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Logarithmic Hybrid Aggregation Operators Based on Single Valued Neutrosophic Sets and Their Applications in Decision Support Systems

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Abstract: Recently, neutrosophic sets are found to be more general and useful to express incomplete, indeterminate and inconsistent information. The purpose of this paper is to introduce new aggregation operators based on logarithmic operations and to develop a multi-criteria decision-making approach to study the interaction between the input argument under the single valued neutrosophic (SVN) environment. The main advantage of the proposed operator is that it can deal with the situations of the positive interaction, negative interaction or non-interaction among the criteria, during decision-making process. In this paper, we also defined some logarithmic operational rules on SVN sets, then we propose the single valued neutrosophic hybrid aggregation operators as a tool for multi-criteria decision-making (MCDM) under the neutrosophic environment and discussd some properties. Finally, the detailed decision-making steps for the single valued neutrosophic MCDM problems were developed, and a practical case was given to check the created approach and to illustrate its validity and superiority. Besides this, a systematic comparison analysis with other existent methods is conducted to reveal the advantages of our proposed method. Results indicate that the proposed method is suitable and effective for decision process to evaluate their best alternative.

Keywords: single valued neutrosophic sets; logarithmic operational laws; logarithmic aggregation operators; MCGDM problems

1. Introduction

The information involves, in most of the real-life decision-making problems are often incomplete, indeterminate and inconsistent. Fuzzy set theory introduced by Zadeh [1] deals with imprecise, inconsistent information. Although fuzzy set information proved to be very handy but it cannot express the information about rejection. Atanassov [2] introduced the intuitionistic fuzzy set (IFS) to bring in non-membership. Non membership function represents degree of rejection. To incorporate indeterminate and inconsistent information, in addition to incomplete information, the concept of neutrosophic set (NS) proposed by Smarandache [3]. A NS generalizes the notion of the classic set, fuzzy set (FS) [1], IFS [2], paraconsistent set [4], dialetheist set, paradoxist set [4], and tautological set [4] to name a few. In NS, indeterminacy is quantified explicitly, and truth, indeterminacy, and falsity memberships are expressed independently. The NS generalizes different types of non-crisp sets but in real scientific and engineering applications the NS and the set-theoretic operators require to be specified. For a detailed study on NS we refer to [5–17].

Related Work

Most of the weighted aggregation operators consider situations in which criteria and preferences of experts are independent, which means that additivity is a main property of these operators. However, in real life decision-making problems, the criteria of the problems are often interdependent or interactive.

Most of the weighted average operators are based on the basic algebraic product and algebraic sum of single valued neutrosophic numbers (SVNNs) which are not the only operations available to model the intersection and union of SVNNs. The logarithmic algebraic product and sum are two good alternatives of algebraic operations which can be used the model intersection and union of SVNNs. Moreover, it is observed that in the literature there is little investigation on aggregation operators utilizing the logarithmic operations on SVNNs. For a detailed review on the applications of logarithmic operations, we refer to [10]. As already mentioned that the single valued neutrosophic set (SVNS) is an effective tool to describe the uncertain, incomplete and indeterminate information. The logarithmic single valued neutrosophic hybrid and logarithmic generalized single valued neutrosophic algebraic operators have the ability to express interactions among the criteria and it can replace the weighted average to aggregate dependent criteria for obtaining more accurate results. Motivated by these, we find it interesting to develop the logarithmic single valued neutrosophic hybrid aggregation operators for decision-making with neutrosophic information.

Also, we proposed the possibility of a degree-ranking technique for SVNNs from the probability point of view, since the ranking of SVNNs is very important for decision-making under the SVN environment. Furthermore, we proposed a multi-criteria decision-making model based on the logarithmic single valued neutrosophic hybrid weighted operators. Forstudy the multi-criteria decision-making models, we refer [18–31].

The aim of writing this paper is to introduce a decision-making method for MCDM problems in which there exist interrelationships among the criteria. The contributions of this research are:

(1) A novel logarithmic operations for neutrosophic information is defined, which can overcome the weaknesses of algebraic operations and obtain the relationship between various SVNNs.

(2) Logarithmic operators for IFSs are extended to logarithmic single-valued neutrosophic hybrid operators and logarithmic generalized single-valued neutrosophic operators, namely, logarithmic single valued neutrosophic hybrid weighted averaging (L-SVNHWA), logarithmic single valued neutrosophic hybrid weighted geometric (L-SVNHWG), logarithmic generalized single-valued neutrosophic weighted averaging (L-GSVNWA) and logarithmic single-valued neutrosophic weighted geometric (L-GSVNWG) to SVNSs, which can overcome the algebraic operators drawbacks.

(3) A decision-making approach to handle the MCDM problems under the neutrosophic informations is introduced.

To attain our research goals which are stated above, the arrangement of the paper is offered as: Section 2 concentrates on basic definitions and operations of existing extensions of fuzzy set theories. In Section 3, some novel logarithmic operational laws of SVNSs are presented. Section 4 defines the logarithmic hybrid aggregation operators for SVNNs. In Section 5, an algorithm for handling the neutrosophic MCDM problem based on the developed logarithmic operators is presented. In Section 5.1, an application to verify the novel method is given and Section 5.2 presents the comparison study about algebraic and logarithmic aggregation operators. Section 6 consists of the conclusion of the study.

2. Preliminaries

This section includes the concepts and basic operations of existing extensions of fuzzy sets to make the study self contained.

Definition 1. *[2] For a set $\Re$, by an intuitionistic fuzzy set in $\Re$, we have a structure*

$$\zeta = \{\langle P_\sigma(r), N_\sigma(r)\rangle \mid r \in \Re\}, \quad (1)$$

in which $P_\sigma : \Re \to \Theta$ and $N_\sigma : \Re \to \Theta$ indicate the membership and non-membership grades in $\Re$, $\Theta = [0,1]$ be the unit interval. Also the following condition is satisfied by P_σ and N_σ, $0 \le P_\sigma(r) + N_\sigma(r) \le 1; \forall\, r \in \Re$. Then ζ is said to be intuitionistic fuzzy set in $\Re$.

Definition 2. *[32] For a set $\Re$, by a neutrosophic set in $\Re$, we have a structure*

$$\zeta = \{\langle P_\sigma(r), I_\sigma(r), N_\sigma(r)\rangle \mid r \in \Re\}, \quad (2)$$

in which $P_\sigma : \Re \to \Theta$, $I_\sigma : \Re \to \Theta$ and $N_\sigma : \Re \to \Theta$ indicate the truth, indeterminacy and falsity memberships in $\Re$, $\Theta =]0^-, 1^+[$. Also the following condition is satisfied by P_σ, I_σ and N_σ, $0^- \le P_\sigma(r) + I_\sigma(r) + N_\sigma(r) \le 3^+; \forall\, r \in \Re$. Then, ζ is said to be neutrosophic set in $\Re$.

Definition 3. *[33] For a set $\Re$, by a single valued neutrosophic set in $\Re$, we mean a structure*

$$\zeta = \{\langle P_\sigma(r), I_\sigma(r), N_\sigma(r)\rangle \mid r \in \Re\}, \quad (3)$$

in which $P_\sigma : \Re \to \Theta$, $I_\sigma : \Re \to \Theta$ and $N_\sigma : \Re \to \Theta$ indicate the truth, indeterminacy and falsity memberships in $\Re$, $\Theta = [0,1]$. Also the following condition is satisfied by P_σ, I_σ and N_σ, $0 \le P_\sigma(r) + I_\sigma(r) + N_\sigma(r) \le 3; \forall r \in \Re$. Then, ζ is said to be a single valued neutrosophic set in $\Re$. We denote this triplet $\zeta = \langle P_\sigma(r), I_\sigma(r), N_\sigma(r)\rangle$, in whole study called SVNN.

Ye [14], Wang et al. [33] and [34] proposed the basic operations of SVNNs, which are as follows:

Definition 4. *[34] For any two SVNNs $\zeta_p = \left\langle P_{\xi_p}(r), I_{\xi_p}(r), N_{\xi_p}(r)\right\rangle$ and $\zeta_q = \left\langle P_{\xi_q}(r), I_{\xi_q}(r), N_{\xi_q}(r)\right\rangle$ in $\Re$. The union, intersection and compliment are proposed as:*
(1) $\zeta_p \subseteq \zeta_q$ iff $\forall r \in \Re$, $P_{\xi_p}(r) \le P_{\xi_q}(r)$, $I_{\xi_p}(r) \ge I_{\xi_q}(r)$ and $N_{\xi_p}(r) \ge N_{\xi_q}(r)$;
(2) $\zeta_p = \zeta_q$ iff $\zeta_p \subseteq \zeta_q$ and $\zeta_q \subseteq \zeta_p$;
(3) $\zeta_p \cup \zeta_q = \left\langle \max\left(P_{\xi_p}, P_{\xi_q}\right),\ \min\left(I_{\xi_p}, I_{\xi_q}\right),\ \min\left(N_{\xi_p}, N_{\xi_q}\right)\right\rangle$;
(4) $\zeta_p \cap \zeta_q = \left\langle \min\left(P_{\xi_p}, P_{\xi_q}\right),\ \max\left(I_{\xi_p}, I_{\xi_q}\right),\ \max\left(N_{\xi_p}, N_{\xi_q}\right)\right\rangle$;
(5) $\zeta_p^c = \left\langle N_{\xi_p}, I_{\xi_p}, P_{\xi_p}\right\rangle$.

Definition 5. *[13,15,33] For any two SVNNs $\zeta_p = \left\langle P_{\xi_p}(r), I_{\xi_p}(r), N_{\xi_p}(r)\right\rangle$ and $\zeta_q = \left\langle P_{\xi_q}(r), I_{\xi_q}(r), N_{\xi_q}(r)\right\rangle$ in $\Re$ and $\beta \ge 0$.Then the operations of SVNNs are proposed as:*
(1) $\zeta_p \oplus \zeta_q = \left\{P_{\xi_p} + P_{\xi_q} - P_{\xi_p} \cdot P_{\xi_q},\ I_{\xi_p} \cdot I_{\xi_q},\ N_{\xi_p} \cdot N_{\xi_q}\right\}$;
(2) $\beta \cdot \zeta_p = \left\{1 - (1 - P_{\xi_p})^\beta,\ (I_{\xi_p})^\beta,\ (N_{\xi_p})^\beta\right\}$;
(3) $\zeta_p \otimes \zeta_q = \left\{P_{\xi_p} \cdot P_{\xi_q},\ I_{\xi_p} + I_{\xi_q} - I_{\xi_p} \cdot I_{\xi_q}, N_{\xi_p} + N_{\xi_q} - N_{\xi_p} \cdot N_{\xi_q}\right\}$;
(4) $\zeta_p^\beta = \left\{(P_{\xi_p})^\beta,\ 1 - (1 - I_{\xi_p})^\beta, 1 - (1 - N_{\xi_p})^\beta\right\}$.
(5) $$\beta^{\zeta_p} = \begin{cases} \left(\beta^{1-P_{\xi_p}}, 1 - \beta^{I_{\xi_p}}, 1 - \beta^{N_{\xi_p}}\right) & if\ \beta \in (0,1) \\ \left(\left(\frac{1}{\beta}\right)^{1-P_{\xi_p}}, 1 - \left(\frac{1}{\beta}\right)^{I_{\xi_p}}, 1 - \left(\frac{1}{\beta}\right)^{N_{\xi_p}}\right) & if\ \beta \ge 1 \end{cases}$$

Definition 6. *[33] For any three SVNNs $\zeta_p = \left\langle P_{\xi_p}(r), I_{\xi_p}(r), N_{\xi_p}(r)\right\rangle$, $\zeta_q = \left\langle P_{\xi_q}(r), I_{\xi_q}(r), N_{\xi_q}(r)\right\rangle$ and $\zeta_l = \langle P_{\sigma_l}(r), I_{\sigma_l}(r), N_{\sigma_l}(r)\rangle$ in $\Re$ and $\beta_1, \beta_2 \ge 0$. Then, we have*

(1) $\zeta_p \oplus \zeta_q = \zeta_q \oplus \zeta_p$;
(2) $\zeta_p \otimes \zeta_q = \zeta_q \otimes \zeta_p$;
(3) $\beta_1(\zeta_p \oplus \zeta_q) = \beta_1\zeta_p \oplus \beta_1\zeta_q,\ \beta_1 > 0$;
(4) $(\zeta_p \otimes \zeta_q)^{\beta_1} = \zeta_p^{\beta_1} \otimes \zeta_q^{\beta_1},\ \beta_1 > 0$;
(5) $\beta_1\zeta_p \oplus \beta_2\zeta_p = (\beta_1 + \beta_2)\zeta_p, \beta_1 > 0, \beta_2 > 0$;
(6) $\zeta_p^{\beta_1} \otimes \zeta_p^{\beta_2} = \zeta_p^{\beta_1+\beta_2}, \beta_1 > 0, \beta_2 > 0$;
(7) $(\zeta_p \oplus \zeta_q) \oplus \zeta_l = \zeta_p \oplus (\zeta_q \oplus \zeta_l)$;
(8) $(\zeta_p \otimes \zeta_q) \otimes \zeta_l = \zeta_p \otimes (\zeta_q \otimes \zeta_l)$.

Definition 7. *[33] For any SVNN $\zeta_p = \left\langle P_{\xi_p}(r), I_{\xi_p}(r), N_{\xi_p}(r)\right\rangle$ in $\Re$. Then score and accuracy values are defined as:*
(1) $\widetilde{S}(\zeta_p) = P_{\xi_p} - I_{\xi_p} - N_{\xi_p}$
(2) $\widetilde{A}(\zeta_p) = P_{\xi_p} + I_{\xi_p} + N_{\xi_p}$

The above definitions of score and accuracy funtions suggest which SVNN is greater than other SVNNs. The comparison technique is defined in following definition.

Definition 8. *[33] For any SVNNs $\zeta_p = \left\langle P_{\xi_p}(r), I_{\xi_p}(r), N_{\xi_p}(r)\right\rangle$ $(p = 1, 2)$ in $\Re$.*

Then comparison techniques are proposed as:
(1) *If* $\widetilde{S}(\zeta_1) < \widetilde{S}(\zeta_2)$, *then* $\zeta_1 < \zeta_2$,
(2) *If* $\widetilde{S}(\zeta_1) > \widetilde{S}(\zeta_2)$, *then* $\zeta_1 > \zeta_2$,
(3) *If* $\widetilde{S}(\zeta_1) = \widetilde{S}(\zeta_2)$, *and*
(a) $\widetilde{A}(\zeta_1) < \widetilde{A}(\zeta_2)$, *then* $\zeta_1 < \zeta_2$,
(b) $\widetilde{A}(\zeta_1) > \widetilde{A}(\zeta_2)$, *then* $\zeta_1 > \zeta_2$,
(c) $\widetilde{A}(\zeta_1) = \widetilde{A}(\zeta_2)$, *then* $\zeta_1 \approx \zeta_2$.

Garg and Nancy [10] proposed some logarithmic-based aggregation operators, which are as follows:

Definition 9. *[10] For any collection of SVNNs $\zeta_p = \left\langle P_{\xi_p}(r), I_{\xi_p}(r), N_{\xi_p}(r)\right\rangle$ $(p = 1, 2, ..., n)$ in $\Re$, with $0 < \sigma_p \leq \min\left\{P_{\xi_p}, 1 - I_{\xi_p}, 1 - N_{\xi_p}\right\} < 1$, $\sigma \neq 1$. Then, the structure of logarithmic single valued neutrosophic weighted averaging (L-SVNWA) operator is defined as:*

$$L - SVNWA(\zeta_1, \zeta_2, ..., \zeta_n) = \begin{pmatrix} 1 - \prod_{p=1}^{n}\left(\ell og_{\sigma_p} P_{\xi_p}\right)^{\beta_p}, \\ \prod_{p=1}^{n}\left(\ell og_{\sigma_p}\left(1 - I_{\xi_p}\right)\right)^{\beta_p}, \\ \prod_{p=1}^{n}\left(\ell og_{\sigma_p}\left(1 - N_{\xi_p}\right)\right)^{\beta_p} \end{pmatrix}, \quad (4)$$

where β_p $(p = 1, 2, ..., n)$ are weight vectors with $\beta_p \geq 0$ and $\sum_{p=1}^{n} \beta_p = 1$.

Definition 10. *[10] For any collection of SVNNs* $\zeta_p = \left\langle P_{\xi_p}(r), I_{\xi_p}(r), N_{\xi_p}(r) \right\rangle$ $(p = 1, 2, ..., n)$ *in* $\Re$, *with* $0 < \sigma_p \leq \min\left\{P_{\xi_p}, 1 - I_{\xi_p}, 1 - N_{\xi_p}\right\} < 1, \sigma \neq 1$. *Then, the structure of the logarithmic single-valued neutrosophic-ordered weighted averaging (L-SVNOWA) operator is defined as:*

$$L - SVNOWA(\zeta_1, \zeta_2, ..., \zeta_n) = \begin{pmatrix} 1 - \prod_{p=1}^{n} \left(\ell og_{\sigma_p} P_{\zeta_{\eta(p)}}\right)^{\beta_p}, \\ \prod_{p=1}^{n} \left(\ell og_{\sigma_p}\left(1 - I_{\zeta_{\eta(p)}}\right)\right)^{\beta_p}, \\ \prod_{p=1}^{n} \left(\ell og_{\sigma_p}\left(1 - N_{\zeta_{\eta(p)}}\right)\right)^{\beta_p} \end{pmatrix}, \tag{5}$$

where $\beta_p\,(p = 1, 2, ..., n)$ *are weighting vector with* $\beta_p \geq 0$, $\sum_{p=1}^{n} \beta_p = 1$ *and pth largest weighted value is* $\zeta_{\eta(p)}$ *consequently by total order* $\zeta_{\eta(1)} \geq \zeta_{\eta(2)} \geq ... \geq \zeta_{\eta(n)}$.

Definition 11. *[10] For any collection of SVNNs* $\zeta_p = \left\langle P_{\xi_p}(r), I_{\xi_p}(r), N_{\xi_p}(r) \right\rangle$ $(p = 1, 2, ..., n)$ *in* $\Re$, *with* $0 < \sigma_p \leq \min\left\{P_{\xi_p}, 1 - I_{\xi_p}, 1 - N_{\xi_p}\right\} < 1, \sigma \neq 1$. *Then, the structure of logarithmic single-valued neutrosophic-weighted geometric (L-SVNWG) operator is defined as:*

$$L - SVNWG(\zeta_1, \zeta_2, ..., \zeta_n) = \begin{pmatrix} \prod_{p=1}^{n} \left(1 - \ell og_{\sigma_p} P_{\xi_p}\right)^{\beta_p}, \\ 1 - \prod_{p=1}^{n} \left(1 - \ell og_{\sigma_p}\left(1 - I_{\xi_p}\right)\right)^{\beta_p}, \\ 1 - \prod_{p=1}^{n} \left(1 - \ell og_{\sigma_p}\left(1 - N_{\xi_p}\right)\right)^{\beta_p} \end{pmatrix}, \tag{6}$$

where $\beta_p\,(p = 1, 2, ..., n)$ *are weight vectors with* $\beta_p \geq 0$ *and* $\sum_{p=1}^{n} \beta_p = 1$.

Definition 12. *[10] For any collection of SVNNs* $\zeta_p = \left\langle P_{\xi_p}(r), I_{\xi_p}(r), N_{\xi_p}(r) \right\rangle$ $(p = 1, 2, ..., n)$ *in* $\Re$, *with* $0 < \sigma_p \leq \min\left\{P_{\xi_p}, 1 - I_{\xi_p}, 1 - N_{\xi_p}\right\} < 1, \sigma \neq 1$. *Then, the structure of logarithmic single valued neutrosophic ordered weighted geometric (L-SVNOWG) operator is defined as:*

$$L - SVNOWG(\zeta_1, \zeta_2, ..., \zeta_n) = \begin{pmatrix} \prod_{p=1}^{n} \left(1 - \ell og_{\sigma_p} P_{\xi_{\eta(p)}}\right)^{\beta_p}, \\ 1 - \prod_{p=1}^{n} \left(1 - \ell og_{\sigma_p}\left(1 - I_{\xi_{\eta(p)}}\right)\right)^{\beta_p}, \\ 1 - \prod_{p=1}^{n} \left(1 - \ell og_{\sigma_p}\left(1 - N_{\xi_{\eta(p)}}\right)\right)^{\beta_p} \end{pmatrix}, \tag{7}$$

where $\beta_p\,(p = 1, 2, ..., n)$ *are weighting vector with* $\beta_p \geq 0$ *and* $\sum_{p=1}^{n} \beta_p = 1$ *and pth are the largest weighted value is* $\zeta_{\eta(p)}$ *consequently by total order* $\zeta_{\eta(1)} \geq \zeta_{\eta(2)} \geq ... \geq \zeta_{\eta(n)}$.

3. Logarithmic Operational Laws

Motivated by the well growing concept of SVNSs, we introduce some novel logarithmic operational laws for single valued neutrosophic numbers. As in real number systems $\ell og_{\sigma} 0$ is meaningless and $\ell og_{\sigma} 1$ is not defined therefore, in our study we take non-empty SVNSs and $\sigma \neq 1$, where σ is any real number.

Definition 13. *For any SVNN* $\zeta_p = \left\langle P_{\xi_p}(r), I_{\xi_p}(r), N_{\xi_p}(r) \right\rangle$ *in* $\Re$. *The logarithmic SVNN is defined as:*

$$\ell og_{\sigma} \zeta_p = \left\{\left\langle 1 - \left(\ell og_{\sigma} P_{\xi_p}(r)\right), \ell og_{\sigma}\left(1 - I_{\xi_p}(r)\right), \ell og_{\sigma}\left(1 - N_{\xi_p}(r)\right)\right\rangle | r \in \Re\right\}, \tag{8}$$

in which $P_\sigma : \Re \to \Theta$, $I_\sigma : \Re \to \Theta$ *and* $N_\sigma : \Re \to \Theta$ *are indicated the truth, indeterminacy and falsity memberships in* $\Re$, $\Theta = [0,1]$ *be the unit interval. Also following condition is satisfied by* P_σ, I_σ *and* N_σ, $0 \le P_\sigma(r) + I_\sigma(r) + N_\sigma(r) \le 3; \forall\, r \in \Re$. *Therefore the truth membership grade is*

$$1 - \left(\ell og_\sigma P_{\xi_p}(r)\right) : \Re \to \Theta, \text{ such that } 0 \le 1 - \left(\ell og_\sigma P_{\xi_p}(r)\right) \le 1, \text{ for all } r \in \Re$$

the indeterminacy membership is

$$\ell og_\sigma \left(1 - I_{\xi_p}(r)\right) : \Re \to \Theta, \text{ such that } 0 \le \ell og_\sigma \left(1 - I_{\xi_p}(r)\right) \le 1, \text{ for all } r \in \Re$$

and falsity membership is

$$\ell og_\sigma \left(1 - N_{\xi_p}(r)\right) : \Re \to \Theta, \text{such that } 0 \le \ell og_\sigma \left(1 - N_{\xi_p}(r)\right) \le 1, \text{ for all } r \in \Re.$$

Therefore

$$\begin{aligned} \ell og_\sigma \zeta_p &= \left\{\left\langle 1 - \left(\ell og_\sigma P_{\xi_p}(r)\right), \ell og_\sigma \left(1 - I_{\xi_p}(r)\right), \ell og_\sigma \left(1 - N_{\xi_p}(r)\right)\right\rangle | r \in \Re\right\} \\ 0 &< \sigma \le \min\left\{P_{\xi_p}, 1 - I_{\xi_p}, 1 - N_{\xi_p}\right\} \le 1,\ \sigma \ne 1 \end{aligned}$$

is SVNS.

Definition 14. *For any SVNN* $\zeta_p = \left\langle P_{\xi_p}(r), I_{\xi_p}(r), N_{\xi_p}(r)\right\rangle$ *in* $\Re$. *If*

$$\ell og_\sigma \zeta_p = \begin{cases} \begin{pmatrix} 1 - \left(\ell og_\sigma P_{\xi_p}(r)\right), \\ \ell og_\sigma \left(1 - I_{\xi_p}(r)\right), \\ \ell og_\sigma \left(1 - N_{\xi_p}(r)\right) \end{pmatrix} & 0 < \sigma \le \min\left\{P_{\xi_p}, 1 - I_{\xi_p}, 1 - N_{\xi_p}\right\} < 1 \\ \begin{pmatrix} 1 - \left(\ell og_{\frac{1}{\sigma}} P_{\xi_p}(r)\right), \\ \ell og_{\frac{1}{\sigma}} \left(1 - I_{\xi_p}(r)\right), \\ \ell og_{\frac{1}{\sigma}} \left(1 - N_{\xi_p}(r)\right) \end{pmatrix} & \begin{matrix} 0 < \frac{1}{\sigma} \le \min\left\{P_{\xi_p}, 1 - I_{\xi_p}, 1 - N_{\xi_p}\right\} < 1, \\ \sigma \ne 1 \end{matrix} \end{cases} \tag{9}$$

then the function $\ell og_\sigma \zeta_p$ *is known to be a logarithmic operator for SVNS, and its value is said to be logarithmic SVNN (L-SVNN). Here, we take* $\ell og_\sigma 0 = 0, \sigma > 0, \sigma \ne 1$.

Theorem 1. *[10] For any SVNN* $\zeta_p = \left\langle P_{\xi_p}(r), I_{\xi_p}(r), N_{\xi_p}(r)\right\rangle$ *in* $\Re$, *then* $\ell og_\sigma \zeta_p$ *is also be SVNN.*

Now, we give some discussion on the basic properties of the L-SVNN.

Definition 15. *For any two L-SVNNs* $\ell og_\sigma \zeta_p = \begin{pmatrix} 1 - \left(\ell og_\sigma P_{\xi_p}(r)\right), \\ \ell og_\sigma \left(1 - I_{\xi_p}(r)\right), \\ \ell og_\sigma \left(1 - N_{\xi_p}(r)\right) \end{pmatrix}$ *and* $\ell og_\sigma \zeta_q = \begin{pmatrix} 1 - \left(\ell og_\sigma P_{\xi_q}(r)\right), \\ \ell og_\sigma \left(1 - I_{\xi_q}(r)\right), \\ \ell og_\sigma \left(1 - N_{\xi_q}(r)\right) \end{pmatrix}$ *in* $\Re$ *and* $\beta \ge 0$.*Then the logarithmic operations of L-SVNNs are propose as*

(1) $\ell og_\sigma \zeta_p \oplus \ell og_\sigma \zeta_q = \begin{Bmatrix} 1 - \left(\ell og_\sigma P_{\xi_p}(r)\right) \cdot \left(\ell og_\sigma P_{\xi_q}(r)\right), \\ \ell og_\sigma \left(1 - I_{\xi_p}(r)\right) \cdot \ell og_\sigma \left(1 - I_{\xi_q}(r)\right), \\ \ell og_\sigma \left(1 - N_{\xi_p}(r)\right) \cdot \ell og_\sigma \left(1 - N_{\xi_q}(r)\right) \end{Bmatrix}$;

(2) $\beta \cdot \ell og_\sigma \zeta_p = \left\{ \begin{array}{c} 1-\left(\ell og_\sigma P_{\xi_p}\left(r\right)\right)^\beta, \\ \left(\ell og_\sigma\left(1-I_{\xi_p}\left(r\right)\right)\right)^\beta, \\ \left(\ell og_\sigma\left(1-N_{\xi_p}\left(r\right)\right)\right)^\beta \end{array} \right\};$

(3) $\ell og_\sigma \zeta_p \otimes \ell og_\sigma \zeta_q = \left\{ \begin{array}{c} 1-\left(\ell og_\sigma P_{\xi_p}\left(r\right)\right)\cdot 1-\left(\ell og_\sigma P_{\xi_q}\left(r\right)\right), \\ 1-\left(1-\ell og_\sigma\left(1-I_{\xi_p}\left(r\right)\right)\right)\cdot\left(1-\ell og_\sigma\left(1-I_{\xi_q}\left(r\right)\right)\right), \\ 1-\left(1-\ell og_\sigma\left(1-N_{\xi_p}\left(r\right)\right)\right)\cdot\left(1-\ell og_\sigma\left(1-N_{\xi_q}\left(r\right)\right)\right) \end{array} \right\};$

(4) $\left(\ell og_\sigma \zeta_p\right)^\beta = \left\{ \begin{array}{c} \left(1-\left(\ell og_\sigma P_{\xi_p}\left(r\right)\right)\right)^\beta, \\ 1-\left(1-\ell og_\sigma\left(1-I_{\xi_p}\left(r\right)\right)\right)^\beta, \\ 1-\left(1-\ell og_\sigma\left(1-N_{\xi_p}\left(r\right)\right)\right)^\beta \end{array} \right\}.$

Theorem 2. *[10] For any two L-SVNNs* $\ell og_\sigma \zeta_p = \left(\begin{array}{c} 1-\left(\ell og_\sigma P_{\xi_p}\left(r\right)\right), \\ \ell og_\sigma\left(1-I_{\xi_p}\left(r\right)\right), \\ \ell og_\sigma\left(1-N_{\xi_p}\left(r\right)\right) \end{array} \right)$ $(p=1,2)$ *in* $\Re$*, with* $0 < \sigma \leq \min\left\{P_{\xi_p}, 1-I_{\xi_p}, 1-N_{\xi_p}\right\} < 1, \sigma \neq 1, \beta, \beta_1, \beta_2 > 0$ *be any real numbers. Then*
(1) $\beta\left(\ell og_\sigma \zeta_1 \oplus \ell og_\sigma \zeta_2\right) = \beta \ell og_\sigma \zeta_1 \oplus \beta \ell og_\sigma \zeta_2$;
(2) $\left(\ell og_\sigma \zeta_1 \otimes \ell og_\sigma \zeta_2\right)^\beta = \left(\ell og_\sigma \zeta_1\right)^\beta \otimes \left(\ell og_\sigma \zeta_2\right)^\beta$;
(3) $\beta_1 \ell og_\sigma \zeta_1 \oplus \beta_2 \ell og_\sigma \zeta_1 = \left(\beta_1+\beta_2\right) \ell og_\sigma \zeta_1$;
(4) $\left(\ell og_\sigma \zeta_1\right)^{\beta_1} \otimes \left(\ell og_\sigma \zeta_1\right)^{\beta_2} = \left(\ell og_\sigma \zeta_1\right)^{\left(\beta_1+\beta_2\right)}$;
(5) $\left(\left(\ell og_\sigma \zeta_1\right)^{\beta_1}\right)^{\beta_2} = \left(\ell og_\sigma \zeta_1\right)^{\beta_1\beta_2}$.

Comparison Technique for L-SVNNs

Definition 16. *[10] For any L-SVNN* $\ell og_\sigma \zeta_p = \left(\begin{array}{c} 1-\left(\ell og_\sigma P_{\xi_p}\left(r\right)\right), \\ \ell og_\sigma\left(1-I_{\xi_p}\left(r\right)\right), \\ \ell og_\sigma\left(1-N_{\xi_p}\left(r\right)\right) \end{array} \right)$ *in* $\Re$*. Then score and accuracy values are define as*
(1) $\widetilde{S}(\ell og_\sigma \zeta_p) = 1-\left(\ell og_\sigma P_{\xi_p}\left(r\right)\right) - \ell og_\sigma\left(1-I_{\xi_p}\left(r\right)\right) - \left(\ell og_\sigma\left(1-N_{\xi_p}\left(r\right)\right)\right)$
(2) $\widetilde{A}(\ell og_\sigma \zeta_p) = 1-\left(\ell og_\sigma P_{\xi_p}\left(r\right)\right) + \ell og_\sigma\left(1-I_{\xi_p}\left(r\right)\right) + \left(\ell og_\sigma\left(1-N_{\xi_p}\left(r\right)\right)\right)$

The above defined score and accuracy values suggest which L-SVNN are greater than other L-SVNNs. The comparison technique is defined in the following definition.

Definition 17. *For any L-SVNNs* $\ell og_\sigma \zeta_p = \left(\begin{array}{c} 1-\left(\ell og_\sigma P_{\xi_p}\left(r\right)\right), \\ \ell og_\sigma\left(1-I_{\xi_p}\left(r\right)\right), \\ \ell og_\sigma\left(1-N_{\xi_p}\left(r\right)\right) \end{array} \right)$ $(p=1,2)$ *in* $\Re$*. Then, comparison technique is proposed as:*
(1) *If* $\widetilde{S}(\ell og_\sigma \zeta_1) < \widetilde{S}(\ell og_\sigma \zeta_2)$ *then* $\ell og_\sigma \zeta_1 < \ell og_\sigma \zeta_2$,
(2) *If* $\widetilde{S}(\ell og_\sigma \zeta_1) > \widetilde{S}(\ell og_\sigma \zeta_2)$ *then* $\ell og_\sigma \zeta_1 > \ell og_\sigma \zeta_2$,
(3) *If* $\widetilde{S}(\ell og_\sigma \zeta_1) = \widetilde{S}(\ell og_\sigma \zeta_2)$ *then*
(a) $\widetilde{A}(\ell og_\sigma \zeta_1) < \widetilde{A}(\ell og_\sigma \zeta_2)$ *then* $\ell og_\sigma \zeta_1 < \ell og_\sigma \zeta_2$,
(b) $\widetilde{A}(\ell og_\sigma \zeta_1) > \widetilde{A}(\ell og_\sigma \zeta_2)$ *then* $\ell og_\sigma \zeta_1 > \ell og_\sigma \zeta_2$,
(c) $\widetilde{A}(\ell og_\sigma \zeta_1) = \widetilde{A}(\ell og_\sigma \zeta_2)$ *then* $\ell og_\sigma \zeta_1 \approx \ell og_\sigma \zeta_2$.

4. Logarithmic Aggregation Operators for L-SVNNs

Now, we propose novel logarithmic hybrid aggregation operators for L-SVNNs based on logarithmic operations laws as follows:

4.1. Logarithmic Hybrid Averaging Operator

Definition 18. *For any collection of SVNNs* $\zeta_p = \left\langle P_{\xi_p}(r), I_{\xi_p}(r), N_{\xi_p}(r) \right\rangle$ $(p = 1, ..., n)$ *in* $\Re$, *with* $0 < \sigma_p \leq \min\left\{P_{\xi_p}, 1 - I_{\xi_p}, 1 - N_{\xi_p}\right\} < 1$, $\sigma \neq 1$. *The structure of logarithmic single valued neutrosophic hybrid weighted averaging (L-SVNHWA) operator is*

$$L - SVNHWA(\zeta_1, \zeta_2, ..., \zeta_n) = \sum_{p=1}^{n} \omega_p \ell og_{\sigma_p} \zeta^*_{\eta(p)}, \tag{10}$$

where β_p $(p = 1, ..., n)$ *is the weighting vector with* $\beta_p \geq 0$ *and* $\sum_{p=1}^{n} \beta_p = 1$ *and pth biggest weighted value is* $\zeta^*_{\eta(p)}$ $\left(\zeta^*_{\eta(p)} = n\beta_p \zeta_{\eta(p)}, \ P \in N\right)$ *consequently by total order* $\zeta^*_{\eta(1)} \geq \zeta^*_{\eta(2)} \geq ... \geq \zeta^*_{\eta(n)}$. *Also, the associated weights are* $\omega = (\omega_1, \omega_2, ..., \omega_n)$ *with* $\omega_p \geq 0$, $\Sigma_{p=1}^{n} \omega_p = 1$.

Theorem 3. *For any collection of SVNNs* $\zeta_p = \left\langle P_{\xi_p}(r), I_{\xi_p}(r), N_{\xi_p}(r) \right\rangle$ $(p = 1, ..., n)$ *in* $\Re$, *with* $0 < \sigma_p \leq \min\left\{P_{\xi_p}, 1 - I_{\xi_p}, 1 - N_{\xi_p}\right\} < 1$, $\sigma \neq 1$. *Then by using logarithmic operations and Definition 18,* $L - SVNHWA$ *is defined as*

$$L - SVNHWA(\zeta_1, \zeta_2, ..., \zeta_n) = \begin{cases} \begin{pmatrix} 1 - \prod_{p=1}^{n} \left(\ell og_{\sigma_p} P^*_{\xi_{\eta(p)}}\right)^{\omega_p}, \\ \prod_{p=1}^{n} \left(\ell og_{\sigma_p} \left(1 - I^*_{\xi_{\eta(p)}}\right)\right)^{\omega_p}, \\ \prod_{p=1}^{n} \left(\ell og_{\sigma_p} \left(1 - N^*_{\xi_{\eta(p)}}\right)\right)^{\omega_p} \end{pmatrix} & 0 < \sigma_p \leq \min\left\{\begin{array}{c} P_{\xi_p}, \\ 1 - I_{\xi_p}, \\ 1 - N_{\xi_p} \end{array}\right\} < 1 \\ \begin{pmatrix} 1 - \prod_{p=1}^{n} \left(\ell og_{\frac{1}{\sigma_p}} P^*_{\xi_{\eta(p)}}\right)^{\omega_p}, \\ \prod_{p=1}^{n} \left(\ell og_{\frac{1}{\sigma_p}} \left(1 - I^*_{\xi_{\eta(p)}}\right)\right)^{\omega_p}, \\ \prod_{p=1}^{n} \left(\ell og_{\frac{1}{\sigma_p}} \left(1 - N^*_{\xi_{\eta(p)}}\right)\right)^{\omega_p} \end{pmatrix} & \begin{array}{c} 0 < \frac{1}{\sigma_p} \leq \min\left\{\begin{array}{c} P_{\xi_p}, \\ 1 - I_{\xi_p}, \\ 1 - N_{\xi_p} \end{array}\right\} < 1, \\ \sigma \neq 1 \end{array} \end{cases} \tag{11}$$

where β_p $(p = 1, ..., n)$ *are weighting vector with* $\beta_p \geq 0$ *and* $\sum_{p=1}^{n} \beta_p = 1$ *and pth biggest weighted value is* $\zeta^*_{\eta(p)}$ $\left(\zeta^*_{\eta(p)} = n\beta_p \zeta_{\eta(p)}, \ P \in N\right)$ *consequently by total order* $\zeta^*_{\eta(1)} \geq \zeta^*_{\eta(2)} \geq ... \geq \zeta^*_{\eta(n)}$. *Also the associated weights are* $\omega = (\omega_1, \omega_2, ..., \omega_n)$ *with* $\omega_p \geq 0$, $\Sigma_{p=1}^{n} \omega_p = 1$.

Proof. Using mathematical induction to prove Equation (3), we proceed as:

(a) For $n = 2$, since

$$\omega_1 \ell og_{\sigma_1} \zeta^*_{\eta(1)} = \begin{pmatrix} 1 - \left(\ell og_{\sigma_1} P^*_{\xi_{\eta(1)}}\right)^{\omega_1}, \\ \left(\ell og_{\sigma_1} \left(1 - I^*_{\xi_{\eta(1)}}\right)\right)^{\omega_1}, \\ \left(\ell og_{\sigma_1} \left(1 - N^*_{\xi_{\eta(1)}}\right)\right)^{\omega_1} \end{pmatrix}$$

and

$$\omega_2 \ell og_{\sigma_2} \zeta^*_{\eta(2)} = \begin{pmatrix} 1 - \left(\ell og_{\sigma_2} P^*_{\xi_{\eta(2)}}\right)^{\omega_2}, \\ \left(\ell og_{\sigma_2}\left(1 - I^*_{\xi_{\eta(2)}}\right)\right)^{\omega_2}, \\ \left(\ell og_{\sigma_2}\left(1 - N^*_{\xi_{\eta(2)}}\right)\right)^{\omega_2} \end{pmatrix}$$

Then

$$\begin{aligned} & L - SVNHWA\left(\zeta_1, \zeta_2\right) = \omega_1 \ell og_{\sigma_1} \zeta^*_{\eta(1)} \oplus \omega_2 \ell og_{\sigma_2} \zeta^*_{\eta(2)} \\ = & \begin{pmatrix} 1 - \left(\ell og_{\sigma_1} P^*_{\xi_{\eta(1)}}\right)^{\omega_1}, \\ \left(\ell og_{\sigma_1}\left(1 - I^*_{\xi_{\eta(1)}}\right)\right)^{\omega_1}, \\ \left(\ell og_{\sigma_1}\left(1 - N^*_{\xi_{\eta(1)}}\right)\right)^{\omega_1} \end{pmatrix} \oplus \begin{pmatrix} 1 - \left(\ell og_{\sigma_2} P^*_{\xi_{\eta(2)}}\right)^{\omega_2}, \\ \left(\ell og_{\sigma_2}\left(1 - I^*_{\xi_{\eta(2)}}\right)\right)^{\omega_2}, \\ \left(\ell og_{\sigma_2}\left(1 - N^*_{\xi_{\eta(2)}}\right)\right)^{\omega_2} \end{pmatrix} \\ = & \begin{pmatrix} 1 - \left(\ell og_{\sigma_1} P^*_{\xi_{\eta(1)}}\right)^{\omega_1} \cdot \left(\ell og_{\sigma_2} P^*_{\xi_{\eta(2)}}\right)^{\omega_2}, \\ \left(\ell og_{\sigma_1}\left(1 - I^*_{\xi_{\eta(1)}}\right)\right)^{\omega_1} \cdot \left(\ell og_{\sigma_2}\left(1 - I^*_{\xi_{\eta(2)}}\right)\right)^{\omega_2}, \\ \left(\ell og_{\sigma_1}\left(1 - N^*_{\xi_{\eta(1)}}\right)\right)^{\omega_1} \cdot \left(\ell og_{\sigma_2}\left(1 - N^*_{\xi_{\eta(2)}}\right)\right)^{\omega_2} \end{pmatrix} \\ = & \begin{pmatrix} 1 - \prod_{p=1}^{2} \left(\ell og_{\sigma_p} P^*_{\xi_{\eta(p)}}\right)^{\omega_p}, \\ \prod_{p=1}^{n} \left(\ell og_{\sigma_p}\left(1 - I^*_{\xi_{\eta(p)}}\right)\right)^{\omega_p}, \\ \prod_{p=1}^{n} \left(\ell og_{\sigma_p}\left(1 - N^*_{\xi_{\eta(p)}}\right)\right)^{\omega_p} \end{pmatrix}. \end{aligned}$$

(b) Now Equation (3) is true for $n = k$,

$$L - SVNHWA\left(\zeta_1, \zeta_2, ..., \zeta_k\right) = \begin{pmatrix} 1 - \prod_{p=1}^{k} \left(\ell og_{\sigma_p} P^*_{\xi_{\eta(p)}}\right)^{\omega_p}, \\ \prod_{p=1}^{k} \left(\ell og_{\sigma_p}\left(1 - I^*_{\xi_{\eta(p)}}\right)\right)^{\omega_p}, \\ \prod_{p=1}^{k} \left(\ell og_{\sigma_p}\left(1 - N^*_{\xi_{\eta(p)}}\right)\right)^{\omega_p} \end{pmatrix}.$$

(c) Now, we prove that Equation (3) for $n = k + 1$, that is

$$L - SVNHWA\left(\zeta_1, \zeta_2, ..., \zeta_k\right) = \sum_{p=1}^{k} \omega_p \ell og_{\sigma_p} \zeta^*_{\eta(p)} + \omega_{k+1} \ell og_{\sigma_{k+1}} \zeta^*_{\eta(k+1)}$$

$$L-SVNHWA\left(\zeta_1,\zeta_2,...,\zeta_k\right)$$
$$=\begin{pmatrix} 1-\prod_{p=1}^{k}\left(\ell og_{\sigma_p}P^*_{\xi_{\eta(p)}}\right)^{\omega_p}, \\ \prod_{p=1}^{k}\left(\ell og_{\sigma_p}\left(1-I^*_{\xi_{\eta(p)}}\right)\right)^{\omega_p}, \\ \prod_{p=1}^{k}\left(\ell og_{\sigma_p}\left(1-N^*_{\xi_{\eta(p)}}\right)\right)^{\omega_p} \end{pmatrix} \oplus \begin{pmatrix} 1-\left(\ell og_{\sigma_{k+1}}P^*_{\xi_{\eta(k+1)}}\right)^{\omega_{k+1}}, \\ \left(\ell og_{\sigma_{k+1}}\left(1-I^*_{\xi_{\eta(k+1)}}\right)\right)^{\omega_{k+1}}, \\ \left(\ell og_{\sigma_{k+1}}\left(1-N^*_{\xi_{\eta(k+1)}}\right)\right)^{\omega_{k+1}} \end{pmatrix}$$
$$=\begin{pmatrix} 1-\prod_{p=1}^{k+1}\left(\ell og_{\sigma_p}P^*_{\xi_{\eta(p)}}\right)^{\omega_p}, \\ \prod_{p=1}^{k+1}\left(\ell og_{\sigma_p}\left(1-I^*_{\xi_{\eta(p)}}\right)\right)^{\omega_p}, \\ \prod_{p=1}^{k+1}\left(\ell og_{\sigma_p}\left(1-N^*_{\xi_{\eta(p)}}\right)\right)^{\omega_p} \end{pmatrix}$$

Thus Equation (3) is true for $n=z+1$. Hence its satisfies for whole n. Therefore

$$L-SVNHWA\left(\zeta_1,\zeta_2,...,\zeta_n\right)=\begin{pmatrix} 1-\prod_{p=1}^{n}\left(\ell og_{\sigma_p}P^*_{\xi_{\eta(p)}}\right)^{\omega_p}, \\ \prod_{p=1}^{n}\left(\ell og_{\sigma_p}\left(1-I^*_{\xi_{\eta(p)}}\right)\right)^{\omega_p}, \\ \prod_{p=1}^{n}\left(\ell og_{\sigma_p}\left(1-N^*_{\xi_{\eta(p)}}\right)\right)^{\omega_p} \end{pmatrix}.$$

In a similarly way, if $0<\frac{1}{\sigma_p}\le\min\left\{P_{\xi_p},1-I_{\xi_p},1-N_{\xi_p}\right\}<1$, $\sigma\neq1$, we can also obtain

$$L-SVNHWA\left(\zeta_1,\zeta_2,...,\zeta_n\right)=\begin{pmatrix} 1-\prod_{p=1}^{n}\left(\ell og_{\frac{1}{\sigma_p}}P^*_{\xi_{\eta(p)}}\right)^{\omega_p}, \\ \prod_{p=1}^{n}\left(\ell og_{\frac{1}{\sigma_p}}\left(1-I^*_{\xi_{\eta(p)}}\right)\right)^{\omega_p}, \\ \prod_{p=1}^{n}\left(\ell og_{\frac{1}{\sigma_p}}\left(1-N^*_{\xi_{\eta(p)}}\right)\right)^{\omega_p} \end{pmatrix}$$

which completes the proof. □

Remark 1. *If $\sigma_1=\sigma_2=\sigma_3=...=\sigma_n=\sigma$, that is $0<\sigma\le\min\left\{P_{\xi_p},1-I_{\xi_p},1-N_{\xi_p}\right\}<1$, $\sigma\neq1$, then $L-SVNHWA$ operator is reduced as follows*

$$L-SVNHWA\left(\zeta_1,\zeta_2,...,\zeta_n\right)=\begin{pmatrix} 1-\prod_{p=1}^{n}\left(\ell og_{\sigma}P^*_{\xi_{\eta(p)}}\right)^{\omega_p}, \\ \prod_{p=1}^{n}\left(\ell og_{\sigma}\left(1-I^*_{\xi_{\eta(p)}}\right)\right)^{\omega_p}, \\ \prod_{p=1}^{n}\left(\ell og_{\sigma}\left(1-N^*_{\xi_{\eta(p)}}\right)\right)^{\omega_p} \end{pmatrix}. \quad (12)$$

Properties

$L-SVNHWA$ operator satisfies some properties are enlist below;

(1) Idempotency: For any collection of SVNNs $\zeta_p=\left\langle P_{\xi_p}(r),I_{\xi_p}(r),N_{\xi_p}(r)\right\rangle$ $(p=1,...,n)$ in $\Re$. Then, if collection of SVNNs $\zeta_p=\left\langle P_{\xi_p}(r),I_{\xi_p}(r),N_{\xi_p}(r)\right\rangle$ $(p=1,...,n)$ are identical, that is

$$L-SVNHWA\left(\zeta_1,\zeta_2,...,\zeta_n\right)=\zeta. \quad (13)$$

(2) Boundedness: for any collection of SVNNs $\zeta_p = \left\langle P_{\xi_p}(r), I_{\xi_p}(r), N_{\xi_p}(r) \right\rangle$ $(p = 1, ..., n)$ in $\Re$. $\zeta_p^- = \left\langle \min_p P^*_{\xi_p}, \max_p I^*_{\xi_p}, \max_p N^*_{\xi_p} \right\rangle$ and $\zeta_p^+ = \left\langle \max_p P^*_{\xi_p}, \min_p I^*_{\xi_p}, \min_p N^*_{\xi_p} \right\rangle$ $(p = 1, ..., n)$ in $\Re$, therefore

$$\zeta_p^- \subseteq L - SVNHWA(\zeta_1, \zeta_2, ..., \zeta_n) \subseteq \zeta_p^+. \tag{14}$$

(3) Monotonically: for any collection of SVNNs $\zeta_p = \left\langle P_{\xi_p}(r), I_{\xi_p}(r), N_{\xi_p}(r) \right\rangle$ $(p = 1, ..., n)$ in $\Re$. If $\zeta_{\eta(p)} \subseteq \zeta^*_{\eta(p)}$ for $(p = 1, ..., n)$, then

$$L - SVNHWA(\zeta_1, \zeta_2, ..., \zeta_n) \subseteq L - SVNHWA(\zeta_1^*, \zeta_2^*, ..., \zeta_n^*). \tag{15}$$

4.2. Logarithmic Hybrid Geometric Operators

Definition 19. *For any collection of SVNNs* $\zeta_p = \left\langle P_{\xi_p}(r), I_{\xi_p}(r), N_{\xi_p}(r) \right\rangle$ $(p = 1, ..., n)$ *in* $\Re$, *with* $0 < \sigma_p \le \min\left\{P_{\xi_p}, 1 - I_{\xi_p}, 1 - N_{\xi_p}\right\} < 1$, $\sigma \neq 1$. *The structure of logarithmic single valued neutrosophic hybrid weighted geometric (L-SVNHWG) operator is*

$$L - SVNHWG(\zeta_1, \zeta_2, ..., \zeta_n) = \prod_{p=1}^{n} \left(\ell og_{\sigma_p} \zeta^*_{\eta(p)}\right)^{\omega_p} \tag{16}$$

where $\beta_p\,(p = 1, ..., n)$ *are weight vectors with* $\beta_p \ge 0$ *and* $\sum_{p=1}^{n} \beta_p = 1$ *and pth biggest weighted value is* $\zeta^*_{\eta(p)} \left(\zeta^*_{\eta(p)} = \left(\zeta_{\eta(p)}\right)^{n\beta_p}, P \in N\right)$ *consequently by total order* $\zeta^*_{\eta(1)} \ge \zeta^*_{\eta(2)} \ge ... \ge \zeta^*_{\eta(n)}$. *Also associated weights are* $\omega = (\omega_1, \omega_2, ..., \omega_n)$ *with* $\omega_p \ge 0$, $\Sigma_{p=1}^{n} \omega_p = 1$.

Theorem 4. *For any collection of SVNNs* $\zeta_p = \left\langle P_{\xi_p}(r), I_{\xi_p}(r), N_{\xi_p}(r) \right\rangle$ $(p = 1, ..., n)$ *in* $\Re$, *with* $0 < \sigma_p \le \min\left\{P_{\xi_p}, 1 - I_{\xi_p}, 1 - N_{\xi_p}\right\} < 1$, $\sigma \neq 1$. *Then by using logarithmic operations and Definition 19, L−SVNHWG define as*

$$L - SVNHWG(\zeta_1, \zeta_2, ..., \zeta_n) = \begin{cases} \begin{pmatrix} \prod_{p=1}^{n} \left(1 - \ell og_{\sigma_p} P^*_{\xi_{\eta(p)}}\right)^{\beta_p} \\ 1 - \prod_{p=1}^{n} \left(1 - \left(\ell og_{\sigma_p}\left(1 - I^*_{\xi_{\eta(p)}}\right)\right)\right)^{\beta_p} \\ 1 - \prod_{p=1}^{n} \left(1 - \left(\ell og_{\sigma_p}\left(1 - n^*_{\xi_{\eta(p)}}\right)\right)\right)^{\beta_p} \end{pmatrix} & 0 < \sigma_p \le \min\left\{\begin{array}{c} P_{\xi_p}, \\ 1 - I_{\xi_p}, \\ 1 - N_{\xi_p} \end{array}\right\} < 1 \\ \begin{pmatrix} \prod_{p=1}^{n} \left(1 - \ell og_{\frac{1}{\sigma_p}} P^*_{\xi_{\eta(p)}}\right)^{\beta_p} \\ 1 - \prod_{p=1}^{n} \left(1 - \left(\ell og_{\frac{1}{\sigma_p}}\left(1 - I^*_{\xi_{\eta(p)}}\right)\right)\right)^{\beta_p} \\ 1 - \prod_{p=1}^{n} \left(1 - \left(\ell og_{\frac{1}{\sigma_p}}\left(1 - n^*_{\xi_{\eta(p)}}\right)\right)\right)^{\beta_p} \end{pmatrix} & 0 < \frac{1}{\sigma_p} \le \min\left\{\begin{array}{c} P_{\xi_p}, \\ 1 - I_{\xi_p}, \\ 1 - N_{\xi_p} \end{array}\right\} < 1, \\ & \sigma \neq 1 \end{cases} \tag{17}$$

where $\beta_p\,(p = 1, ..., n)$ *are weight vectors with* $\beta_p \ge 0$ *and* $\sum_{p=1}^{n} \beta_p = 1$ *and pth biggest weighted value is* $\zeta^*_{\eta(p)} \left(\zeta^*_{\eta(p)} = \left(\zeta_{\eta(p)}\right)^{n\beta_p}, P \in N\right)$ *consequently by total order* $\zeta^*_{\eta(1)} \ge \zeta^*_{\eta(2)} \ge ... \ge \zeta^*_{\eta(n)}$. *Also associated weights are* $\omega = (\omega_1, \omega_2, ..., \omega_n)$ *with* $\omega_p \ge 0$, $\Sigma_{p=1}^{n} \omega_p = 1$.

Proof. Using mathematical induction to prove Equation (4), we proceed as:

(a) For $n = 2$, since

$$(\ell og_{\sigma_1}\zeta_1^*)^{\omega_1} = \begin{pmatrix} \left(1-\ell og_{\sigma_1}P^*_{\xi_1}\right)^{\omega_1} \\ 1-\left(1-\left(\ell og_{\sigma_1}\left(1-I^*_{\xi_1}\right)\right)\right)^{\omega_1} \\ 1-\left(1-\left(\ell og_{\sigma_1}\left(1-N^*_{\xi_1}\right)\right)\right)^{\omega_1} \end{pmatrix}$$

and

$$(\ell og_{\sigma_2}\zeta_2^*)^{\omega_2} = \begin{pmatrix} \left(1-\ell og_{\sigma_2}P^*_{\xi_2}\right)^{\omega_2} \\ 1-\left(1-\left(\ell og_{\sigma_2}\left(1-I^*_{\xi_2}\right)\right)\right)^{\omega_2} \\ 1-\left(1-\left(\ell og_{\sigma_2}\left(1-N^*_{\xi_2}\right)\right)\right)^{\omega_2} \end{pmatrix}.$$

Then

$$\begin{aligned} & L-SVNHWG\,(\zeta_1,\zeta_2) = (\ell og_{\sigma_1}\zeta_1^*)^{\omega_1} \otimes (\ell og_{\sigma_2}\zeta_2^*)^{\omega_2} \\ = & \begin{pmatrix} \left(1-\ell og_{\sigma_1}P^*_{\xi_1}\right)^{\omega_1} \\ 1-\left(1-\left(\ell og_{\sigma_1}\left(1-I^*_{\xi_1}\right)\right)\right)^{\omega_1} \\ 1-\left(1-\left(\ell og_{\sigma_1}\left(1-N^*_{\xi_1}\right)\right)\right)^{\omega_1} \end{pmatrix} \otimes \begin{pmatrix} \left(1-\ell og_{\sigma_2}P^*_{\xi_2}\right)^{\omega_2} \\ 1-\left(1-\left(\ell og_{\sigma_2}\left(1-I^*_{\xi_2}\right)\right)\right)^{\omega_2} \\ 1-\left(1-\left(\ell og_{\sigma_2}\left(1-N^*_{\xi_2}\right)\right)\right)^{\omega_2} \end{pmatrix} \\ = & \left\{ \begin{matrix} \left(1-\ell og_{\sigma_1}P^*_{\xi_1}\right)^{\omega_1} \cdot \left(1-\ell og_{\sigma_2}P^*_{\xi_2}\right)^{\omega_2} \\ 1-\left(1-\left(\ell og_{\sigma_1}\left(1-I^*_{\xi_1}\right)\right)\right)^{\omega_1} \cdot \left(1-\left(\ell og_{\sigma_2}\left(1-I^*_{\xi_2}\right)\right)\right)^{\omega_2} \\ 1-\left(1-\left(\ell og_{\sigma_1}\left(1-N^*_{\xi_1}\right)\right)\right)^{\omega_1} \cdot \left(1-\left(\ell og_{\sigma_2}\left(1-N^*_{\xi_2}\right)\right)\right)^{\omega_2} \end{matrix} \right\} \\ = & \begin{pmatrix} \prod_{p=1}^{2}\left(1-\ell og_{\sigma_p}P^*_{\xi_p}\right)^{\omega_p}, \\ 1-\prod_{p=1}^{2}\left(1-\left(\ell og_{\sigma_p}\left(1-I^*_{\xi_p}\right)\right)\right)^{\omega_p} \\ 1-\prod_{p=1}^{2}\left(1-\left(\ell og_{\sigma_p}\left(1-N^*_{\xi_p}\right)\right)\right)^{\omega_p} \end{pmatrix}. \end{aligned}$$

(b) Now Equation (4) is true for $n = k$,

$$L-SVNHWG\,(\zeta_1,\zeta_2,...,\zeta_k) = \begin{pmatrix} \prod_{p=1}^{k}\left(1-\ell og_{\sigma_p}P^*_{\xi_p}\right)^{\omega_p} \\ 1-\prod_{p=1}^{k}\left(1-\left(\ell og_{\sigma_p}\left(1-I^*_{\xi_p}\right)\right)\right)^{\omega_p} \\ 1-\prod_{p=1}^{k}\left(1-\left(\ell og_{\sigma_p}\left(1-N^*_{\xi_p}\right)\right)\right)^{\omega_p} \end{pmatrix},$$

(c) Now, we prove that Equation (4) for $n = k+1$, that is

$$L-SVNHWG\,(\zeta_1,\zeta_2,...,\zeta_k,\zeta_{k+1}) = \prod_{p=1}^{k}\left(\ell og_{\sigma_p}\zeta_p\right)^{\omega_p} \otimes \left(\ell og_{\sigma_{k+1}}\zeta_{k+1}\right)^{\omega_{k+1}}$$

$$L-SVNHWG\left(\zeta_1,\zeta_2,...,\zeta_k,\zeta_{k+1}\right)$$
$$=\begin{pmatrix}\prod_{p=1}^{k}\left(1-\ell og_{\sigma_p}P^*_{\zeta_p}\right)^{\omega_p}\\ 1-\prod_{p=1}^{k}\left(1-\left(\ell og_{\sigma_p}\left(1-I^*_{\zeta_p}\right)\right)\right)^{\omega_p}\\ 1-\prod_{p=1}^{k}\left(1-\left(\ell og_{\sigma_p}\left(1-N^*_{\zeta_p}\right)\right)\right)^{\omega_p}\end{pmatrix}\otimes\begin{pmatrix}\left(1-\ell og_{\sigma_p}P^*_{\zeta_{k+1}}\right)^{\omega_{k+1}}\\ 1-\left(1-\left(\ell og_{\sigma_p}\left(1-I^*_{\zeta_{k+1}}\right)\right)\right)^{\omega_{k+1}}\\ 1-\left(1-\left(\ell og_{\sigma_p}\left(1-N^*_{\zeta_{k+1}}\right)\right)\right)^{\omega_{k+1}}\end{pmatrix}$$
$$=\begin{pmatrix}\prod_{p=1}^{k+1}\left(1-\ell og_{\sigma_p}P^*_{\zeta_p}\right)^{\omega_p}\\ 1-\prod_{p=1}^{k+1}\left(1-\left(\ell og_{\sigma_p}\left(1-I^*_{\zeta_p}\right)\right)\right)^{\omega_p}\\ 1-\prod_{p=1}^{k+1}\left(1-\left(\ell og_{\sigma_p}\left(1-N^*_{\zeta_p}\right)\right)\right)^{\omega_p}\end{pmatrix}$$

Thus Equation (4) is true for $n=z+1$. Hence it is satisfied for all n. Therefore

$$L-SVNHWG\left(\zeta_1,\zeta_2,...,\zeta_n\right)=\begin{pmatrix}\prod_{p=1}^{n}\left(1-\ell og_{\sigma_p}P^*_{\zeta_p}\right)^{\omega_p}\\ 1-\prod_{p=1}^{n}\left(1-\left(\ell og_{\sigma_p}\left(1-I^*_{\zeta_p}\right)\right)\right)^{\omega_p}\\ 1-\prod_{p=1}^{n}\left(1-\left(\ell og_{\sigma_p}\left(1-N^*_{\zeta_p}\right)\right)\right)^{\omega_p}\end{pmatrix}.$$

In a similar way, if $0<\frac{1}{\sigma_p}\leq\min\left\{P_{\zeta_p},1-I_{\zeta_p},1-N_{\zeta_p}\right\}<1$, $\sigma\neq1$, we can also obtain

$$L-SVNHWG\left(\zeta_1,\zeta_2,...,\zeta_n\right)=\begin{pmatrix}\prod_{p=1}^{n}\left(1-\ell og_{\frac{1}{\sigma_p}}P^*_{\zeta_p}\right)^{\omega_p}\\ 1-\prod_{p=1}^{n}\left(1-\left(\ell og_{\frac{1}{\sigma_p}}\left(1-I^*_{\zeta_p}\right)\right)\right)^{\omega_p}\\ 1-\prod_{p=1}^{n}\left(1-\left(\ell og_{\frac{1}{\sigma_p}}\left(1-N^*_{\zeta_p}\right)\right)\right)^{\omega_p}\end{pmatrix}$$

which completes the proof. □

Remark 2. *If* $\sigma_1=\sigma_2=\sigma_3=...=\sigma_n=\sigma$, *that is* $0<\sigma\leq\min\left\{P_{\zeta_p},1-I_{\zeta_p},1-N_{\zeta_p}\right\}<1$, $\sigma\neq1$, *then* $L-SVNHWG$ *operator reduced as follows*

$$L-SVNHWG\left(\zeta_1,\zeta_2,...,\zeta_n\right)=\begin{pmatrix}\prod_{p=1}^{n}\left(1-\ell og_{\sigma}P^*_{\zeta_p}\right)^{\omega_p}\\ 1-\prod_{p=1}^{n}\left(1-\left(\ell og_{\sigma}\left(1-I^*_{\zeta_p}\right)\right)\right)^{\omega_p}\\ 1-\prod_{p=1}^{n}\left(1-\left(\ell og_{\sigma}\left(1-N^*_{\zeta_p}\right)\right)\right)^{\omega_p}\end{pmatrix}. \tag{18}$$

Properties

$L-SVNHWG$ operator satisfies some properties are enlist below;

(1) Idempotency: for any collection of SVNNs $\zeta_p=\left\langle P_{\zeta_p}\left(r\right),I_{\zeta_p}\left(r\right),N_{\zeta_p}\left(r\right)\right\rangle$ $(p=1,...,n)$ in $\Re$. Then, if collection of SVNNs $\zeta_p=\left\langle P_{\zeta_p}\left(r\right),I_{\zeta_p}\left(r\right),N_{\zeta_p}\left(r\right)\right\rangle$ $(p=1,...,n)$ are identical, that is

$$L-SVNHWG\left(\zeta_1,\zeta_2,...,\zeta_n\right)=\zeta. \tag{19}$$

(2) Boundedness: for any collection of SVNNs $\zeta_p = \left\langle P_{\xi_p}(r), I_{\xi_p}(r), N_{\xi_p}(r) \right\rangle$ $(p = 1, ..., n)$ in $\Re$. $\zeta_p^- = \left\langle \min_p P_{\xi_p}, \max_p I_{\xi_p}, \max_p N_{\xi_p} \right\rangle$ and $\zeta_p^+ = \left\langle \max_p P_{\xi_p}, \min_p I_{\xi_p}, \min_p N_{\xi_p} \right\rangle$ $(p = 1, ..., n)$ in $\Re$, therefore

$$\zeta_p^- \subseteq L - SVNHWG(\zeta_1, \zeta_2, ..., \zeta_n) \subseteq \zeta_p^+. \tag{20}$$

(3) Monotonically: for any collection of SVNNs $\zeta_p = \left\langle P_{\xi_p}(r), I_{\xi_p}(r), N_{\xi_p}(r) \right\rangle$ $(p = 1, ..., n)$ in $\Re$. If $\zeta_p \subseteq \zeta_p^*$ for $(p = 1, ..., n)$, then

$$L - SVNHWG(\zeta_1, \zeta_2, ..., \zeta_n) \subseteq L - SVNHWG(\zeta_1^*, \zeta_2^*, ..., \zeta_n^*). \tag{21}$$

4.3. Generalized Logarithmic Averaging Operator

Definition 20. *For any collection of SVNNs* $\zeta_p = \left\langle P_{\xi_p}(r), I_{\xi_p}(r), N_{\xi_p}(r) \right\rangle$ $(p = 1, ..., n)$ *in* $\Re$*, with* $0 < \sigma_p \leq \min\left\{P_{\xi_p}, 1 - I_{\xi_p}, 1 - N_{\xi_p}\right\} < 1$, $\sigma \neq 1$*. The structure of logarithmic generalized single-valued neutrosophic weighted averaging (L-GSVNWA) operator is*

$$L - GSVNWA(\zeta_1, \zeta_2, ..., \zeta_n) = \left(\sum_{p=1}^{n} \beta_p \ell og_{\sigma_p} (\zeta_p)^{\gamma} \right)^{\frac{1}{\gamma}} \tag{22}$$

where β_p $(p = 1, ..., n)$ *are weighting vector with* $\beta_p \geq 0$ *and* $\sum_{p=1}^{n} \beta_p = 1$.

Theorem 5. *For any collection of SVNNs* $\zeta_p = \left\langle P_{\xi_p}(r), I_{\xi_p}(r), N_{\xi_p}(r) \right\rangle$ $(p = 1, ..., n)$ *in* $\Re$*, with* $0 < \sigma_p \leq \min\left\{P_{\xi_p}, 1 - I_{\xi_p}, 1 - N_{\xi_p}\right\} < 1, \sigma \neq 1, \gamma \geq 1$*. Then by using logarithmic operations and Definition 20,* $L - GSVNWA$ *define as*

$$L - GSVNWA(\zeta_1, \zeta_2, ..., \zeta_n)$$
$$\begin{cases} \begin{pmatrix} \left(1 - \prod_{p=1}^{n} \left(1 - \left(1 - \left(\ell og_{\sigma_p} P_{\xi_p}\right)\right)^{\gamma}\right)^{\beta_p}\right)^{\frac{1}{\gamma}}, \\ 1 - \left[1 - \prod_{p=1}^{n} \left(1 - \left(1 - \ell og_{\sigma_p}\left(1 - I_{\xi_p}\right)\right)^{\gamma}\right)^{\beta_p}\right]^{\frac{1}{\gamma}} \\ 1 - \left[1 - \prod_{p=1}^{n} \left(1 - \left(1 - \ell og_{\sigma_p}\left(1 - N_{\xi_p}\right)\right)^{\gamma}\right)^{\beta_p}\right]^{\frac{1}{\gamma}} \end{pmatrix} & 0 < \sigma_p \leq \min\begin{Bmatrix} P_{\xi_p}, \\ 1 - I_{\xi_p}, \\ 1 - N_{\xi_p} \end{Bmatrix} < 1 \\ \begin{pmatrix} \left(1 - \prod_{p=1}^{n} \left(1 - \left(1 - \left(\ell og_{\frac{1}{\sigma_p}} P_{\xi_p}\right)\right)^{\gamma}\right)^{\beta_p}\right)^{\frac{1}{\gamma}}, \\ 1 - \left[1 - \prod_{p=1}^{n} \left(1 - \left(1 - \ell og_{\frac{1}{\sigma_p}}\left(1 - I_{\xi_p}\right)\right)^{\gamma}\right)^{\beta_p}\right]^{\frac{1}{\gamma}} \\ 1 - \left[1 - \prod_{p=1}^{n} \left(1 - \left(1 - \ell og_{\frac{1}{\sigma_p}}\left(1 - N_{\xi_p}\right)\right)^{\gamma}\right)^{\beta_p}\right]^{\frac{1}{\gamma}} \end{pmatrix} & 0 < \frac{1}{\sigma_p} \leq \min\begin{Bmatrix} P_{\xi_p}, \\ 1 - I_{\xi_p}, \\ 1 - N_{\xi_p} \end{Bmatrix} < 1, \\ & \sigma \neq 1 \end{cases} \tag{23}$$

where β_p $(p = 1, ..., n)$ *are weighting vector with* $\beta_p \geq 0$ *and* $\sum_{p=1}^{n} \beta_p = 1$.

Apparently, if we use $\gamma = 1$, then the $L - GSVNWA$ operator is becomes into $L - SVNWA$ operator.

Proof. Theorem 5 take the form by utilized the technique of mathematical induction and procedure is eliminate here. □

Remark 3. *If $\sigma_1 = \sigma_2 = \sigma_3 = ... = \sigma_n = \sigma$, that is $0 < \sigma \leq \min\left\{P_{\xi_p}, 1 - I_{\xi_p}, 1 - N_{\xi_p}\right\} < 1, \sigma \neq 1$, then $L - GSVNWA$ operator reduced as follows*

$$L - GSVNWA\left(\zeta_1, \zeta_2, ..., \zeta_n\right) = \begin{pmatrix} \left(1 - \prod_{p=1}^{n}\left(1 - \left(1 - \left(\ell og_{\sigma} P_{\xi_p}\right)\right)^{\gamma}\right)^{\beta_p}\right)^{\frac{1}{\gamma}}, \\ 1 - \left[1 - \prod_{p=1}^{n}\left(1 - \left(1 - \ell og_{\sigma}\left(1 - I_{\xi_p}\right)\right)^{\gamma}\right)^{\beta_p}\right]^{\frac{1}{\gamma}} \\ 1 - \left[1 - \prod_{p=1}^{n}\left(1 - \left(1 - \ell og_{\sigma}\left(1 - N_{\xi_p}\right)\right)^{\gamma}\right)^{\beta_p}\right]^{\frac{1}{\gamma}} \end{pmatrix}. \quad (24)$$

Properties

$L - GSVNWA$ operator satisfies some properties are enlist below;

(1) Idempotency: For any collection of SVNNs $\zeta_p = \left\langle P_{\xi_p}(r), I_{\xi_p}(r), N_{\xi_p}(r)\right\rangle$ $(p = 1, ..., n)$ in $\Re$. Then, if collection of SVNNs $\zeta_p = \left\langle P_{\xi_p}(r), I_{\xi_p}(r), N_{\xi_p}(r)\right\rangle$ $(p = 1, ..., n)$ are identical, that is

$$L - GSVNWA\left(\zeta_1, \zeta_2, ..., \zeta_n\right) = \zeta. \quad (25)$$

(2) Boundedness: for any collection of SVNNs $\zeta_p = \left\langle P_{\xi_p}(r), I_{\xi_p}(r), N_{\xi_p}(r)\right\rangle$ $(p = 1, ..., n)$ in $\Re$. $\zeta_p^- = \left\langle \min_p P_{\xi_p}, \max_p I_{\xi_p}, \max_p N_{\xi_p}\right\rangle$ and $\zeta_p^+ = \left\langle \max_p P_{\xi_p}, \min_p I_{\xi_p}, \min_p N_{\xi_p}\right\rangle$ $(p = 1, ..., n)$ in $\Re$, therefore

$$\zeta_p^- \subseteq L - GSVNWA\left(\zeta_1, \zeta_2, ..., \zeta_n\right) \subseteq \zeta_p^+. \quad (26)$$

(3) Monotonically: for any collection of SVNNs $\zeta_p = \left\langle P_{\xi_p}(r), I_{\xi_p}(r), N_{\xi_p}(r)\right\rangle$ $(p = 1, ..., n)$ in $\Re$. If $\zeta_p \subseteq \zeta_p^*$ for $(p = 1, ..., n)$, then

$$L - GSVNWA\left(\zeta_1, \zeta_2, ..., \zeta_n\right) \subseteq L - GSVNWA\left(\zeta_1^*, \zeta_2^*, ..., \zeta_n^*\right). \quad (27)$$

4.4. Generalized Logarithmic Geometric Operator

Definition 21. *For any collection of SVNNs $\zeta_p = \left\langle P_{\xi_p}(r), I_{\xi_p}(r), N_{\xi_p}(r)\right\rangle$ $(p = 1, ..., n)$ in $\Re$, with $0 < \sigma_p \leq \min\left\{P_{\xi_p}, 1 - I_{\xi_p}, 1 - N_{\xi_p}\right\} < 1, \sigma \neq 1$. The structure of logarithmic generalized single valued neutrosophic weighted geometric (L-GSVNWG) operator is*

$$L - GSVNWG\left(\zeta_1, \zeta_2, ..., \zeta_n\right) = \left(\sum_{p=1}^{n}\left(\ell og_{\sigma_p}\left(\zeta_p\right)^{\gamma}\right)^{\beta_p}\right)^{\frac{1}{\gamma}} \quad (28)$$

where β_p $(p = 1, ..., n)$ are weighting vector with $\beta_p \geq 0$ and $\sum_{p=1}^{n} \beta_p = 1$.

Theorem 6. *For any collection of SVNNs* $\zeta_p = \left\langle P_{\xi_p}(r), I_{\xi_p}(r), N_{\xi_p}(r) \right\rangle$ $(p = 1, ..., n)$ *in* $\Re$, *with* $0 < \sigma_p \leq \min\left\{P_{\xi_p}, 1 - I_{\xi_p}, 1 - N_{\xi_p}\right\} < 1, \sigma \neq 1, \gamma \geq 1$. *Then by using logarithmic operations and definition (21), $L - GSVNWG$ define as*

$$L - GSVNWG(\zeta_1, \zeta_2, ..., \zeta_n)$$
$$\begin{cases} \begin{pmatrix} 1 - \left[1 - \prod_{p=1}^{n}\left(1 - \left(\ell og_{\sigma_p} P_{\xi_p}\right)^{\gamma}\right)^{\beta_p}\right]^{\frac{1}{\gamma}}, \\ \left(1 - \prod_{p=1}^{n}\left(1 - \left(\ell og_{\sigma_p}\left(1 - I_{\xi_p}\right)\right)^{\gamma}\right)^{\beta_p}\right)^{\frac{1}{\gamma}} \\ \left(1 - \prod_{p=1}^{n}\left(1 - \left(\ell og_{\sigma_p}\left(1 - N_{\xi_p}\right)\right)^{\gamma}\right)^{\beta_p}\right)^{\frac{1}{\gamma}} \end{pmatrix} & 0 < \sigma_p \leq \min\begin{Bmatrix} P_{\xi_p}, \\ 1 - I_{\xi_p}, \\ 1 - N_{\xi_p} \end{Bmatrix} < 1 \\ \begin{pmatrix} 1 - \left[1 - \prod_{p=1}^{n}\left(1 - \left(\ell og_{\frac{1}{\sigma_p}} P_{\xi_p}\right)^{\gamma}\right)^{\beta_p}\right]^{\frac{1}{\gamma}}, \\ \left(1 - \prod_{p=1}^{n}\left(1 - \left(\ell og_{\frac{1}{\sigma_p}}\left(1 - I_{\xi_p}\right)\right)^{\gamma}\right)^{\beta_p}\right)^{\frac{1}{\gamma}} \\ \left(1 - \prod_{p=1}^{n}\left(1 - \left(\ell og_{\frac{1}{\sigma_p}}\left(1 - N_{\xi_p}\right)\right)^{\gamma}\right)^{\beta_p}\right)^{\frac{1}{\gamma}} \end{pmatrix} & 0 < \frac{1}{\sigma_p} \leq \min\begin{Bmatrix} P_{\xi_p}, \\ 1 - I_{\xi_p}, \\ 1 - N_{\xi_p} \end{Bmatrix} < 1, \\ & \sigma \neq 1 \end{cases} \tag{29}$$

where β_p $(p = 1, ..., n)$ *is the weighting vector with* $\beta_p \geq 0$ *and* $\sum_{p=1}^{n} \beta_p = 1$.

Apparently, if we use $\gamma = 1$, then the $L - GSVNWG$ operator is becomes into $L - SVNWG$ operator.

Proof. Theorem 6 takes the form by utilizing the technique of mathematical induction and the procedure is eliminated here. □

Remark 4. *If* $\sigma_1 = \sigma_2 = \sigma_3 = ... = \sigma_n = \sigma$, *that is* $0 < \sigma \leq \min\left\{P_{\xi_p}, 1 - I_{\xi_p}, 1 - N_{\xi_p}\right\} < 1$, $\sigma \neq 1$, *then $L - GSVNWG$ operator reduced as follows*

$$L - GSVNWG(\zeta_1, \zeta_2, ..., \zeta_n) = \begin{pmatrix} 1 - \left[1 - \prod_{p=1}^{n}\left(1 - \left(\ell og_{\sigma} P_{\xi_p}\right)^{\gamma}\right)^{\beta_p}\right]^{\frac{1}{\gamma}}, \\ \left(1 - \prod_{p=1}^{n}\left(1 - \left(\ell og_{\sigma}\left(1 - I_{\xi_p}\right)\right)^{\gamma}\right)^{\beta_p}\right)^{\frac{1}{\gamma}} \\ \left(1 - \prod_{p=1}^{n}\left(1 - \left(\ell og_{\sigma}\left(1 - N_{\xi_p}\right)\right)^{\gamma}\right)^{\beta_p}\right)^{\frac{1}{\gamma}} \end{pmatrix}. \tag{30}$$

Properties

$L - GSVNWG$ operator satisfies some properties are enlist below;

(1) Idempotency: For any collection of SVNNs $\zeta_p = \left\langle P_{\xi_p}(r), I_{\xi_p}(r), N_{\xi_p}(r) \right\rangle$ $(p = 1, ..., n)$ in $\Re$. Then, if collection of SVNNs $\zeta_p = \left\langle P_{\xi_p}(r), I_{\xi_p}(r), N_{\xi_p}(r) \right\rangle$ $(p = 1, ..., n)$ are identical, that is

$$L - GSVNWG(\zeta_1, \zeta_2, ..., \zeta_n) = \zeta. \tag{31}$$

(2) Boundedness: for any collection of SVNNs $\zeta_p = \left\langle P_{\zeta_p}(r), I_{\zeta_p}(r), N_{\zeta_p}(r) \right\rangle$ $(p = 1, ..., n)$ in $\Re$. $\zeta_p^- = \left\langle \min_p P_{\zeta_p}, \max_p I_{\zeta_p}, \max_p N_{\zeta_p} \right\rangle$ and $\zeta_p^+ = \left\langle \max_p P_{\zeta_p}, \min_p I_{\zeta_p}, \min_p N_{\zeta_p} \right\rangle$ $(p = 1, ..., n)$ in $\Re$, therefore

$$\zeta_p^- \subseteq L - GSVNWG\left(\zeta_1, \zeta_2, ..., \zeta_n\right) \subseteq \zeta_p^+. \tag{32}$$

(3) Monotonically: for any collection of SVNNs $\zeta_p = \left\langle P_{\zeta_p}(r), I_{\zeta_p}(r), N_{\zeta_p}(r) \right\rangle$ $(p = 1, ..., n)$ in $\Re$. If $\zeta_p \subseteq \zeta_p^*$ for $(p = 1, ..., n)$, then

$$L - GSVNWG\left(\zeta_1, \zeta_2, ..., \zeta_n\right) \subseteq L - GSVNWG\left(\zeta_1^*, \zeta_2^*, ..., \zeta_n^*\right). \tag{33}$$

5. Proposed Technique for Solving Decision-Making Problems

This section includes the new approach to decision-making based on the single-valued neutrosophic sets, and we will propose a decision-making matrix as indicated below.

Let $H = (h_1, h_2, ..., h_m)$ be a distinct collection of m probable alternatives and $Y = (y_1, y_2, ..., y_n)$ be a finite collection of n criteria, where h_i indicate the i-th alternatives and y_j indicate the j-th criteria. Let $D = (d_1, d_2, ..., d_t)$ be a finite set of t experts, where d_k indicate the k-th expert. The expert d_k supply her appraisal of an alternative h_i on an attribute y_j as a SVNNs $(i = 1, ..., m; j = 1, ..., n)$. The expert's information is represented by the SVNS decision-making matrix $D^s = \left[E_{ip}^{(s)}\right]_{m\times n}$. Assume that $\beta_p(p = 1, ..., m)$ is the weight vector of the attribute y_j, where $0 \leq \beta_p \leq 1$, $\sum\limits_{p=1}^{n} \beta_p = 1$ and $\psi = (\psi_1, \psi_2, ..., \psi_m)$ be the weights of the decision makers d_k such that $\psi_k \leq 1$, $\sum\limits_{k=1}^{n} \psi_k = 1$.

When we construct the SVNS decision-making matrices, $D^s = \left[E_{ip}^{(s)}\right]_{m\times n}$ for decision. Basically, criteria have two types, one is benefit criteria and other one is cost criteria. If the SVNS decision matrices have cost-type criteria metrics$D^s = \left[E_{ip}^{s}\right]_{m\times n}$ can be converted into the normalized SVNS decision matrices, $R^s = \left[r_{ip}^{(s)}\right]_{m\times n}$, where $r_{ip}^s = \begin{cases} E_{ip}^s, \text{ for benefit criteria } A_p \\ \overline{E}_{ip}^s, \text{ for cost criteria } A_p, \end{cases}$ $j = 1, ..., n$, and $\overline{E}_{ip}^s$ is the complement of E_{ip}^s. The normalization is not required, if the criteria have the same type.

Step 1: In this step, we get the neutrosophic information, using the all proposed logarithmic aggregation operators to evolute the alternative preference values with associated weights, which are $\omega = (\omega_1, \omega_2, ..., \omega_n)$ with $\omega_p \geq 0$, $\Sigma_{p=1}^{n} \omega_p = 1$.

Step 2: We find the score value $\widetilde{S}(\ell og_\sigma \zeta_p)$ and the accuracy value $\widetilde{A}(\ell og_\sigma \zeta_p)$ of the cumulative total preference value h_i $(i = 1, ..., m)$.

Step 3: By definition, we give ranking to the alternatives h_i $(i = 1, ..., m)$ and choose the best alternative which has the maximum score value.

5.1. Numerical Example

Assume that there is a committee which selects five applicable emerging technology enterprises $H_g(g = 1, ..., 5)$, which are given as follows.
(1) Augmented reality (H_1),
(2) Personalized medicine (H_2),
(3) Artificial intelligence (H_3),
(4) Gene drive (H_4) and
(5) Quantum computing (H_5).

They assess the possible rising technology enterprises according to the five attributes, which are
(1) Advancement (D_1),
(2) Market risk (D_2),
(3) Financial investments (D_3),

(4) Progress of science and technology (D_4) and
(5) Designs (D_5) .

To avoid the conflict between them, the decision makers take the attribute weights as $\beta = (0.15, 0.28, 0.20, 0.22, 0.15)^T$. They construct the SVNS decision-making matrix given in Table 1.

Table 1. Emerging Technology Enterprises D^1.

	D_1	D_2	D_3	D_4	D_5
H_1	$(0.5, 0.3, 0.4)$	$(0.3, 0.2, 0.5)$	$(0.2, 0.2, 0.6)$	$(0.4, 0.2, 0.3)$	$(0.3, 0.3, 0.4)$
H_2	$(0.7, 0.1, 0.3)$	$(0.3, 0.2, 0.7)$	$(0.6, 0.3, 0.2)$	$(0.2, 0.4, 0.6)$	$(0.7, 0.1, 0.2)$
H_3	$(0.5, 0.3, 0.4)$	$(0.4, 0.2, 0.6)$	$(0.6, 0.1, 0.2)$	$(0.3, 0.1, 0.5)$	$(0.6, 0.4, 0.3)$
H_4	$(0.7, 0.3, 0.2)$	$(0.2, 0.2, 0.7)$	$(0.4, 0.5, 0.2)$	$(0.2, 0.2, 0.5)$	$(0.4, 0.5, 0.4)$
H_5	$(0.4, 0.1, 0.3)$	$(0.2, 0.1, 0.5)$	$(0.4, 0.1, 0.5)$	$(0.6, 0.3, 0.4)$	$(0.3, 0.2, 0.4)$

Since D_1, D_3 are benefit-type criteria and D_2, D_4 is cost type criteria, the normalization is required for these decision matrices. Normalized decision matrices are shown in Table 2.

Table 2. Emerging Technology Enterprises R^1.

	D_1	D_2	D_3	D_4	D_5
H_1	$(0.5, 0.3, 0.4)$	$(0.5, 0.2, 0.3)$	$(0.2, 0.2, 0.6)$	$(0.3, 0.2, 0.4)$	$(0.3, 0.3, 0.4)$
H_2	$(0.7, 0.1, 0.3)$	$(0.7, 0.2, 0.3)$	$(0.6, 0.3, 0.2)$	$(0.6, 0.4, 0.2)$	$(0.7, 0.1, 0.2)$
H_3	$(0.5, 0.3, 0.4)$	$(0.6, 0.2, 0.4)$	$(0.6, 0.1, 0.2)$	$(0.5, 0.1, 0.3)$	$(0.6, 0.4, 0.3)$
H_4	$(0.7, 0.3, 0.2)$	$(0.7, 0.2, 0.2)$	$(0.4, 0.5, 0.2)$	$(0.5, 0.2, 0.2)$	$(0.4, 0.5, 0.4)$
H_5	$(0.4, 0.1, 0.3)$	$(0.5, 0.1, 0.2)$	$(0.4, 0.1, 0.5)$	$(0.4, 0.3, 0.6)$	$(0.3, 0.2, 0.4)$

Step 1: Now, we apply all the proposed logarithmic aggregation operators to collective neutrosophic information as follows.

Case 1: Using logarithmic single-valued neutrosophic hybrid weighted averaging aggregation operator, we obtained the results shown in Table 3.

Table 3. Aggregated information using the logarithmic single valued neutrosophic hybrid weighted averaging (L-SVNHWA) operator for $\sigma = 0.3$.

H_1	$(0.17624, 0.23432, 0.43885)$
H_2	$(0.66164, 0.16229, 0.21840)$
H_3	$(0.52788, 0.18347, 0.32224)$
H_4	$(0.49410, 0.30962, 0.20985)$
H_5	$(0.22496, 0.12393, 0.39318)$

Case 2: Using Logarithmic single valued neutrosophic hybrid weighted geometric aggregation operator, we obtainedthe results shown in Table 4.

Table 4. Aggregated information using logarithmic single valued neutrosophic hybrid weighted geometric (L-SVNHWG) operator for $\sigma = 0.1$.

H_1	$(0.52472, 0.12638, 0.24189)$
H_2	$(0.81968, 0.10633, 0.11764)$
H_3	$(0.74946, 0.11782, 0.17620)$
H_4	$(0.70685, 0.18942, 0.11685)$
H_5	$(0.58497, 0.07427, 0.23305)$

Step 2: We find the score index $\widetilde{S}(\ell og_{\sigma}\zeta_p)$ and the accuracy index $\widetilde{A}(\ell og_{\sigma}\zeta_p)$ of the cumulative overall preference value h_i $(i = 1, 2, 3, 4, 5)$.

Case 1: Using the score of aggregated information for L-SVNHWA operator, we obtained the results shown in Table 5.

Table 5. Score of aggregated information for L-SVNHWA operator.

$\widetilde{S}(\ell og_{0.3}H_1)$	-1.14345	$\widetilde{A}(\ell og_{0.3}H_1)$	0.25985
$\widetilde{S}(\ell og_{0.3}H_2)$	0.30519	$\widetilde{A}(\ell og_{0.3}H_2)$	1.0087
$\widetilde{S}(\ell og_{0.3}H_3)$	-0.02207	$\widetilde{A}(\ell og_{0.3}H_3)$	0.96078
$\widetilde{S}(\ell og_{0.3}H_4)$	-0.08895	$\widetilde{A}(\ell og_{0.3}H_4)$	0.91781
$\widetilde{S}(\ell og_{0.3}H_5)$	-0.76389	$\widetilde{A}(\ell og_{0.3}H_5)$	0.28571

Case 2: Score of Aggregated information for L-SVNHWG Operator, we obtained the results shown in Table 6.

Table 6. Score of aggregated information for L-SVNHWG operator.

$\widetilde{S}(\ell og_{0.1}H_1)$	0.540979	$\widetilde{A}(\ell og_{0.1}H_1)$	0.89888
$\widetilde{S}(\ell og_{0.1}H_2)$	0.810463	$\widetilde{A}(\ell og_{0.1}H_2)$	1.01683
$\widetilde{S}(\ell og_{0.1}H_3)$	0.736126	$\widetilde{A}(\ell og_{0.1}H_3)$	1.01338
$\widetilde{S}(\ell og_{0.1}H_4)$	0.704159	$\widetilde{A}(\ell og_{0.1}H_4)$	0.994506
$\widetilde{S}(\ell og_{0.1}H_5)$	0.618387	$\widetilde{A}(\ell og_{0.1}H_5)$	0.903179

Step 3: We find the best (suitable) alternative which has the maximum score value from the set of alternatives h_i $(i = 1, 2, 3, 4, 5)$. Overall preference value and ranking of the alternatives are summarized in Table 7.

Table 7. Overall preference value and ranking of the alternatives.

	$\widetilde{S}(H_1)$	$\widetilde{S}(H_2)$	$\widetilde{S}(H_3)$	$\widetilde{S}(H_4)$	$\widetilde{S}(H_5)$	**Ranking**
$L-SVNHWA$	-1.143	0.305	-0.022	-0.088	-0.763	$\boldsymbol{H_2} > H_3 > H_4 > H_5 > H_1$
$L-SVNHWG$	0.540	0.810	0.736	0.704	0.618	$\boldsymbol{H_2} > H_3 > H_4 > H_5 > H_1$

5.2. Comparison with Existing Methods

This section consists of the comparative analysis of several existing aggregation operators of neutrosophic information with the proposed logarithmic single valued hybrid weighted aggregation operators. Existing methods for aggregated neutrosophic information are shown in Table 8–11.

Table 8. Average aggregated SVN information.

	SVNWA [35]	**SVNOWA [35]**	**NWA [14]**
H_1	$(0.3779, 0.2259, 0.4002)$	$(0.3820, 0.2449, 0.4071)$	$(0.3779, 0.2314, 0.4223)$
H_2	$(0.6615, 0.2052, 0.2381)$	$(0.6663, 0.1801, 0.2430)$	$(0.6615, 0.2426, 0.2446)$
H_3	$(0.5656, 0.1763, 0.3131)$	$(0.5597, 0.1838, 0.3122)$	$(0.5656, 0.2109, 0.3272)$
H_4	$(0.5722, 0.2929, 0.2219)$	$(0.5706, 0.3145, 0.2219)$	$(0.5722, 0.3348, 0.2338)$
H_5	$(0.4165, 0.1413, 0.3607)$	$(0.3960, 0.1373, 0.3696)$	$(0.4165, 0.1633, 0.4131)$

Table 9. Average aggregated SVN information.

	SVNFWA [12]	**SVNHWA [11]** $\gamma = 2$
H_1	$(0.3755, 0.2262, 0.4018)$	$(0.3725, 0.2264, 0.4033)$
H_2	$(0.6611, 0.2072, 0.2385)$	$(0.6608, 0.2086, 0.2388)$
H_3	$(0.5652, 0.1779, 0.3141)$	$(0.5648, 0.1790, 0.3149)$
H_4	$(0.5692, 0.2956, 0.2225)$	$(0.5663, 0.2978, 0.2230)$
H_5	$(0.4159, 0.1422, 0.3646)$	$(0.4151, 0.1427, 0.3680)$

Table 10. Average aggregated SVN information.

	SVNHWA [11] $\gamma = 3$	L-SVNWA [10]
H_1	$(0.3693, 0.2266, 0.4048)$	$(0.3130, 0.1753, 0.3544)$
H_2	$(0.6604, 0.2099, 0.2390)$	$(0.6486, 0.1989, 0.2313)$
H_3	$(0.5645, 0.1800, 0.3157)$	$(0.4989, 0.1733, 0.3321)$
H_4	$(0.5635, 0.3000, 0.2234)$	$(0.5585, 0.2736, 0.1942)$
H_5	$(0.4143, 0.1432, 0.3714)$	$(0.2849, 0.1249, 0.3758)$

Table 11. Average aggregated SVN information.

	L-SVNOWA [10]
H_1	$(0.3229, 0.1926, 0.3607)$
H_2	$(0.6549, 0.1719, 0.2368)$
H_3	$(0.4896, 0.1823, 0.3303)$
H_4	$(0.5561, 0.2975, 0.1942)$
H_5	$(0.2442, 0.1209, 0.3834)$

Now, we analyze the ranking of the alternatives according to their aggregated information (in Table 12).

Table 12. Overall ranking of the alternatives.

Existing Operators	Ranking
NWA [14]	$\boldsymbol{H_2} > H_3 > H_4 > H_5 > H_1$
SVNWA [35]	$\boldsymbol{H_2} > H_3 > H_4 > H_5 > H_1$
SVNOWA [35]	$\boldsymbol{H_2} > H_3 > H_4 > H_5 > H_1$
SVNWG [35]	$\boldsymbol{H_2} > H_3 > H_4 > H_5 > H_1$
SVNOWG [35]	$\boldsymbol{H_2} > H_3 > H_4 > H_5 > H_1$
SVNFWA [12]	$\boldsymbol{H_2} > H_3 > H_4 > H_5 > H_1$
SVNHWA [11] $\gamma = 2$	$\boldsymbol{H_2} > H_3 > H_4 > H_5 > H_1$
SVNHWA [11] $\gamma = 3$	$\boldsymbol{H_2} > H_3 > H_4 > H_5 > H_1$
NWG [14]	$\boldsymbol{H_2} > H_3 > H_4 > H_5 > H_1$
SVNFWG [12]	$\boldsymbol{H_2} > H_3 > H_4 > H_5 > H_1$
SVNHWG [11] $\gamma = 2$	$\boldsymbol{H_2} > H_3 > H_4 > H_5 > H_1$
SVNHWG [11] $\gamma = 3$	$\boldsymbol{H_2} > H_3 > H_4 > H_5 > H_1$
SNWEA [15]	$\boldsymbol{H_2} > H_3 > H_5 > H_4 > H_1$
L-SVNWA [10]	$\boldsymbol{H_2} > H_4 > H_3 > H_5 > H_1$
L-SVNOWA [10]	$\boldsymbol{H_2} > H_4 > H_3 > H_5 > H_1$
L-SVNWG [10]	$\boldsymbol{H_2} > H_4 > H_3 > H_1 > H_5$
L-SVNOWG [10]	$\boldsymbol{H_2} > H_3 > H_4 > H_5 > H_1$
Proposed Operators	**Ranking**
L-SVNHWA	$\boldsymbol{H_2} > H_3 > H_4 > H_5 > H_1$
L-SVNHWG	$\boldsymbol{H_2} > H_3 > H_4 > H_5 > H_1$
L-GSVNWA	$\boldsymbol{H_2} > H_4 > H_3 > H_5 > H_1$
L-GSVNWG	$\boldsymbol{H_2} > H_4 > H_3 > H_1 > H_5$

The bast alternative was H_2. The obtained results utilizing logarithmic single valued neutrosophic hybrid weighted operators and logarithmic generalized single valued neutrosophic weighted operators were same as results shows existing methods. Hence, this study proposed novel logarithmic aggregation operators to aggregate the neutrosophic information more effectively and efficiently. Utilizing the proposed logarithmic aggregation operators, we sound the best alternative from a set of alternatives given by the decision maker. Hence the proposed MCDM technique based on logarithmic operators lets us find the best alternative as an applications in decision support systems.

6. Conclusions

In this work, an attempt has been made to present different kinds of logarithmic weighted averaging and geometric aggregation operators based on the single-valued neutrosophic set environment. Earlier, it has been observed that the various aggregation operators are defined under the SVNSs environment where the aggregation operators based on the algebraic or Einstein t-norm and t-conorm. In this paper, we proposed novel logarithmic hybrid aggregation operators and also logarithmic generalized averaging and geometric aggregation operators. Aggregation operators, namely L-SVNHWA, L-SVNHWG, L-GSVNWA and L-GSVNWA are developed under the SVNSs environment and we have studied their properties in detail. Further, depending on the standardization of the decision matrix and the proposed aggregation operators, a decision-making approach is presented to find the best alternative to the SVNSs environment. An illustrative example is taken for illustrating the developed approach, and their results are compared with some of the existing approaches of the SVNSs environment to show the validity of it. From the studies, we conclude that the proposed approach is more generic and suitable for solving the stated problem.

In the future, we shall link the proposed operators with some novel fuzzy sets, like as type 2 fuzzy sets, neutrosophic sets, and so on. Moreover, we may examine if our constructed approach can also be applied in different areas, such as personal evaluation, medical artificial intelligence, energy management and supplier selection evaluation.

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