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The Three-Way Decisions Method Based on Theory of Reliability with SV-Triangular Neutrosophic Numbers

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Abstract: Three-way decisions, as a decision-making mode which is consistent with human cognition, have been widely used in various fields. In this paper, we fuse the theory of reliability into the three-way decisions method, replace the conditional probability in the three-way decisions method with reliability, and then propose a novel three-way decisions method. We also describe the loss functions with single-valued triangular neutrosophic numbers (SVTNNs) and propose an operator to calculate the score function of triangular neutrosophic numbers. Then, the result of decision is attained according to the principle of minimizing loss. Finally, we apply this method to the overhaul of machines in a factory, which proves the practicability and effectiveness of the proposed methods.

Keywords: three-way decisions; theory of reliability; single-valued triangular neutrosophic numbers (SVTNNs); loss functions

1. Introduction

When people make a decision about project selection, they are faced with three choices: accepting the project, rejecting the project or delaying the decision. Research on three-way decisions problems has drawn much attention. In 1982, the rough set theory [1,2] was proposed by Pawlak, which uses lower and upper approximations to divide the universal set into three pair-wise disjoint regions: the positive region, negative region and boundary region. After that, three-way decisions were proposed by Yao et al. [3,4], which link the three regions to the three choices of the decision. With the development of three-way decisions theory, three-way decisions have been applied in various fields, such as risk decision-making [5,6], cluster analysis [7,8], credit scoring [9], etc. Based on all the above papers, conditional probability [10] and loss function [11–13] have always been the two main points of three-way decisions methods. Then the question arises: How should the loss functions occurred by the decisions be described? In real decision making, due to the lack of information and uncertainty of people's cognition, it is often difficult for decision-makers to evaluate the alternatives by real numbers. Therefore, Zadeh [14] firstly proposed the fuzzy set (FS) theory, which is a useful analysis tool to deal with fuzzy information. Since then, many research achievements on FS have been made [15–18]. Nevertheless, there is a disadvantage in FS that it only has a membership and cannot express non-membership. To solve this problem, Atanassov [19] proposed the intuitionistic fuzzy set (IFS) by adding a non-membership. Since then, IFS has been favored by many scholars for a long time and developed rapidly [20,21]. However, FS and IFS can only process incomplete information but not the indeterminate information and inconsistent information, which exist commonly in belief systems [22]. Therefore, Smarandache [23] proposed the concept of the neutrosophic set (NS) by adding an independent indeterminacy-membership on the basis of IFS. Wang et al. [24] developed an instance of the NS, called the single-valued neutrosophic set (SVNS). Liu and Wang [25] presented the

concept of single valued triangular neutrosophic numbers, which is appropriate to describe the degree of losses and the assessor's belief.

The three-way decisions method can also be applied to the engineering fields. With the development of our industry, more and more factories are set up. Each factory needs a large number of machines to operate, thus deciding whether to overhaul the machines has become a necessary problem to solve. As everyone knows, the mathematical theory of reliability [26–29] was first studied in the field of machines's maintenance [30]. In this paper, we are committed to fusing the theory of reliability into three-way decisions method, and we replace conditional probability in three-way decisions method with reliability. Next, based on the multiple attributes of the machines, all machines are classified into three types, and then a novel three-way decisions method is proposed. Finally, the method of three-way decisions is applied to the overhaul of machines in factories.

Based on the above analysis, the highlights of this paper mainly include the following points: (1) We fuse the theory of reliability into the three-way decisions method, replace the conditional probability in the three-way decisions method with reliability, and then apply the novel three-way decisions method to the overhaul of machines in factories. (2) We describe the loss function with single-valued triangular neutrosophic numbers (SVTNNs) and propose an operator to calculate the score function of triangular neutrosophic numbers. (3) These machines are classified into different equivalence classes according to their importance and their stages of life. (4) When the machines of different importance fail, they will cause different losses. Therefore, we propose different loss function matrices for them. Such a design is in line with the practical application of engineering.

The remainder of this paper is organized as follows. In Section 2, we provide the basic concepts of three-way decisions theory, SVTNNs and reliability theory. In Section 3, we develop a novel three-way decision model based on the reliability theory. In Section 4, we elaborate the specific decision-making processes of the three-way method. In Section 5, we give a case study of the overhaul of the factory's machines to demonstrate our proposed method. In Section 6, we conclude the importance of our research.

2. Preliminaries

In this part, we carry out a brief introduction to some basic concepts of the three-way decisions [3], single-valued triangular neutrosophic numbers (SVTNNs) [25] and theory of reliability [29].

2.1. Three-Way Decisions

Let U be a finite nonempty set, the theory of three-way decisions aim at dividing U into three disjoint regions. There are two states and three actions in the decision-making process. $\Omega = \{X, X^c\}$ denotes the set of states which indicates that an element is in X or not in X . In terms of the three regions, $\Lambda = \{a_P, a_B, a_N\}$ denotes the set of actions, where the three actions are used to classify an object x . The three actions actually correspond to three regions of rough sets [1], i.e., deciding $x \in POS(X)$, $x \in BND(X)$ and $x \in NEG(X)$, respectively. In different states, the loss function regarding the risk of actions is denoted by Table 1.

Table 1. The risk of actions in different states.

	$X(P)$	$X^c(N)$
a_P	λ_{PP}	λ_{PN}
a_B	λ_{BP}	λ_{BN}
a_N	λ_{NP}	λ_{NN}

In Table 1, the losses incurred for taking actions P , B and N are given by λ_{PP} , λ_{BP} , and λ_{NP} , respectively, when an object x belongs to X . Namely, the losses incurred for taking actions are given by λ_{PN} , λ_{BN} , and λ_{NN} when the object x does not belong to X . $Pr(X|[x])$ denotes the conditional probability of an object x belonging to X given that the object is described by its equivalence class

[x]. In terms of an object x , we can express the expected loss $E(a_i|[x])$ incurred for taking the different action as follows:

$$E(a_P|[x]) = \lambda_{PP}Pr(X|[x]) + \lambda_{PN}Pr(X^c|[x]), \tag{1}$$

$$E(a_B|[x]) = \lambda_{BP}Pr(X|[x]) + \lambda_{BN}Pr(X^c|[x]), \tag{2}$$

$$E(a_N|[x]) = \lambda_{NP}Pr(X|[x]) + \lambda_{NN}Pr(X^c|[x]). \tag{3}$$

Based on the results given in [3], we can construct the following minimum-cost decision rules:

- (P_0) If $E(a_P|[x]) \leq E(a_B|[x])$ and $E(a_P|[x]) \leq E(a_N|[x])$, decide $x \in POS(X)$;
- (B_0) If $E(a_B|[x]) \leq E(a_P|[x])$ and $E(a_B|[x]) \leq E(a_N|[x])$, decide $x \in BND(X)$; and
- (N_0) If $E(a_N|[x]) \leq E(a_P|[x])$ and $E(a_N|[x]) \leq E(a_B|[x])$, decide $x \in NEG(X)$.

Since $Pr(X|[x]) + Pr(X^c|[x]) = 1$, the rules can be simplified based only on the probability $Pr(X|[x])$ and the loss function. According to the semantics of DTRSs [3], we consider a reasonable kind of losses satisfying the conditions:

$$\lambda_{PP} \leq \lambda_{BP} \leq \lambda_{NP}, \tag{4}$$

$$\lambda_{NN} \leq \lambda_{BN} \leq \lambda_{PN}. \tag{5}$$

Then, we can express the decision-making rules $P_1 \sim N_1$ briefly as follows:

- (P_1) If $Pr(X|[x]) \geq \gamma$ and $Pr(X|[x]) \geq \alpha$, decide $x \in POS(X)$;
- (B_1) If $Pr(X|[x]) \geq \beta$ and $Pr(X|[x]) \leq \alpha$, decide $x \in BND(X)$; and
- (N_1) If $Pr(X|[x]) \leq \beta$ and $Pr(X|[x]) \leq \gamma$, decide $x \in NEG(X)$.

The threshold values α, β, γ could be deduced in the form:

$$\alpha = \frac{(\lambda_{PN} - \lambda_{BN})}{(\lambda_{PN} - \lambda_{BN}) + (\lambda_{BP} - \lambda_{PP})}, \tag{6}$$

$$\beta = \frac{(\lambda_{BN} - \lambda_{NN})}{(\lambda_{BN} - \lambda_{NN}) + (\lambda_{NP} - \lambda_{BP})}, \tag{7}$$

$$\gamma = \frac{(\lambda_{PN} - \lambda_{NN})}{(\lambda_{PN} - \lambda_{NN}) + (\lambda_{NP} - \lambda_{PP})}. \tag{8}$$

When $(\lambda_{PN} - \lambda_{BN})(\lambda_{NP} - \lambda_{BP}) > (\lambda_{BP} - \lambda_{PP})(\lambda_{BN} - \lambda_{NN})$, $\alpha > \gamma > \beta$ could be deduced, the decision-making rules $P_2 \sim N_2$ could be described as follows:

- (P_2) If $Pr(X|[x]) \geq \alpha$, decide $x \in POS(X)$;
- (B_2) If $\beta < Pr(X|[x]) < \alpha$, decide $x \in BND(X)$; and
- (N_2) If $Pr(X|[x]) \leq \beta$, decide $x \in NEG(X)$.

2.2. Single-Valued Triangular Neutrosophic Numbers

The single-valued triangular neutrosophic number (SVTNN) is composed of triangular fuzzy number, truth-membership, indeterminacy-membership, and falsity-membership. It overcomes the limitation of traditional fuzzy set [14] and intuitionistic fuzzy set [19], which is a very useful tool to character the information of evaluation by decision makers on alternatives under the indeterminate or inconsistent conditions.

Definition 1. [23] Let X be a space of points and $x \in X$, $A = \{ \langle x : T_A(x), I_A(x), F_A(x) \rangle, x \in X \}$ be a neutrosophic set, where the functions $T, I, F : X \rightarrow]0^-, 1^+[$ denote the truth-membership function, an indeterminacy-membership function and a falsity-membership function of the element $x \in X$ to the set A , respectively. Therefore, $0^- \leq \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \leq 3^+$.

Definition 2. [24] Let $\hat{A} = \{ \langle x : T_A(x), I_A(x), F_A(x) \rangle, x \in X \}$ be a single valued neutrosophic set, where the functions $T, I, F : X \rightarrow [0, 1]$ with $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$.

Definition 3. [25] Let $\tilde{A} = \langle (a_1, a_2, a_3); t_{\tilde{a}}, i_{\tilde{a}}, f_{\tilde{a}} \rangle$ be a single valued triangular neutrosophic number, which is a neutrosophic set on the real line set R , with $a_1, a_2, a_3 \in R$, $a = (a_1, a_2, a_3)$ denotes a triangular fuzzy number, $a_1 \leq a_2 \leq a_3$, and $t_{\tilde{a}}, i_{\tilde{a}}, f_{\tilde{a}} \in [0, 1]$. Then, we could define the truth-membership, the indeterminacy-membership and the falsity-membership functions of this neutrosophic number as follows:

$$T_{\tilde{a}}(x) = \begin{cases} t_{\tilde{a}} \left(\frac{x-a_1}{a_2-a_1} \right), & (a_1 \leq x \leq a_2) \\ t_{\tilde{a}}, & (x = a_2) \\ t_{\tilde{a}} \left(\frac{a_3-x}{a_3-a_2} \right), & (a_2 \leq x \leq a_3) \\ 0, & (\text{otherwise}) \end{cases} \quad (9)$$

$$I_{\tilde{a}}(x) = \begin{cases} \frac{(a_2-x+i_{\tilde{a}}(x-a_1))}{a_2-a_1}, & (a_1 \leq x \leq a_2) \\ i_{\tilde{a}}, & (x = a_2) \\ \frac{(x-a_2+i_{\tilde{a}}(x-a_1))}{a_3-a_2}, & (a_2 \leq x \leq a_3) \\ 1, & (\text{otherwise}) \end{cases} \quad (10)$$

$$F_{\tilde{a}}(x) = \begin{cases} \frac{(a_2-x+f_{\tilde{a}}(x-a_1))}{a_2-a_1}, & (a_1 \leq x \leq a_2) \\ f_{\tilde{a}}, & (x = a_2) \\ \frac{(x-a_2+f_{\tilde{a}}(a_3-x))}{a_3-a_2}, & (a_2 \leq x \leq a_3) \\ 1, & (\text{otherwise}) \end{cases} \quad (11)$$

where $t_{\tilde{a}}, i_{\tilde{a}}$ and $f_{\tilde{a}}$ denote the maximum degree of truth-membership, the minimum degree of indeterminacy and the falsity-membership, respectively. A single valued triangular neutrosophic number $\tilde{A} = \langle (a_1, a_2, a_3); t_{\tilde{a}}, i_{\tilde{a}}, f_{\tilde{a}} \rangle$ may express an indeterminate quantity about a , which is approximately equal to a .

Liu and Wang [25] defined the SVTNNs, but did not give the method to calculate the score function of SVTNNs. Therefore, we propose a novel operator for calculating the score function as follows:

Definition 4. Let $\tilde{A} = \langle (a_1, a_2, a_3); t_{\tilde{a}}, i_{\tilde{a}}, f_{\tilde{a}} \rangle$ be a single valued triangular neutrosophic number. Then, the score function $s(\tilde{A})$ of a SVTNN is defined as follows:

$$s(\tilde{A}) = \frac{1}{9} [a_1 + a_2 + a_3] \times [2 + t_{\tilde{a}} - i_{\tilde{a}} - f_{\tilde{a}}] \quad (12)$$

Example 1. Assuming $\tilde{A} = \langle (2, 4, 5); 0.9, 0.1, 0.2 \rangle$ is a single valued triangular neutrosophic number, we can calculate its score function as follows:

$$s(\tilde{A}) = \frac{1}{9} (2 + 4 + 5) \times (2 + 0.9 - 0.1 - 0.2) = \frac{1}{9} \times 11 \times 2.6 \approx 3.18 \quad (13)$$

2.3. Theory of Reliability

Reliability theory is a science that takes the life characteristics of products as the main research object. It has important value to reduce the loss caused by equipment's malfunction in engineering.

Definition 5. [29] The loss of a specified function of a product is called malfunction. We usually describe a product's life with a nonnegative random variable T , the corresponding distribution function of T is

$$F(t) = P\{T \leq t\}, t \geq 0 \quad (14)$$

Then, $F(t)$ denotes the probability of occurring malfunction before the time t .

Definition 6. [29] Let $R(t)$ be reliability of product, which represents the probability of a product performing a specified function under specified conditions and within a specified time. Then, $R(t)$ could be expressed as follows:

$$R(t) = P\{T > t\} = 1 - F(t) \quad (15)$$

Then, $R(t)$ denotes the probability of working normally before the time t .

Definition 7. [29] For a product, the typical function of malfunction's rate is shown in Figure 1. It takes on the shape of a bathtub, thus we call it a bathtub curve. As shown in Figure 1, when the product is in the period of Stage I, it shows a downward trend, which is the period of early malfunction. Besides, it is also a stage for the new product to blend in with the working environment. When the product is in the period of Stage II, it basically stays constant, and this is the period of accidental malfunction, which is the best time for the product to work. When the product is in the period of Stage III, it has a rising trend, which is the period of wear malfunction. Because of aging, wear and so on, the product performance gradually becomes worse.

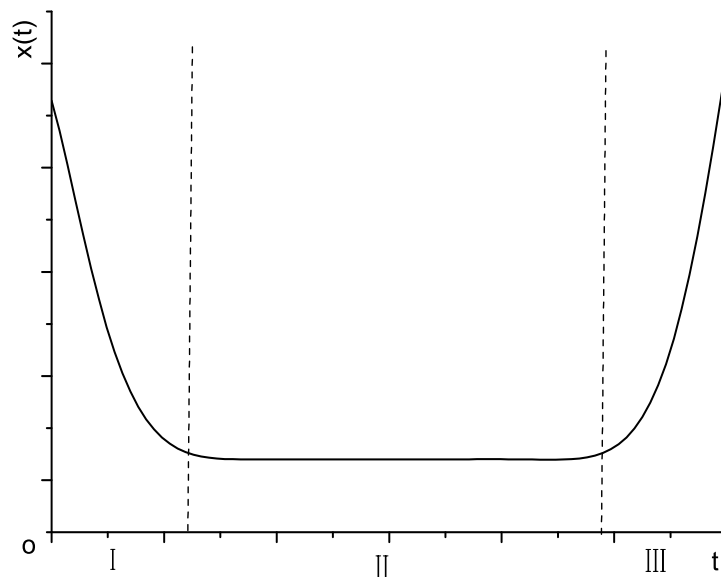


Figure 1. The typical function of malfunction's rate.

3. Three-Way Decisions Based on the Theory of Reliability

In this section, we fuse the theory of reliability into the three-way decisions method, and then we replace conditional probability in three-way decisions method with reliability. Afterwards, the loss function of three-way decision is described by a SVTNN. In the end, a new model of three-way decision is proposed by us. Based on the decision-making procedure of three-way decisions, the new model of three-way decisions conclude three main steps: (a) constructing loss function matrix with SVTNNs; (b) calculating its thresholds according to the Bayesian decision-making procedure; and (c) forming decision-making rules.

3.1. Loss Function Matrix with SVTNNs

There are two states and three actions in the decision-making process, and then a corresponding loss will be incurred when you take an action in a state. To ensure the integrity of the information, a decision maker evaluates these losses with its SVTNN value, which is given in Table 2.

Table 2. Loss function with SVTNNs in different states.

	$X(P)$	$X^c(N)$
a_P	\tilde{A}_{PP}	\tilde{A}_{PN}
a_B	\tilde{A}_{BP}	\tilde{A}_{BN}
a_N	\tilde{A}_{NP}	\tilde{A}_{NN}

In Table 2,

$$\tilde{A}_{PP} = \langle (a_{PP_1}, a_{PP_2}, a_{PP_3}); t_{\tilde{P}\tilde{P}}, i_{\tilde{P}\tilde{P}}, f_{\tilde{P}\tilde{P}} \rangle$$

$$\tilde{A}_{BP} = \langle (a_{BP_1}, a_{BP_2}, a_{BP_3}); t_{\tilde{B}\tilde{P}}, i_{\tilde{B}\tilde{P}}, f_{\tilde{B}\tilde{P}} \rangle$$

$$\tilde{A}_{NP} = \langle (a_{NP_1}, a_{NP_2}, a_{NP_3}); t_{\tilde{N}\tilde{P}}, i_{\tilde{N}\tilde{P}}, f_{\tilde{N}\tilde{P}} \rangle$$

where $(a_{PP_1}, a_{PP_2}, a_{PP_3})$ denotes the losses degrees that is incurred to take actions of a_P , a_B , and a_N , respectively, when an object belong to X . $t_{\tilde{P}\tilde{P}}$ denotes the indeterminacy-membership function of $(a_{PP_1}, a_{PP_2}, a_{PP_3})$. $i_{\tilde{P}\tilde{P}}$ denotes the truth-membership function of $(a_{PP_1}, a_{PP_2}, a_{PP_3})$. $f_{\tilde{P}\tilde{P}}$ denotes the falsity-membership function of $(a_{PP_1}, a_{PP_2}, a_{PP_3})$.

$$\tilde{A}_{PN} = \langle (a_{PN_1}, a_{PN_2}, a_{PN_3}); t_{\tilde{P}\tilde{N}}, i_{\tilde{P}\tilde{N}}, f_{\tilde{P}\tilde{N}} \rangle$$

$$\tilde{A}_{BN} = \langle (a_{BN_1}, a_{BN_2}, a_{BN_3}); t_{\tilde{B}\tilde{N}}, i_{\tilde{B}\tilde{N}}, f_{\tilde{B}\tilde{N}} \rangle$$

$$\tilde{A}_{NN} = \langle (a_{NN_1}, a_{NN_2}, a_{NN_3}); t_{\tilde{N}\tilde{N}}, i_{\tilde{N}\tilde{N}}, f_{\tilde{N}\tilde{N}} \rangle$$

where $(a_{PN_1}, a_{PN_2}, a_{PN_3})$ denotes the losses degrees that is incurred to take actions of a_P , a_B , and a_N , respectively, when an object belong to X^c . $t_{\tilde{P}\tilde{N}}$ denotes the indeterminacy-membership function of $(a_{PN_1}, a_{PN_2}, a_{PN_3})$. $i_{\tilde{P}\tilde{N}}$ denotes the truth-membership function of $(a_{PN_1}, a_{PN_2}, a_{PN_3})$. $f_{\tilde{P}\tilde{N}}$ denotes the falsity-membership function of $(a_{PN_1}, a_{PN_2}, a_{PN_3})$.

Example 2. \tilde{A}_{PP} denotes the loss of keep working when there is no fault with the machines, which is measured by a SVTNN, e.g. $\tilde{A}_{PP} = \langle (0, 0, 0); 1, 0, 0 \rangle$. \tilde{A}_{BP} represents the loss of not making an immediate decision when there is no fault with the machines, e.g. $\tilde{A}_{BP} = \langle (1, 1.6, 2); 0.86, 0.15, 0.1 \rangle$. \tilde{A}_{PP} denotes the loss of choosing to overhaul the machine when there is no fault with the machines, which is measured by a SVTNN, e.g. $\tilde{A}_{NP} = \langle (3, 3.5, 4); 0.9, 0.2, 0.2 \rangle$. \tilde{A}_{PN} denotes the loss of keep working when there is fault with the machines, which is measured by a SVTNN, e.g. $\tilde{A}_{PP} = \langle (6.5, 7, 8); 0.9, 0.1, 0.1 \rangle$. \tilde{A}_{BN} represents the loss of not making an immediate decision when there is fault with the machines, e.g. $\tilde{A}_{BN} = \langle (3, 3.5, 4); 0.85, 0.4, 0.3 \rangle$. \tilde{A}_{NN} denotes the loss of choosing to overhaul the machines when there is fault with the machines, which is measured by a SVTNN, e.g. $\tilde{A}_{NN} = \langle (1.2, 1.5, 2); 0.95, 0.3, 0.2 \rangle$.

Based on Equations (4) and (5), a reasonable relationship for the losses could be obtained as follows:

$$s(\tilde{A}_{PP}) \leq s(\tilde{A}_{BP}) < s(\tilde{A}_{NP})$$

$$s(\tilde{A}_{NN}) \leq s(\tilde{A}_{BN}) < s(\tilde{A}_{PN}) \tag{16}$$

Definition 8. $R(t)$ denotes reliability of machines, which represents the probability of machines performing a specified function under specified conditions and within a specified time. $F(t)$ denotes the probability of

occurring malfunction before the time t . Then, based on Equations (1)–(3), we replace the conditional probability with reliability, and the expected loss based on reliability could be defined as follows:

$$\begin{aligned} E(a_P|[x]) &= s(\tilde{A}_{PP})R(t) + s(\tilde{A}_{PN})F(t), \\ E(a_B|[x]) &= s(\tilde{A}_{BP})R(t) + s(\tilde{A}_{BN})F(t), \\ E(a_N|[x]) &= s(\tilde{A}_{NP})R(t) + s(\tilde{A}_{NN})F(t). \end{aligned} \quad (17)$$

For a shipment of machines $[x]$, $E(a_P|[x])$ denotes the expected loss when you decide to keep working without overhaul, $E(a_B|[x])$ denotes the expected loss when you decide to delay the decision, and $E(a_N|[x])$ denotes the expected loss when you decide to overhaul the machines.

3.2. The Calculation of Three Threshold Values with SVTNNs

Calculating the threshold values is an important step of decision making in three-way decision method. The score function is a useful tool during aggregating the SVTNNs information, which maps the SVTNNs into real numbers.

Based on Equations (6)–(8), we can calculate the threshold values α_m , β_m , and γ_m with SVTNNs as follows:

$$\begin{aligned} \alpha_m &= \frac{s(\tilde{A}_{PN}) - s(\tilde{A}_{BN})}{\{s(\tilde{A}_{PN}) - s(\tilde{A}_{BN})\} + \{s(\tilde{A}_{BP}) - s(\tilde{A}_{PP})\}}, \\ \beta_m &= \frac{s(\tilde{A}_{BN}) - s(\tilde{A}_{NN})}{\{s(\tilde{A}_{BN}) - s(\tilde{A}_{NN})\} + \{s(\tilde{A}_{NP}) - s(\tilde{A}_{BP})\}}, \\ \gamma_m &= \frac{s(\tilde{A}_{BN}) - s(\tilde{A}_{NN})}{\{s(\tilde{A}_{BN}) - s(\tilde{A}_{NN})\} + \{s(\tilde{A}_{NP}) - s(\tilde{A}_{PP})\}}. \end{aligned} \quad (18)$$

3.3. Decision-Making Rules Based on Theory of Reliability

According to the decision-making rules $P_1 \sim N_1$, the corresponding decision-making rules based on reliability $P_3 \sim N_3$ can be deduced as follows:

(P₃) If $R(t) \geq \alpha_m$ and $R(t) \geq \gamma_m$, decide $x \in POS(X)$, that is we decide to keep working without overhaul;

(B₃) If $R(t) \leq \alpha_m$ and $R(t) \geq \beta_m$, decide $x \in BND(X)$, that is we decide to delay the decision; and

(N₃) If $R(t) \leq \beta_m$ and $R(t) \leq \gamma_m$, decide $x \in NEG(X)$, that is we decide to overhaul the machines.

4. Applications in the Overhaul of Factory's Machines

With the development of our industry, more and more factories are set up. Each factory needs a large number of machines to be operated. Every time we start work, we have to make an important decision: whether to overhaul the machines or not. If we decide to keep working without overhaul when there is fault with the machines, once they break down in the working process, the working process of the whole factory will be delayed, the raw materials will be wasted, all workers in the factory will stop working, and it will cause great loss to the factory. If we decide to overhaul the machines when there is no fault with the machines, it will cause a waste of human resources and time. Therefore, to minimize the loss of each factory, deciding whether to overhaul the machines becomes a problem that we have to solve.

The Decision-Making Process of Three-Way Decisions

According to the above-studied result, a new method of three-way decisions based on the reliability in the SVTNNs environment is developed by us. The detailed steps of our proposed method are as follows:

Step 1: Based on the attributes of substitutability, quantity and the difficulty of maintenance, all machines are classified into three types. A_1 : important, their substitutability is very low, they are few in number and their difficulty of maintenance is very high; A_2 : medium, their substitutability is medium, they are medium in number and their difficulty of maintenance is medium; and A_3 : general, their substitutability is high, there are a lot of them and their difficulty of maintenance is very low.

Step 2: For three different types of machines, different loss function matrices with SVTNNs are given by decision maker, respectively. The importance of three different types of machines is different, thus the loss caused by the same action in the same state is not the same.

(1) Machines A_1 : They are important types of machines, and their substitutability is very low, in the working process. Once they break down, the working process of the whole factory will be delayed, the raw materials will be wasted, and then they are not easy to repair. Therefore, the loss functions \tilde{A}_{PN} of them will be large.

(2) Machines A_2 : They are medium types of machines, and their substitutability is medium, therefore the loss functions \tilde{A}_{PN} of them will be medium.

(3) Machines A_3 : They are general types of machines, and their substitutability is very high. In the working process, if they break down, we could use other machines to replace them, and they are easy to repair. Therefore, the loss functions \tilde{A}_{PN} of them will be small.

Step 3: According to the stages of their life, we continue to subdivide the three types of machines further. In Definition 7, we define Stages *I*, *II*, and *III* based on the malfunction's rate of products. Therefore, we could divide Machines A_1 into three types, namely A_{11} , A_{12} , and A_{13} . A_{11} refers to the important machines which are in the period of Stage *I*. Similarly, Machines A_2 could be divided into three types: A_{21} , A_{22} , and A_{23} . Machines A_3 could also be divided into three types: A_{31} , A_{32} , and A_{33} .

Step 4: Based on the historical data of the such machines, the function of malfunction's rate of such machines is fitted. Then, we calculate the reliability of each machines based on Equations (14) and (15). The reliability of a type of machines could be calculated as follows:

$$R(A_{ij}) = \frac{\sum R(a_{ij})}{|A_{ij}|} \quad (i, j = 1, 2, 3)$$

where a_{ij} is the element of A_{ij} , $R(A_{ij})$ is the reliability of Machines A_{ij} , $R(a_{ij})$ is the reliability of machine a_{ij} , and $|A_{ij}|$ denotes the number of elements in Machines A_{ij} .

Step 5: Based on the loss function matrices gained from Step 2 and Equation (12), we convert the loss function matrices with SVTNNs to the loss function matrices with real numbers. Then, we could calculate the threshold values α_m , β_m , and γ_m of three types of Machines A_1 – A_3 based on Equation (18).

Step 6: Based on the decision rules $(P_3) \sim (N_3)$, we can determine the decision-making result for each object by comparing the reliability with the threshold values.

With the aid of Steps 1–6, the decision result for each object can be determined by us. Next, we apply this method to the following case to prove its practicability.

5. A Case Study about the Overhaul of the Factory's Machines

With the rise of the construction industry, many factories producing building materials have been set up. There is a glazed tile factory which has one blank extruder, two slurry agitators, four ball grinding mills and eight tile pressing machines. We need to decide whether to overhaul the machines before they start working in the factory. If we decide to keep working without overhaul when there is

fault with the machines, once they break down in the working process, the working process of the whole factory will be delayed, the raw materials will be wasted, all the workers in the factory will stop working, and it will cause great loss to the factory. If we decide to overhaul the machines when there is no fault with the machines, it will cause a waste of human resources and time. Therefore, to minimize the loss of factory, in this section, our proposed method (see Section 4) is adopted to make decisions. For the sake of clarity, the decision-making process for factory is shown in Figure 2.

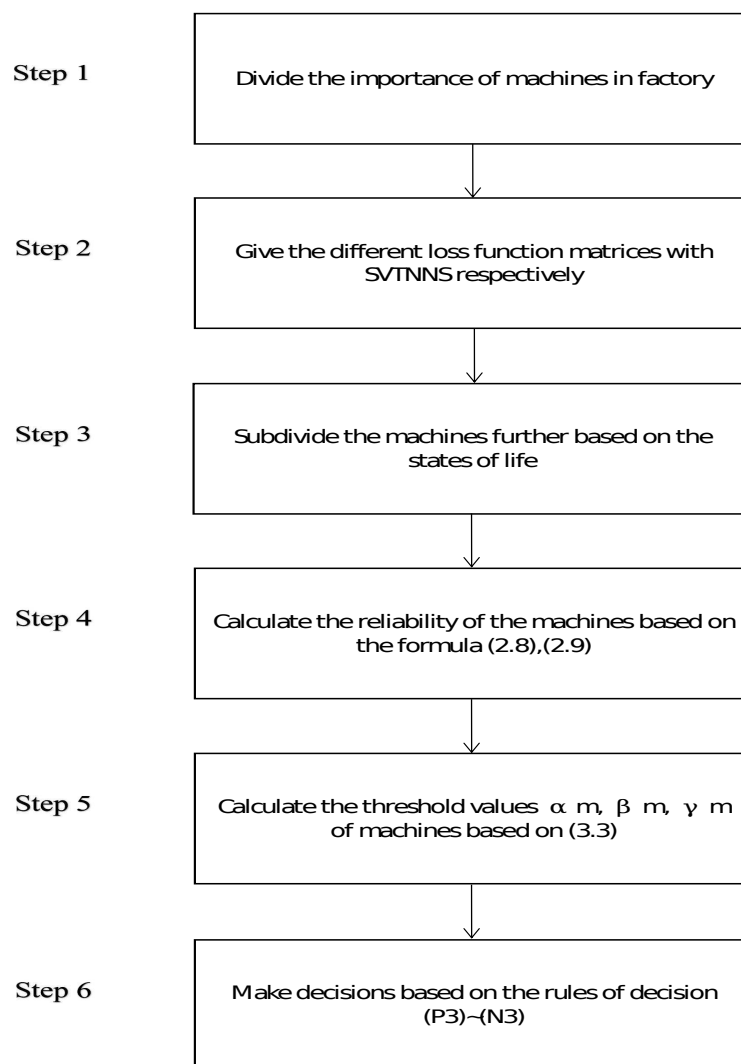


Figure 2. The decision-making process for factory.

Step 1: There are four kinds of machines in this glazed tile factory: blank extruder, slurry agitator, ball grinding mill and tile pressing machine. The blank extruder is the most important; all materials must be processed to form by it. It is the core machine for the production of glazed tiles, with a small number. Next, the ball grinding mill and the slurry agitator are the initial tools for processing raw materials; their substitutability is medium. Finally, the number of tile pressing machines is a little large, and we can replace these machines with manual operation. Therefore, we could divide the machines into three types:

- A_1 : blank extruder;
- A_2 : ball grinding mill, slurry agitator; and
- A_3 : tile press.

Step 2: For three different types of machines, different loss function matrices with SVTNNS are given in the form of Table 2 by decision maker, respectively.

For Machines A_1 (Table 3):

Table 3. Loss function with SVTNNs for A_1 .

	$X(P)$	$X^c(N)$
a_P	$\{(0, 0, 0); 1, 0, 0\}$	$\{(6.5, 7, 8); 0.9, 0.1, 0.1\}$
a_B	$\{(1, 1.6, 2); 0.86, 0.12, 0.2\}$	$\{(3, 3.5, 4); 0.82, 0.4, 0.3\}$
a_N	$\{(3, 3.5, 4); 0.9, 0.2, 0.24\}$	$\{(1.4, 1.6, 2); 0.96, 0.3, 0.2\}$

For Machines A_2 (Table 4):

Table 4. Loss function with SVTNNs for A_2 .

	$X(P)$	$X^c(N)$
a_P	$\{(0, 0, 0); 1, 0, 0\}$	$\{(7, 7.5, 8.4); 0.92, 0.12, 0.1\}$
a_B	$\{(1.5, 2, 2.2); 0.9, 0.2, 0.3\}$	$\{(3.6, 4, 4.4); 0.86, 0.2, 0.2\}$
a_N	$\{(4, 4.6, 5); 0.88, 0.3, 0.1\}$	$\{(2, 2.5, 2.8); 0.98, 0.1, 0.16\}$

For Machines A_3 (Table 5):

Table 5. Loss function with SVTNNs for A_3 .

	$X(P)$	$X^c(N)$
a_P	$\{(0, 0, 0); 1, 0, 0\}$	$\{(5, 5.6, 6.4); 0.9, 0.2, 0.15\}$
a_B	$\{(1.4, 2, 2.1); 0.88, 0.2, 0.1\}$	$\{(2.8, 3.2, 3.6); 0.88, 0.1, 0.12\}$
a_N	$\{(2.4, 2.8, 3); 0.92, 0.1, 0.3\}$	$\{(1.2, 1.5, 1.8); 0.92, 0.2, 0.1\}$

Step 3: In Machine A_1 , the blank extruder is in the period of Stage II, therefore we put it in the A_{12} category. In Machines A_2 , the ball grinding mills are in the period of Stage I, while the slurry agitators are in the period of Stage II, therefore we put the ball grinding mills in the A_{21} category, and the slurry agitators in the A_{22} category. In Machines A_3 , four tile press machines are in the period of Stage II, while four tile press machines are in the period of Stage III, therefore we put four tile press machines in the A_{32} category, and the others in the A_{33} category.

Step 4: Based on Step 4 of the decision-making process in Section 4, we can attain the reliability of equivalence classes shown as Table 6.

Table 6. Loss function with SVTNNs for A_3 .

$R(A_{12})$	$R(A_{21})$	$R(A_{22})$	$R(A_{32})$	$R(A_{33})$
0.84	0.5	0.8	0.64	0.45

Step 5: Based on Equation (12) and the loss functions attained from Step 2, we convert the loss function matrices with SVTNNs to the loss function matrices with real numbers. For Machines A_1 :

$$s(\tilde{A}_{PP}) = \frac{1}{9}(0 + 0 + 0) \times (2 + 1 - 0 - 0) = \frac{1}{9} \times 0 \times 3 = 0$$

$$s(\tilde{A}_{BP}) = \frac{1}{9}(1 + 1.6 + 2) \times (2 + 0.86 - 0.12 - 0.2) = \frac{1}{9} \times 4.6 \times 2.54 \approx 1.30$$

$$s(\tilde{A}_{NP}) = \frac{1}{9}(3 + 3.5 + 4) \times (2 + 0.9 - 0.2 - 0.24) = \frac{1}{9} \times 10.5 \times 2.46 \approx 2.87$$

The transformed result is shown in Table 7.

Table 7. Loss function with real numbers for A_1 .

	$X(P)$	$X^c(N)$
a_P	0	6.45
a_B	1.30	2.47
a_N	2.87	1.37

Then, we could calculate the threshold values α_m , β_m , and γ_m of three types of Machines A_1 – A_3 based on Equation (18). The result of threshold values is shown in Table 8.

Table 8. The threshold values of three-way decision.

U	α_m	β_m	γ_m
A_1	0.754	0.412	0.639
A_2	0.703	0.324	0.554
A_3	0.556	0.492	0.604

Step 6: At last, according to the decision rules $P_3 \sim N_3$, we can compare the value of reliability to the threshold values. Then, the following results can be deduced: $POS(X) = \{A_{12}, A_{22}, A_{32}\}$, $BND(X) = \{A_{21}\}$, and $NEG(X) = \{A_{33}\}$. The results we attain imply that we should decide to make Machines A_{12}, A_{22}, A_{32} keep working without overhaul and overhaul Machines A_{33} , while Machines A_{21} need to be further investigated for their judgements.

6. Conclusions

In this paper, we fuse the theory of reliability into the three-way decisions method, replace the conditional probability in the three-way decisions method with reliability, and then propose a novel three-way decisions method. We also describe the loss function with single-valued triangular neutrosophic numbers (SVTNNs) and propose an operator to calculate the score function of triangular neutrosophic numbers. Finally, the results of decisions are attained according to the principle of minimizing loss. According to the existing literature, no scholar has applied three-way decisions into the maintenance strategy of factory equipment, thus the research in this paper has a certain degree of innovation. The method proposed in this paper belongs to the maintenance based on status in the common maintenance mode. Compared with the traditional post-fault maintenance mode in the factory, it can help the factory to reduce losses. Compared with periodic preventive maintenance mode, it is more efficient and can save time and resources. Therefore, the method proposed in this paper has relatively high application value.

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