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A Decision-Making Algorithm Based on the Average Table and Antitheses Table for Interval-Valued Fuzzy Soft Set

Xiuqin Ma *, **Yanan Wang**, **Hongwu Qin** and **Jin Wang**

College of Computer Science and Engineering, Northwest Normal University, Lanzhou 730070, China; 2018221819@nwnu.edu.cn (Y.W.); qinhongwu@nwnu.edu.cn (H.Q.); 2018221822@nwnu.edu.cn (J.W.)
* Correspondence: maxiuqin@nwnu.edu.cn

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Abstract: Interval-valued fuzzy soft set is one efficient mathematical model employed to handle the uncertainty of data. At present, there exist two interval-valued fuzzy soft set-based decision-making algorithms. However, the two existing algorithms are not applicable in some cases. Therefore, for the purpose of working out this problem, we propose a new decision-making algorithm, based on the average table and the antitheses table, for this mathematical model. Here, the antitheses table has symmetry between the objects. At the same time, an example is designed to prove the availability of our algorithm. Later, we compare our proposed algorithm with the two existing decision-making algorithms in several cases. The comparison result shows that only our proposed algorithm can make an effective decision in exceptional cases, and the other two methods cannot make decisions. It is therefore obvious that our algorithm has a stronger decision-making ability, thus further demonstrating the feasibility of our algorithm. In addition, a real data set of the homestays in Siming District, Xiamen is provided to further corroborate the practicability of our algorithm in a realistic situation.

Keywords: fuzzy soft sets; interval-valued fuzzy soft set; the average table; the antitheses table; decision-making

1. Introduction

With different kinds of uncertainties and ambiguities in economics, society, science and engineering, traditional mathematical models can no longer meet the increasing demands, so soft sets and related models have been proposed in order to dispose of these complex problems. Since Molodtsov [1] put forward the soft set theory in 1999, increasing numbers scholars have begun to study new models [2–33] based on soft sets and their applications. In this article, we focus on the interval-valued fuzzy soft set model, which was initiated by Yang et al. [4]. This model was created byway of combining interval-valued fuzzy sets and soft sets. The combined nature of the model indicates why it has the powerful function of processing dubious, blurry data: this model has the strong points of both interval-valued fuzzy set and soft set models. At present, the model is mainly applied in the two domains of decision-making and parameter reduction. For instance, Feng et al. [19] proposed the elastic scheme for decision-making, which is based on (weighted) interval-valued fuzzy soft sets, with the support of the concepts of reduced fuzzy soft sets and level soft sets. Yang et al. [4] proposed another decision-making algorithm based on the interval fuzzy choice values and scores for this model. Due to the lack of an entire evaluation solution based on the interval-valued fuzzy soft set model, Qin et al. [20] provided a straightforward evaluation system for this model. Moreover, when we need to do data analysis of this model with partial information, we could use several feasible methods, as outlined in [21]. In addition, in order to decrease the excrescent parameters so as to cater to different needs, Ma et al. [22] presented four different algorithms of parameter reduction for this model, and compared the four algorithms in terms of computation complexity, exact level of reduction, applicability, multi-usability, reduction results, etc. Later, Peng et al. [23] not only introduced the calculation formula for distance and similarity measurements, entropy, and the mutual conversion relationship, they also exploited the combined weights of both the subjective and objective information. As far as the interval-valued fuzzy soft set-based decision-making algorithms are concerned, there exist two methods, which were initiated by Yang et al. in [4], and Feng et al. in [19], which were mentioned above. However, the two existing algorithms are not applicable in some cases. Therefore, for the purpose of working out this problem, we express a new decision-making algorithm based on the average table and the antitheses table for this model, which has a stronger decision-making ability compared with the two existing methods.

The remainder of this paper is structured as follows. In Section 2, we recall some basic definitions of this model. At the same time, we outline two existing decision-making algorithms which are based on this model, and give their simple instances. In Section 3, we propose some new, related concepts. Then, a novel algorithm for decision-making based on this model is proposed at the end of the part. In Section 4, we compare our proposed method with those of Yang et al. and Feng et al. in some special cases, in order to prove that the algorithm we propose is more feasible and efficient. Later, we provide a set of real data sets from the homestays in Siming District, Xiamen, to further corroborate the practicability of our algorithm in realistic situations, in Section 5. Finally, we summarize the paper in Section 6.

2. Preliminary and Related Work

In this section, we simply recollect some concepts involving this model. At the same time, two existing decision-making algorithms, based on the model of interval-valued fuzzy soft sets, are represented.

Definition 1. *Let U be a non-empty initial universe of objects, and E be a set of parameters in relation to objects in U. Let P(U) be the power set of U, and A be a subset of E. A pair (F, A) is called a soft set over U, where F is a mapping given by F: E → P(U) [1].*

Definition 2. *An interval-valued fuzzy set $\hat{X}$ on a universe U is a mapping such that $\hat{X}$: U → Int([0, 1]), where Int ([0, 1]) represents the set of all closed sub-intervals of [0, 1], and the set of all interval-valued fuzzy sets on U is denoted by $\tilde{\psi}(U)$. Let $\hat{X} \in \tilde{\psi}(U)$ for every $x \in U$, where $\tilde{\psi}(U)$ represents the set of all interval-valued fuzzy sets on U. $\mu^-_{\hat{X}}(x)$ and $\mu^+_{\hat{X}}(x)$ denote the lower and upper degrees of the membership of x to $\hat{X}$ ($0 \le \mu^-_{\hat{X}}(x) \le \mu^+_{\hat{X}}(x) \le 1$), respectively, while $\mu_{\hat{X}}(x) = [\mu^-_{\hat{X}}(x), \mu^+_{\hat{X}}(x)]$ is referred to as the degree of membership of an element x to $\hat{X}$ [3].*

Definition 3. *Let U be an initial universe of objects and E be a set of parameters in relation to objects in U. A pair ($\tilde{\omega}$, E) is called an interval-valued fuzzy soft set over $\tilde{\psi}(U)$, where $\tilde{\omega}$ is a mapping given by $\tilde{\omega}$: E → $\tilde{\psi}(U)$ [4].*

We establish the following instance, with the purpose of demonstrating this model:

- U is the set including six car candidates, and then U = $\{h_1, h_2, h_3, h_4, h_5, h_6\}$;
- A is the set of parameters, and A = $\{\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4, \varepsilon_6\}$ = {power, cheap, security, ride comfort, braking performance}.

A table expressing an interval-valued fuzzy soft set (F, A) is represented in Table 1, in which the lower and upper limits of such an evaluation are shown. For instance, we cannot provide the accuracy of the power of the car h_1, while the degree of the power of the car h_1 is at least 0.3, and at most 0.5.

Two existing algorithms were described for decision-making applications, based on the model. It is clear that Yang et al.'s algorithm [4] is based on interval fuzzy choice values and scores. At the same time, Feng et al.'s algorithm [19] is based on the level soft set and the opinion weighting vector.

Table 1. The interval-valued fuzzy soft set (F, A).

U	ε_1	ε_2	ε_3	ε_4	ε_5
h_1	[0.3,0.5]	[0.5,0.6]	[0.4,0.6]	[0.3,0.5]	[0.7,0.9]
h_2	[0.5,0.7]	[0.6,0.7]	[0.6,0.7]	[0.1,0.2]	[0.5,0.8]
h_3	[0.4,0.5]	[0.1,0.3]	[0.4,0.5]	[0.6,0.7]	[0.2,0.4]
h_4	[0.5,0.6]	[0.2,0.3]	[0.7,0.9]	[0.2,0.4]	[0.1,0.3]
h_5	[0.8,1.0]	[0.0,0.2]	[0.7,0.8]	[0.7,0.9]	[0.4,0.6]
h_6	[0.5,0.8]	[0.5,0.7]	[0.5,0.8]	[0.4,0.5]	[0.3,0.4]

Algorithm 1 ([4])

1. Give the interval-valued fuzzy soft set (F, A).
2. $\forall$ $h_i \in$ U, figure out the choice value c_i for each object h_i such that $c_i = [c_i^-, c_i^+] = [\sum_{p\in P} \mu^-_{\tilde{H}(p)}(h_i), \sum_{p\in P} \mu^+_{\tilde{H}(p)}(h_i)]$.
3. $\forall$ $h_i \in$ U, achieve the score r_i of h_i such that $r_i = \sum_{h_j \in U} ((c_i^- - c_j^-) + (c_i^+ - c_j^+))$.
4. Choose any one of the objects $h_k \in$ U such that $r_k = \max_{h_i\in U}\{r_i\}$ as the best candidate.

The specific usage of the above algorithm was clarified in [4] by an application. Let us briefly review the instance here. In this instance, the interval-valued fuzzy soft set (F, A) is shown in Table 1. Via the above algorithm, we work out the interval fuzzy choice value c_i and the score r_i for all $h_i \in$ U. In the end, the relevant outcome is shown in Table 2, from which we can conclude that h_5 is the best option, in line with Yang et al.'s algorithm, because it has the maximum score r_5 = 6.1.

Table 2. Interval-valued fuzzy soft set (F, A) with interval fuzzy choice values and scores.

U	ε_1	ε_2	ε_3	ε_4	ε_5	c_i	r_i
h_1	[0.3,0.5]	[0.5,0.6]	[0.4,0.6]	[0.3,0.5]	[0.7,0.9]	[2.2,3.1]	1.3
h_2	[0.5,0.7]	[0.6,0.7]	[0.6,0.7]	[0.1,0.2]	[0.5,0.8]	[2.3,3.1]	1.9
h_3	[0.4,0.5]	[0.1,0.3]	[0.4,0.5]	[0.6,0.7]	[0.2,0.4]	[1.7,2.4]	−5.9
h_4	[0.5,0.6]	[0.2,0.3]	[0.7,0.9]	[0.2,0.4]	[0.1,0.3]	[1.7,2.5]	−5.3
h_5	[0.8,1.0]	[0.0,0.2]	[0.7,0.8]	[0.7,0.9]	[0.4,0.6]	[2.6,3.5]	6.1
h_6	[0.5,0.8]	[0.5,0.7]	[0.5,0.8]	[0.4,0.5]	[0.3,0.4]	[2.2,3.2]	1.9

Algorithm 2 ([19])

1. Input the (resultant) interval-valued fuzzy soft set $(\tilde{F}, A)$ and an opinion weighting vector $W = (\alpha, \beta)$;
2. Work out Weighted Reduct Fuzzy Soft Set(WRFS) $\Phi_W = (\tilde{F}_W, A)$ of $(\tilde{F}, A)$ with respect to W;
3. Choose an aggregation operation G;
4. Figure out and display the level soft set $L(\Phi_W; G)$ in tables;
5. Work out the choice value ci of oi, $\forall$i;
6. Select h_k if $c_k = \max_i c_i$ as the optimal choice.

In the same way, the interval-valued fuzzy soft set under consideration is (F, A), which is shown in Table 1. Suppose that the car buyer circumspectly wants to choose the option that meets the criterion most strongly. In such a case, we adopt the "Pre-Top" scheme to handle the decision difficulty in this situation. First of all, we ought to obtain the pessimistic reduct fuzzy soft set of (F, A) on the basis of

Table 1, called $\delta_- = (F_-, A)$, as in Table 3. Then, according to the above algorithm requirements, it is time to choose the aggregation operator G. Due to the scheme we adopted, the aggregation operator G=max will generate a threshold fuzzy soft set, which is worked out as follows:

$$\max_{\delta_-} = \begin{pmatrix} \epsilon_1 & \epsilon_2 & \epsilon_3 & \epsilon_4 & \epsilon_5 \\ 0.8 & 0.6 & 0.7 & 0.7 & 0.7 \end{pmatrix}$$

At last, the relevant top-level soft set $L(\delta_-, \max)$ is presented in Table 4. From Table 4, we can determine that the choice value of object h_5 is the largest, and the maximum value is 3. Based on this result, the car buyer ought to choose h_5 as the best option, in line with Feng et al.'s algorithm.

Although the above two algorithms have the best decision-making ability in some situation, in several special cases, they cannot be effectively used for decision-making. With regards to working out this problem, we depict our method as follows. Then, we compare the three methods in Section 4.

Table 3. Pessimistic reduct fuzzy soft set $\delta_- = (F_-, A)$ of the interval-valued fuzzy soft set (F, A).

U	ε_1	ε_2	ε_3	ε_4	ε_5
h_1	0.3	0.5	0.4	0.3	0.7
h_2	0.5	0.6	0.6	0.1	0.5
h_3	0.4	0.1	0.4	0.6	0.2
h_4	0.5	0.2	0.7	0.2	0.1
h_5	0.8	0.0	0.7	0.7	0.4
h_6	0.5	0.5	0.5	0.4	0.3

Table 4. The level soft set $L(\delta_-, \max)$ with choice values.

U	ε_1	ε_2	ε_3	ε_4	ε_5	c_i
h_1	0	0	0	0	1	1
h_2	0	1	0	0	0	1
h_3	0	0	0	0	0	0
h_4	0	0	1	0	0	1
h_5	1	0	1	1	0	3
h_6	0	0	0	0	0	0

3. The Proposed Decision-Making Algorithm

In this part, firstly, we propose some new related definitions. Then, a new decision-making algorithm, based on the average table and the antitheses table, is depicted at the end of the part.

3.1. The Related Definitions

Definition 4. *For interval-valued fuzzy soft set* $\left(\tilde{S}, E\right)$, $U = \{h_1, h_2, \cdots, h_n\}$ *and* $E = \{e_1, e_2, \cdots, e_m\}$. $\mu_{\tilde{S}(e_j)}(h_i) = [\mu^-_{\tilde{S}(e_j)}(h_i), \mu^+_{\tilde{S}(e_j)}(h_i)]$ *is the degree of membership of an element* h_i *to* $\tilde{S}(e_j)$ *. Then, the concept of the mean degree of membership, which is used in the average table, is defined as follows:*

$$\overline{\mu}_{\tilde{S}(e_j)}(h_i) = (\mu^-_{\tilde{S}(e_j)}(h_i) + \mu^+_{\tilde{S}(e_j)}(h_i))/2 \quad (1)$$

Definition 5. b_{ij} *in the table is defined as the sum of the non-negative values of the below-definedlimited list:*

$$\frac{a_{i1} - a_{j1}}{Q_1}, \frac{a_{i2} - a_{j2}}{Q_2}, \frac{a_{i3} - a_{j3}}{Q_3}, \ldots\ldots, \frac{a_{im} - a_{jm}}{Q_m}$$

Here, Q_j *(j = 1, 2, 3, . . . , m) is the maximum mean membership value for each parameter in each column in the average table.*

Definition 6. *The calculation formula of the row-sum M_i of an object h_i is as follows:*

$$M_i = \sum_{j=1}^{n} b_{ij} \tag{2}$$

In the same way, the column-sum N_j of an object h_j can be calculated as follows:

$$N_j = \sum_{i=1}^{n} b_{ij} \tag{3}$$

Definition 7. *The score of an object h_i is S_i, which may be computed as follows:*

$$S_i = M_i - N_i \tag{4}$$

3.2. The Proposed Algorithm

1. Input the interval-valued fuzzy soft set (F, A).
2. Obtain the average table, in which entry is denoted as a_{ij}, by calculating the mean degree of membership given by the above definition.
3. Find the maximum mean membership value, called Q_j (j = 1, 2, 3, ... , m), for each parameter in each column in the average table.
4. Construct the antitheses table. Each element b_{ij} in the table is defined as the sum of the non-negative values of the below-mentioned limited list: $\frac{a_{i1}-a_{j1}}{Q_1}, \frac{a_{i2}-a_{j2}}{Q_2}, \frac{a_{i3}-a_{j3}}{Q_3}, \ldots \ldots, \frac{a_{im}-a_{jm}}{Q_m}$
5. Compute the row-sum M_i and column-sum N_i in the antitheses table and the score S_i for each object h_i, i = 1, 2, 3, ... , n.
6. The final decision is any object h_k which has the highest score value, i.e., any h_k such that S_k= maxi S_i.

3.3. Example

In order to demonstrate this method, the following example is shown.

Let U be the set of the cars under consideration, and U = {h_1, h_2, h_3, h_4, h_5, h_6}. A is the set of parameters and A = {ε_1, ε_2, ε_3, ε_4, ε_5} = {power, cheap, security, ride comfort, braking performance}. Let (F, A) be an interval-valued fuzzy soft set over the universe U, as shown in Table 1. First of all, according to the above definition of the mean degree of membership, it is easy to compute the average table of (F, A), as shown in Table 5 below.

Table 5. The average table of the interval-valued fuzzy soft set (F, A).

U	ε_1	ε_2	ε_3	ε_4	ε_5
h_1	0.40	0.55	0.50	0.40	0.80
h_2	0.60	0.65	0.65	0.15	0.65
h_3	0.45	0.20	0.45	0.65	0.30
h_4	0.55	0.25	0.80	0.30	0.20
h_5	0.90	0.10	0.75	0.80	0.50
h_6	0.65	0.60	0.65	0.45	0.35

Next, choose the maximum mean membership value, which is called Q_j (j = 1, 2, 3, 4, 5), for the ε_1, ε_2, ε_3, ε_4 and ε_5 parameters in each column of the average table. The results are shown in Table 6 below.

Once more, it is time to construct the antitheses table according to the algorithm, which is given in Table 7.

Here, in order to facilitate the reader's understanding, we give the specific solution method of b_{12} according to the above Definition 5.

Table 6. The average table with the maximum mean membership value.

U	ε_1	ε_2	ε_3	ε_4	ε_5
h_1	0.40	0.55	0.50	0.40	0.80
h_2	0.60	0.65	0.65	0.15	0.65
h_3	0.45	0.20	0.45	0.65	0.30
h_4	0.55	0.25	0.80	0.30	0.20
h_5	0.90	0.10	0.75	0.80	0.50
h_6	0.65	0.60	0.65	0.45	0.35
Q_j	0.90	0.65	0.80	0.80	0.80

From b_{12}, we can get $i = 1$, and $j = 2$.
Next, b_{12} is the sum of the non-negative values of the below-mentioned limited list:

$$\frac{a_{11}-a_{21}}{Q_1}, \frac{a_{12}-a_{22}}{Q_2}, \frac{a_{13}-a_{23}}{Q_3}, \frac{a_{14}-a_{24}}{Q_4}, \frac{a_{15}-a_{25}}{Q_5}$$

Then, bring the corresponding data from Table 6 into the following list.

$$\tfrac{a_{11}-a_{21}}{Q_1} = \tfrac{0.40-0.60}{0.90} < 0, \tfrac{a_{13}-a_{23}}{Q_3} = \tfrac{0.50-0.65}{0.80} < 0, \tfrac{a_{14}-a_{24}}{Q_4} = \tfrac{0.40-0.15}{0.80} = 0.3125 > 0,$$
$$\tfrac{a_{15}-a_{25}}{Q_5} = \tfrac{0.80-0.65}{0.80} = 0.1875 > 0$$

As such, we can obtain the value of b_{12} according to the above definition of b_{12}, which is

$$b_{12} = \frac{a_{14}-a_{24}}{Q_4} + \frac{a_{15}-a_{25}}{Q_5} = 0.4995 \approx 0.50$$

Similarly, we can get the other related data from Table 7 as the antitheses table of (F, A).

Table 7. The antitheses table of the interval-valued fuzzy soft set (F, A).

	h_1	h_2	h_3	h_4	h_5	h_6
h_1	0	0.50	1.23	1.34	1.07	0.56
h_2	0.56	0	1.55	1.23	1.03	0.45
h_3	0.37	0.63	0	0.56	0.15	0.25
h_4	0.54	0.38	0.63	0	0.29	0.19
h_5	1.37	1.27	1.31	1.39	0	1.03
h_6	0.60	0.43	1.15	1.02	0.77	0

Finally, on the basis of the antitheses table, we can figure out the row-sum and the column-sum of each object, in order to further obtain the score S_i for each object h_i, which scores are illustrated in Table 8.

From Table 8, we can sort the scores as $h_5 > h_2 > h_6 > h_1 > h_4 > h_3$, and determine that h_5 has the highest score value 3.06. Consequently, we decide to choose h_5 as the best solution.

Table 8. The score table with the row-sum and the column-sum.

	Row-Sum M_i	Column-Sum N_i	Score S_i
h_1	4.7	3.44	1.26
h_2	4.82	3.21	1.61
h_3	1.96	5.87	−3.91
h_4	2.03	5.54	−3.51
h_5	6.37	3.31	3.06
h_6	3.97	2.48	1.49

4. Comparison with the Method

Algorithm 1 [4] by Yang et al. and Algorithm 2 [19] by Feng et al. each have some decision-making ability, however in some cases they cannot be successfully used for decision-making. Let us look at the following case, shown in Table 9.

Table 9. The interval-valued fuzzy soft set (Z, B).

U	e_1	e_2	e_3	e_4	e_5
h_1	[0.6,0.8]	[0.4,0.5]	[0.3,0.5]	[0.7,0.9]	[0.1,0.5]
h_2	[0.2,0.7]	[0.3,0.5]	[0.5,0.6]	[0.4,0.8]	[0.6,0.7]

We apply Yang et al.'s algorithm, Feng et al.'s method and our method to make decision for Table 9.

4.1. Using Yang et al.'s Algorithm

First, let us take this method to work out the below decision-making problem. Table 10 presents its tabular representation with choice values and scores. According to Table 10, what we can know is that the score of h_1 is identical to the score of h_2, namely $r_1 = r_2 = 0$. Consequentially, we can determine that both of the objects can be regarded as the best option by means of Yang et al.'s algorithm, which cannot triumphantly address this decision's difficulty.

Table 10. The interval-valued fuzzy soft set (Z, B) with interval fuzzy choice values and scores.

U	e_1	e_2	e_3	e_4	e_5	c_i	r_i
h_1	[0.6,0.8]	[0.4,0.5]	[0.3,0.5]	[0.7,0.9]	[0.1,0.5]	[2.1,3.2]	0
h_2	[0.2,0.7]	[0.3,0.5]	[0.5,0.6]	[0.4,0.8]	[0.6,0.7]	[2.0,3.3]	0

4.2. Using Feng et al.'s Algorithm

Then, let us take Algorithm 2 to work out the above decision-making problem. To acquire the choice value, we decide to adopt the "Opt-Top" scheme here, as proposed in article [19], which is suitable for this situation. Accordingly, we ought to work out the optimistic reduct fuzzy soft set of (Z, B), i.e., $\xi_+ = (Z_+, B)$.This can be understood as W = (0, 1). Its tabular representation is shown in Table 11. What is more, it is primitively known that the aggregation operator G=max will generate a threshold fuzzy soft set, which is determined as follows:

$$\max_{\xi_+} = \left(\begin{array}{ccccc} e_1 & e_2 & e_3 & e_4 & e_5 \\ 0.8 & 0.5 & 0.6 & 0.9 & 0.7 \end{array} \right)$$

Then, the relevant top-level soft set $L(\xi_+, \max)$ is presented in Table 12. From Table 12, what we can know is that the choice value of h_1 is identical to the choice value of h_2, namely $c_1 = c_2 = 3$. Therefore, we see that both of the objects can be regarded as the best option according to Feng et al.'s method, which cannot effectively make a decision between the two objects.

Table 11. Optimistic reduct fuzzy soft set $\xi_+ = (Z_+, B)$ of the interval-valued fuzzy soft set (Z, B).

U	e_1	e_2	e_3	e_4	e_5
h_1	0.80	0.50	0.50	0.90	0.50
h_2	0.70	0.50	0.60	0.80	0.70

Table 12. The level soft set $L(\xi_+, \max)$ with choice values.

U	e_1	e_2	e_3	e_4	e_5	c_1
h_1	1	1	0	1	0	3
h_2	0	1	1	0	1	3

4.3. Using Our Proposed Algorithm

Next, let us use our proposed algorithm to work out the decision-making problem of Table 10.

Step 1: Input (Z, B), as shown in Table 10, according to the algorithm.

Step 2: According to the above definition of the mean degree of membership, compute the average table of (Z, B), as shown in Table 13 below.

Step 3: Select the maximum mean membership value for each parameter, as shown in Table 13 below.

Step 4: Construct the antitheses table according to our proposed algorithm, which is shown as Table 14 below.

Step 5: On the basis of the above antitheses table, we can figure out the row-sum and the column-sum of each object in order to further obtain the score S_i for each object h_i, which scores are illustrated in Table 15.

Step 6: Find the object with the largest score value from Table 15, which is the best choice for decision-making.

Table 13. The average table with the maximum mean membership values.

U	e_1	e_2	e_3	e_4	e_5
h_1	0.70	0.45	0.40	0.80	0.30
h_2	0.45	0.40	0.55	0.60	0.65
Q_j	0.70	0.45	0.55	0.80	0.65

Table 14. The antitheses table of the interval-valued fuzzy soft set (Z, B).

	h_1	h_2
h_1	0	0.72
h_2	0.81	0

Table 15. The score table with the row-sum and the column-sum.

	Row-Sum M_i	Column-Sum N_i	Score S_i
h_1	0.72	0.81	−0.09
h_2	0.81	0.72	0.09

In summary, from Table 15, we can determine that h_2 has a higher score than h_1. For this reason, we ought to choose h_2 as the best option. Therefore, our proposed algorithm is more feasible and efficient, and the results show that our algorithm has a stronger decision-making ability compared to the two existing methods.

5. Application of Our Algorithm in a Practical Situation

In this part, we provide a real data set of the homestays in Siming District, Xiamen, which is from the website www.agoda.com, to further corroborate the practicability and power of our algorithm in a realistic situation.

A graduating senior student is preparing for a graduation trip to Xiamen. It is known that Xiamen's homestays are very wonderful and distinctive, so many tourists choose to stay in a homestay.

Therefore, he wants to seek out a good homestay in this area of Siming District, Xiamen. We can get the review data of these homestays from www.agoda.com. Through the survey of these homestays, we know that the sojourners who stayed here usually scored the homestays in the following categories, including "Environment and cleanliness", "Position", "Comfort level", "Cost performance", and so on. At the same time, these sojourners are composed of five types: sweet couples, solo travelers, families with infants and young children, families with teenagers and groups of friends. Each type of accommodation group provided the homestay with an average score. Next, we take the minimum score value and the maximum score value, from the average score value given by the five types of sojourners, as the lower degree of membership and the upper degree of membership, respectively, which are expounded and normalized by the interval-valued fuzzy soft set (F, A). At present, we have collected 19 alternative homestays as, follows, U = $\{h_1, h_2, h_3, \ldots, h_{19}\}$ = {Xiamen Aishang Inn, That Year Yishe Guest House, Liangzhu Lifestyle Hotel, Xiamen Shibajian Inn, Meng Shi Guang Homestay, Sunny Sea House, Logom Xiamen Moonwatcher Seascape Inn, Xiamen Sunshine Beach Inn, Xiamen Banpo Inn, Fenghuang Mu Coffee Guest House, Xiamen Into Spring Hometel, Garden Dreamer, Xiamen Chenxi Garden, Xia Men Jia No.17, Xiamen Bloom Pinellia Holiday Home, Xiamen Slow Life Hotel, Youran Hotel, Mansion 1929, Seclusion light luxury Guesthouse}, and there is a related set of six parameters, that is, A = {"Environment and cleanliness", "Position", "Service", "Facilities", "Comfort level", "Cost performance"}. Table 16 below shows the interval-valued fuzzy soft set (F, A) as a tabular form of the 19 homestays in Siming District, Xiamen.

Table 16. The interval-valued fuzzy soft set (F, A) for homestays in Siming District, Xiamen.

U	e_1	e_2	e_3	e_4	e_5	e_6
h_1	[0.72, 0.88]	[0.78, 0.89]	[0.84, 0.96]	[0.79, 0.88]	[0.75, 0.85]	[0.75, 0.85]
h_2	[0.78, 0.91]	[0.88, 0.89]	[0.93, 0.97]	[0.80, 0.92]	[0.78, 0.89]	[0.77, 0.86]
h_3	[0.90, 0.93]	[0.89, 0.93]	[0.95, 0.98]	[0.85, 0.92]	[0.88, 0.92]	[0.84, 0.89]
h_4	[0.85, 0.95]	[0.83, 0.95]	[0.84, 0.96]	[0.81, 0.94]	[0.82, 0.93]	[0.78, 0.94]
h_5	[0.67, 1.00]	[0.60, 1.00]	[0.85, 1.00]	[0.70, 1.00]	[0.70, 1.00]	[0.70, 1.00]
h_6	[0.79, 1.00]	[0.75, 0.88]	[0.85, 1.00]	[0.83, 1.00]	[0.79, 0.88]	[0.75, 0.85]
h_7	[0.89, 1.00]	[0.85, 0.94]	[0.93, 1.00]	[0.96, 1.00]	[0.89, 1.00]	[0.89, 0.97]
h_8	[0.67, 0.96]	[0.58, 0.96]	[0.92, 1.00]	[0.50, 0.95]	[0.58, 0.96]	[0.58, 0.96]
h_9	[0.54, 0.83]	[0.58, 0.84]	[0.67, 0.94]	[0.58, 0.83]	[0.54, 0.85]	[0.58, 0.83]
h_{10}	[0.25, 1.00]	[0.75, 1.00]	[0.50, 1.00]	[0.50, 0.96]	[0.25, 1.00]	[0.25, 0.92]
h_{11}	[0.84, 0.92]	[0.86, 0.90]	[0.89, 0.99]	[0.89, 0.92]	[0.86, 0.93]	[0.85, 0.89]
h_{12}	[0.84, 1.00]	[0.79, 1.00]	[0.81, 1.00]	[0.84, 1.00]	[0.81, 1.00]	[0.81, 1.00]
h_{13}	[0.38, 0.75]	[0.38, 0.75]	[0.75, 0.75]	[0.50, 0.75]	[0.50, 0.75]	[0.38, 0.75]
h_{14}	[0.67, 0.80]	[0.83, 0.94]	[0.67, 0.85]	[0.67, 0.80]	[0.67, 0.75]	[0.67, 0.80]
h_{15}	[0.75, 0.88]	[0.80, 0.95]	[0.81, 1.00]	[0.74, 1.00]	[0.72, 0.92]	[0.68, 0.88]
h_{16}	[0.73, 0.93]	[0.83, 0.96]	[0.79, 0.96]	[0.67, 0.93]	[0.65, 0.89]	[0.65, 0.93]
h_{17}	[0.75, 0.89]	[0.83, 0.91]	[0.80, 0.89]	[0.73, 0.85]	[0.70, 0.83]	[0.68, 0.83]
h_{18}	[0.71, 0.89]	[0.86, 0.93]	[0.88, 0.96]	[0.79, 0.93]	[0.75, 0.89]	[0.67, 0.88]
h_{19}	[0.70, 0.92]	[0.78, 0.83]	[0.80, 0.92]	[0.78, 0.92]	[0.73, 0.85]	[0.65, 0.84]

Let us use the algorithm we proposed to solve the following practical decision-making problem: one traveler needs to select the most suitable homestay from these 19 homestays. Therefore, the final decision result will be obtained step by step, according to the following steps.

Step 1: Input the interval-valued fuzzy soft set (F, A).

Step 2: Compute the average table of (F, A) in line with the definition of the mean degree of membership, as shown in Table 17 below.

Step 3: Select the maximum mean membership value for each parameter, as shown in Table 17 below.

Step 4: Construct the antitheses table according to our proposed algorithm, which is shown as Table 18 below.

Step 5: Figure out the row-sum and the column-sum of each object, in order to further obtain the score S_i for each object h_i, which are illustrated in 19.

Step 6: find the object with the largest score valued from Table 19, which is the best option for decision-making.

From the final result, we can see that the seventh homestay, called Logom Xiamen Moonwatcher Seascape Inn, is the best homestay option, which has the highest score among the 19 homestays. What is more, the priority order of the whole homestays is as follows:

$$h_7 > h_{12} > h_3 > h_{11} > h_4 > h_2 > h_6 > h_5 > h_{18} > h_{15} > h_1 > h_{16} > h_{19} > h_{17} > h_8 > h_{14} > h_9 > h_{10} > h_{13}.$$

As a result, the validity and the reliability of our algorithm have been further corroborated through this practical case regarding homestay evaluation and selection in Siming District, Xiamen.

Table 17. The average table with the maximum mean membership value.

U	e_1	e_2	e_3	e_4	e_5	e_6
h_1	0.80	0.84	0.90	0.84	0.80	0.80
h_2	0.85	0.89	0.95	0.86	0.84	0.82
h_3	0.92	0.91	0.97	0.89	0.90	0.87
h_4	0.90	0.89	0.90	0.88	0.88	0.86
h_5	0.84	0.80	0.93	0.85	0.85	0.85
h_6	0.90	0.82	0.93	0.92	0.84	0.80
h_7	0.95	0.90	0.97	0.98	0.95	0.93
h_8	0.82	0.77	0.96	0.73	0.77	0.77
h_9	0.69	0.71	0.81	0.71	0.70	0.71
h_{10}	0.63	0.88	0.75	0.73	0.63	0.59
h_{11}	0.88	0.88	0.94	0.91	0.90	0.87
h_{12}	0.92	0.90	0.91	0.92	0.91	0.91
h_{13}	0.57	0.57	0.75	0.63	0.63	0.57
h_{14}	0.74	0.89	0.76	0.74	0.71	0.74
h_{15}	0.82	0.88	0.91	0.87	0.82	0.78
h_{16}	0.83	0.90	0.88	0.80	0.77	0.79
h_{17}	0.82	0.87	0.85	0.79	0.77	0.76
h_{18}	0.80	0.90	0.92	0.86	0.82	0.78
h_{19}	0.81	0.81	0.86	0.85	0.79	0.75
Q_j	0.95	0.91	0.97	0.98	0.95	0.93

Table 18. The antitheses table of the interval-valued fuzzy soft set (F, A).

U	h_1	h_2	h_3	h_4	h_5	h_6	h_7	h_8	h_9	h_{10}	h_{11}	h_{12}	h_{13}	h_{14}	h_{15}	h_{16}	h_{17}	h_{18}	h_{19}
h_1	0.00	0.00	0.00	0.00	0.04	0.02	0.00	0.25	0.69	0.85	0.00	0.00	1.33	0.47	0.02	0.10	0.18	0.02	0.14
h_2	0.24	0.00	0.00	0.05	0.14	0.12	0.00	0.42	0.93	1.05	0.02	0.04	1.58	0.66	0.15	0.26	0.37	0.15	0.36
h_3	0.51	0.26	0.00	0.16	0.36	0.30	0.01	0.68	1.19	1.31	0.11	0.07	1.84	0.92	0.40	0.51	0.63	0.40	0.62
h_4	0.35	0.16	0.00	0.00	0.24	0.18	0.00	0.58	1.04	1.16	0.03	0.00	1.68	0.76	0.25	0.37	0.47	0.27	0.47
h_5	0.19	0.04	0.00	0.03	0.00	0.06	0.00	0.35	0.83	1.04	0.00	0.02	1.48	0.66	0.15	0.26	0.35	0.16	0.27
h_6	0.26	0.11	0.03	0.07	0.16	0.00	0.00	0.44	0.92	1.11	0.03	0.02	1.57	0.73	0.20	0.33	0.42	0.22	0.36
h_7	0.74	0.49	0.24	0.39	0.59	0.50	0.00	0.91	1.42	1.54	0.32	0.22	2.07	1.15	0.63	0.74	0.86	0.63	0.85
h_8	0.08	0.01	0.00	0.06	0.03	0.03	0.00	0.00	0.52	0.76	0.02	0.05	1.16	0.39	0.05	0.08	0.12	0.06	0.14
h_9	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.33	0.00	0.00	0.65	0.05	0.00	0.00	0.00	0.00	0.00
h_{10}	0.04	0.00	0.00	0.00	0.09	0.07	0.00	0.12	0.21	0.00	0.00	0.00	0.53	0.00	0.00	0.00	0.01	0.00	0.08
h_{11}	0.42	0.20	0.02	0.10	0.28	0.21	0.00	0.61	1.11	1.23	0.00	0.03	1.76	0.85	0.32	0.45	0.54	0.34	0.54
h_{12}	0.52	0.32	0.08	0.17	0.39	0.30	0.00	0.74	1.20	1.32	0.13	0.00	1.85	0.93	0.41	0.52	0.64	0.42	0.64
h_{13}	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
h_{14}	0.05	0.00	0.00	0.00	0.10	0.08	0.00	0.14	0.32	0.39	0.01	0.00	0.92	0.00	0.01	0.00	0.02	0.00	0.09
h_{15}	0.13	0.01	0.00	0.01	0.11	0.07	0.00	0.33	0.79	0.91	0.00	0.00	1.44	0.53	0.00	0.15	0.23	0.03	0.22
h_{16}	0.10	0.01	0.00	0.01	0.11	0.09	0.00	0.25	0.68	0.80	0.02	0.00	1.33	0.41	0.04	0.00	0.12	0.04	0.18
h_{17}	0.05	0.00	0.00	0.00	0.08	0.05	0.00	0.17	0.56	0.69	0.00	0.00	1.21	0.31	0.00	0.00	0.00	0.02	0.09
h_{18}	0.13	0.01	0.00	0.03	0.12	0.09	0.00	0.34	0.79	0.91	0.02	0.01	1.44	0.52	0.03	0.16	0.25	0.00	0.23
h_{19}	0.02	0.00	0.00	0.00	0.01	0.00	0.00	0.19	0.57	0.77	0.00	0.00	1.22	0.38	0.00	0.07	0.09	0.01	0.00

Table 19. The score table with the row-sum and the column-sum.

U	Row-Sum M_i	Column-Sum N_i	Score S_i
h_1	4.11	3.83	0.28
h_2	6.54	1.62	4.92
h_3	10.28	0.37	9.91
h_4	8.01	1.08	6.93
h_5	5.89	2.85	3.04
h_6	6.98	2.17	4.81
h_7	14.29	0.01	14.28
h_8	3.56	6.52	−2.96
h_9	1.03	13.77	−12.74
h_{10}	1.15	16.17	−15.02
h_{11}	9.01	0.71	8.30
h_{12}	10.58	0.46	10.12
h_{13}	0.00	25.06	−25.06
h_{14}	2.13	9.72	−7.59
h_{15}	4.96	2.66	2.30
h_{16}	4.19	4.00	0.19
h_{17}	3.23	5.30	−2.07
h_{18}	5.08	2.77	2.31
h_{19}	3.33	5.28	−1.95

6. Conclusions

Since the two existing algorithms have no decision-making ability in some cases, in this article, for the purpose of solving this problem, we propose a new decision-making algorithm based on the average table and the antitheses table for interval-valued fuzzy soft set. Next, in order to explain this new algorithm, we give an example which is easy to understand. By comparing the algorithm we proposed with the two existing methods in some special cases, we have proven that our algorithm has a stronger decision-making ability, thus further demonstrating the superiority of our algorithm. What is more, a related application of our algorithm in real life is given. That is, we offer a real data set of the homestays in Siming District, Xiamen, in order to conclusively corroborate the validity and availability of our algorithm in a practical situation. As a result, it can be concluded that our proposed algorithm is viable and conducive to further research on the decision-making issue, based on this model. The future scope of this research might reach to applying our decision-making methods to real applications as diverse as evaluation systems, recommender systems and conflict handling, etc., and providing the complete solution.

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