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The Theoretical Relationship between the CCR Model and the Two-Stage DEA Model with an Application in the Efficiency Analysis of the Financial Industry

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Abstract: Since Charnes, Cooper, and Rhodes introduced data envelopment analysis (DEA) in 1978, later called the DEA-CCR model, many studies applied this technique to different fields. Based on the original CCR model, many modified DEA models were developed by researchers. Since 1999, Seiford and Zhu presented a two-stage DEA model. Later, these models were widely used in many studies. However, the relationship between the efficiency scores that are obtained from the original CCR model and the two-stage DEA model remains unknown. To fill this gap, this study proposed a theoretical relationship between the efficiency scores that are calculated from the two-stage DEA model and those that are obtained from the original CCR model. How the sets of nonsymmetrical weights affected the efficiency scores were also investigated. Theorems regarding the relationship were developed, and then the model was utilized to evaluate the two-stage efficiency scores of the insurance companies (non-life) and bank branches. The results show that using a two-stage DEA model can get more information about operational efficiency than the traditional CCR model does. The findings from this study about the two-stage DEA technique can provide significant reasons for using this model to evaluate performance efficiency.

Keywords: data envelopment analysis; CCR model; two-stage DEA model; financial industry

1. Introduction

Since Charnes, Cooper, and Rhodes [1] first presented data envelopment analysis (DEA), a methodology commonly called the DEA-CCR model, to identify the relative efficiency of decision-making units (DMUs), this approach has been widely employed in many fields and industries [2–5].

Along with the rapid pace of development of the global economy, international enterprises are now developing stronger than ever before. The scale of organizations and businesses has also become larger and more complicated. Hence, intensification of corporate operations and improving service quality is becoming critical for their survival [6]. The traditional one-stage DEA technique can measure management performance in the industry before assessing the industry's advantages and disadvantages. It is also able to determine the factors that affect managerial efficiency.

Nevertheless, this model can only be used to measure overall performance with initial inputs and outputs. It treats the DMU as a black box and ignores all the transformation process. What if an organization with a complicated system were to divide its performance into many kinds of segments and wants to measure the managerial abilities of each segment separately, such as profitability, marketability, or financial abilities? The traditional one-stage DEA cannot provide sufficient management information for managers and decision makers to assess the advantages and disadvantages of their competitive strategies [6]. To overcome this problem, Seiford and Zhu [7] introduced the two-stage DEA model for providing more information of the inefficiency DMUs. I.e., in the two-stage model, a mediating factor will be applied to split the traditional one-stage DEA CCR model into two stages; the mediating factor is the output of the first stage and the input of the second stage.

The two-stage DEA can appropriately assess the managerial abilities of each segment and has been mentioned and applied in some previous studies. In the study of [8], the authors used a two-stage benchmarked DEA to measure the performance efficiency of US national banks. Deyneli [9] applied a two-stage DEA method to determine the relationship between the efficiency of justice service and the salaries of judges. Hwang and Kao [6] used a two-stage DEA to evaluate the managerial efficiency of Taiwan's 20 non-life insurance companies. An et al. [10] used a two-stage DEA to measure the efficiency of Chinese commercial banks. Bian et al. [11] used a two-stage DEA to measure the efficiency of Chinese industrial systems. Fatimah and Mahmudah [12] used a two-stage DEA to measure the efficiency of elementary schools in Indonesia. Raheli et al. [13] used the model to compute the energy efficiency and sustainability of tomato production.

However, these studies mainly focus on applying DEA models. Only a few studies have mentioned the reasons why they employ a two-stage DEA method to evaluate operational efficiency or focus on the model's development [14–21]. Guo et al. [22] investigated the factors affecting the overall efficiency of the additive network DEA model. This study found that the two-stage DEA identified more sources of inefficiency than the CCR model because the discriminating power of the two-stage DEA is better than that of the traditional DEA. However, few studies have theoretically examined the relationship between the efficiency scores in one-stage and two-stage DEA models. It will be very useful if a bridge between these two models can be built to help decision-makers to flexibly apply the DEA technique to evaluate performance flexibly in their organizations.

Moreover, the basis of a DEA model is that it computes the weights of the inputs and outputs for each DMU individually to obtain the highest efficiency [23]. Hence, the sets of weights are the major factors that determine the efficiency scores. Therefore, it is a very interesting issue to explore how the sets of weights affect on efficiency scores when sets of weights are shifted among stages.

To answer the above questions, there is an urgent need to conduct a study to address these issues. With respect to the CCR model, this study expects to clarify these problems by developing and proving some mathematical theorems that represent the relationship between the efficiency scores and how the set of weights affects them. In addition, this study assesses the importance and advantages of a two-stage DEA model in evaluating the operational efficiency of organizations and companies. Moreover, the different effects of the sets of weights on the efficiency scores when they are shifted among stages are also revealed.

Real data are also provided to illustrate the theorems and show the managerial impacts of managers in terms of applying the proposed models. In particular, the data of twenty insurance companies (non-life) in Taiwan within the period from 2001–2002 will be treated as the inputs, mediators, and outputs of the DMUs. Based on the results, decision makers and investors could make better decisions in their businesses and improve their managerial abilities. This will help decision makers gain an objective perspective on using a two-stage CCR model to measure performance efficiency. Moreover, the study proposes suggestions for further studies on the DEA technique in terms of developing and expanding the CCR model and other DEA models.

2. Literature Review

As mentioned above, the original one-stage DEA model only can characterize operational performance; it does not reflect the profitability or market valuation of companies and organizations [7]. Because a one-stage DEA model will produce many efficient DMUs, it loses distinguishability in an evaluation. Therefore, many kinds of relative techniques have been developed, such as the super efficiency (SE) model of [24], the cross efficiency (CE) model [25], and the stochastic frontier approach (SFA) [26]. In addition, SFA within the Bayesian frameworks enables one to obtain estimates of inefficiency indices and provides information about statistical uncertainty of estimates. Among these models, within the context of this paper, the traditional DEA-CCR model is chosen and examined. Moreover, due to fluctuations and changes in the global economy, companies and organizations have experienced significantly changing structures and scales. Therefore, there is an urgent need for organizations to seek more effective performance measurement methods. Hwang and Kao [6] also judged that the original one-stage DEA technique provides insufficient management information for the production process of a company. As a result, a two-stage DEA model was first introduced by [7], and it has been adopted in many kinds of industries to provide managers with more precise and sufficient management information about their organizations. In a two-stage DEA model, there are inputs, mediators, and outputs. In stage 1, the inputs and mediators (as outputs) are employed to calculate the efficiency score, and then the mediators (as inputs) and outputs are used to compute the efficiency score of stage 2.

In their study, Seiford and Zhu [7] applied the two-stage DEA model to measure the production progress of the top 55 U.S. commercial banks. They measured the marketability in the first stage and the profitability in the second stage. The inputs of the first stage are the following: employees, assets, and shareholders' equity. The mediating factors connecting these two stages are revenues and profits. The three factors that serve as the outputs of the second stage are earnings per share, market value, and total investment return.

By applying the same method of the two-stage DEA method that was introduced by [7], Zhu [27] analyzed the financial efficiency of the top 500 companies as ranked by Fortune magazine. The measurement methods that were used in the first and second stages were unanimous those of [7]. In particular, the researchers employed three input factors—employees, assets, and shareholders' equity. The other three factors, which play the role as the outputs, are market value, total investment returns, and earnings per share (EPS). Subsequently, Chen [28] applied a similar method to evaluate the managerial performances of 22 banks in Taiwan from 1996 to 2000. However, in this study, the researcher divided the banking services of these companies into three stages to evaluate the financial efficiency, business operations, and the marketability separately.

Moreover, Sexton, and Lewis [29] utilized the two-stage DEA model to evaluate the efficiency of the production activities of the American Major League Baseball. The researchers used the resources in the first stage, which include the salaries of all players, as the inputs. The mediating factors are the total gained bases and total surrendered bases in the games. In the second stage, game victories are the outputs factors that can be achieved from the mediating factors. Hwang and Kao [6] followed up the two-stage DEA method to evaluate the managerial efficiency of Taiwan's 20 non-life insurance companies. In this study, marketability was evaluated in the first stage, and profitability was evaluated in the second stage. First, the researchers selected business and administrative expenses, and commissions and acquisition expenses as the inputs; and direct written premiums and reinsurance premiums were selected as the outputs. Second, they treated the first stage outputs as the inputs in the second stage, and chose net underwriting income and investment income as the outputs to measure profitability. In addition, the data of these companies were used one more time in their study in 2008 to decompose the efficiency using a two-stage network structure.

Besides a typical series-structure two-stage DEA model, Fare, and Grosskopf [30] mentioned that there are parallel structures and/or mixed structures, which are named network structures. The DEA technique used to evaluate the efficiency of a network structure is called network DEA [30–32].

The above literature analyzes the production processes using mediators as the outputs of the first stage and the inputs of the second stage. The same idea can be applied to a more than two-stage DEA model, where the outputs of the previous stage can be considered as the inputs of the next stage to enhance a production process, and this could be called a multiple-stage DEA model. Based on the above discussion, it can be observed that a two-stage DEA model can assess the managerial efficiency of organizations more than the traditional one-stage model can. The main point is that all the studies mentioned above only focused on applying the model, and there is no study that examines the relationship between the efficiency scores in one-stage and two-stage DEA models. Is there any connection, and what can the results reveal? Moreover, the set of weights is the major element that determines the efficiency scores. However, few studies attempt to explore the effect of the set of weights on the efficiency scores. This paper is going to discuss those issues and focus on the series structure two-stage DEA model.

3. Model Construction

3.1. Model Explanation

The two-stage DEA model was first presented by [7]. This model can be applied in many fields to measure different managerial abilities separately in each stage, including marketability, profitability, financial abilities, etc. In this model, a mediating factor will be employed to split the traditional one-stage DEA CCR model into two stages. This mediating factor is the output of the first stage, which is the input of the second stage. With that idea, an illustration of a complete two-stage DEA model can be shown in Figure 1.

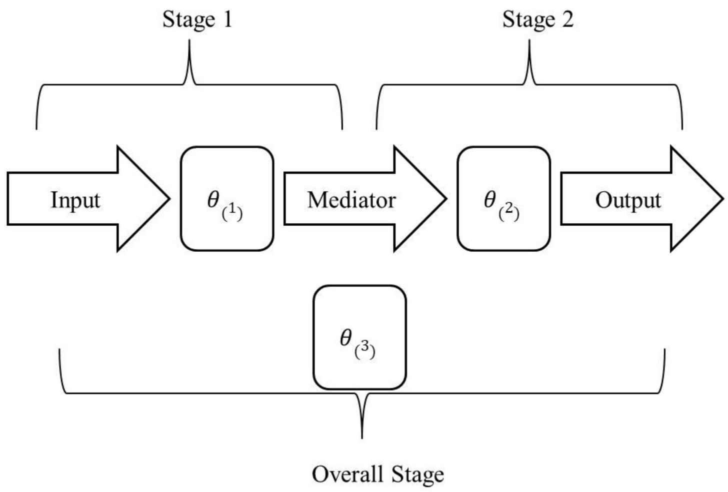

Figure 1. A two-stage DEA model. ($\theta_{(1)}$: the relative efficiency of a DMU in stage one. $\theta_{(2)}$: the elative efficiency of a DMU in stage two. $\theta_{(3)}$: the relative efficiency of a DMU in the overall stage.)

3.1.1. Notation Explanation

In this study, mediating factors are introduced along with the inputs and outputs; hence, there are numerous notations that are used, and they are explained as follows:

Let

- n: the number of evaluated decision-making units (DMUs).
- m: the number of inputs of DMUs.
- s: the number of mediators of DMUs.
- t: the number of outputs of DMUs.
- z_{hk}: h^{th} output of the k^{th} DMU.
- y_{rk}: r^{th} mediator of the k^{th} DMU.
- x_{ik}: i^{th} input of the k^{th} DMU.
- u_{rk}: the weight for r^{th} mediator of the k^{th} DMU in stage one.
- v_{ik}: the weight for i^{th} input of the k^{th} DMU in stage one.

- u^*_{rk}: the weight for r^{th} mediator of the k^{th} DMU in stage one when V' replaces V.
- u'_{rk}: the weight for r^{th} mediator of the k^{th} DMU in stage two.
- w_{hk}: the weight for h^{th} output of the k^{th} DMU in stage two.
- w^*_{hk}: the weight for h^{th} output of the k^{th} DMU in stage two when U replaces U'.
- w'_{hk}: the weight for h^{th} output of the k^{th} DMU in the overall stage.
- v'_{ik}: the weight for i^{th} input of the k^{th} DMU in the overall stage.
- ε: a arbitrary small positive real number.
- $U = (u_{1k}, u_{2k}, \ldots, u_{sk})$, the set of weights for mediator of the k^{th} DMU in stage one.
- $U' = (u'_{1k}, u'_{2k}, \ldots, u'_{sk})$, the set of weights for mediator of the k^{th} DMU in stage two.
- $U^* = (u^*_{1k}, u^*_{2k}, \ldots, u^*_{sk})$, the set of weights for the mediator of the k^{th} DMU in stage one when V' replaces V.
- $W = (w_{1k}, w_{2k}, \ldots, w_{tk})$, the set of weights for output of the k^{th} DMU in stage two.
- $W' = (w'_{1k}, w'_{2k}, \ldots, w'_{tk})$, the set of weights for output of the k^{th} DMU in overall stage.
- $W^* = (w^*_{1k}, w^*_{2k}, \ldots, w^*_{tk})$, the set of weights for output of the k^{th} DMU in stage two when U replaces U'.
- $V = (v_{1k}, v_{2k}, \ldots, v_{mk})$, the set of weights for input of the k^{th} DMU in stage one.
- $V' = (v'_{1k}, v'_{2k}, \ldots, v'_{mk})$, the set of weights for the input of the k^{th} DMU in the overall stage.
- $\theta_{k(1)}$: the relative efficiency score of the k^{th} DMU in stage one, $k = 1, 2, \ldots, n$.
- $\theta_{k(2)}$: the relative efficiency score of the k^{th} DMU in stage two, $k = 1, 2, \ldots, n$.
- $\theta_{k(3)}$: the relative efficiency score of the k^{th} DMU in overall stage, $k = 1, 2, \ldots, n$.
- $\theta^*_{k(1)}$: the relative efficiency score of the k^{th} DMU in stage one when V' replaces V; $k = 1, 2, \ldots, n$.
- $\theta^*_{k(2)}$: the relative efficiency score of the k^{th} DMU in stage two when U replaces U'; $k = 1, 2, \ldots, n$.

3.1.2. Mathematical Model

With all the notation given above, the relative efficiency score of the k^{th} DMU in stage one can be computed as follows:

$$\begin{array}{c} Max.\ \theta_{k(1)} = \sum_{r=1}^{s} u_{rk} y_{rk} \\ s.t. \quad \sum_{i=1}^{m} v_{ik} x_{ik} = 1 \\ \sum_{r=1}^{s} u_{rk} y_{rj} - \sum_{i=1}^{m} v_{ik} x_{ij} \le 0,\ j = 1, 2, \ldots, n \\ u_{rk}, v_{ik} \ge \varepsilon > 0 \end{array} \tag{1}$$

The relative efficiency score of the k^{th} DMU in stage two can be computed as follows:

$$\begin{array}{c} Max.\ \theta_{k(2)} = \sum_{h=1}^{t} w_{hk} z_{hk} \\ s.t. \quad \sum_{r=1}^{s} u'_{rk} y_{rk} = 1 \\ \sum_{h=1}^{t} w_{hk} z_{hj} - \sum_{r=1}^{s} u'_{rk} y_{rj} \le 0, j = 1, 2, \ldots, n \\ w_{hk}, u'_{rk} \ge \varepsilon > 0 \end{array} \tag{2}$$

The relative efficiency score of the k^{th} DMU in the overall stage can be computed as follows:

$$\begin{aligned} Max.\ \theta_{k(3)} &= \sum_{h=1}^{t} w'_{hk} z_{hk} \\ s.t. \quad & \sum_{i=1}^{m} v'_{ik} x_{ik} = 1 \\ \sum_{h=1}^{t} w'_{hk} z_{hj} - \sum_{i=1}^{m} v'_{ik} x_{ij} &\leq 0,\ j = 1, 2, \ldots, n \\ w'_{hk}, v'_{ik} &\geq \varepsilon > 0 \end{aligned} \tag{3}$$

It seems that the efficiency scores are obtained from Equations (1) and (2) should have some relationships to the efficiency scores that are obtained from Equation (3). However, after adding the set of weights into the equations, the results are different from the initial expectation.

3.2. Proposed Theoretical Relationship

The relationship among the efficiency scores is determined by the set of weights in each stage. Owing of that reason, the set of weights are the major factors that connect the stages, and all the theorems that are developed in this study are proved based on them.

At first glance, it seems to be logical to state that if a DMU is efficient in stage one and stage two (the efficiency scores equal 1), then this DMU should be efficient in the overall stage (the overall efficiency scores equal 1). However, in fact, it is possible that the efficiency score in the overall stage could be less than 1. This issue is proven in the following theorem:

Theorem 1. *If a DMU is not efficient in the overall stage ($\theta_{k(3)} < 1$), then it is possible that it is efficient in stage one and/or stage two, which means that $\theta_{k(1)} = 1$ and/or $\theta_{k(2)} = 1$.*

Proof. If $\theta_{k(3)} < 1$, then, from Equation (3), we get the following:

$$\begin{aligned} &\sum_{h=1}^{t} w'_{hk} z_{hj} - \sum_{i=1}^{m} v'_{ik} x_{ij} \leq 0,\ j = 1, 2, \ldots, n \\ &\sum_{i=1}^{m} v'_{ik} x_{ik} = 1 \text{ and} \\ &\sum_{h=1}^{t} w'_{hk} z_{hk} < 1 \end{aligned} \tag{4}$$

Comparing Equation (4) with Equation (1), $\sum_{h=1}^{t} w'_{hk} z_{hk} < 1$ will not affect Equation (1) and its result. Therefore, it is possible that $\theta_{k(1)} = \sum_{r=1}^{s} u_{rk} y_{rk} = 1$.

Comparing Equation (4) with Equation (2), it can be seen that $\theta_{k(2)}$ and $\theta_{k(3)}$ have a certain relationship since they share a common output factor (z_{hk}). However, since the set of weight W is different from W', it is also possible that $\theta_{k(2)} = \sum_{h=1}^{t} w_{hk} z_{hk} = 1$. □

Theorem 1 clarifies the relationship between the efficiency scores from the original one-stage DEA model and the two-stage DEA model. The theorem indicates that there is no strict relationship between the values of these stages, and an efficiency score of 1 in the overall stage of a DMU does not guarantee that the DMU is also efficient in sectional stages. Generally, the efficiency scores in the one-stage DEA model cannot predict the efficiency scores in the two-stage DEA model.

The second objective of this study is to explore the effect of a set of nonsymmetrical weights on the efficiency scores. In the real world, to simplify the evaluating process, some managers prefer to use existing weights to evaluate their subordinate's performance. To investigate the consequence of using existing weights, researchers replace the set of weights of a stage with the equivalent ones of another stage to observe the change of the relative efficiency scores. The exploration processes and obtained results are stated in the three following theorems.

Theorem 2. *If the set of weights U of a DMU that is obtained in stage one is used to replace the set of weights U′ that is obtained in stage two, and we recompute its new set of weights for output (W*) and efficiency score, then the new efficiency score* $\theta^*_{k(2)}$ *will not be greater than* $\theta_{k(2)}$.

Proof. To examine the influence of replacing U' with U, there are two situations that need to be considered:

(a) When $\theta_{k(1)} = 1$, it means that $\sum_{r=1}^{s} u_{rk}y_{rk} = 1$. From Equation (2), we get the following:

$$\theta_{k(2)} = \sum_{h=1}^{t} w_{hk}z_{hk} \leq \sum_{r=1}^{s} u'_{rk}y_{rk} = 1 \tag{5}$$

When the new efficiency score $\theta^*_{k(2)}$ is obtained by replacing the set of weights U' with the set of weights U, then we get the following:

$$\theta{*}k_{(2)} = \sum_{h=1}^{t} w^*_{hk}z_{hk} \leq \sum_{r=1}^{s} u_{rk}y_{rk} = \sum_{r=1}^{s} u'_{rk}y_{rk} = 1 \tag{6}$$

This is because U' is the best set of weights for computing the value of $\theta_{k(2)}$ while U is not in this case. Therefore, from Equations (5) and (6), it is obvious that $\theta^*_{k(2)} \leq \theta_{k(2)}$.

(b) When $\theta_{k(1)} < 1$, it means that $\sum_{r=1}^{s} u_{rk}y_{rk} < 1$. When the new efficiency score $\theta^*_{k(2)}$ is obtained by replacing the set of weights U' with the set of weights U, then we get the following:

$$\theta{*}k_{(2)} = \sum_{h=1}^{t} w^*_{hk}z_{hk} \leq \sum_{r=1}^{s} u_{rk}y_{rk} < \sum_{r=1}^{s} u'_{rk}y_{rk} = 1. \tag{7}$$

From Restraints (5) and (7), it is obvious that $\theta^*_{k(2)} \leq \theta_{k(2)}$.

By combining (a) and (b), the theorem is proved. □

Theorem 3. *If the set of weights U of a DMU that is obtained in stage one is used to replace the set of weights U′ that is obtained in stage two, and we recompute its new efficiency score without recomputing the new set of weights for output, then the new efficiency score* $\theta^*_{k(2)}$ *will not be less than* $\theta_{k(2)}$.

Proof. From Equation (2), when the set of weights U' is replaced by the set of weights U and only the efficiency score of stage two is computed, then the new efficiency score $\theta^*_{k(2)}$ can be calculated as follows:

$$\theta^*_{k(2)} = \frac{\sum_{h=1}^{t} w_{hk}z_{hk}}{\sum_{r=1}^{s} u_{rk}y_{rk}} \tag{8}$$

There are two situations that need to be considered:

(a) If $\theta_{k(1)} = 1$, it means that $\sum_{r=1}^{s} u_{rk}y_{rk} = 1$ and $\sum_{i=1}^{m} v_{ik}x_{ik} = 1$, then:

$$\frac{\sum_{h=1}^{t} w_{hk}z_{hk}}{\sum_{r=1}^{s} u'_{rk}y_{rk}} = \frac{\sum_{h=1}^{t} w_{hk}z_{hk}}{\sum_{r=1}^{s} u_{rk}y_{rk}}, \text{ since } \sum_{r=1}^{s} u_{rk}y_{rk} = \sum_{r=1}^{s} u'_{rk}y_{rk} = 1$$

It can be concluded that $\theta_{k(2)} = \theta*_{k(2)}$

(b) If $\theta_{k(1)} < 1$, it means that $\sum_{r=1}^{s} u_{rk}y_{rk} < 1$ and $\sum_{i=1}^{m} v_{ik}x_{ik} = 1$, then:

$$\frac{\sum_{h=1}^{t} w_{hk}z_{hk}}{\sum_{r=1}^{s} u'_{rk}y_{rk}} < \frac{\sum_{h=1}^{t} w_{hk}z_{hk}}{\sum_{r=1}^{s} u_{rk}y_{rk}} \text{ since } \sum_{r=1}^{s} u_{rk}y_{rk} < \sum_{r=1}^{s} u'_{rk}y_{rk} = 1$$

It can be concluded that $\theta_{k(2)} < \theta*_{k(2)}$

By combining (a) and (b), the theorem is proved. □

Theorem 4. *If the set of weights V′ of a DMU that are obtained in the overall stage is used to replace the set of weights V that is obtained in stage one, and we recompute its new set of weights for the mediator (U*) and the efficiency score, then the new efficiency score* $\theta*_{k(1)}$ *will not be greater than* $\theta_{k(1)}$.

Proof. From Equation (1), we get the following:

$$\theta_{k(1)} = \sum_{r=1}^{s} u_{rk}y_{rk} \leq \sum_{i=1}^{m} v_{ik}x_{ik} = 1 \quad (9)$$

When the new efficiency score $\theta*_{k(1)}$ is obtained by replacing the set of weights V with the set of weights V', then we get the following:

$$\theta*_{k(1)} = \sum_{r=1}^{s} u^*_{rk}y_{rk} \leq \sum_{i=1}^{m} v'_{ik}x_{ik} = \sum_{i=1}^{m} v_{ik}x_{ik} = 1 \quad (10)$$

Although $\sum_{i=1}^{m} v'_{ik}x_{ik} = \sum_{i=1}^{m} v_{ik}x_{ik} = 1$, V is the best set of weights for computing the value of $\theta_{k(1)}$, while V' is not in this case. Therefore, from Equations (9) and (10), it is obvious that $\theta*_{k(1)} \leq \theta_{k(1)}$. □

4. Real Case Application

In this section, the real data will be provided to illustrate and realistically test the theories in the previous section. The results are analyzed to provide detailed information about the efficiency scores. In addition, it also reveals the advantages of the two-stage DEA model compared to the original model.

The study recalls the data from twenty insurance companies (non-life) in Taiwan, which were examined in the research of [6] to illustrate the theorems, as shown in Table 1. In that research, the authors set the inputs and outputs in each stage as follows:

Input factors: business and administrative expenses (I_1) and commissions and acquisition expenses (I_2); mediator factors: direct written premiums (M_1) and reinsurance premiums received (M_2); and output factors: net underwriting income (O_1) and investment income (O_2).

Table 1. Data of 20 decision-making units (DMUs).

No.	DMU	Inputs		Mediator		Outputs	
		I_1	I_2	M_1	M_2	O_1	O_2
1	Taiwan-Fire	1,178,744	673,512	7,451,757	856,735	984,143	681,687
2	Chung-Kuo	1,381,822	1,352,755	10,020,274	1,812,894	1,228,502	834,754
3	Tai-Ping	1,177,494	592,790	4,776,548	560,244	293,613	658,428
4	China-Mriners	601,320	594,259	3,147,851	371,863	248,709	177,331
5	Fubon	6,699,063	3,531,614	37,392,862	1,753,794	7,851,229	3,925,272
6	Zurich	2,627,707	668,363	9,747,908	952,326	1,713,598	415,058
7	Taian	1,942,833	1,443,100	10,685,457	643,412	2,239,593	439,039
8	Ming-Tai	3,789,001	1,873,530	17,267,266	1,134,600	3,899,530	622,868
9	Central	1,567,746	950,432	11,473,162	546,337	1,043,778	264,098
10	First	1,303,249	1,298,470	8,210,389	504,528	1,697,941	554,806
11	Kuo-Hua	1,962,448	672,414	7,222,378	643,178	1,486,014	18,259
12	Union	2,592,790	650,952	9,434,406	118,489	1,574,191	909,295
13	Shing-Kong	2,609,941	1,368,802	13,921,467	811,343	3,609,236	223,047
14	South-China	1,396,002	988,888	7,396,396	465,509	1,401,200	332,283
15	Cathay-Century	2,184,944	651,063	10,422,297	749,893	3,355,197	555,482
16	Allianz-President	1,211,716	415,071	5,606,013	402,881	854,054	197,947
17	Newa	1,453,797	1,085,019	7,695,461	342,489	3,144,484	371,984
18	AIU	757,515	547,997	3,631,484	995,620	692,371	163,927
19	North-America	159,422	182,338	1,141,950	483,291	519,121	46,857
20	Federal	145,442	53,518	316,829	131,920	355,624	26,537

Unit: NT$.

4.1. Relationship between the Efficiency Scores

From Theorem 1, this study shows that: if a DMU is not efficient in the overall stage, then it is possible that it is efficient in stage one and/or stage two. The efficiency scores of stage one (marketability), stage two (profitability), and the overall stage as evaluated by the original one-stage DEA method, which is summarized in Table 2, correspond to this theorem.

Table 2. Operating efficiency scores of the 20 DMUs.

No.	Stage 1	Stage 2	Overall Stage
1	0.993	0.738	0.984
2	0.998	0.635	1
3	0.690	1	0.988
4	0.718	0.451	0.488
5	0.838	1	1
6	1	0.466	0.602
7	0.752	0.577	0.538
8	0.726	0.563	0.495
9	1	0.294	0.371
10	0.862	0.721	0.811
11	0.741	0.416	0.333
12	0.905	1	1
13	0.811	0.596	0.530
14	0.725	0.561	0.518
15	1	0.796	1
16	0.907	0.432	0.499
17	0.723	1	0.838
18	0.794	0.389	0.468
19	1	0.455	1
20	0.934	1	1

According to the result, it can be clearly observed that there are four DMUs that are efficient with respect to marketability (Stage 1), and they are the following: DMU_6, DMU_9, DMU_{15}, and DMU_{19}; and five DMUs that are efficient in profitability (Stage 2), they are: DMU_3, DMU_5, DMU_{12}, DMU_{17}, and DMU_{20}. It is obvious that none of the DMUs in this example are efficient in both stages: marketability and profitability. Moreover, the results also show that some of these DMUs are not efficient in the overall stage using the original DEA models, such as DMU_3, DMU_6, DMU_9, and DMU_{17}. These results are solid evidence that reinforces the reliability of Theorem one. From the results, it seems that if a DMU has an efficiency score of 1 in the one-stage DEA model (overall stage), then it could not have an efficiency score of 1 in both stages of the two-stage DEA model, as we can see in Table 2.

Notably, DMU_2 has an efficiency score of 1 using the one-stage DEA and different efficiency score in stage one (score of 0.998) and stage two (score of 0.635). While the difference between the marketability efficiency and the one-stage efficiency is small, there is a significant difference between the efficiency score of profitability and the one-stage efficiency score. Based on this argument, it can be stated that the analysis using the one-stage DEA model is insufficient for reflecting the management of the mediating factors (direct written premiums and reinsurance premiums received) of DMU_2.

From the analysis, it can be realized that each DMU has its own strengths and weaknesses, and the two-stage DEA model can clearly reflect these characteristics more clearly and evaluate the managerial efficiency of each DMU in more detail than the original one-stage DEA model can. For instance, DMU_5 and DMU_{15} have efficiency scores of 1 using the one-stage DEA model, and it can only be concluded that these two DMUs were performing well compared to the others in the field. In terms of business, this implies that they are better at making profits than the others. Nevertheless, once the two-stage DEA model is adopted for the analysis, the results show something different: while DMU_5 is efficient with respect to profitability, DMU_{15} is efficient with respect to the marketability aspect.

One more thing that needs to be considered is the difference between the efficiency scores of these two DEA models. The bigger the difference is, the better at indicating the advantages and disadvantages of the two-stage DEA model are compared to the one-stage DEA model. This helps managers more clearly observe the status of their companies and identify exactly where the problems come from to provide proper solutions. For example, DMU_{19} has an efficiency score of one in the one-stage DEA model, but when the two-stage model is used, it provides a score of 0.455 for profitability, and it is significantly lower than their marketability efficiency, which also has a score of one. This suggests that this DMU suffered big problems with respect to making profits, and the manager of this company should implement solutions to increase productivity using mediating factors (direct written premiums and reinsurance premiums received).

4.2. How the Weights Affect Efficiency Scores

To determine how the set of nonsymmetrical weights affects the efficiency score, first, the set of weights of the DMUs that are obtained in stages 1 and 2, and the overall stages, are listed in Tables 3–5, respectively. From Theorem 2, if the set of weights U that is obtained in stage one is used to replace the set of weights U' that is obtained in stage two. Its new efficiency score is recomputed, then the new efficiency score $\theta^*_{k(2)}$ will not be greater than $\theta_{k(2)}$, which means that: $\theta^*_{k(2)} \leq \theta_{k(2)}$. The original scores (refer to Table 2) and new efficiency scores of stage two that correspond to Theorem 2 are summarized in Table 6.

Table 3. Set of weights of the DMUs in stage 1.

No.	Inputs		Mediators	
	v_1	v_2	u_1	u_2
1	3.21×10^{-7}	9.23×10^{-7}	1.11×10^{-7}	1.91×10^{-7}
2	7.24×10^{-7}	1×10^{-11}	9.86×10^{-8}	5.70×10^{-9}
3	3.47×10^{-7}	9.97×10^{-7}	1.20×10^{-7}	2.07×10^{-7}
4	1.66×10^{-6}	1×10^{-11}	2.27×10^{-7}	1.31×10^{-8}
5	5.17×10^{-8}	1.85×10^{-7}	2.24×10^{-8}	1×10^{-11}
6	1×10^{-11}	1.50×10^{-6}	6.80×10^{-8}	3.54×10^{-7}
7	5.15×10^{-7}	1×10^{-11}	7.01×10^{-8}	4.06×10^{-9}
8	1.09×10^{-7}	3.13×10^{-7}	3.78×10^{-8}	6.49×10^{-8}
9	6.38×10^{-7}	1×10^{-11}	8.72×10^{-8}	1×10^{-11}
10	7.67×10^{-7}	1×10^{-11}	1.05×10^{-7}	6.05×10^{-9}
11	2.57×10^{-7}	7.38×10^{-7}	8.89×10^{-8}	1.53×10^{-7}
12	1×10^{-11}	1.54×10^{-6}	9.60×10^{-8}	1×10^{-11}
13	1.53×10^{-7}	4.39×10^{-7}	5.29×10^{-8}	9.10×10^{-8}
14	7.16×10^{-7}	1×10^{-11}	9.76×10^{-8}	5.65×10^{-9}
15	2.21×10^{-7}	7.93×10^{-7}	9.59×10^{-8}	1×10^{-11}
16	4.16×10^{-7}	1.19×10^{-6}	1.44×10^{-7}	2.48×10^{-7}
17	6.88×10^{-7}	1×10^{-11}	9.40×10^{-8}	1×10^{-11}
18	4.29×10^{-7}	1.23×10^{-6}	1.49×10^{-7}	2.55×10^{-7}
19	6.27×10^{-6}	1×10^{-11}	8.52×10^{-7}	4.94×10^{-8}
20	1×10^{-11}	1.87×10^{-5}	7.70×10^{-7}	5.23×10^{-6}

Table 4. Set of weights of the DMUs in Stage 2.

No.	Mediators		Outputs	
	u'_1	u'_2	w_1	w_2
1	1.10×10^{-7}	2.10×10^{-7}	1.06×10^{-7}	9.30×10^{-7}
2	9.98×10^{-8}	1×10^{-11}	3.61×10^{-8}	7.08×10^{-7}
3	2.09×10^{-7}	1×10^{-11}	7.57×10^{-8}	1.49×10^{-6}
4	2.59×10^{-7}	4.94×10^{-7}	2.51×10^{-7}	2.19×10^{-6}
5	2.46×10^{-8}	4.67×10^{-8}	2.37×10^{-8}	2.07×10^{-7}
6	7.03×10^{-8}	3.31×10^{-7}	1.47×10^{-7}	5.18×10^{-7}
7	7.29×10^{-8}	3.43×10^{-7}	1.52×10^{-7}	5.38×10^{-7}
8	4.42×10^{-8}	2.08×10^{-7}	9.23×10^{-8}	3.26×10^{-7}
9	7.12×10^{-8}	3.35×10^{-7}	1.49×10^{-7}	5.25×10^{-7}
10	9.45×10^{-8}	4.45×10^{-7}	1.97×10^{-7}	6.97×10^{-7}
11	9.06×10^{-8}	5.38×10^{-7}	2.80×10^{-7}	1×10^{-11}
12	4.56×10^{-8}	4.81×10^{-6}	6.35×10^{-7}	1×10^{-11}
13	5.34×10^{-8}	3.17×10^{-7}	1.65×10^{-7}	1×10^{-11}
14	1.04×10^{-7}	4.91×10^{-7}	2.18×10^{-7}	7.69×10^{-7}
15	7.17×10^{-8}	3.38×10^{-7}	1.50×10^{-7}	5.28×10^{-7}
16	1.33×10^{-7}	6.28×10^{-7}	2.78×10^{-7}	9.83×10^{-7}
17	1.03×10^{-7}	6.10×10^{-7}	3.18×10^{-7}	1×10^{-11}
18	2.75×10^{-7}	1×10^{-11}	9.96×10^{-8}	1.95×10^{-6}
19	8.76×10^{-7}	1×10^{-11}	3.17×10^{-7}	6.21×10^{-6}
20	3.16×10^{-6}	1×10^{-11}	2.63×10^{-6}	1×10^{-11}

Table 5. Set of weights of the DMUs in the overall stage.

No.	**Inputs**		**Outputs**	
	v'_1	v'_2	w_1	w_2
1	8.15×10^{-8}	5.80×10^{-8}	1×10^{-11}	1.44×10^{-6}
2	7.24×10^{-7}	1×10^{-11}	7.02×10^{-8}	1.09×10^{-6}
3	2.05×10^{-7}	1.28×10^{-6}	1×10^{-11}	1.50×10^{-6}
4	1.66×10^{-6}	1×10^{-11}	1×10^{-11}	2.75×10^{-6}
5	2.92×10^{-8}	2.28×10^{-7}	3.14×10^{-8}	1.92×10^{-7}
6	1×10^{-11}	1.50×10^{-6}	1.58×10^{-7}	7.97×10^{-7}
7	3.52×10^{-7}	2.19×10^{-7}	1.38×10^{-7}	5.22×10^{-7}
8	2.02×10^{-7}	1.26×10^{-7}	7.91×10^{-8}	2.99×10^{-7}
9	4.63×10^{-7}	2.88×10^{-7}	1.82×10^{-7}	6.87×10^{-7}
10	7.67×10^{-7}	1×10^{-11}	1.43×10^{-7}	1.02×10^{-6}
11	1×10^{-11}	1.49×10^{-6}	2.24×10^{-7}	1×10^{-11}
12	1×10^{-11}	1.54×10^{-6}	1.63×10^{-7}	8.18×10^{-7}
13	3.03×10^{-7}	1.54×10^{-7}	1.47×10^{-7}	1×10^{-11}
14	4.97×10^{-7}	3.10×10^{-7}	1.95×10^{-7}	7.37×10^{-7}
15	1×10^{-11}	1.54×10^{-6}	1.76×10^{-7}	7.36×10^{-7}
16	2.25×10^{-7}	1.75×10^{-6}	2.42×10^{-7}	1.48×10^{-6}
17	4.70×10^{-7}	2.93×10^{-7}	1.84×10^{-7}	6.96×10^{-7}
18	9.10×10^{-7}	5.67×10^{-7}	3.57×10^{-7}	1.35×10^{-6}
19	6.27×10^{-6}	1×10^{-11}	1.93×10^{-6}	1×10^{-11}
20	5.79×10^{-6}	2.94×10^{-6}	2.81×10^{-6}	1×10^{-11}

Table 6. The efficiency scores when replacing U' with U and recomputed W^{*}.

No.	**Original Efficiency Scores $\theta_{k(2)}$**	**New Efficiency Scores $\theta^*_{k(2)}$**
1	0.738	0.728
2	0.635	0.633
3	1	0.689
4	0.451	0.309
5	1	0.687
6	0.466	0.444
7	0.577	0.279
8	0.563	0.329
9	0.294	0.198
10	0.721	0.483
11	0.416	0.202
12	1	0.674
13	0.596	0.292
14	0.561	0.283
15	0.796	0.655
16	0.432	0.349
17	1	0.355
18	0.389	0.296
19	0.455	0.454
20	1	0.873

When using a two-stage DEA model to evaluate the performance of an organization, what would happen if the set of weights that was obtained in stage one was utilized to compute the efficiency scores of the DMUs in stage two rather than recomputing these values again? Therefore, there is a need to examine the results that are obtained from these two methods in terms of using the two-stage DEA model. Theorem 2 provides strong evidence that the efficiency score in stage two cannot be increased when replacing the set of weights of the inputs in this stage with the set of weights of the outputs that are obtained in stage one. In other words, if the same set of weights of the mediating factors is used for both stages, it may cause a decrease in the efficiency score of stage two.

Moreover, from Theorem 3, if a new weight is not calculated after replacing the weight U' from stage two with U from stage one, and only the efficiency score of stage two is recomputed, then the "new" efficiency score $\theta^*_{k(2)}$ will always greater than or equal to $\theta_{k(2)}$, which means that: $\theta_{k(2)} \leq \theta^*_{k(2)}$. The original scores (refer to Table 2) and new efficiency scores of stage two corresponding to Theorem 3 are summarized in Table 7.

Table 7. The efficiency scores when replacing U' with U without recomputed W^*.

No.	Original Efficiency Scores $\theta_{k(2)}$	New Efficiency Scores $\theta^*_{k(2)}$	Difference
1	0.738	0.744	0.006
2	0.635	0.636	0.001
3	1	1.449	0.449
4	0.451	0.627	0.177
5	1	1.194	0.194
6	0.466	0.466	0
7	0.577	0.767	0.190
8	0.563	0.776	0.213
9	0.294	0.294	0
10	0.721	0.837	0.116
11	0.416	0.562	0.146
12	1	1.105	0.105
13	0.596	0.735	0.139
14	0.561	0.774	0.213
15	0.796	0.796	0
16	0.432	0.476	0.044
17	1	1.383	0.383
18	0.389	0.490	0.101
19	0.455	0.455	0
20	1	1.071	0.071

The results clearly indicate clearly that the new efficiency scores of stage two are always greater than or, at least, equal to the original ones. Theorem two and three are similar, but while the new set of weights W^* and new efficiency score $\theta^*_{k(2)}$ are calculated after replacing U' with U, in Theorem 2, only the new efficiency score $\theta^*_{k(2)}$ is calculated in Theorem 3. With this difference, the new efficiency scores are totally reversed from being less than the original ones in Theorem 2 to be greater than those in Theorem 3. The difference between the original and the new values according to the heorem supports Theorem 3 in that only the DMUs (DMU_6, DMU_9, DMU_{15}, and DMU_{19}) that are efficient in stage one retain the same scores after U replaces U'.

Theorem 4 applies the same method as Theorem 2 to examine the change in the relative efficiency scores in stage one. If the set of weights V' of a DMU that is obtained in the overall stage is used to replace the set of weights V that is obtained in stage one, and we recompute its new set of weights U^* and efficiency score, then the new efficiency score $\theta^*_{k(1)}$ will not greater than $\theta_{k(1)}$. The original scores (refer to Table 2) and new efficiency scores of stage one corresponding to Theorem 4 are summarized in Table 8.

The results in the table provide strong evidence that supports the propriety of Theorem 4. One interesting point in the results is that the original and new scores are only equal when they have values of 1. This is feasible since V and V' serve as the sets of weights of the input factors in stage one and the overall stage, respectively. Based on the results of Theorems 2 and 4, it can be concluded that, when a set of weights in a stage is replaced by the equivalent one from another stage, then the new relative efficiency score will not greater than the original value. All of these arguments are illustrated in Table 8.

Table 8. The efficiency scores of Stage 1 when replacing V with $V^{'}$.

No.	Original Efficiency Scores $\theta_{k(1)}$	New Efficiency Scores $\theta^*_{k(1)}$
1	0.993	0.873
2	0.998	0.982
3	0.690	0.655
4	0.718	0.709
5	0.838	0.761
6	1	1
7	0.752	0.714
8	0.726	0.665
9	1	1
10	0.862	0.859
11	0.741	0.716
12	0.905	0.858
13	0.811	0.759
14	0.725	0.7
15	1	1
16	0.907	0.877
17	0.723	0.681
18	0.794	0.723
19	1	1
20	0.934	0.41

Theorem 4 provides strong evidence that the efficiency score in stage one cannot be increased when replacing the set of weights of the inputs in this stage with the set of weights of the inputs that are obtained in the overall stage.

4.3. Application to the Bank

To illustrate the proposed theorems more clearly, data of ten bank branches in [33] published in 2019 is used as an example. The data is listed in Table 9. Employees (EMs)(10^3), fixed assets (FAs)(¥10^8), and expenses (EXs)(¥10^8) are inputs; credit (CR)(¥10^8) and interbank loans (ILs)(¥10^8) are the mediator in the first stage. Outputs loan (Lo)(¥10^8) and profit (PR)(¥10^8) are outputs in the second stage.

Table 9. Data of 10 Bank Branches.

No.	DMU	Inputs			Mediator		Outputs	
		I_1(EMs)	I_2(FAs)	I_3(EXs)	M_1(CR)	M_2(ILs)	O_1(Lo)	O_2(PR)
1	Maanshan	0.478	0.526	0.385	49.917	5.461	34.990	0.843
2	Anqing	1.236	0.713	0.555	37.495	4.083	20.601	0.486
3	Huangshan	0.446	0.443	0.342	20.985	0.690	8.633	0.129
4	Fuyang	1.248	0.638	0.457	45.051	1.724	9.235	0.302
5	Suzhou	0.705	0.575	0.404	38.163	2.249	12.017	0.314
6	Chuzhou	0.645	0.432	0.401	30.168	2.335	13.813	0.377
7	Luan	0.724	0.510	0.371	26.539	1.342	5.096	0.145
8	Chizhou	0.336	0.322	0.233	16.124	0.489	5.980	0.093
9	Chaozhou	0.668	0.423	0.347	22.185	1.177	9.235	0.200
10	Bozhou	0.342	0.256	0.159	13.436	0.406	2.533	0.006

Unit: ¥.

The efficiency score of Stages 1, 2, and overall stage of the ten bank branches are computed and listed in Table 10.

Table 10. Operating efficiency scores of the 10 Bank Branches.

No.	Stage 1	Stage 2	Overall Stage
1	1	1	1
2	0.554	0.786	0.434
3	0.499	1	0.293
4	0.759	0.970	0.301
5	0.729	0.833	0.355
6	0.736	1	0.545
7	0.552	0.632	0.179
8	0.533	1	0.282
9	0.553	1	0.328
10	0.650	0.498	0.175

Again, from Theorems 2 and 3, if the set of weights U that is obtained in stage one is used to replace the set of weights U' that is obtained in stage two, and its new efficiency score is recomputed—with or without recomputing the set of weights for output—the results are summarized in Table 11. The first column in Table 11 is similar to column 2 in Table 10, while columns 2 and 3 are the results of efficiency scores of the ten bank branches with or without recomputing the set of weights for output.

Table 11. The efficiency scores of the 10 bank branches when replacing U' with U.

No.	Original Efficiency Scores $\theta_{k(2)}$	New Efficiency Scores $\theta^*_{k(2)}$	
		Recomputed W^*	Not Recomputed Weights
1	1	1	1
2	0.786	0.434	1.419
3	1	0.293	2.003
4	0.970	0.301	1.277
5	0.833	0.355	1.143
6	1	0.545	1.359
7	0.632	0.179	1.145
8	1	0.282	1.878
9	1	0.328	1.809
10	0.498	0.175	0.766

The efficiency scores of all DMUs in column 2 are less than or equal to the efficiency scores in column 1, which corresponds to Theorem 2, while the efficiency scores of all DMUs in column 3 are greater than or equal to the efficiency scores in column 1, which corresponds to Theorem 3. It is easy to verify Theorem 3: that the new efficiency score of stage 2 without recomputing the set of weights for the mediator is just equal to the original efficiency score of stage 2 divided by the efficiency score of stage 1.

To verify Theorem 4, we used the set of weights of input in the overall stage (V') to replace the set of weights of input in stage 1 (V) and then computed the efficiency score; the results are listed in Table 12. In this case, the set of weights of input in the overall stage (V') is the same as the set of weights of input in stage 1 (V). Therefore, all efficiency scores remain the same.

Table 12. The efficiency scores of 10 bank branches in stage 1 when replacing V with V'.

No.	Original Efficiency Scores $\theta_{k(1)}$	New Efficiency Scores $\theta^*_{k(1)}$
1	1	1
2	0.554	0.554
3	0.499	0.499
4	0.759	0.759
5	0.729	0.729
6	0.736	0.736
7	0.552	0.552
8	0.533	0.533
9	0.553	0.553
10	0.650	0.650

To verify Theorem 1, using the data in Table 9, change the second input (FAs) of third DMU (Huangshan) from 0.443 to 0.221; then compute the efficiency scores of all DMUs in stage 1 and 2, and the overall Stage. The results are listed in Table 13. It is obvious that the efficiency score in the overall stage of DMU3 is 0.587, and the efficiency scores in stage 1 and 2 are both equal to 1. These results correspond to Theorem 1.

Table 13. Operating efficiency scores of the 10 bank branches with modified data.

No.	Stage 1	Stage 2	Overall Stage
1	1	1	1
2	0.554	0.786	0.434
3	1	1	0.587
4	0.759	0.970	0.301
5	0.729	0.833	0.355
6	0.736	1	0.545
7	0.552	0.632	0.179
8	0.533	1	0.282
9	0.553	1	0.328
10	0.650	0.498	0.175

5. Conclusions and Suggestions

This section summarizes the research findings and draws conclusions. Suggestions for further research are also provided.

5.1. Conclusions

This study has focused on examining the two-stage DEA model to assess the relationship between efficiency scores. The main purpose of this study was to obtain a better understanding of the advantages and disadvantages of using a two-stage DEA model to evaluate the performances of organizations. In addition, this study also answers three questions. Why is a two-stage DEA model used? What is the relationship between the efficiency scores? How does the set of weights affect the efficiency scores?

The original one-stage DEA model with initial inputs and outputs only can characterize operational performance; it does not reflect the profitability or market valuations of companies. They are different aspects of the operational performance of a company. While managers focus on profitability, investors pay their attention to the marketability of that company. That is the reason why the original one-stage DEA technique is stated to be not able to provide adequate management information about a firm's production process. Nevertheless, these kinds of issues can be overcome by using a two-stage DEA model.

Once a two-stage DEA model is applied to provide more information for managers and investors, the first theorem in this study claims that there is no strong relationship between the efficiency scores of the stages. The results from the data of twenty non-life insurance companies in Taiwan listed in Table 2 also support this argument, and an efficiency score of 1 in the overall stage of a DMU does not guarantee that the DMU is also efficient in the sectional stages. In other words, the efficiency scores in the two-stage DEA model are independent of those in the one-stage DEA model.

In terms of the set of nonsymmetrical weights that compute the efficiency scores of the stages, the results from the data of twenty non-life insurance companies in Taiwan show different effects of the set of weights on the efficiency scores when different tests are given. In particular, the same idea was applied in Theorems 2 and 3; however, a minor difference in the calculations generated opposite results in these two theorems. Theorem 2 proved that the efficiency score of a DMU in stage two would be decreased when the set of weights in this stage was replaced by the equivalent one from stage one. When the same calculation method is used in Theorem 4, the obtained results correspond to those in Theorem 2 in that the new scores cannot be greater than the original scores.

5.2. Suggestions

From the conclusion above, it is clear that a two-stage DEA model has advantages in evaluating the operating performance compared to the one-stage DEA model. Hence, a two-stage DEA model is highly recommended for use if managers and decision makers want to have a better understanding of the profitability and market valuations of their companies.

One more suggestion is based on the results that are summarized in Tables 6 and 8, which reflect Theorems 2 and 4, respectively. According to these theorems, the new efficiency scores, which are computed using an equivalent set of weights from another stage, are not greater than the original efficiency scores. This argument implies that when using a two-stage DEA model, managers should not utilize the set of weights that are obtained from a stage to compute efficiency scores for another stage. It would cause a decrease in the relative efficiency scores and cause incorrect adjustments of the inputs and outputs of the examining stage.

This study is the first of its kind that attempted to explore the efficiency scores of a two-stage DEA model; hence, only the DEA-CCR model was used in both stages. There are numerous types of DEA-based models that have been being developed in recent decades, including the BCC model, cross efficiency, super efficiency, stochastic frontier, etc. Hence, researchers can combine other DEA-based models to discover the different aspects and applications of a two-stage DEA model.

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