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The Quantum Cosmological Constant

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Abstract: We present an extension of general relativity in which the cosmological constant becomes dynamical and turns out to be conjugate to the Chern–Simons invariant of the Ashtekar connection on a spatial slicing. The latter has been proposed Soo and Smolin as a time variable for quantum gravity: the Chern–Simons time. In the quantum theory, the inverse cosmological constant and Chern–Simons time will then become conjugate operators. The “Kodama state” gets a new interpretation as a family of transition functions. These results imply an uncertainty relation between Λ and Chern–Simons time; the consequences of which will be discussed elsewhere.

Keywords: Cosmological Constant, Quantum Gravity, Varying Constants

1. Introduction

The long-standing cosmological constant problem comes in many guises. Before the observation of cosmic acceleration, the search had been to seek a theory where all contributions to the cosmological constant summed up to zero. If the data on acceleration continues to be consistent with a plain cosmological constant, rather than a more general form of dark energy, then the new cosmological constant problem becomes a question of extreme fine-tuning. In this work we take an agnostic position on this matter and are instead motivated by insights from nonperturbative quantum gravity.

In quantum mechanics, while perturbation theory is a powerful tool in unveiling numerous physical observables, it can often times obscure or be blind to important nonperturbative information. For example, quantum tunneling is a nonperturbative process and any perturbative expansion in the vicinity of the tunneling barrier would be blind to tunneling. It is in this context, we would like to address the issue of both the IR and UV versions of the cosmological constant problems (We will not address the issue of phase transitions in this work, and how it might impart on the problem). In both cases, the evaluation of the vacuum energy arises from choosing a classical space time background and calculating the relevant perturbative vacuum diagrams, which are divergent and, after imposing regularization, deemed to be highly fine tuned.

Given the difficulty of the problem, perhaps a new approach is needed. We explore the idea that the cosmological constant is a dynamical variable, not in the sense of a dynamical field, as in quintessence models, but as a degree of freedom for the entire spatial or space-time manifold. Even though similar ideas have been explored before [1–6], the novelty of this paper is that it examines the issue from the unique vantage point of the Ashtekar variables [7] and its associated quantization [8]. In this opening article we focus on the classical formulation of the theory. This will give us guidance

for turning the cosmological constant into an operator in the quantum theory, which we develop in later papers in this series.

In this work, it is convenient to formulate general relativity by gauging the complexified Lorentz group $SL(2, \mathbb{C})_{\mathbb{C}}$ on a four dimensional manifold \mathcal{M} . The space-time connection $A^{ab} = -A^{ba}$ is a one-form, valued in $\mathfrak{sl}(2, \mathbb{C})_{\mathbb{C}}$, the Lie algebra of $SL(2, \mathbb{C})_{\mathbb{C}}$. This Lie algebra is represented by complex antisymmetric 4×4 matrices, $M^{ab} = -M^{ba}$, where $a, b, c = 0, 1, 2, 3$ are internal Lorentz indices. The resulting gravitational dynamics is determined by a connection A^{ab} , a two-form Σ^{ab} , valued in the Lie algebra of $SL(2, \mathbb{C})$ and a scalar field which provides a map $\Psi : \mathfrak{sl}(2, \mathbb{C})_{\mathbb{C}} \rightarrow \mathfrak{sl}(2, \mathbb{C})_{\mathbb{C}}$, which is written as Ψ_{abcd} with the following symmetries and constraints.

$$\Psi_{abcd} = \Psi_{cdab} = -\Psi_{bacd}, \quad \varepsilon^{abcd}\Psi_{abcd} = 0 \quad (1)$$

It is then convenient to change to two-component spinor indices, where $A, B = 0, 1$ are left-handed spinor indices, while $A', B' = 0', 1'$ are right-handed spinor indices. The connection then decomposes into

$$A^{ab} = A^{AA'BB'} = \varepsilon^{AB}A^{A'B'} + A^{AB}\varepsilon^{A'B'} \quad (2)$$

and the two-form Σ^{ab} similarly decomposes. Very importantly, the curvature two-form decomposes the same way

$$R^{ab} = R^{AA'BB'} = \varepsilon^{AB}R^{A'B'} + R^{AB}\varepsilon^{A'B'} \quad (3)$$

where

$$R^{AB} = dA^{AB} + A^{AC} \wedge A^B_C \quad (4)$$

is the left-handed part of the curvature tensor (in much of the literature on LQG, R^{AB} is denoted F^{AB}).

The scalar fields Ψ_{abcd} decompose into pure spin-two fields, represented by Ψ_{ABCD} and $\Psi_{A'B'C'D'}$, which are both totally symmetric, and mixed components $\Psi_{ABA'B'}$ on symmetric pairs of indices. Thus,

$$\Psi_{ABCD} = \Psi_{(ABCD)} \quad (5)$$

and the same for primed indices represents the spin-two field.

In our work, we formulate an extension of general relativity in which the cosmological constant, Λ , varies (prior works in which Λ varies include works by the authors of [2,3,6,9–11]), and is determined by the solution of an equation of motion. The theory has contributions proportional to both Λ and $\frac{1}{\Lambda}$ so the process that determines its value is nonperturbative in Λ . The term in Λ denotes space-time volume:

$$S^V = -\frac{1}{8\pi G} \int_{\mathcal{M}} \frac{\Lambda}{6} \Sigma^{AB} \wedge \Sigma_{AB}, \quad (6)$$

where $\Sigma^{AB} = e^{A'A} \wedge e^B_{A'}$ is the self-dual two-form of the space-time metric, $e^B_{A'}$. We propose a novel coupling of $\frac{1}{\Lambda}$ to the topological invariant $\int_{\mathcal{M}} R^{AB} \wedge R_{AB}$, where R_{AB} is the left-handed part of the curvature tensor. In Euclidean theory, which, for simplicity, we will be studying in this paper, this reads

$$S^{new} = -\frac{1}{8\pi G} \int_{\mathcal{M}} \frac{3}{2\Lambda} R^{AB} \wedge R_{AB}. \quad (7)$$

This is motivated by a key property of deSitter space-time, which is that it is a self-dual solution, in the sense that

$$R_{AB} = \frac{\Lambda}{3} \Sigma_{AB}. \quad (8)$$

Thus, a formal duality transformation that has deSitter space-time as a fixed point must have symmetry

$$R^{AB} \rightarrow \frac{\Lambda}{3} \Sigma^{AB} \quad \Sigma^{AB} \rightarrow \frac{3}{\Lambda} R^{AB} \quad (9)$$

and therefore it must take the usual cosmological constant term (6) to its dual, which is S^{new} , given by Equation (7).

Immediately we find an interesting quantum implication. A reflection of this duality is that the solution to the Hamilton–Jacobi equation that corresponds to deSitter space-time is [12]

$$S^{HJ} = \frac{3}{2\Lambda} \int_{\Sigma} Y_{CS}(A) \tag{10}$$

where $Y_{CS}(A)$ is the Chern–Simons invariant of the Ashtekar connection. This is of course closely related to S^{new} , as we will discuss below. It leads to to a semiclassical state:

$$\Psi_K(A) = e^{\frac{i}{\hbar} S^{HJ}} \tag{11}$$

which is called the Kodama state [13]. Remarkably in some regularizations and ordering prescriptions this is an exact solution to all the constraints of quantum gravity [12]. There have, however, been issues concerning the physical adequacy and interpretation of the Kodama state [14]. Below we propose a new interpretation for this state, stemming from our proposal.

The form of the new term (7), and particularly that it is CP odd, suggests an analogy between $\frac{3}{\Lambda}$ and the theta angle in QCD [15]. Relating the cosmological constant problem to the theta vacuum was considered in works by the authors of [16–18]. This further suggests treating Λ as a dynamical degree of freedom. We explore three versions of this idea, in which Λ is chosen to be a dynamical field, or a function of time in a preferred 3 + 1 slicing, or a single variable for all space-time (as in the work by the authors of [2]). We discover in each case that the Λ equation of motion determines its value.

Finally, in one case for the realization of this theory, we show how the Hamiltonian formalism might be set up and lead to the canonical quantization of the theory. Even ignoring details of the dynamics at the classical level, we can see that a quantum uncertainty principle would always be in action, rendering Λ and Chern–Simons time [1], complementary variables. This may possibly leads to deep implications for quantum cosmology and quantum gravity, which we outline in the concluding Section, and take up again elsewhere.

Before beginning, we note that the idea that the cosmological constant is conjugate to a measure of time has appeared before, in the context of unimodular gravity [10,11]. There the conjugate measure of time is four volume to the past of a three slice.

2. A Plebanski Formulation for Our Proposal

We work first in the Euclidean case. We fix a topology $\mathcal{M} = I \times \Sigma$, where I is the interval and we take Σ compact. We start with an action for general relativity coupled to N chiral fermion fields, all expressed in terms (where P_+^{abcd} is the projection operator onto self-dual two-forms) of Ashtekar variables [19,20],

$$\begin{aligned} \frac{1}{\hbar} S = & \int_{\mathcal{M}} \frac{1}{8\pi G \hbar} \left\{ -P_+^{abcd} e_a \wedge e_b \wedge R_{cd}^+(A^+) + 2\Lambda e^{abcd} e_a \wedge e_b \wedge e^c \wedge e^d + \frac{3}{2\Lambda} R^{AB} \wedge R_{AB} \right\} \\ & + \int_{\Sigma} \bar{\Psi}_{A'} \sigma_a^{A'B} e^a \wedge (D\Psi)_B, \end{aligned} \tag{12}$$

but with the addition of a new, third term, which will suffice to make Λ dynamical, as we shall presently see. We divide the action by Planck’s constant for reasons that will become apparent below.

We then rewrite this action, with its new term, in the Plebanski formulation [21–23]

$$\frac{1}{\hbar} S^{Pleb} = \int_{\mathcal{M}} \frac{1}{8\pi G \hbar} \left(\Sigma^{AB} \wedge R_{AB} - \frac{\Lambda}{6} \Sigma^{AB} \wedge \Sigma_{AB} - \frac{1}{2} \Phi_{ABCD} \Sigma^{AB} \wedge \Sigma^{CD} - \frac{3}{2\Lambda} R^{AB} \wedge R_{AB} \right) + \mathcal{L}^{matter} \quad (13)$$

The new term can be rewritten as

$$S^{CS} = -\frac{1}{16\pi G \hbar} \int_{\mathcal{M}} \frac{3}{\Lambda} R^{AB} \wedge R_{AB} = -\frac{1}{16\pi G \hbar} \int_{\mathcal{M}} \frac{3}{\Lambda} dY_{CS} \quad (14)$$

$$= -\frac{1}{8\pi G \hbar} \int_{\Sigma_{final}} \frac{3}{2\Lambda} Y_{CS} + \frac{1}{8\pi G \hbar} \int_{\Sigma_{initial}} \frac{3}{2\Lambda} Y_{CS} + \frac{1}{16\pi G \hbar} \int_{\mathcal{M}} d\left(\frac{3}{\Lambda}\right) Y_{CS} \quad (15)$$

We note that if we exponentiate this action (divided by \hbar), the first and second terms give a generalization of the Kodama state on the initial and final slice. The last term can be written as

$$S^{CS} = \frac{3}{16\pi \hbar G} \int dt \frac{\dot{\Lambda}}{\Lambda^2} \int_{\Sigma} Y_{CS} \quad (16)$$

and vanishes if Lambda is forced to be a constant.

The field equations for our theory, in the absence of matter, are

$$0 = \frac{\delta S}{\delta \Phi_{ABCD}} \rightarrow \Sigma^{(AB} \wedge \Sigma^{CD)} = 0 \quad (17)$$

$$0 = \frac{\delta S}{\delta \Sigma_{AB}} \rightarrow R_{AB} = \frac{\Lambda}{3} \Sigma_{AB} + \Phi_{ABCD} \Sigma^{CD} \quad (18)$$

$$0 = \frac{\delta S}{\delta A_{AB}} \rightarrow \mathcal{D} \wedge \Sigma^{AB} \equiv S^{AB} \quad (19)$$

with

$$S^{AB} = d\left(\frac{3}{2\Lambda}\right) \wedge R^{AB} = -\frac{3}{2\Lambda^2} d\Lambda \wedge R^{AB} \quad (20)$$

The solution to (17) is that there exists a frame field $e^{AA'}$ such that

$$\Sigma^{AB} = e^{A'A} \wedge e_{A'}^B \quad (21)$$

is the self-dual two form of the metric made from $e^{AA'}$. We also note that if

$$\mathcal{D}e^{BA'} = T^{BA'} \quad (22)$$

is the torsion, then

$$S^{AB} = 2e_{A'}^{(A} \wedge \mathcal{D}e^{B)A'} = 2e_{A'}^{(A} \wedge T^{B)A'} \quad (23)$$

A new feature is then a contribution to the torsion (20) related to the derivative of the cosmological constant.

3. The Underlying Duality

We can see that the terms that involve Λ :

$$S^{\Lambda} = \frac{-1}{16\pi \hbar G} \int_{\mathcal{M}} \left\{ \frac{\Lambda}{3} \Sigma_{AB} \wedge \Sigma^{AB} + \frac{3}{\Lambda} R_{AB} \wedge R^{AB} \right\} \quad (24)$$

have an interesting structure: they have a formal duality symmetry under

$$R^{AB} \rightarrow \frac{\Lambda}{3} \Sigma^{AB} \quad \Sigma^{AB} \rightarrow \frac{3}{\Lambda} R^{AB}. \quad (25)$$

The self-dual solutions, including deSitter space-time, are the self-dual points

$$R^{AB} = \frac{\Lambda}{3} \Sigma^{AB}, \quad \Phi_{ABCD} = 0, \quad (26)$$

where Φ_{ABCD} is a Lagrange multiplier.

We say this symmetry is formal because on shell Σ^{AB} satisfies (17), which is typically not satisfied by R^{AB} .

Thus, we can extend the theory to one that has (25) as an exact symmetry:

$$\begin{aligned} \frac{1}{\hbar} S^{Plcb} = & \int_{\mathcal{M}} \frac{1}{8\pi G \hbar} \left(\Sigma^{AB} \wedge R_{AB} - \frac{\Lambda}{6} \Sigma^{AB} \wedge \Sigma_{AB} - \frac{1}{2} \Phi_{ABCD} (\Sigma^{AB} + \frac{3}{\Lambda} R^{AB}) \wedge (\Sigma^{CD} + \frac{3}{\Lambda} R^{CD}) \right. \\ & \left. - \frac{3}{2\Lambda} R^{AB} \wedge R_{AB} \right) + \mathcal{L}^{matter}. \end{aligned} \quad (27)$$

Instead of (21), this says that there is a frame field such that

$$e^{A'A} \wedge e_{A'}^B = \Sigma^{AB} + \frac{3}{\Lambda} R^{AB}. \quad (28)$$

When we write the action and field equations in terms of this new $e^{AA'}$ we find this yields again an action for the Einstein equations.

4. Three Cases for the Realization of the Theory

Duality (25) results in the determination of Λ as a function of the other fields. To see this we study the field equations for varying Λ . There are three cases, depending on what we choose Λ to be a function of.

- Case I: $\Lambda(x^\mu)$ is a function of space and time.

Varying by $\Lambda(x^\mu)$ we find an equation for $\Lambda(x^\mu)$:

$$\frac{\Lambda(x^\mu)}{3} = \sqrt{\frac{R_{AB} \wedge R^{AB}}{\Sigma_{AB} \wedge \Sigma^{AB}}} \quad (29)$$

Plugging this back into the action, we find

$$S^\Lambda = \frac{-1}{8\pi \hbar G} \int_{\mathcal{M}} \sqrt{R_{AB} \wedge R^{AB}} \sqrt{\Sigma_{AB} \wedge \Sigma^{AB}} \quad (30)$$

This gives an interesting set of equations, the question is whether they are consistent. Further study of this case is left to a future publication.

- Case II: Λ is a function of time on some preferred 3 + 1 slicing.

We fix an explicit slicing such as constant mean curvature slicing. This gives a time coordinate t . We also define Chern–Simons time by an integral over this slicing, leading to $\tau_{CS}(t)$. We fix Λ to be a function of the slicing.

Varying by $\Lambda(t)$ we find an equation for $\Lambda(t)$:

$$\frac{\Lambda(t)}{3} = \sqrt{\frac{\int_{\Sigma_t} R_{AB} \wedge R^{AB}}{\int_{\Sigma_t} \Sigma_{AB} \wedge \Sigma^{AB}}} \tag{31}$$

Plugging this back into the action, we find

$$S^\Lambda = \frac{-1}{8\pi\hbar G} \int dt \sqrt{\int_{\Sigma_t} R_{AB} \wedge R^{AB}} \sqrt{\int_{\Sigma_t} \Sigma_{AB} \wedge \Sigma^{AB}} \tag{32}$$

This is similar to the theory described by the authors of [6], only rather than being conjugate to Newton’s constant, G , it appears the cosmological constant is conjugate to the Chern–Simons time τ_{CS} in the preferred slicing.

The equation of motion (18) becomes nonlocal

$$R_{AB} = +\Phi_{ABCD}\Sigma^{CD} + \sqrt{\frac{\int_{\Sigma_t} R_{AB} \wedge R^{AB}}{\int_{\Sigma_t} \Sigma_{AB} \wedge \Sigma^{AB}}}\Sigma_{AB} \tag{33}$$

This theory is also under investigation.

We can check the homogeneous solutions, with (note that the covariant curl of (18) vanishes because the torsion is (20))

$$R^{AB} = f(t)\Sigma^{AB}, \quad \phi_{ABCD} = 0 \tag{34}$$

which yields

$$f = \frac{\Lambda(t)}{3} \tag{35}$$

and the torsion (20).

It is an important open question whether there are nontrivial solutions where $\Lambda(t)$ varies, with matter or non-zero Weyl tensor, which are not equivalent to deSitter space-time.

- Case III: Λ is one variable over all of space-time [2].

Varying by Λ we find an equation for Λ :

$$\frac{\Lambda}{3} = \sqrt{\frac{\int_{\mathcal{M}} R_{AB} \wedge R^{AB}}{\int_{\mathcal{M}} \Sigma_{AB} \wedge \Sigma^{AB}}} \tag{36}$$

Plugging this back into the action, we find

$$S^\Lambda = \frac{-1}{8\pi\hbar G} \sqrt{\int_{\mathcal{M}} R_{AB} \wedge R^{AB}} \sqrt{\int_{\mathcal{M}} \Sigma_{AB} \wedge \Sigma^{AB}} \tag{37}$$

This is a version of the Kalapor–Padilla theory [2,3].

We can write out all the components of R_{AB} as

$$R_{AB} = \Psi_{ABCD}\Sigma^{CD} + R\Sigma_{AB} + \Phi_{AA'BB'}\bar{\Sigma}^{A'B'} \tag{38}$$

where $\Phi_{AA'BB'}$ and R are the trace-free part of the Ricci tensor and its trace, respectively. Then, the Poyntriagin density is

$$R^{AB} \wedge R_{AB} = 40te^4 \left(R^2 + \Psi^{ABCD}\Psi_{ABCD} - \Phi^{AA'BB'}\Phi_{AA'BB'} \right). \tag{39}$$

It is interesting to note that in all cases the cosmological constant is proportional to the square root Pontryagin density. It was shown that in metric variables the Pontryagin density vanishes in most cosmological space-times. However, in approximate de-Sitter space-times, while the background Pontryagin density is zero, backreacted chiral gravitational waves are sourced by a dynamical field that is coupled to the density and this will yield a nonvanishing Pontryagin density [24,25]. It is interesting to expect that such a mechanism could generate a small cosmological constant today and we leave this for a future work [26].

For the remainder of this paper we study Case II. Whether these cases are all separate, or indeed different phases of the same theory, remains a matter for speculation. We refer the reader to follow up papers for a study of the phenomenological implications of these various possibilities [27–30].

5. The Basis for a Hamiltonian Treatment

Let us now apply a 3 + 1 decomposition to the action, which yields

$$S = \int dt \left[\int_{\Sigma} \frac{1}{8\pi G} (E^{ai} \dot{A}_{ai} - \mathcal{N}\mathcal{H} - \mathcal{N}^a \mathcal{D}_a - \mu_i \mathcal{G}^i) + \frac{3}{16\pi\hbar G} \frac{\dot{\Lambda}}{\Lambda^2} \int_{\Sigma} Y_{CS}(A) \right]. \quad (40)$$

As explained above, we assume that Λ is a function of time, t , alone. Then, we have, as usual, the canonical brackets

$$\{A_a^i(x), E_k^b(y)\} = 8\pi G \delta_a^b \delta_k^i \delta^3(\tilde{y}, x). \quad (41)$$

In addition Λ has a momentum P such that

$$\{\Lambda, P\} = 1 \quad (42)$$

and there is a new primary constraint

$$\mathcal{W} = P - \frac{3}{16\pi G} \frac{1}{\Lambda^2} \int_{\Sigma} Y_{CS}(A) = 0. \quad (43)$$

We add \mathcal{W} to the Hamiltonian with a new lagrange multiplier, ϕ .

$$H = \int_{\Sigma} (\mathcal{N}\mathcal{H} + \mathcal{N}^a \mathcal{D}_a + \mu_i \mathcal{G}^i) + \phi \mathcal{W}. \quad (44)$$

In a future paper we will study further the algebra and other properties of this system of constraints. For the purpose of this paper, it is sufficient to note that Λ , or a function thereof, appears to be conjugate to a function of the Chern–Simons time [1] once the new primary constraint is taken into account. This suggests at once a quantum theory containing a Heisenberg uncertainty principle involve Λ and CS time, something we start to explore here.

6. Towards a Quantum Theory

Given the classical canonical structures unveiled in Section 5 we can now lay down the basis of the quantum theory, resulting in a new interpretation for the Kodama state. Combining (42) and (43) we can infer the Poisson bracket:

$$\{\Lambda, \int_{\Sigma} Y_{CS}(A)\} = \frac{16\pi G \Lambda^2}{3}. \quad (45)$$

Obtaining this classical structure was the ultimate goal of this first paper. It suggests that in a quantum theory we can elevate Λ and $\tau_{CS} = \int_{\Sigma} Y_{CS}(A)$ to operators with commutation relations

$$[\hat{\Lambda}, \hat{\tau}_{CS}] = i \frac{16\pi\hbar G \hat{\Lambda}^2}{3} \quad (46)$$

We note that the commutator of Λ is proportional to Λ^2 , so the larger Λ is in Planck units, the less classical it is. Specifically, using purely kinematical arguments, we can derive an uncertainty principle of the form

$$\Delta\Lambda\Delta\tau_{CS} \geq \frac{8\pi\hbar G}{3} \langle \hat{\Lambda}^2 \rangle. \quad (47)$$

If the expectation of Λ is large in Planck units, then Λ and CS time are complementary or incompatible variables. If it is not, as seems to be the case in the “current” universe, they can be treated as classical variables. In our theory, the onset of classicality in cosmology is therefore related to the observed smallness of Λ (for similar constructions involving G and Λ we refer the reader to the work by the authors of [31]). It would also be interesting to promote G to an operator here so as to close the algebra. Various realizations of this idea in a different context were examined in the work by the authors of [32]).

Note that since $[q, p] = i$ implies $[f(q), p] = if'(q)$, we can re-express the commutator (46) in the more canonical form

$$\left[\frac{\hat{1}}{\Lambda}, \hat{\tau}_{CS} \right] = -i \frac{16\pi\hbar G}{3}. \quad (48)$$

It is then natural to consider representations that diagonalize either $\frac{\hat{1}}{\Lambda}$ or $\hat{\tau}_{CS}$, i.e., one of the two complementary variables. In the CS time representation we have

$$\hat{\tau}_{CS}\Psi(\tau_{CS}) = \tau_{CS}\Psi(\tau_{CS}) \quad (49)$$

$$\frac{\hat{1}}{\Lambda}\Psi(\tau_{CS}) = -i \frac{4\pi\hbar G}{3} \frac{\delta}{\delta\tau_{CS}} \Psi(\tau_{CS}). \quad (50)$$

The Kodama state then appears as an eigenstate of $\frac{\hat{1}}{\Lambda}$ in the CS time basis

$$\langle \tau_{CS} | \frac{1}{\Lambda} \rangle = \Psi_{CS}^* = e^{-i \frac{3}{2\Lambda G \hbar} \frac{1}{8\pi} \tau_{CS}} = e^{-i \frac{3}{2\Lambda G \hbar} \frac{1}{8\pi} \int_{\Sigma} Y_{CS}}. \quad (51)$$

More generally, the Kodama state can be seen as a transition amplitude between eigenstates of $\hat{\tau}_{CS}$ and those of $\frac{\hat{1}}{\Lambda}$:

$$\langle \frac{1}{\Lambda} | \tau_{CS} \rangle = \Psi_{CS} = e^{i \frac{3}{2\Lambda G \hbar} \frac{1}{8\pi} \tau_{CS}} = e^{i \frac{3}{2\Lambda G \hbar} \frac{1}{8\pi} \int_{\Sigma} Y_{CS}}. \quad (52)$$

Within a variable Λ theory, the Kodama state therefore receives a new interpretation as a transition amplitude. We further note that it satisfies a new operator self-dual equation with a quantum Λ operator:

$$\left(\hat{E}_i^a - 3\epsilon^{abc} R_{bc}^i \hat{\Lambda}^{-1} \right) \Psi_{CS} = 0. \quad (53)$$

7. Outlook

In this paper, we have introduced the term S^{new} , given by Equation (7), aiming at introducing a variable or dynamical Λ , and we have just begun a study of the implications for quantum gravity and cosmology. At the level of the classical theory, we have left open important questions. To begin with, we want to extend these results to the Lorentzian theory, in which case the Chern–Simons time becomes the imaginary part of the Chern–Simons invariant of the Ashtekar connection [1].

In each of the three cases we have considered in Section 4 we need to establish whether or not the field equations force Λ to be constant. If the field equations allow, we will want to study classical solutions where Λ varies classically.

We note that matter couplings may play a key role, as they may introduce terms in the torsion that compensate those due to derivatives of Λ , given by Equation (20). One reason to expect this will be noted below.

However, even if Λ is constrained to be constant at the classical level, there may be allowed transitions in the quantum theory in which the value of Λ changes. These would represent new kind of tunneling, which may be important in the early universe. Naively, the amplitude for such a transition would be proportional to

$$\mathcal{A} = e^{\frac{i}{\hbar} S^\Lambda}. \quad (54)$$

A quantum uncertainty principle between Λ and a measure of time could also have deep cosmological consequences. These will be the subject of separate papers [33].

Further novelties in the the quantum theory would result from a possible chiral gravitational anomaly. Let us write the quantum partition function as

$$Z = \int DeDAD\Lambda D\Psi D\bar{\Psi} e^{\frac{i}{\hbar} S}. \quad (55)$$

We propose that under the integral $D\Psi D\bar{\Psi}$ for a set of chiral fermions, there will be a chiral anomaly,

$$\text{Tr} F \wedge F = dJ \quad (56)$$

where J is the dual of the chiral current $J_{abc} = \epsilon_{abcd} \tilde{J}^d$, where

$$\tilde{J}^a = \bar{\Psi}_{A'} e^{A'Ca} \Psi_C \quad (57)$$

Note that in ordinary, perturbative quantum gravity, there is a chiral anomaly of the form of (57), so it is natural to propose that the anomaly appears here as well. On the basis of this conjecture, we can write the action as

$$S = \int_{\mathcal{M}} \left\{ \frac{1}{8\pi G} (-\epsilon^{abcd} e_a \wedge e_b \wedge R_{cd}^+(A^+) + 2\Lambda \epsilon^{abcd} e_a \wedge e_b \wedge e^c \wedge e^d) + \Sigma_N \bar{\Psi}_{A'} \sigma_a^{A'B} e^a \wedge (D\Psi)_B \right\} + \frac{3}{16\pi G\hbar} \int_{\mathcal{M}} \frac{1}{\Lambda} dJ \quad (58)$$

Therefore, in our theory, we can write the CS phase as

$$S^{\text{CS}} = \frac{3}{16\pi G\hbar} \int_{\mathcal{M}} \frac{1}{\Lambda} dJ = -\frac{3}{16\pi G\hbar} \int_{\mathcal{M}} \frac{1}{\Lambda^2} d\Lambda J = +\frac{3}{16\pi G\hbar} \int ds \frac{1}{\Lambda^2} \dot{\Lambda} \int_{\Sigma} J \quad (59)$$

We then have a torsion added to the connection

$$T_i^a = \frac{\delta S^{\text{CS}}}{\delta A_a^i} = (d(\frac{1}{\Lambda}) \wedge R^i)^{*a} = ((d(\frac{1}{\Lambda}) \wedge J^i)^{*a}) \quad (60)$$

where

$$\tilde{J}_i^a = \frac{\delta S^{\Psi}}{\delta A_a^i(x)} = \frac{1}{4\pi} \bar{\Psi}_{A'} e^{A'Ba} \sigma_{iB}^C \Psi_C \quad (61)$$

This modifies the curvature tensor and the Einstein Equation. These aspects of quantum theory will be explored in companion papers. In particular it would be interesting to analyze Black Hole evaporation for the Schwarzschild de-Sitter space from the perspective of this proposal (e.g., in the work by the authors of [34]).

To conclude, we have laid the basis for a new theory of a “quantum cosmological constant” that could address the nagging problems Λ leads to in cosmology and quantum gravity. The glimpses obtained already shed new light on outstanding issues, such as how to interpret the Kodama state in quantum gravity. Finally, as we will argue elsewhere, the quantum theory also hints at how a nonperturbative approach might resolve the problem of the smallness of Λ in our universe.

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