle$
y_4	$\langle 0.6, 0.2, 0.4\rangle$	$\langle 0.6, 0.3, 0.4\rangle$	$\langle 0.6, 0.3, 0.3\rangle$

Let $\phi = (0.3, 0.2, 0.5)^T$ and $\theta = (0.2, 0.1, 0.3, 0.4)^T$ be the weight vectors for the parameters and experts respectively. Then, we get by using Theorem 6 as:
$SVNSWGA(\tilde{C}_{e_{11}}, \tilde{C}_{e_{12}}, \ldots, \tilde{C}_{e_{43}})$

$$= \left\langle \prod_{t=1}^{3}\left(\prod_{s=1}^{4}(T_{st})^{\theta_s}\right)^{\phi_t}, 1-\prod_{t=1}^{3}\left(\prod_{s=1}^{4}(1-I_{st})^{\theta_s}\right)^{\phi_t}, 1-\prod_{t=1}^{3}\left(\prod_{s=1}^{4}(1-F_{st})^{\theta_s}\right)^{\phi_t} \right\rangle$$

$$\Big\langle \Big(\Big\{(0.6)^{0.2}(0.5)^{0.1}(0.4)^{0.3}(0.6)^{0.4}\Big\}^{0.3} \Big\{(0.7)^{0.2}(0.5)^{0.1}(0.7)^{0.3}(0.6)^{0.4}\Big\}^{0.2}$$
$$\Big\{(0.4)^{0.2}(0.8)^{0.1}(0.5)^{0.3}(0.6)^{0.4}\Big\}^{0.5}\Big),$$
$$1-\Big\{(1-0.3)^{0.2}\,(1-0.4)^{0.1}\,(1-0.1)^{0.3}\,(1-0.2)^{0.4}\Big\}^{0.3}$$
$$\Big\{(1-0.2)^{0.2}\,(1-0.3)^{0.1}\,(1-0.1)^{0.3}\,(1-0.3)^{0.4}\Big\}^{0.2}$$
$$\Big\{(1-0.2)^{0.2}\,(1-0.1)^{0.1}\,(1-0.2)^{0.3}\,(1-0.3)^{0.4}\Big)^{0.5}, 1-\Big\{(1-0.3)^{0.2}\,(1-0.2)^{0.1}\,(1-0.2)^{0.3}\,(1-0.4)^{0.4}\Big\}^{0.3}$$
$$\Big\{(1-0.5)^{0.2}\,(1-0.2)^{0.1}\,(1-0.5)^{0.3}\,(1-0.4)^{0.4}\Big\}^{0.2}$$
$$\Big\{(1-0.2)^{0.2}\,(1-0.1)^{0.1}\,(1-0.3)^{0.3}\,(1-0.3)^{0.4}\Big)^{0.5}\Big\rangle$$
$$= \Big\langle (0.5518, 0.2261, 0.3138) \Big\rangle.$$

SVNSWGA operator satisfies the following properties as similar as SVNSWA operator

- **(Idempotency Property)** If $\tilde{C}_{e_{st}} = \tilde{C}_e = \langle T, I, F\rangle$ for all s, t, then

$$SVNSWGA(\tilde{C}_{e_{11}}, \tilde{C}_{e_{12}}, \ldots, \tilde{C}_{e_{mn}}) = \tilde{C}_e.$$

- **(Boundedness Property)** If $\tilde{C}^-_{e_{st}} = \langle \min_t \min_s\{T_{st}\}, \max_t \max_s\{I_{st}\}, \max_t \max_s\{F_{st}\}\rangle$ and if

$$\tilde{C}^+_{e_{st}} = \langle \max_t \max_s\{T_{st}\}, \min_t \min_s\{I_{st}\}, \min_t \min_s\{F_{st}\}\rangle,$$

then

$$\tilde{C}^{-}_{e_{st}} \leq SVNSWGA(\tilde{C}_{e_{11}}, \tilde{C}_{e_{12}}, \ldots, \tilde{C}_{e_{mn}}) \leq \tilde{C}^{+}_{e_{st}}.$$

- (**Shift-invariance Property**) Let $\tilde{C}_e = \langle T, I, F \rangle$ be another SVNSN then

$$SVNSWGA(\tilde{C}_{e_{11}} \otimes \tilde{C}_e, \tilde{C}_{e_{12}} \otimes \tilde{C}_e, \ldots, \tilde{C}_{e_{mn}} \otimes \tilde{C}_e) = SVNSWGA(\tilde{C}_{e_{11}}, \tilde{C}_{e_{12}}, \ldots, \tilde{C}_{e_{mn}}) \otimes \tilde{C}_e.$$

- (**Homogeneity Property**) For any real number $\lambda > 0$, we have

$$SVNSWGA(\tilde{C}^{\lambda}_{e_{11}}, \tilde{C}^{\lambda}_{e_{12}}, \ldots, \tilde{C}^{\lambda}_{e_{mn}}) = \Big(SVNSWGA(\tilde{C}_{e_{11}}, \tilde{C}_{e_{12}}, \ldots, \tilde{C}_{e_{mn}})\Big)^{\lambda}.$$

5. Model for MCDM Method Using Single-Valued Soft Information

In this Section, we shall present multi-criteria decision making (MCDM) method using single-valued neutrosophic soft weighted averaging operator (SVNSWA) and single-valued neutrosophic soft weighted geometric (SVNSWGA) operator in the environment of single-valued neutrosophic soft numbers.

An Approach Based on Proposed Operators

Let $\tilde{Y} = \{\tilde{Y}_1, \tilde{Y}_2, \ldots, \tilde{Y}_l\}$ be the discrete set of alternatives is evaluated by the set of m experts $\{y_1, y_2, \ldots, y_m\}$ under the constraints of n parameters $E = \{e_1, e_2, \ldots, e_n\}$. Let $\theta = (\theta_1, \theta_2, \ldots, \theta_m)^T$ and $\phi = (\phi_1, \phi_2, \ldots, \phi_n)^T$ are respectively denote the weight vectors of the m experts $x'_s s$ and n parameters $e'_t s$ that $\theta_s > 0, \theta \in [0,1]$ such that $\sum_{s=1}^{m} \theta_s = 1$ and $\phi_t > 0, \phi \in [0,1]$ such that $\sum_{t=1}^{n} \phi_s = 1$. In order to choice the best l alternates by the preference of n experts in the form of SVNSNs $\tilde{C}_{e_{st}} = \langle T_{st}, I_{st}, F_{st} \rangle$ where $0 \leq T_{st} + I_{st} + F_{st} \leq 3$ and collective over all decision matrix is expressed as $\tilde{M} = (\tilde{C}_{e_{st}})_{m \times n}$. By these preference values of the experts, the aggregated SVNSN $\tilde{C}_{e_k}$ for the alternatives $\tilde{p}_k$ $(k = 1, 2, \ldots, l)$ is $\tilde{C}_{e_k} = \langle T_k, I_k, F_k \rangle$ by applying weighted averaging or geometric averaging operators which is given in Equations (5) and (8). Ranking order of the alternatives is determine based on the score function of the aggregated values of SVNSNs $\tilde{C}_{e_k}$ $(k = 1, 2, \ldots, l)$.

In the following algorithm we propose to solve MCDM problem with single-valued neutrosophic soft information using SVNSWA and SVNSWGA operators.

Step 1. Collect all the information in the form of single-valued neutrosophic soft matrix $C = \langle T_{st}, I_{st}, F_{st} \rangle$ $(s = 1, 2, \ldots, m; t = 1, 2, \ldots, n)$ related to each alternatives under different parameters e_k $(k = 1, 2, \ldots, l)$ as

$$\tilde{C}_{m \times n} = M = \begin{bmatrix} (T_{11}, I_{11}, F_{st}) & (T_{12}, I_{12}, F_{12}) & \ldots & (T_{1n}, I_{1n}, F_{1n}) \\ (T_{21}, I_{21}, F_{21}) & (T_{22}, I_{22}, F_{22}) & \ldots & (T_{2n}, I_{2n}, F_{2n}) \\ \vdots & \vdots & \ddots & \vdots \\ (T_{m1}, I_{m1}, F_{m1}) & (T_{m2}, I_{m2}, F_{m2}) & \ldots & (T_{mn}, I_{mn}, F_{mn}) \end{bmatrix}$$

Step 2. To normalize the aggregated decision matrix by transforming values of benefit type (B) into cost (C) type by using the formula depicted in [61].

$$g_{ij} = \begin{cases} \tilde{C}^{c}_{e_{st}}, & \text{if} e_t \in \tilde{B} \\ \tilde{C}_{e_{st}}, & \text{if} e_t \in \tilde{C} \end{cases}$$

where $\tilde{C}^{c}_{e_{st}} = \langle 1 - F_{st}, I_{st}, T_{st} \rangle$ is the complement of $\tilde{C}_{e_{st}} = \langle T_{st}, I_{st}, F_{st} \rangle$.

Step 3. Aggregate the SVNSNs $\tilde{C}_{e_{st}}$ $(s = 1, 2, \ldots, m; t = 1, 2, \ldots, n)$ for each alternatives Y_k $(k = 1, 2, \ldots, l)$ into collective decision matrix Ψ_k using SVNSWA or (SVNSWGA) operators.

Step 4. Using Equation (1) we get the score value of Ψ_k $(k = 1, 2, \ldots, l)$ for each alternatives A_k $(k = 1, 2, \ldots, l)$.

Step 5. Rank all the alternative A_k $(k = 1, 2, \ldots, l)$ in order to choice the best one(s) in accordance with Ψ_k $(k = 1, 2, \ldots, l)$.

Step 6. End.

6. Numerical Example

In the above described decision-making method has been demonstrated with a practical example about the Medical diagnosis. The experts of five doctors m_1, m_2, m_3, m_4, m_5 whose weight vector is $\theta = (0.2, 0.15, 0.2, 0.3, 0.15)^T$, will give their judgement based on the diagnosis of four patients Y_1, Y_2, Y_3, Y_4 under the parameters

$$E = \{Temparature(e_1), Headache(e_2), Stomachpain(e_3), Cough(e_4), Chestpain(e_5)\}$$

with weight vector $\phi = (0.2, 0.1, 0.3, 0.15, 0.25)^T$. Then, we utilize the developed method to get most desirable candidate(s).

6.1. By SVNSWA Operator

The steps of the proposed approach performed and their corresponding details are reviewed as follows:

Step 1. The given patients are being evaluated by five experts doctors to give their grades in terms of SVNSNs and are found in Tables 3–6 respectively for each candidate.

Step 2. All the parameters are of same type, so, there is no required for normalization.

Table 3. Single-valued neutrosophic soft matrix for the patient Y_1.

Experts	e_1	e_2	e_3	e_4	e_5
m_1	$\langle 0.6, 0.2, 0.3\rangle$	$\langle 0.5, 0.5, 0.5\rangle$	$\langle 0.6, 0.3, 0.2\rangle$	$\langle 0.3, 0.5, 0.7\rangle$	$\langle 0.3, 0.4, 0.4\rangle$
m_2	$\langle 0.6, 0.4, 0.5\rangle$	$\langle 0.3, 0.2, 0.4\rangle$	$\langle 0.5, 0.4, 0.5\rangle$	$\langle 0.4, 0.6, 0.4\rangle$	$\langle 0.4, 0.1, 0.2\rangle$
m_3	$\langle 0.7, 0.1, 0.4\rangle$	$\langle 0.4, 0.3, 0.4\rangle$	$\langle 0.3, 0.1, 0.4\rangle$	$\langle 0.7, 0.3, 0.2\rangle$	$\langle 0.6, 0.2, 0.6\rangle$
m_4	$\langle 0.4, 0.5, 0.3\rangle$	$\langle 0.7, 0.2, 0.1\rangle$	$\langle 0.2, 0.6, 0.4\rangle$	$\langle 0.6, 0.1, 0.5\rangle$	$\langle 0.5, 0.1, 0.5\rangle$
m_5	$\langle 0.5, 0.2, 0.2\rangle$	$\langle 0.6, 0.1, 0.6\rangle$	$\langle 0.6, 0.2, 0.2\rangle$	$\langle 0.4, 0.1, 0.1\rangle$	$\langle 0.4, 0.2, 0.3\rangle$

Table 4. Single-valued neutrosophic soft matrix for the patient Y_2.

Experts	e_1	e_2	e_3	e_4	e_5
m_1	$\langle 0.3, 0.4, 0.4\rangle$	$\langle 0.8, 0.1, 0.1\rangle$	$\langle 0.7, 0.1, 0.1\rangle$	$\langle 0.4, 0.1, 0.3\rangle$	$\langle 0.2, 0.3, 0.4\rangle$
m_2	$\langle 0.5, 0.1, 0.3\rangle$	$\langle 0.5, 0.2, 0.2\rangle$	$\langle 0.4, 0.2, 0.3\rangle$	$\langle 0.6, 0.1, 0.2\rangle$	$\langle 0.2, 0.1, 0.3\rangle$
m_3	$\langle 0.2, 0.1, 0.2\rangle$	$\langle 0.4, 0.1, 0.4\rangle$	$\langle 0.5, 0.4, 0.5\rangle$	$\langle 0.4, 0.2, 0.6\rangle$	$\langle 0.5, 0.2, 0.2\rangle$
m_4	$\langle 0.7, 0.2, 0.3\rangle$	$\langle 0.5, 0.1, 0.4\rangle$	$\langle 0.6, 0.2, 0.4\rangle$	$\langle 0.4, 0.2, 0.1\rangle$	$\langle 0.7, 0.1, 0.1\rangle$
m_5	$\langle 0.5, 0.2, 0.2\rangle$	$\langle 0.5, 0.3, 0.2\rangle$	$\langle 0.4, 0.1, 0.5\rangle$	$\langle 0.3, 0.2, 0.2\rangle$	$\langle 0.6, 0.4, 0.2\rangle$

Table 5. Single-valued neutrosophic soft matrix for the patient Y_3.

Experts	e_1	e_2	e_3	e_4	e_5
m_1	$\langle 0.4, 0.3, 0.2\rangle$	$\langle 0.8, 0.1, 0.4\rangle$	$\langle 0.5, 0.2, 0.3\rangle$	$\langle 0.6, 0.1, 0.2\rangle$	$\langle 0.2, 0.3, 0.3\rangle$
m_2	$\langle 0.5, 0.1, 0.2\rangle$	$\langle 0.4, 0.2, 0.3\rangle$	$\langle 0.3, 0.2, 0.4\rangle$	$\langle 0.4, 0.2, 0.3\rangle$	$\langle 0.5, 0.2, 0.2\rangle$
m_3	$\langle 0.2, 0.1, 0.1\rangle$	$\langle 0.4, 0.2, 0.3\rangle$	$\langle 0.4, 0.2, 0.3\rangle$	$\langle 0.5, 0.1, 0.4\rangle$	$\langle 0.5, 0.1, 0.1\rangle$
m_4	$\langle 0.7, 0.2, 0.4\rangle$	$\langle 0.5, 0.2, 0.2\rangle$	$\langle 0.2, 0.1, 0.5\rangle$	$\langle 0.6, 0.2, 0.5\rangle$	$\langle 0.6, 0.2, 0.2\rangle$
m_5	$\langle 0.5, 0.3, 0.3\rangle$	$\langle 0.5, 0.4, 0.4\rangle$	$\langle 0.4, 0.1, 0.2\rangle$	$\langle 0.4, 0.1, 0.5\rangle$	$\langle 0.7, 0.2, 0.3\rangle$

Table 6. Single-valued neutrosophic soft matrix for the patient Y_4.

Experts	e_1	e_2	e_3	e_4	e_5
m_1	$\langle 0.4, 0.1, 0.2\rangle$	$\langle 0.5, 0.4, 0.4\rangle$	$\langle 0.5, 0.2, 0.1\rangle$	$\langle 0.5, 0.1, 0.3\rangle$	$\langle 0.2, 0.3, 0.7\rangle$
m_2	$\langle 0.5, 0.1, 0.3\rangle$	$\langle 0.3, 0.2, 0.2\rangle$	$\langle 0.3, 0.2, 0.3\rangle$	$\langle 0.4, 0.2, 0.2\rangle$	$\langle 0.3, 0.2, 0.2\rangle$
m_3	$\langle 0.5, 0.3, 0.4\rangle$	$\langle 0.5, 0.1, 0.5\rangle$	$\langle 0.4, 0.2, 0.2\rangle$	$\langle 0.2, 0.2, 0.4\rangle$	$\langle 0.5, 0.2, 0.4\rangle$
m_4	$\langle 0.6, 0.2, 0.3\rangle$	$\langle 0.4, 0.5, 0.2\rangle$	$\langle 0.3, 0.2, 0.4\rangle$	$\langle 0.7, 0.2, 0.1\rangle$	$\langle 0.3, 0.1, 0.1\rangle$
m_5	$\langle 0.5, 0.3, 0.1\rangle$	$\langle 0.4, 0.6, 0.1\rangle$	$\langle 0.4, 0.2, 0.3\rangle$	$\langle 0.3, 0.1, 0.4\rangle$	$\langle 0.6, 0.3, 0.3\rangle$

Step 3. The opinion of doctors for each patient Y_k $(k = 1, 2, 3, 4)$ are aggregated by using Equation (5) given as follows: $\Omega_1 = \langle 0.4918, 0.2326, 0.3404\rangle$, $\Omega_2 = \langle 0.5154, 0.1700, 0.2522\rangle$, $\Omega_3 = \langle 0.4800, 0.1656, 0.2753\rangle$ and $\Omega_4 = \langle 0.4319, 0.1942, 0.2444\rangle$.

Step 4. The values of score functions are: $\Psi(\Omega_1) = 0.1514$, $\Psi(\Omega_2) = 0.2632$, $\Psi(\Omega_3) = 0.2047$ and $\Psi(\Omega_4) = 0.1875$.

Step 5. Ranking all the patients Y_k $(k = 1, 2, 3, 4)$ in accordance with the value of the score $\Psi(\Omega_k)$ $(k = 1, 2, 3, 4)$ of the overall single-valued neutrosophic soft numbers as $Y_2 \succ Y_3 \succ Y_4 \succ Y_1$.

Step 6. Therefore, Y_2 is the more illness patient than other patients.

6.2. By Using SVNSWGA Operator

If we apply SVNSWGA operator on the proposed problem for the selection of appropriate candidate(s) that follows the following steps:

Step 3. The aggregated values for each patients Y_k $(k = 1, 2, 3, 4)$ using SVNSWGA operator are as follows from Equation (10): $\Omega_1 = \langle 0.4432, 0.3084, 0.3913\rangle$, $\Omega_2 = \langle 0.4515, 0.1999, 0.3044\rangle$, $\Omega_3 = \langle 0.4224, 0.1825, 0.3079\rangle$ and $\Omega_4 = \langle 0.3960, 0.2204, 0.3030\rangle$.

Step 4. The values of score functions are: $\Psi(\Omega_1) = 0.0519$, $\Psi(\Omega_2) = 0.1471$, $\Psi(\Omega_3) = 0.1145$ and $\Psi(\Omega_4) = 0.0930$.

Step 5. Ranking all the candidates Y_k $(k = 1, 2, 3, 4)$ in accordance with the value of the score $\Psi(\Omega_k)$ $(k = 1, 2, 3, 4)$ of the overall single-valued neutrosophic soft numbers as $Y_2 \succ Y_3 \succ Y_4 \succ Y_1$.

Step 6. Hence, Y_2 is the most illness patient diagnosed by the expert doctors .

From the above analysis, it is clear that although overall rating values of the alternatives are different by using two operators, but ranking order of the alternatives are similar, the most illness patient is Y_2 among four patients.

7. Comparative Analysis

To compare the proposed work with the existing approach, an analysis has been made based on aggregation operator (see [25,60,62]). In that reason, the different parameters of single-valued neutrosophic soft numbers are aggregated by using weighted averaging operator corresponding to the weighted vector $(0.2, 0.1, 0.3, 0.15, 0.25)^T$ and then obtained aggregated single-valued neutrosophic soft matrix for the different candidates Y_k $(k = 1, 2, 3, 4)$ given in Table 7. From this evaluated matrix, a comparative study has been established with the existing work based on weighted aggregation operator on simplified neutrosophic numbers, single-valued neutrosophic Domi weighted aggregation operators and single-valued neutrosophic weighted averaging operators developed by researchers [25,60,62] for each candidate are shown in Table 8. It also shows that proposed method is stable with compare the existing methods. From the Table 7, we can see that the candidate Y_2 is most illness person diagnosed by the experts doctors. The characteristic comparison between propose study with existing methods are given in Table 9. The propose method utilize advance technique to compare the existing works [56,57] where a decision making method has been develop based on some soft algebraic operations in neutrosophic soft environment but present paper leads a decision making method based on aggregating single-valued neutrosophic soft arguments in the environment of SVN soft numbers.The advantages of this paper is that they are capable to facilitate the descriptions of the real-world problems situation with the help of their parameterizations property. Therefore, proposed method can be utilize to solve decision-making problems instead of other existing operator in the environment of SVN soft numbers.

Table 7. Aggregated value of single-valued neutrosophic soft matrix for the patients.

Experts	C_1	C_2	C_3	C_4
m_1	$\langle 0.4884, 0.3378, 0.3411\rangle$	$\langle 0.5161, 0.1737, 0.2200\rangle$	$\langle 0.4854, 0.2018, 0.2679\rangle$	$\langle 0.4168, 0.1861, 0.2531\rangle$
m_2	$\langle 0.4680, 0.2805, 0.3761\rangle$	$\langle 0.4256, 0.1320, 0.2711\rangle$	$\langle 0.4211, 0.1741, 0.2725\rangle$	$\langle 0.3605, 0.1741, 0.2449\rangle$
m_3	$\langle 0.5545, 0.1565, 0.3990\rangle$	$\langle 0.4251, 0.2000, 0.3327\rangle$	$\langle 0.4092, 0.1320, 0.1911\rangle$	$\langle 0.4333, 0.2024, 0.3322\rangle$
m_4	$\langle 0.4513, 0.2531, 0.3594\rangle$	$\langle 0.6182, 0.1569, 0.2169\rangle$	$\langle 0.5246, 0.1625, 0.3470\rangle$	$\langle 0.4573, 0.1843, 0.2024\rangle$
m_5	$\langle 0.5081, 0.1682, 0.2226\rangle$	$\langle 0.4747, 0.2012, 0.2633\rangle$	$\langle 0.5223, 0.1702, 0.2952\rangle$	$\langle 0.4650, 0.2415, 0.2253\rangle$

Table 8. Comparison analysis with the existing method.

Methods	$\Psi(\Omega_1)$	$\Psi(\Omega_2)$	$\Psi(\Omega_3)$	$\Psi(\Omega_4)$	**Ranking Order**
Proposed SVNSWA operator	0.1514	0.2632	0.2047	0.1875	$Y_2 \succ Y_3 \succ Y_4 \succ Y_1$
Proposed SVNSWGA operator	0.0519	0.1417	0.1145	0.0930	$Y_2 \succ Y_3 \succ Y_4 \succ Y_1$
Ye [25] by SNWAA operator	0.1440	0.2583	0.1969	0.1822	$Y_2 \succ Y_3 \succ Y_4 \succ Y_1$
Ye [25] by SNWGA operator	0.1487	0.2506	0.1999	0.1852	$Y_2 \succ Y_3 \succ Y_4 \succ Y_1$
Chen and Ye [62] SVNDWA operator	0.1594	0.2732	0.2131	0.1915	$Y_2 \succ Y_3 \succ Y_4 \succ Y_1$
Chen and Ye [62] SVNDWG operator	0.1378	0.2383	0.1875	0.1760	$Y_2 \succ Y_3 \succ Y_4 \succ Y_1$
Sahin [60] SVNWAA operator	0.1515	0.2632	0.2047	0.1869	$Y_2 \succ Y_3 \succ Y_4 \succ Y_1$
Sahin [60] SVNWGA operator	0.1412	0.2445	0.1921	0.1791	$Y_2 \succ Y_3 \succ Y_4 \succ Y_1$

Table 9. Characteristic comparisons of different methods.

Methods	*Fuzzy Information Easier*	*Weather Aggregate Parameter Information*
Ye [25]	Yes	No
Chen and Ye [62]	Yes	No
Sahin [60]	Yes	No
Proposed operators	Yes	Yes

8. Conclusions

In this article, we have studied multi-criteria group decision-making problem using in the environment of single-valued neutrosophic soft information. We have introduced two new operators namely (SVNSWA) operator, (SVNSWGA) operators in SVN soft environment. The different features of those recommended operators is deliberated. For this purpose, firstly some algebraic structures of two SVNSNs are given and their operational rules are defined. The two aggregation operators have been proposed in the environment of SVNS numbers. Some properties of these two kinds of operators have been established. We justify the propose method with the existing methods and a characteristic comparison also shown to demonstrate advantage and applicability of the proposed method. A Medical decision-making problems has been studied based on SVNSWA and SVNSWGA operators under the environment of SVN soft information. The main advantages of these operators is that they are able to make smooth description of the real-world problems by the use of parameterizations factor. Ultimately, a realistic example for the selection of most illness patient is provided to develop a strategy and in accordance with expounding the utility and effectiveness of the proposed method. In the future work, the propose model further develop new soft aggregation operators for simplified neutrosophic sets and apply them to solve practical applications like engineering [63], group decision-making [64], expert system, information fusion system, fault diagnosis, robotics design [65] and other domains under different fuzzy soft environments.

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