



Article Risky Multi-Attribute Decision-Making Method Based on the Interval Number of Normal Distribution

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Abstract: Focusing on risky decision-making problems taking the interval number of normal distribution as the information environment, this paper proposes a decision-making method based on the interval number of normal distribution. Firstly, the normalized matrix based on the decision maker's attitude is obtained through analysis and calculation. Secondly, according to the existing properties of standard normal distribution, the risk preference factors of the decision makers are considered to confirm the possibility degree of each scheme. The possibility degree is then used for establishing a possibility degree matrix and, consequently, sequencing of all schemes is conducted according to existing theories of possibility degree meaning and the value size of possibility degree. Finally, the feasibility and validity of this method is verified through calculation example analysis.

Keywords: normal distribution; interval number; risky decision-making; multi-attribute decision-making; possibility degree

1. Introduction

Decision-making generally exists in politics, economics, technology, and the daily life of humans. With the continuous development of social economy, decision-making with a single target and attribute is applied less and less in actual economic and management activities. In real multi-attribute decision-making, the circumstance of an uncertain natural state occurs frequently, namely risky multi-attribute decision-making. Since risky multi-attribute decision-making has an extensive practical background in the field of new product development, investment project selection, and engineering project development, the question of how to solve risky multi-attribute decision-making problems is therefore an important topic with academic research value and practical significance.

In regard to multi-attribute decision-making problems in which the attribute value is an interval number, scholars generally agree that the distribution rule of the interval number is uniform distribution [1,2], however, there are also studies [3–6] that consider the distribution rule of the interval number is normal distribution, which is more reasonable. To give a few examples, the distribution of students' examination results, the life distribution of a species, and the height distribution of people. In recent years, the research into the interval number of normal distribution has attracted the attention of experts and scholars, and has been widely used in the field of multi-attribute decision-making problems, but the research has not been mature nor perfectly optimized. For example, Liu et al. [3] analyzed the possibility measure of interval numbers, and studied the interval number complying with the normal distribution rule. They focused on the limitation that the interval number of uniform distribution was adopted to describe fuzzy evaluation value in decision-making, and proposed the method of conducting multi-attribute decision-making by applying the interval number of normal distribution. Wang and Xiao [4] provided several aggregation operators, and proposed a multi-attribute

group decision-making method with incomplete information based on the interval number of normal distribution in group decision-making situations. Concentrating on the existing intersectional situation of two interval numbers, Xu and Lv [5] put forward the intercomparable concept of the possibility degree between interval numbers in normal distribution and a comparative method of interval numbers; according to the principle of maximum deviation, they obtained the method of ascertaining attribute weights and thus provided a multiple attribute decision-making method. Yang et al. [6] were mainly concerned with the effective supplement of incomplete information and the full utilization of uncertain information in grey number sequence prediction, and conducted the random implementation of true value to interval grey numbers of normal distribution under effective numerical coverage. Focusing on multi-attribute decision-making problems in the interval number of normal distribution, Ding and Mao [7] proposed the aggregated method for the interval number of normal distribution, established a possibility degree matrix by using possibility degree, and obtained the optimal decision by using an ordering vector method. For multi-attribute decision-making problems taking the interval number of normal distribution as the information environment, Mao et al. [8] provided the concept and related properties of cross entropy for the interval number of normal distribution, and proposed a decision-making method based on cross entropy and score function. Zhang et al. [9] used the Choquet integral to propose the normal distribution interval number for a Choquet ordered averaging operator. Chiranjibe and Madhumangal [10] attempted to lay a foundation for providing a new approach of a single-valued neutrosophic soft tool which considers many problems that contain uncertainties. In the present study, new aggregation operators of single-valued neutrosophic soft numbers have so far not yet been applied for ranking of the alternatives in decision-making problems. Song et al. [11] focused on and identified both primary strategic and operational elements that will aid managers in evaluating and making risky multi-criteria decisions on green capacity investment projects. In relevant research fields, Liu and Ren [12] only considered the deviation between membership degree and non-membership degree of the existing intuitionistic fuzzy entropy, and excluded the self-contained hesitation information in the intuitionistic fuzzy set, proposing a new class of multiple attribute decision method for intuitionistic fuzzy entropy. Shao and Zhao [13] studied the multi-criteria decision-making (MCDM) problem with completely unknown weights and evaluation information of interval-valued intuitionistic fuzzy number (IVIFN). Considering the influence of the hesitancy degree, a vector representation derived from alternative schemes and positive ideal schemes, negative ideal schemes were proposed, and a method of vector projection measure was put forward for interval intuitionistic fuzzy information. Fu et al. [14] discussed the multi-attribute decision-making (MADM) problem for attribute value with the form of IVIFN carrying incomplete attribute weight information, and comprehensively considered the correlations among attributes, proposing a decision method to address such problems. Novak Zagradjanin et al. [15] considered the multi-robot system based on the cloud technology with a high level of autonomy, which is intended for the execution of tasks in a complex and crowded environment. The proposed concept uses a multi-robot path-planning algorithm that can operate in an environment that is unknown in advance. With the aim of improving the efficiency of path planning, the implementation of multi-criteria decision-making while using the full consistency method is proposed.

In conclusion, this study considers that the attribute value is the interval number of normal distribution, and the occurrence probability of each attribute in a natural state, and in order to solve multi-attribute decision-making problems, puts forward a risky decision-making method based on the interval number of normal distribution. In theoretical research, the interval number of normal distribution is more in accordance with social and natural laws than with the interval number of uniform distribution, but it has been less researched than the interval number of uniform distribution, so the paper will help to strengthen the research in this field. In terms of practical application, the method mentioned in the paper can normalize the decision-making matrix according to different degrees of recognition of decision makers to attributes, and thus make it more widely applicable.

The result of this study will help decision makers to make more reasonable decisions in social and economic activities, and thus bring more economic and social benefits.

2. Theoretical Basis

2.1. Interval Number

Suppose $\tilde{a} = [a^L, a^U] = \{x | a^L \le x \le a^U\}, a^L, a^U \in R; \text{ wherein, } a^L \text{ represents the lower limit value, } a^U \text{ represents the upper limit value, thus a closed interval in real number axis } \tilde{a} = [a^L, a^U] \text{ will be referred to as the interval number. In addition, if } a^L = a^U, \tilde{a} \text{ will degenerate into a certain number, namely the ordinary real number will be regarded as a special interval number [2].$

Definition 1. Suppose interval number $\tilde{a} = [a^L, a^U]$, $\tilde{b} = [b^L, b^U]$, if and only if $a^L = a^L$, $a^U = b^U$, there is $\tilde{a} = \tilde{b}$

Definition 2. Suppose interval number $\tilde{a} = [a^L, a^U], \tilde{b} = [b^L, b^U]$, if and only if $a^L \ge b^U$, there is $\tilde{a} > \tilde{b}$

2.2. Interval Number of Normal Distribution

In the multi-attribute decision-making process, the attribute value provided by decision makers is usually stable, and the attribute value tends towards a certain point, namely, an attribute value with maximum possibility. At this point, the interval number of normal distribution can be used to represent this attribute value [3].

Definition 3. Suppose $\tilde{a} = [a^L, a^U]$ as an interval number in a set of real numbers, if the attribute value $r \in [a^L, a^U]$ complies with normal distribution $N(\mu_a, \sigma_a)$, \tilde{a} will thus be referred to as the interval number of normal distribution, wherein μ_a is the mean value of normal distribution and σ_a is the variance of normal distribution.

According to 3σ principle of normal distribution, μ_a and σ_a in the above definitions can be determined by the following formula [4]:

$$\mu_a = \left(a^L + a^U\right)/2, \ \sigma_a = \left(a^U - a^L\right)/6.$$
(1)

2.3. Sequencing for the Interval Number of Normal Distribution

For the comparison of two interval numbers of normal distribution without intersection, the sequencing can be conducted according to Definition 2, and while the intersection exists between two interval numbers of normal distribution, the sequencing can be conducted according to the following method [16]:

Theorem 1. Suppose random variable A, B is intervals $[a^L, a^U]$, $[b^L, b^U]$ respectively and complying with normal distribution, record as $A \sim N(\mu_a, \sigma_a)$, $B \sim N(\mu_b, \sigma_b)$, thus,

$$P(A > B) = \phi \left(\frac{\mu_a - \mu_b}{\sqrt{\sigma_a + \sigma_b}} \right)$$
⁽²⁾

Definition 4. Suppose interval number $\tilde{a} = [a^L, a^U], \tilde{b} = [b^L, b^U], call P(A > B) = \phi\left(\frac{\mu_a - \mu_b}{\sqrt{\sigma_a + \sigma_b}}\right)$ as possibility degree of A > B, wherein the value of $\phi\left(\frac{\mu_a - \mu_b}{\sqrt{\sigma_a + \sigma_b}}\right)$ is the corresponding function value of standard normal distribution [17,18].

Definition 5. If the possibility degree of the corresponding interval number A > B in scheme S_a , S_b is P(A > B) = 1, then scheme S_a is totally superior to scheme S_b , record as $S_a \xrightarrow{>} 1_1 S_b$; If the possibility degree of

A > B is P(A > B) = 0.5, thus scheme S_a is equivalent to scheme S_b , record as $S_a \xrightarrow{>} S_b$. If the possibility degree of A > B is 0 < P(A > B) < 0.5, thus the possibility degree in which scheme S_a is superior to scheme S_b is P(A > B), record as $S_a \xrightarrow{>} S_b$ and, at this point, the sequencing is $S_a < S_b$. If the possibility degree of A > B is 0.5 < P(A > B) < 1, thus the possibility degree in which scheme S_a is superior to scheme S_b is P(A > B), record as $S_a \xrightarrow{>} S_b$ and, at this point, the sequencing is $S_a < S_b$. If the possibility degree of A > B is 0.5 < P(A > B) < 1, thus the possibility degree in which scheme S_a is superior to scheme S_b is P(A > B), record as $S_a \xrightarrow{>} P(A > B) < 1$, thus the possibility degree in which scheme $S_a > S_b$ [19,20].

Theorem 2. For the decision-making scheme S_a , S_b , S_c , if $S_a \xrightarrow{} P(A > B) = S_b$, $S_b \xrightarrow{} P(B > C) = S_c$ exists and P(A > B) > 0.5, P(B > C) > 0.5, thus $S_a \xrightarrow{} P(A > C) = S_c$ and P(A > C) > 0.5

Through Definition 5 and Theorems 1 and 2, the sequencing method for the interval number of normal distribution can be obtained, namely, the possibility degree matrix with pairwise comparison can be established by a pairwise comparison of all interval numbers of normal distribution.

$$P = \begin{bmatrix} p_{11} & p_{12} & \cdots & p_{1m} \\ p_{21} & p_{22} & \cdots & p_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ p_{m1} & p_{m2} & \cdots & p_{mm} \end{bmatrix}$$

wherein P_{ij} represents the possibility degree of $S_i > S_j$, if $P_{ij} > 0.5$, scheme *i* will be superior to scheme *j* and, otherwise, scheme *j* will be superior to scheme *i* [21].

3. Risky Multi-Attribute Decision-Making Method

3.1. Problem Description

Consider a certain risky multi-attribute decision-making problem, as a matter of convenience, record as $A = \{A_1, A_2, \dots, A_m\}$ which represents the set of *m* alternative schemes, wherein A_i represents the alternative scheme of number *i*; $C = \{C_1, C_2, \dots, C_n\}$ represents the set of number *n* attributes, wherein C_j represents the attribute of number *j*; $w = \{w_1, w_2, \dots, w_n\}$ represents the weight vector of the attribute, wherein w_j is the weight or importance degree of attribute C_j which meets $w_j \ge 0$ and $\sum_{j=1}^{n} W_j = 1$; $\theta = \{\theta_1, \theta_2, \dots, \theta_h\}$ represents the set of natural states, wherein θ_t represents the state of number *t* and p_t represents the occurrence possibility of state θ_t , which meets $p_t \ge 0$ and $\sum_{t=1}^{h} p_t = 1$.

Suppose the interval number $r_{xyz} = [r_{xyz}^L, r_{xyz}^U]$ represents the attribute value in natural state θ_z for scheme A_x in allusion to attribute C_y , thus the risk decision-making matrix R is established [22].

3.2. Normalization of Decision-Making Matrix

In solving risky multi-attribute decision-making problems, generally, the attributes can be divided into benefit-type and cost-type, where the bigger benefit-type attribute value will be better, and the smaller cost-type attribute value will be better. When both appear in a decision-making process at the same time, in order to eliminate the influence on decision-making results caused by different dimensions of different attributes, the normalization of each attribute value should be processed. There are numerous methods for the normalization of the attribute values of interval values, and each has its own pertinence, for example, the extreme value in the range transformation method significantly influences the specification results. This study adopts the range transformation method to process the normalization of the decision-making matrix, and the specific normalization formulas are shown below.

Benefit-type attribute:

$$a_{xyz}^{L} = \frac{r_{xyz}^{L} - minr_{xyz}}{\underset{x}{maxr_{xyz} - minr_{xyz}}}, a_{xyz}^{U} = \frac{r_{xyz}^{U} - minr_{xyz}}{\underset{x}{maxr_{xyz} - minr_{xyz}}}$$
(3)

Cost-type attribute:

$$a_{xyz}^{L} = \frac{\max_{x} r_{xyz} - r_{xyz}^{U}}{\max_{x} r_{xyz} - \min_{x} r_{xyz}}, a_{xyz}^{U} = \frac{\max_{x} r_{xyz} - r_{xyz}^{L}}{\max_{x} r_{xyz} - \min_{x} r_{xyz}}$$
(4)

wherein $\max_{x} r_{xyz} = \max\left(\max_{x} r_{xyz}^{L}, \max_{x} r_{xyz}^{U}\right); \min_{x} r_{xyz} = \min\left(\min_{x} r_{xyz}^{L}, \min_{x} r_{xyz}^{U}\right).$ Concentrating on risky multi-attribute decision-making problems, in view of the limitations of

Concentrating on risky multi-attribute decision-making problems, in view of the limitations of common normalization methods, the normalization result can be amended according to the preference of the decision makers. According to Formula (5), transform the results of the above steps again and obtain the normalization results:

$$b_{xyz}^{L} = exp[(a_{xyz}^{L} - 1)/k_i], b_{xyz}^{U} = exp[(a_{xyz}^{U} - 1)/k_i]$$
(5)

wherein the value of k_i is based on the judgment of decision makers, take $k_i = 1, 2, 3$, and the value of i is the sequence number of attribute C_j , $1 \le i \le n$. If the value of k_i is bigger, the degree of recognition for its corresponding attribute will be lower [23,24].

3.3. Decision-Making Steps

In regard to the above problem, the specific steps of the risky multi-attribute decision-making method based on the interval number of normal distribution mentioned in the paper are as follows:

Step 1 According to Formulas (3) and (4), conduct the normalization to risky decision-making matrix by using the range transformation method, and obtain the decision-making matrix *R*.

Step 2 In view of the limitations of the common normalization method, according to Formula (5), transform the attribute value again based on the preference of the decision makers, and obtain the decision-making matrix R_1 .

Step 3 For the decision-making matrix after normalization, according to Formula (6), conduct weighting operation to attribute value through occurrence probability P_k in different natural states θ_k , obtain decision-making matrix $R_2 = \left(r_{xy}^L, r_{xy}^U\right)$ [25].

$$r_{xy}^{L} = \sum_{k=1}^{m} r_{xyz}^{L} p_{k}, r_{xy}^{U} = \sum_{k=1}^{m} r_{xyz}^{U} p_{k}$$
(6)

Step 4 Combine with the weight vector of attribute w, calculate the deviation V_x of each scheme according to Formula (7).

$$V_x = \sum_{y=1}^m w_y r_{xy} = \left[\sum_{y=1}^m w_y r_{xy}^L, \sum_{y=1}^m w_y r_{xy}^U \right]$$
(7)

Step 5 According to the sequencing method for the interval value of normal distribution derived from the above steps, establish the possibility degree matrix of pairwise comparison *P*, and hereafter, conduct the sequencing of all schemes.

4. Calculating Example Analysis

Consider a selection problem of a new product development project [1]. A company proposes to develop an electronic product; where there are 5 schemes (A_1, A_2, \dots, A_5) available, the main attributes to be considered include developing cost C_1 , sales quality of product C_2 , and rate of return C_3 . In these three attributes, C_1 is cost-type attribute, C_2 and C_3 are benefit-type attributes, and the attribute values of all attributes are interval numbers. Suppose the value located in the interval complies with normal distribution, furthermore, $(\theta_1, \theta_2, \theta_3)$ exist in the future market environment, representing excellent, average and poor states respectively, and their probability of occurrence is p = (0.3, 0.4, 0.3) respectively. Suppose the attribute weight vector provided by the decision makers is $w = (0.35, 0.25, 0.4)^T$, the risky decision-making matrix is shown in Table 1.

Table 1. Risky Decision-Making Matrix.

Scheme	C1			C2			C3		
	θ_1	θ_2	θ_3	$\boldsymbol{ heta}_1$	θ_2	θ_3	θ_1	θ_2	θ_3
A1	[80,90]	[90,100]	[90,110]	[100,120]	[80,100]	[70,80]	[12,16]	[9,12]	[6,8]
A2	[90,100]	[100,110]	[110,120]	[110,120]	[90,100]	[80,90]	[12,18]	[10,15]	[7,10]
A3	[90,110]	[100,120]	[110,130]	[120,130]	[100,110]	[80,100]	[15,22]	[13,20]	[8,12]
A4	[100,110]	[110,130]	[120,130]	[100,110]	[80,90]	[60,80]	[18,23]	[15,20]	[6,10]
A5	[110,120]	[115,130]	[120,140]	[120,150]	[100,120]	[90,100]	[20,25]	[12,18]	[8,10]

(1) According to Formulas (3) and (4), the range transformation method is adopted for processing the normalization of risky decision-making matrix to get decision-making matrix *R*, shown in Table 2.

C alt arm a		C1			C2	C2 C3 θ_2 θ_3 θ_1 θ_2 θ_3 $[0,0.5]$ $[0.25,0.5]$ $[0,0.308]$ $[0,0.273]$ $[0,0.333]$ $0.25,0.5]$ $[0.5,0.75]$ $[0,0.462]$ $[0.091,0.545]$ $[0.167,0.667]$ $0.5,0.75]$ $[0.5,1]$ $[0.231,0.769]$ $[0.364,1]$ $[0.333,1]$ $[0,0.25]$ $[0,0.5]$ $[0.462,846]$ $[0.545,1]$ $[0,0.33,067]$ $[0,5,1]$ $[0,75,1]$ $[0.615,1]$ $[0.233,0667]$ $[0.333,067]$			
Scheme	θ_1	θ_2	θ_3	$\boldsymbol{ heta}_1$	θ_2	θ_3	$\boldsymbol{ heta}_1$	θ_2	θ_3
A1	[0.75,1]	[0.75,1]	[0.6,1]	[0,0.4]	[0,0.5]	[0.25,0.5]	[0,0.308]	[0,0.273]	[0,0.333]
A2	[0.5,0.75]	[0.5,0.75]	[0.4,0.6]	[0.2,0.4]	[0.25,0.5]	[0.5,0.75]	[0,0.462]	[0.091,0.545]	[0.167,0.667]
A3	[0.25,0.75]	[0.25,0.75]	[0.2,0.6]	[0.4,0.6]	[0.5 <i>,</i> 0.75]	[0.5,1]	[0.231,0.769]	[0.364,1]	[0.333,1]
A4	[0.25,0.5]	[0,0.5]	[0.2,0.4]	[0,0.2]	[0,0.25]	[0,0.5]	[0.462,846]	[0.545,1]	[0,0.667]
A5	[0,0.25]	[0,0.375]	[0,0.4]	[0.4,1]	[0.5,1]	[0.75,1]	[0.615,1]	[0.273,0.818]	[0.333,0.667]

Table 2. Risky Decision-Making Matrix R after Normalized Processing.

(2) According to Formula (5), transform attribute value again based on the preferences of the decision makers, suppose decision makers attach importance to each attribute comparably, take $k_1 = 1$, $k_2 = 1$, $k_3 = 1$, and obtain the decision-making matrix R_1 , shown in Table 3.

Table 3. Risky Decision-Making Matrix *R*₁ after Transforming Again.

C .1	C1			C2			C3		
Scheme	θ_1	θ_2	θ_3	$\boldsymbol{ heta}_1$	θ_2	θ_3	$\boldsymbol{ heta}_1$	θ_2	θ_3
A1	[0.779,1.000]	[0.779,1.000]	[0.670,1.000]	[0.368,0.549]	[0.368,0.607]	[0.472,0.607]	[0.368,0.500]	[0.368,0.0.483]	[0.368,0.513]
A2	[0.607,0.779]	[0.607,0.779]	[0.549,0.670]	[0.449,0.549]	[0.472,0.607]	[0.607,0.779]	[0.368,0.584]	[0.403,0.635]	[0.435,0.717]
A3	[0.472,0.779]	[0.472,0.779]	[0.449,0.670]	[0.549,0.670]	[0.607,0.779]	[0.607,1.000]	[0.463,0.794]	[0.529,1.000]	[0.513,1.000]
A4	[0.472,0.607]	[0.368,0.607]	[0.449,0.549]	[0.368,0.449]	[0.368,0.472]	[0.368,0.607]	[0.584,0.857]	[0.635,1.000]	[0.368,0.717]
A5	[0.368,0.472]	[0.368,0.535]	[0.368,0.549]	[0.549,1.000]	[0.607,1.000]	[0.779,1.000]	[0.681,1.000]	[0.483,0.834]	[0.513,0.717]

(3) According to Formula (6), conduct the weighting operation to attribute value through the probability p_k in every natural state θ_k , and then obtain the decision-making matrix R_2 .

$$R_2 = \begin{bmatrix} 0.746, 1.000 & [0.399, 0.589] & [0.368, 0.497] \\ [0.589, 0.746] & [0.506, 0.641] & [0.402, 0.644] \\ [0.465, 0.746] & [0.589, 0.813] & [0.505, 0.938] \\ [0.424, 0.589] & [0.368, 0.506] & [0.539, 0.872] \\ [0.368, 0.520] & [0.641, 1.000] & [0.552, 0.848] \end{bmatrix}$$

(4) Combine the known attribute weight vector $w = (0.35, 0.25, 0.4)^T$, and calculate deviation of each scheme V_x according to Formula (7).

$$V_1 = [0.508, 0.696], V_2 = [0.493, 0.679], V_3 = [0.512, 0.840],$$

 $V_4 = [0.456, 0.682], V_5 = [0.510, 0.772],$

According to Formula (1), transform deviation V_x to the form of (μ_a, σ_a) .

$$V_1' = [0.602, 0.031], V_2' = [0.586, 0.031], V_3' = [0.676, 0.055],$$

$$V'_4 = [0.569, 0.038], V'_5 = [0.641, 0.044],$$

(5) According to the sequencing method for the interval number of normal distribution and theorem 2, a possibility degree matrix with pairwise comparison P can be established, and hereby, all schemes can be sequenced.

	0.5000	0.5255	0.4013	0.5505	0.4443
	0.4745	0.5000	0.3783	0.5267	0.4207
$\mathbf{P} =$	0.5987	0.6217	0.5000	0.6380	0.5450
	0.4495	0.4733	0.3620	0.5000	0.4013
1	0.5557	0.5793	0.4550	0.5987	0.5000

From this, the sequencing result of each scheme is $A_3 > A_5 > A_1 > A_2 > A_4$, therefore, when the value of k_i is 1, Scheme A_3 will be optimal.

(6) In order to verify the influence on decision-making result caused by parameter k_i , suppose the corresponding values of three attributes are k_1 , k_2 , k_3 respectively, now the sequencing result of each scheme is shown in Table 4.

	k _i Value		Sequencing Result of Each Scheme
K1=1 K1=1 K1=1	K2=1 K2=1 K2=3	K3=1 K3=3 K3=1	A3> A5> A1> A2> A4 A3> A5> A1> A4> A2 A3> A5> A1> A4> A2
K1=1 K1=3 K1=3 K1=3 K1=3	K2=3 K2=1 K2=1 K2=3 K2=3	K3=3 K3=1 K3=3 K3=1 K3=3	A3> A5> A1> A2> A4 A5> A3> A2> A4 A5> A3> A2> A1> A4 A5> A3> A2> A1> A4 A3> A5> A2> A1> A4 A3> A5> A2> A4> A1 A3> A5> A1> A2> A4

Table 4. Influence on Decision-Making Result Caused by *k_i* Value.

According to the decision-making method considering regret aversion mentality in Reference [1], its final sequencing result is the same as the result of this research, in which $k_1 = 3$, $k_2 = 3$, and $k_3 = 1$. From the above table, when decision makers are more concerned about cost and rate of return, Scheme 3 will be superior to Scheme 5; when decision makers are more concerned about sales volume and rate of return, Scheme 5 will be superior to Scheme 3.

The characteristics of the method mentioned in [1] are that it considers the regret aversion mentality and behavior of decision makers and obtains the sequencing result by calculating the utility value of the attribute as well as the regret and delight values between schemes. From Table 4, it can be seen that this method tends toward scheme A_3 with lower cost and higher rate of return; for scheme A_5 with higher cost and rate of return, its sequencing result is lower due to higher risk and regret aversion mentality. From original data, it can be seen that both the cost and the sales value of Scheme 5 are higher than those of Scheme 3. In conclusion, the optimal scheme using method in this reference tends to be conservative. The method mentioned in this paper can be adapted to the requirements of various decision makers for decision making, since k_i can take different values. Conservative decision makers can take a smaller corresponding value of cost-type attribute k_i , while optimistic decision makers can take a smaller corresponding value of benefit-type attribute k_i . Therefore, the decision-making method mentioned in this paper is closer to actual production and life in the application of multi-attribute decision-making problems, can serve decision makers with different personalities, and can be more widely applied.

5. Conclusions

For interval number multi-attribute decision-making problems, the interval number of normal distribution undoubtedly has great practical application value, since the distribution rule of interval numbers receives less attention. The paper utilizes the basic theory of normal distribution, transforms the sequencing of decision-making schemes into the comparison of possibility degrees, and thus obtains the optimal decision after normalizing the initial matrix based on the attitude of decision makers. As a result, it provides a multi-attribute decision-making approach based on the interval number of normal distribution and verifies the feasibility and effectiveness of the aforementioned methods by combining calculation example analysis, and enriches the application of decision-making methods [26]. The method mentioned in this study is appropriate for fuzzy decision-making environments in view of its superiority in a number of respects. Not only does the interval number of normal distribution but, furthermore, the decision-making problems described by the interval number of normal distribution are closer to real life, and the influence on decision-making behavior caused by risk preference factors is taken into full consideration. Therefore, this method can be widely applied, and has both promotional and actual decision-making value.

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