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Combination of the Single-Valued Neutrosophic Fuzzy Set and the Soft Set with Applications in Decision-Making

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Abstract: In this article, we propose a novel concept of the single-valued neutrosophic fuzzy soft set by combining the single-valued neutrosophic fuzzy set and the soft set. For possible applications, five kinds of operations (e.g., subset, equal, union, intersection, and complement) on single-valued neutrosophic fuzzy soft sets are presented. Then, several theoretical operations of single-valued neutrosophic fuzzy soft sets are given. In addition, the first type for the fuzzy decision-making based on single-valued neutrosophic fuzzy soft set matrix is constructed. Finally, we present the second type by using the AND operation of the single-valued neutrosophic fuzzy soft set for fuzzy decision-making and clarify its applicability with a numerical example.

Keywords: single-valued neutrosophic fuzzy set; soft set; Algorithm 1; Algorithm 2; decision-making

1. Introduction

Many areas (e.g., physics, social sciences, computer sciences, and medicine) work with vague data that require fuzzy sets [1], intuitionistic fuzzy sets [2], picture fuzzy sets [3], and other mathematical tools. Molodtsov [4] presented a novel approach termed "soft set theory", which plays a very significant role in different fields. Therefore, several researchers have developed some methods and operations of soft set theory. For instance, Maji et al. [5] introduced some notions of and operations on soft sets. In addition, Maji et al. [6] gave an application of soft sets to solve fuzzy decision-making. Maji et al. [7] proposed the notion of fuzzy soft sets, followed by studies on inverse fuzzy soft sets [8], belief interval-valued soft sets [9], interval-valued intuitionistic fuzzy soft sets [10], interval-valued picture fuzzy soft sets [11], interval-valued neutrosophic soft sets [12], and generalized picture fuzzy soft sets [13]. Furthermore, several expansion models of soft sets have been developed very quickly, such as possibility Pythagorean fuzzy soft sets [14], possibility m-polar fuzzy soft sets [15], possibility neutrosophic soft sets [16], and possibility multi-fuzzy soft sets [17]. Karaaslan and Hunu [18] defined the notion of type-2 single-valued neutrosophic sets and gave several distance measure methods: Hausdorff, Hamming, and Euclidean distances for Type-2 single-valued neutrosophic sets. Al-Quran et al. [19] presented the notion of fuzzy parameterized complex neutrosophic soft expert sets and gave a novel approach by transforming from the complex case to the real case for decision-making. Qamar and Hassan [20] proposed a novel approach to Q-neutrosophic soft sets and studied several operations of Q-neutrosophic soft sets. Further, they generalized Q-neutrosophic soft expert sets based on uncertainty for decision-making [21]. On the other hand, Uluçay et al. [22] presented the concept of generalized neutrosophic soft expert sets and applied a novel algorithm for multiple-criteria decision-making. Zhang et al. [23] gave novel algebraic operations of totally dependent neutrosophic sets and totally dependent neutrosophic soft sets. In 2018, Smarandache [24] generalized the soft set to the hypersoft set by transforming the function F into a multi-argument function.

Fuzzy sets are used to tackle uncertainty using the membership grade, whereas neutrosophic sets are used to tackle uncertainty using the truth, indeterminacy, and falsity membership grades, which are considered as independent. As the motivation of this article, we present a novel notion of the single-valued neutrosophic fuzzy soft set, which can be seen as a novel single-valued neutrosophic fuzzy soft set model, which gives rise to some new concepts. Since neutrosophic fuzzy soft sets have some difficulties in dealing with some real-life problems due to the nonstandard interval of neutrosophic components, we introduce the single-valued neutrosophic fuzzy soft set (i.e., the single-valued neutrosophic set has a symmetric form, since the membership (T) and nonmembership (F) are symmetric with each other, while indeterminacy (I) is in the middle), which is considered as an instance of neutrosophic fuzzy soft sets. The structural operations (e.g., subset, equal, union, intersection, and complement) on single-valued neutrosophic fuzzy soft sets, and several fundamental properties of the five operations above are introduced. Lastly, two novel approaches (i.e., Algorithms 1 and 2) to fuzzy decision-making depending on single-valued neutrosophic fuzzy soft sets are discussed, in addition to a numerical example to show the two approaches we have developed.

The rest of this article is arranged as follows. Section 2 briefly introduces several notions related to fuzzy sets, neutrosophic sets, single-valued neutrosophic sets, neutrosophic fuzzy sets, single-valued neutrosophic fuzzy sets, soft sets, fuzzy soft sets, and neutrosophic soft sets. Section 3 discusses single-valued neutrosophic fuzzy soft sets (along with their basic operations and structural properties). Section 4 gives two algorithms for single-valued neutrosophic fuzzy soft sets for decision-making. Lastly, the conclusions are given in Section 5.

2. Preliminaries

In the following, we present a short survey of seven definitions which are necessary to this paper.

2.1. Fuzzy Set

Definition 1 (cf. [1])**.** *Assume that X (i.e., $X = \{x_1, x_2, ..., x_p\}$) is a set of elements and $\mu(x_p)$ is a membership function of element $x_p \in X$. Then*

(1) The following mapping (called fuzzy set), is given by

$$\mu : X \longrightarrow [0,1]$$

and $[0,1]^X$ is a set of whole fuzzy subset over X.

(2) Let

$$\mu = \left\{ \frac{\mu(x_1)}{x_1}, \frac{\mu(x_2)}{x_2}, \cdots, \frac{\mu(x_p)}{x_p} \middle| x_p \in X \right\} \in [0,1]^X$$

and

$$\nu = \left\{ \frac{\nu(x_1)}{x_1}, \frac{\nu(x_2)}{x_2}, \cdots, \frac{\nu(x_p)}{x_p} \middle| x_p \in X \right\} \in [0,1]^X.$$

Then

(1) *The union $\mu \cup \nu$, is defined as*

$$\mu \cup \nu = \left\{ \frac{\mu(x_1) \vee \nu(x_1)}{x_1}, \frac{\mu(x_2) \vee \nu(x_2)}{x_2}, \cdots, \frac{\mu(x_p) \vee \nu(x_p)}{x_p} \middle| x_p \in X \right\}.$$

(2) *The intersection $\mu \cap \nu$, is defined as*

$$\mu \cap \nu = \left\{ \frac{\mu(x_1) \wedge \nu(x_1)}{x_1}, \frac{\mu(x_2) \wedge \nu(x_2)}{x_2}, \cdots, \frac{\mu(x_p) \wedge \nu(x_p)}{x_p} \middle| x_p \in X \right\}.$$

2.2. Neutrosophic Set and Single-Valued Neutrosophic Set

Definition 2 (cf. [25,26]). *Assume that X (i.e., $X = \{x_1, x_2, ..., x_p\}$) is a set of elements and*

$$\Phi = \left\{ \frac{(T_{\Phi}(x_p), I_{\Phi}(x_p), F_{\Phi}(x_p))}{x_p} \middle| x_p \in X,\ 0 \le T_{\hat{\Phi}}(x_p) + I_{\hat{\Phi}}(x_p) + F_{\hat{\Phi}}(x_p) \le 3 \right\}.$$

(1) *If $T_{\Phi}(x_p) \in]0^-, 1^+[$ (i.e., the degree of truth membership), $I_{\Phi}(x_p) \in]0^-, 1^+[$ (i.e., the degree of indeterminacy membership), and $F_{\Phi}(x_p)$ (i.e., the degree of falsity membership), then Φ is called a neutrosophic set on X, denoted by $(\mathbb{NS})^X$.*

(2) *If $T_{\Phi}(x_p) \in [0,1]$ (i.e., the degree of truth membership), $I_{\Phi}(x_p) \in [0,1]$ (i.e., the degree of indeterminacy membership), and $F_{\Phi}(x_p) \in [0,1]$ (i.e., the degree of falsity membership), then Φ is called a single-valued neutrosophic set on X, denoted by $(\mathbb{SVNS})^X$.*

2.3. Neutrosophic Fuzzy Set and Single-Valued Neutrosophic Fuzzy Set

Definition 3 (cf. [27]). *Assume that X (i.e., $X = \{x_1, x_2, ..., x_p\}$) is a set of elements and*

$$\hat{\Phi} = \left\{ \frac{(T_{\hat{\Phi}}(x_p), I_{\hat{\Phi}}(x_p), F_{\hat{\Phi}}(x_p), \mu(x_p))}{x_p} \middle| x_p \in X,\ 0 \le T_{\hat{\Phi}}(x_p) + I_{\hat{\Phi}}(x_p) + F_{\hat{\Phi}}(x_p) \le 3 \right\}.$$

(1) *If $T_{\hat{\Phi}}(x_p) \in]0^-, 1^+[$ (i.e., the degree of truth membership), $I_{\hat{\Phi}}(x_p) \in]0^-, 1^+[$ (i.e., the degree of indeterminacy membership), and $F_{\hat{\Phi}}(x_p)$ (i.e., the degree of falsity membership), then $\hat{\Phi}$ is called a neutrosophic fuzzy set on X, denoted by $(\mathbb{NFS})^X$.*

(2) *If $T_{\hat{\Phi}}(x_p) \in [0,1]$ (i.e., the degree of truth membership), $I_{\hat{\Phi}}(x_p) \in [0,1]$ (i.e., the degree of indeterminacy membership), and $F_{\hat{\Phi}}(x_p) \in [0,1]$ (i.e., the degree of falsity membership), then $\hat{\Phi}$ is called a single-valued neutrosophic fuzzy set on X, denoted by $(\mathbb{SVNFS})^X$.*

Definition 4 (cf. [27]). *Let $\hat{\Phi}, \hat{\Psi} \in (\mathbb{SVNFS})^X$, where*

$$\hat{\Phi} = \left\{ \frac{(T_{\hat{\Phi}}(x_p), I_{\hat{\Phi}}(x_p), F_{\hat{\Phi}}(x_p), \mu(x_p))}{x_p} \middle| x_p \in X,\ 0 \le T_{\hat{\Phi}}(x_p) + I_{\hat{\Phi}}(x_p) + F_{\hat{\Phi}}(x_p) \le 3 \right\}$$

and

$$\hat{\Psi} = \left\{ \frac{(T'_{\hat{\Psi}}(x_p), I'_{\hat{\Psi}}(x_p), F'_{\hat{\Psi}}(x_p), \mu'(x_p))}{x_p} \middle| x_p \in X,\ 0 \le T_{\hat{\Psi}}(x_p) + I_{\hat{\Psi}}(x_p) + F_{\hat{\Psi}}(x_p) \le 3 \right\}.$$

The following operations (i.e., complement, inclusion, equal, union, and intersection) are defined by

(1) $\hat{\Phi}^c = \left\{ \frac{(F_{\hat{\Phi}}(x_p), 1 - I_{\hat{\Phi}}(x_p), T_{\hat{\Phi}}(x_p), 1 - \mu(x_p))}{x_p} \middle| x_p \in X \right\}.$

(2) $\hat{\Phi} \subseteq \hat{\Psi} \iff T_{\hat{\Phi}}(x_p) \leq T'_{\hat{\Psi}}(x_p), I_{\hat{\Phi}}(x_p) \geq I'_{\hat{\Psi}}(x_p), F_{\hat{\Phi}}(x_p) \geq F'_{\hat{\Psi}}(x_p)$ *and* $\mu(x_p) \leq \mu'(x_p)\ (\forall x_p \in X)$.

(3) $\hat{\Phi} = \hat{\Psi} \iff \hat{\Phi} \subseteq \hat{\Psi}$ *and* $\hat{\Psi} \subseteq \hat{\Phi}$.

(4) $\hat{\Phi} \cup \hat{\Psi} = \left\{ \frac{(F_{\hat{\Phi}}(x_p) \vee F'_{\hat{\Psi}}(x_p), I_{\hat{\Phi}}(x_p) \wedge I'_{\hat{\Psi}}(x_p), T_{\hat{\Phi}}(x_p) \wedge T'_{\hat{\Psi}}(x_p), \mu(x_p) \vee \mu'(x_p))}{x_p} \middle| x_p \in X \right\}$.

(5) $\hat{\Phi} \cap \hat{\Psi} = \left\{ \frac{(F_{\hat{\Phi}}(x_p) \wedge F'_{\hat{\Psi}}(x_p), I_{\hat{\Phi}}(x_p) \vee I'_{\hat{\Psi}}(x_p), T_{\hat{\Phi}}(x_p) \vee T'_{\hat{\Psi}}(x_p), \mu(x_p) \wedge \mu'(x_p))}{x_p} \middle| x_p \in X \right\}$.

2.4. Soft Set, Fuzzy Soft Set, and Neutrosophic Soft Set

Definition 5 (cf. [4,7,28]). *Assume that X (i.e., $X = \{x_1, x_2, ..., x_p\}$) is a set of elements and I (i.e., $I = \{i_1, i_2, ..., i_q\}$) is a set of parameters, where $(p, q \in N, N$ are natural numbers). Then*

(1) The following mapping (called a soft set), is given by

$$S : I \to P(X),$$

where $P(X)$ is a set of all subsets over X.

(2) The following mapping (called a fuzzy soft set), is given by

$$\widetilde{S} : I \to [0,1]^X,$$

where $[0,1]^X$ is a set of whole fuzzy subset over X.

(3) The following mapping (called a neutrosophic soft set), is given by

$$\widetilde{\hat{S}} : I \to (\mathbb{NS})^X,$$

where $(\mathbb{NS})^X$ is a set of whole neutrosophic subset over X.

Example 1. *Assume that the two brothers Mr. Z and Mr. M plan to go the car dealership office to purchase a new car. Suppose that the car dealership office contains types of new cars $X = \{x_1, x_2, x_3, x_4\}$ and $I = \{i_1, i_2, i_3\}$ characterize three parameters, where i_1 is "cheap", i_2 is "expensive", and i_3 is "beautiful". Then*

(1) *By Definition 5(1) we can describe the soft sets as $S_{(i_1)} = \{x_1, x_3\}, S_{(i_2)} = \{x_3, x_4\}$, and $S_{(i_3)} = \{x_2\}$. Therefore,*

$$\mathcal{S} = \left\{ \frac{\{x_1, x_3\}}{i_1}, \frac{\{x_3, x_4\}}{i_2}, \frac{\{x_2\}}{i_3} \right\}.$$

(2) *It is obvious to replace the crisp number 0 or 1 by a membership of fuzzy information. Therefore, by Definition 5(2) we can describe the fuzzy soft sets by $\widetilde{S}_{(i_1)} = \left\{ \frac{0.3}{x_1}, \frac{0.4}{x_2}, \frac{0.6}{x_3}, \frac{0.5}{x_4} \right\}$, $\widetilde{S}_{(i_2)} = \left\{ \frac{0.6}{x_1}, \frac{0.9}{x_2}, \frac{0.1}{x_3}, \frac{0.2}{x_4} \right\}$, $\widetilde{S}_{(i_3)} = \left\{ \frac{0.7}{x_1}, \frac{0.5}{x_2}, \frac{0.2}{x_3}, \frac{0.9}{x_4} \right\}$. Then,*

$$\widetilde{S} = \left\{ \frac{\left\{ \frac{0.3}{x_1}, \frac{0.4}{x_2}, \frac{0.6}{x_3}, \frac{0.5}{x_4} \right\}}{i_1}, \frac{\left\{ \frac{0.6}{x_1}, \frac{0.9}{x_2}, \frac{0.1}{x_3}, \frac{0.2}{x_4} \right\}}{i_2}, \frac{\left\{ \frac{0.7}{x_1}, \frac{0.5}{x_2}, \frac{0.2}{x_3}, \frac{0.9}{x_4} \right\}}{i_3} \right\}.$$

(3) *By Definition 5(3) we can describe the neutrosophic soft sets as*

$$\widetilde{\hat{S}}_{(i_1)} = \left\{ \frac{(0.3, 0.7, 0.5)}{x_1}, \frac{(0.1, 0.8, 0.5)}{x_2}, \frac{(0.2, 0.6, 0.8)}{x_3}, \frac{(0.4, 0.7, 0.6)}{x_4} \right\},$$

$$\widetilde{\hat{S}}_{(i_2)} = \left\{ \frac{(0.3, 0.7, 0.5)}{x_1}, \frac{(0.1, 0.8, 0.5)}{x_2}, \frac{(0.2, 0.6, 0.8)}{x_3}, \frac{(0.5, 0.8, 0.3)}{x_4} \right\},$$

and

$$\widetilde{\hat{S}}_{(i_3)} = \left\{ \frac{(0.3, 0.7, 0.5)}{x_1}, \frac{(0.1, 0.8, 0.5)}{x_2}, \frac{(0.2, 0.6, 0.8)}{x_3}, \frac{(0.8, 0.9, 0.2)}{x_4} \right\}.$$

3. Single-Valued Neutrosophic Fuzzy Soft Set

In the following, we propose the concept of a single-valued neutrosophic fuzzy soft set and study some definitions, propositions, and examples.

Definition 6. *Assume that X (i.e., $X = \{x_1, x_2, ..., x_p\}$) is a set of elements, I (i.e., $I = \{i_1, i_2, ..., i_q\}$) is a set of parameters, and $\mathbb{S}^{XI}$ is called a soft universe. A single-valued neutrosophic fuzzy soft set $\hat{\Phi}_{(i_q)}$ over X, denoted by $(\mathbb{SVNFS})^{XI}$, is defined by*

$$\hat{\Phi}_{(i_q)} = \left\{ \frac{(T_{\hat{\Phi}_{(i_q)}}(x_p), I_{\hat{\Phi}_{(i_q)}}(x_p), F_{\hat{\Phi}_{(i_q)}}(x_p), \mu(x_p))}{x_p} \,\middle|\, i_q \in I,\ x_p \in X,\ 0 \le T_{\hat{\Phi}_{(i_q)}}(x_p) + I_{\hat{\Phi}_{(i_q)}}(x_p) + F_{\hat{\Phi}_{(i_q)}}(x_p) \le 3 \right\},$$

where $p, q \in N$ (N are natural numbers) and $\mu(x_p) \in [0,1]$. For each parameter $i_q \in I$ and for each $x_p \in X$, $T_{\hat{\Phi}_{(i_q)}}(x_p) \in [0,1]$ (i.e., the degree of truth membership), $I_{\hat{\Phi}_{(i_q)}}(x_p) \in [0,1]$ (i.e., the degree of indeterminacy membership), and $F_{\hat{\Phi}_{(i_q)}}(x_p) \in [0,1]$ (i.e., the degree of falsity membership).

Example 2. *Assume that $X = \{x_1, x_2, x_3\}$ are three kinds of novel cars and $I = \{i_1, i_2, i_3\}$ are three parameters, where i_1 is "cheap", i_2 is "expensive", and i_3 is "beautiful". Let $\mu \in [0,1]^X$ and $\hat{\Phi}_{(i_q)} \in (\mathbb{SVNFS})^{XI}$ are defined as follows ($q = 1, 2, 3$):*

$$\hat{\Phi}_{(i_1)} = \left\{ \frac{(0.3, 0.7, 0.5, 0.2)}{x_1}, \frac{(0.1, 0.8, 0.5, 0.5)}{x_2}, \frac{(0.2, 0.6, 0.8, 0.7)}{x_3} \right\},$$

$$\hat{\Phi}_{(i_2)} = \left\{ \frac{(0.9, 0.4, 0.5, 0.7)}{x_1}, \frac{(0.3, 0.7, 0.5, 0.4)}{x_2}, \frac{(0.8, 0.2, 0.6, 0.8)}{x_3} \right\},$$

$$\hat{\Phi}_{(i_3)} = \left\{ \frac{(0.6, 0.3, 0.5, 0.6)}{x_1}, \frac{(0.3, 0.5, 0.6, 0.4)}{x_2}, \frac{(0.7, 0.1, 0.6, 0.3)}{x_3} \right\}.$$

Additionally, we can write by matrix form as

$$\hat{\Phi} = \left(\begin{array}{c|ccc} I & x_1 & x_2 & x_3 \\ \hline i_1 & (0.3, 0.7, 0.5, 0.2) & (0.1, 0.8, 0.5, 0.5) & (0.2, 0.6, 0.8, 0.7) \\ i_2 & (0.9, 0.4, 0.5, 0.7) & (0.3, 0.7, 0.5, 0.4) & (0.8, 0.2, 0.6, 0.8) \\ i_3 & (0.6, 0.3, 0.5, 0.6) & (0.3, 0.5, 0.6, 0.4) & (0.7, 0.1, 0.6, 0.3) \end{array} \right).$$

Definition 7. *Let $\hat{\Phi}_{(i_q)}, \hat{\Psi}_{(i_q)} \in (\mathbb{SVNFS})^{XI}$ over $\mathbb{S}^{XI}$ and $\mu, \mu' \in [0,1]^X$, where*

$$\hat{\Phi}_{(i_q)} = \left\{ \frac{(T_{\hat{\Phi}_{(i_q)}}(x_p), I_{\hat{\Phi}_{(i_q)}}(x_p), F_{\hat{\Phi}_{(i_q)}}(x_p), \mu(x_p))}{x_p} \,\middle|\, i_q \in I,\ x_p \in X,\ 0 \le T_{\hat{\Phi}_{(i_q)}}(x_p) + I_{\hat{\Phi}_{(i_q)}}(x_p) + F_{\hat{\Phi}_{(i_q)}}(x_p) \le 3 \right\}$$

and

$$\hat{\Psi}_{(i_q)} = \left\{ \frac{(T'_{\hat{\Psi}_{(i_q)}}(x_p), I'_{\hat{\Psi}_{(i_q)}}(x_p), F'_{\hat{\Psi}_{(i_q)}}(x_p), \mu'(x_p))}{x_p} \,\middle|\, i_q \in I,\ x_p \in X,\ 0 \le T'_{\hat{\Psi}_{(i_q)}}(x_p) + I'_{\hat{\Psi}_{(i_q)}}(x_p) + F'_{\hat{\Psi}_{(i_q)}}(x_p) \le 3 \right\}.$$

Then, $\hat{\Phi}_{(i_q)} \Subset \hat{\Psi}_{(i_q)}$ (i.e., $\hat{\Phi}_{(i_q)}$ is a single-valued neutrosophic fuzzy soft subset of $\hat{\Psi}_{(i_q)}$) if

(1) $\mu(x_p) \le \mu'(x_p)\ \forall x_p \in X$;

(2) *For all* $i_q \in I, x_p \in X, T_{\hat{\Phi}_{(i_q)}}(x_p) \le T'_{\hat{\Psi}_{(i_q)}}(x_p), I_{\hat{\Phi}_{(i_q)}}(x_p) \ge I'_{\hat{\Psi}_{(i_q)}}(x_p), F_{\hat{\Phi}_{(i_q)}}(x_p) \ge F'_{\hat{\Psi}_{(i_q)}}(x_p)$.

Example 3. *(Continued from Example 2). Let* $\hat{\Psi}_{(i_q)} \in (\mathbb{SVNFS})^{XI}$ *be defined as follows* $(q = 1, 2, 3)$*:*

$$\hat{\Psi} = \left(\begin{array}{c|ccc} I & x_1 & x_2 & x_3 \\ \hline i_1 & (0.4, 0.6, 0.4, 0.4) & (0.2, 0.7, 0.3, 0.5) & (0.3, 0.4, 0.7, 1) \\ i_2 & (1, 0.3, 0.5, 0.8) & (0.4, 0.6, 0.4, 0.6) & (0.9, 0.2, 0.4, 0.9) \\ i_3 & (0.7, 0.2, 0.4, 0.7) & (0.4, 0.5, 0.6, 0.6) & (0.8, 0.1, 0.5, 0.5) \end{array} \right).$$

Thus, $\hat{\Phi}_{(i_q)} \Subset \hat{\Psi}_{(i_q)}$ $(\forall i_q \in I)$.

Definition 8. *Let* $\hat{\Phi}_{(i_q)}, \hat{\Psi}_{(i_q)} \in (\mathbb{SVNFS})^{XI}$ *over* $\mathbb{S}^{XI}$ *and* $\mu, \mu' \in [0,1]^X$*, where*

$$\hat{\Phi}_{(i_q)} = \left\{ \frac{(T_{\hat{\Phi}_{(i_q)}}(x_p), I_{\hat{\Phi}_{(i_q)}}(x_p), F_{\hat{\Phi}_{(i_q)}}(x_p), \mu(x_p))}{x_p} \middle| i_q \in I,\ x_p \in X,\ 0 \leq T_{\hat{\Phi}_{(i_q)}}(x_p) + I_{\hat{\Phi}_{(i_q)}}(x_p) + F_{\hat{\Phi}_{(i_q)}}(x_p) \leq 3 \right\}$$

and

$$\hat{\Psi}_{(i_q)} = \left\{ \frac{(T'_{\hat{\Psi}_{(i_q)}}(x_p), I'_{\hat{\Psi}_{(i_q)}}(x_p), F'_{\hat{\Psi}_{(i_q)}}(x_p), \mu'(x_p))}{x_p} \middle| i_q \in I,\ x_p \in X,\ 0 \leq T'_{\hat{\Psi}_{(i_q)}}(x_p) + I'_{\hat{\Psi}_{(i_q)}}(x_p) + F'_{\hat{\Psi}_{(i_q)}}(x_p) \leq 3 \right\}.$$

Then, $\hat{\Phi}_{(i_q)} = \hat{\Psi}_{(i_q)}$ *(i.e.,* $\hat{\Phi}_{(i_q)}$ *is a single-valued neutrosophic fuzzy soft equal to* $\hat{\Psi}_{(i_q)}$*) if* $\hat{\Phi}_{(i_q)} \Subset \hat{\Psi}_{(i_q)}$ *and* $\hat{\Phi}_{(i_q)} \Supset \hat{\Psi}_{(i_q)}$.

Definition 9. *Let* $\hat{\Phi}_{(i_q)} \in (\mathbb{SVNFS})^{XI}$ *over* $\mathbb{S}^{XI}$ *and* $\mu \in [0,1]^X$*, where*

$$\hat{\Phi}_{(i_q)} = \left\{ \frac{(T_{\hat{\Phi}_{(i_q)}}(x_p), I_{\hat{\Phi}_{(i_q)}}(x_p), F_{\hat{\Phi}_{(i_q)}}(x_p), \mu(x_p))}{x_p} \middle| i_q \in I,\ x_p \in X,\ 0 \leq T_{\hat{\Phi}_{(i_q)}}(x_p) + I_{\hat{\Phi}_{(i_q)}}(x_p) + F_{\hat{\Phi}_{(i_q)}}(x_p) \leq 3 \right\}$$

over $\mathbb{S}^{XI}$*. Then,*

(1) $\hat{\Phi}_{(i_q)}$ *is called a single-valued neutrosophic fuzzy soft null set (denoted by* $\hat{\emptyset}_{(i_q)}$*), defined as*

$$\hat{\emptyset}_{(i_q)} = \left\{ \frac{(0, 1, 1, 0)}{x_p} | i_q \in I, x_p \in X \right\}.$$

(2) $\hat{\Phi}_{(i_q)}$ *is called a single-valued neutrosophic fuzzy soft universal set (denoted by* $\hat{X}_{(i_q)}$*), defined as*

$$\hat{X}_{(i_q)} = \left\{ \frac{(1, 0, 0, 1)}{x_p} | i_q \in I, x_p \in X \right\}.$$

Example 4. *(Continued from Example 2). Then,* $\hat{\emptyset}_{(i_q)}, \hat{X}_{(i_q)} \in (\mathbb{SVNFS})^{XI}$ *are defined as follows:*

$$\hat{\emptyset} = \left(\begin{array}{c|ccc} I & x_1 & x_2 & x_3 \\ \hline i_1 & (0, 1, 1, 0) & (0, 1, 1, 0) & (0, 1, 1, 0) \\ i_2 & (0, 1, 1, 0) & (0, 1, 1, 0) & (0, 1, 1, 0) \\ i_3 & (0, 1, 1, 0) & (0, 1, 1, 0) & (0, 1, 1, 0) \end{array} \right)$$

and

$$\hat{X} = \left(\begin{array}{c|ccc} I & x_1 & x_2 & x_3 \\ \hline i_1 & (1, 0, 0, 1) & (1, 0, 0, 1) & (1, 0, 0, 1) \\ i_2 & (1, 0, 0, 1) & (1, 0, 0, 1) & (1, 0, 0, 1) \\ i_3 & (1, 0, 0, 1) & (1, 0, 0, 1) & (1, 0, 0, 1) \end{array} \right).$$

Definition 10. *Let $\hat{\Phi}_{(i_q)}, \hat{\Psi}_{(i_q)} \in (\mathbb{SVNFS})^{XI}$ over $\mathbb{S}^{XI}$ and $\mu, \mu' \in [0,1]^X$, where*

$$\hat{\Phi}_{(i_q)} = \left\{ \frac{(T_{\hat{\Phi}_{(i_q)}}(x_p), I_{\hat{\Phi}_{(i_q)}}(x_p), F_{\hat{\Phi}_{(i_q)}}(x_p), \mu(x_p))}{x_p} \;\middle|\; i_q \in I,\ x_p \in X,\ 0 \leq T_{\hat{\Phi}_{(i_q)}}(x_p) + I_{\hat{\Phi}_{(i_q)}}(x_p) + F_{\hat{\Phi}_{(i_q)}}(x_p) \leq 3 \right\}$$

and

$$\hat{\Psi}_{(i_q)} = \left\{ \frac{(T'_{\hat{\Psi}_{(i_q)}}(x_p), I'_{\hat{\Psi}_{(i_q)}}(x_p), F'_{\hat{\Psi}_{(i_q)}}(x_p), \mu'(x_p))}{x_p} \;\middle|\; i_q \in I,\ x_p \in X,\ 0 \leq T'_{\hat{\Psi}_{(i_q)}}(x_p) + I'_{\hat{\Psi}_{(i_q)}}(x_p) + F'_{\hat{\Psi}_{(i_q)}}(x_p) \leq 3 \right\}.$$

Then,

(1) The union $\hat{\Phi}_{(i_q)} \Cup \hat{\Psi}_{(i_q)}$ is defined as

$$\hat{\Phi}_{(i_q)} \Cup \hat{\Psi}_{(i_q)} = \left\{ \frac{(T_{\hat{\Phi}_{(i_q)}}(x_p) \circ T'_{\hat{\Psi}_{(i_q)}}(x_p), I_{\hat{\Phi}_{(i_q)}}(x_p) * I'_{\hat{\Psi}_{(i_q)}}(x_p), F_{\hat{\Phi}_{(i_q)}}(x_p) * F'_{\hat{\Psi}_{(i_q)}}(x_p), \mu(x_p) \circ \mu'(x_p))}{x_p} \middle| i_q \in I, x_p \in X \right\}.$$

(2) The intersection $\hat{\Phi}_{(i_q)} \Cap \hat{\Psi}_{(i_q)}$ is defined as

$$\hat{\Phi}_{(i_q)} \Cap \hat{\Psi}_{(i_q)} = \left\{ \frac{(T_{\hat{\Phi}_{(i_q)}}(x_p) * T'_{\hat{\Psi}_{(i_q)}}(x_p), I_{\hat{\Phi}_{(i_q)}}(x_p) \circ I'_{\hat{\Psi}_{(i_q)}}(x_p), F_{\hat{\Phi}_{(i_q)}}(x_p) \circ F'_{\hat{\Psi}_{(i_q)}}(x_p), \mu(x_p) * \mu'(x_p))}{x_p} \middle| i_q \in I, x_p \in X \right\}.$$

Example 5. *(Continued from Examples 2 and 3). For $\alpha, \beta \in [0,1]$, let the t-norm (i.e., given as $\alpha * \beta = \alpha \wedge \beta$) and the t-conorm (i.e., given as $\alpha \circ \beta = \alpha \vee \beta$). Then,*

$$\hat{\Phi} \Cup \hat{\Psi} = \left(\begin{array}{c|ccc} I & x_1 & x_2 & x_3 \\ \hline i_1 & (0.4, 0.6, 0.4, 0.4) & (0.2, 0.7, 0.3, 0.5) & (0.3, 0.4, 0.7, 1) \\ i_2 & (1, 0.3, 0.5, 0.8) & (0.4, 0.6, 0.4, 0.6) & (0.9, 0.2, 0.4, 0.9) \\ i_3 & (0.7, 0.2, 0.4, 0.7) & (0.4, 0.5, 0.6, 0.6) & (0.8, 0.1, 0.5, 0.5) \end{array} \right)$$

and

$$\hat{\Phi} \Cap \hat{\Psi} = \left(\begin{array}{c|ccc} I & x_1 & x_2 & x_3 \\ \hline i_1 & (0.3, 0.7, 0.5, 0.2) & (0.1, 0.8, 0.5, 0.5) & (0.2, 0.6, 0.8, 0.7) \\ i_2 & (0.9, 0.4, 0.5, 0.7) & (0.3, 0.7, 0.5, 0.4) & (0.8, 0.2, 0.6, 0.8) \\ i_3 & (0.6, 0.3, 0.5, 0.6) & (0.3, 0.5, 0.6, 0.4) & (0.7, 0.1, 0.6, 0.3) \end{array} \right).$$

Proposition 1. *Let $\hat{\emptyset}_{(i_q)}, \hat{X}_{(i_q)}, \hat{\Phi}_{(i_q)} \in (\mathbb{SVNFS})^{XI}$ over $\mathbb{S}^{XI}$ and $\mu \in [0,1]^X$. Then the following hold:*

(1) $\hat{\Phi}_{(i_q)} \Cup \hat{\Phi}_{(i_q)} = \hat{\Phi}_{(i_q)}$;
(2) $\hat{\Phi}_{(i_q)} \Cap \hat{\Phi}_{(i_q)} = \hat{\Phi}_{(i_q)}$;
(3) $\hat{\Phi}_{(i_q)} \Cup \hat{\emptyset}_{(i_q)} = \hat{\Phi}_{(i_q)}$;
(4) $\hat{\Phi}_{(i_q)} \Cap \hat{\emptyset}_{(i_q)} = \hat{\emptyset}_{(i_q)}$;
(5) $\hat{\Phi}_{(i_q)} \Cup \hat{X}_{(i_q)} = \hat{X}_{(i_q)}$;
(6) $\hat{\Phi}_{(i_q)} \Cap \hat{X}_{(i_q)} = \hat{\Phi}_{(i_q)}$.

Proof. Follows from Definitions 9 and 10. □

Proposition 2. *Let $\hat{\Phi}_{(i_q)}, \hat{\Psi}_{(i_q)}, \hat{\Gamma}_{(i_q)} \in (\mathbb{SVNFS})^{XI}$ over $\mathbb{S}^{XI}$ and $\mu, \mu', \mu'' \in [0,1]^X$. Then the following hold:*

(1) $\hat{\Phi}_{(i_q)} \Cup \hat{\Psi}_{(i_q)} = \hat{\Psi}_{(i_q)} \Cup \hat{\Phi}_{(i_q)}$;
(2) $\hat{\Phi}_{(i_q)} \Cap \hat{\Psi}_{(i_q)} = \hat{\Psi}_{(i_q)} \Cap \hat{\Phi}_{(i_q)}$;
(3) $\hat{\Phi}_{(i_q)} \Cup (\hat{\Psi}_{(i_q)} \Cup \hat{\Gamma}_{(i_q)}) = (\hat{\Phi}_{(i_q)} \Cup \hat{\Psi}_{(i_q)}) \Cup \hat{\Gamma}_{(i_q)}$;

(4) $\hat{\Phi}_{(i_q)} \Cap (\hat{\Psi}_{(i_q)} \Cap \hat{\Gamma}_{(i_q)}) = (\hat{\Phi}_{(i_q)} \Cap \hat{\Psi}_{(i_q)}) \Cap \hat{\Gamma}_{(i_q)}$;

(5) $\hat{\Phi}_{(i_q)} \Cap (\hat{\Psi}_{(i_q)} \Cup \hat{\Gamma}_{(i_q)}) = (\hat{\Phi}_{(i_q)} \Cap \hat{\Psi}_{(i_q)}) \Cup (\hat{\Phi}_{(i_q)} \Cap \hat{\Gamma}_{(i_q)})$;

(6) $\hat{\Phi}_{(i_q)} \Cup (\hat{\Psi}_{(i_q)} \Cap \hat{\Gamma}_{(i_q)}) = (\hat{\Phi}_{(i_q)} \Cup \hat{\Psi}_{(i_q)}) \Cap (\hat{\Phi}_{(i_q)} \Cup \hat{\Gamma}_{(i_q)})$.

Proof. Follows from Definition 10. □

Proposition 3. *Let $\hat{\Phi}_{(i_q)}, \hat{\Psi}_{(i_q)} \in (\mathbb{SVNFS})^{XI}$ over $\mathbb{S}^{XI}, \mu, \mu' \in [0,1]^X$, and $\hat{\Psi}_{(i_q)} \Subset \hat{\Phi}_{(i_q)}$. Then the following hold:*

(1) $\hat{\Phi}_{(i_q)} \Cup \hat{\Psi}_{(i_q)} = \hat{\Phi}_{(i_q)}$;

(2) $\hat{\Phi}_{(i_q)} \Cap \hat{\Psi}_{(i_q)} = \hat{\Psi}_{(i_q)}$.

Proof. Follows from Definitions 7 and 10. □

Next, we propose a definition, example, remark, and two propositions on the complement of $(\mathbb{SVNFS})^{XI}$ over $\mathbb{S}^{XI}$.

Definition 11. *Let $\hat{\Phi}_{(i_q)} \in (\mathbb{SVNFS})^{XI}$ over $\mathbb{S}^{XI}$ and $\mu \in [0,1]^X$, where*

$$\hat{\Phi}_{(i_q)} = \left\{ \frac{(T_{\hat{\Phi}_{(i_q)}}(x_p), I_{\hat{\Phi}_{(i_q)}}(x_p), F_{\hat{\Phi}_{(i_q)}}(x_p), \mu(x_p))}{x_p} \,\middle|\, i_q \in I,\ x_p \in X,\ 0 \leq T_{\hat{\Phi}_{(i_q)}}(x_p) + I_{\hat{\Phi}_{(i_q)}}(x_p) + F_{\hat{\Phi}_{(i_q)}}(x_p) \leq 3 \right\}.$$

Then, the complement $\hat{\Phi}^c_{(i_q)}$ of $\hat{\Phi}_{(i_q)}$ is defined as

$$\hat{\Phi}^c_{(i_q)} = \left\{ \frac{\left(F_{\hat{\Phi}_{(i_q)}}(x_p), 1 - I_{\hat{\Phi}_{(i_q)}}(x_p), T_{\hat{\Phi}_{(i_q)}}(x_p), 1 - \mu(x_p)\right)}{x_p} \,\middle|\, i_q \in I,\ \ x_p \in X \right\}.$$

Example 6. *(Continued from Example 2). The complement $\hat{\Phi}^c_{(i_q)}$ of $\hat{\Phi}_{(i_q)}$ is calculated by*

$$\hat{\Phi}^c = \left(\begin{array}{c|ccc} I & x_1 & x_2 & x_3 \\ \hline i_1 & (0.5, 0.3, 0.3, 0.8) & (0.5, 0.2, 0.1, 0.5) & (0.8, 0.4, 0.2, 0.3) \\ i_2 & (0.5, 0.6, 0.9, 0.3) & (0.5, 0.3, 0.3, 0.6) & (0.6, 0.8, 0.8, 0.2) \\ i_3 & (0.5, 0.7, 0.6, 0.4) & (0.6, 0.5, 0.3, 0.6) & (0.6, 0.9, 0.7, 0.7) \end{array} \right).$$

Proposition 4. *Let $\hat{\varnothing}_{(i_q)}, \hat{X}_{(i_q)}, \hat{\Phi}_{(i_q)} \in (\mathbb{SVNFS})^{XI}$ over $\mathbb{S}^{XI}$, and $\mu \in [0,1]^X$. Then, the following hold:*

(1) $\hat{\varnothing}^c_{(i_q)} = \hat{X}_{(i_q)}$;

(2) $\hat{X}^c_{(i_q)} = \hat{\varnothing}_{(i_q)}$;

(3) $(\hat{\Phi}^c_{(i_q)})^c = \hat{\Phi}^c_{(i_q)}$.

Proof. Follows from Definitions 9 and 11. □

Remark 1. *The equality of $\hat{\Phi}_{(i_q)} \Cup \hat{\Phi}^c_{(i_q)} = \hat{X}_{(i_q)}$ and $\hat{\Phi}_{(i_q)} \Cap \hat{\Phi}^c_{(i_q)} = \hat{\varnothing}_{(i_q)}$ does not hold by the following example.*

Example 7. *(Continued from Examples 2 and 6). Then, $\hat{\Phi}^c_{(i_q)}$ of $\hat{\Phi}_{(i_q)}$ is calculated by*

$$\hat{\Phi} \Cup \hat{\Phi}^c = \left(\begin{array}{c|ccc} I & x_1 & x_2 & x_3 \\ \hline i_1 & (0.5, 0.3, 0.3, 0.8) & (0.5, 0.2, 0.1, 0.5) & (0.8, 0.4, 0.2, 0.3) \\ i_1 & (0.5, 0.6, 0.9, 0.3) & (0.5, 0.3, 0.3, 0.6) & (0.6, 0.8, 0.8, 0.2) \\ i_1 & (0.5, 0.7, 0.6, 0.4) & (0.6, 0.5, 0.3, 0.6) & (0.6, 0.9, 0.7, 0.7) \end{array} \right)$$

and

$$\hat{\Phi} \Cap \hat{\Phi}^c = \left(\begin{array}{c|ccc} I & x_1 & x_2 & x_3 \\ \hline i_1 & (0.3, 0.7, 0.5, 0.2) & (0.1, 0.8, 0.5, 0.5) & (0.2, 0.6, 0.8, 0.7) \\ i_2 & (0.9, 0.4, 0.5, 0.7) & (0.3, 0.7, 0.5, 0.4) & (0.8, 0.2, 0.6, 0.8) \\ i_3 & (0.6, 0.3, 0.5, 0.6) & (0.3, 0.5, 0.6, 0.4) & (0.7, 0.1, 0.6, 0.3) \end{array} \right).$$

This shows that $\hat{\Phi}_{(i_q)} \Cup \hat{\Phi}^c_{(i_q)} \neq \hat{X}_{(i_q)}$ *and* $\hat{\Phi}_{(i_q)} \Cap \hat{\Phi}^c_{(i_q)} \neq \hat{\emptyset}_{(i_q)}$.

Proposition 5. *Let* $\hat{\Phi}_{(i_q)}, \hat{\Psi}_{(i_q)} \in (\mathbb{SVNFS})^{XI}$ *over* $\mathbb{S}^{XI}$ *and* $\mu, \mu' \in [0,1]^X$. *Then, the following hold:*

(1) $(\hat{\Phi}_{(i_q)} \Cup \hat{\Psi}_{(i_q)})^c = \hat{\Phi}^c_{(i_q)} \Cap \hat{\Psi}^c_{(i_q)}$;

(2) $(\hat{\Phi}_{(i_q)} \Cap \hat{\Psi}_{(i_q)})^c = \hat{\Phi}^c_{(i_q)} \Cup \hat{\Psi}^c_{(i_q)}$.

Proof. Consider $a * b = a \wedge b$ (t-norm) and $\alpha \circ \beta = \alpha \vee \beta$ (t-conorm) ($\forall \alpha, \beta \in [0,1]$). We have

(1) $(\hat{\Phi}_{(i_q)} \Cup \hat{\Psi}_{(i_q)})^c(x_p)$

$$= \left(\left\{ \frac{\left(T_{\hat{\Phi}_{(i_q)}}(x_p) \circ T'_{\hat{\Psi}_{(i_q)}}(x_p), I_{\hat{\Phi}_{(i_q)}}(x_p) * I'_{\hat{\Psi}_{(i_q)}}(x_p), F_{\hat{\Phi}_{(i_q)}}(x_p) * F'_{\hat{\Psi}_{(i_q)}}(x_p), \mu(x_p) \circ \mu'(x_p)\right)}{x_p} \Bigg| i_q \in I, x_p \in X \right\} \right)^c$$

$$= \left\{ \frac{\left(F_{\hat{\Phi}_{(i_q)}}(x_p) * F'_{\hat{\Psi}_{(i_q)}}(x_p), 1 - (I_{\hat{\Phi}_{(i_q)}}(x_p) * I'_{\hat{\Psi}_{(i_q)}}(x_p)), T_{\hat{\Phi}_{(i_q)}}(x_p) \circ T'_{\hat{\Psi}_{(i_q)}}(x_p), 1 - (\mu(x_p) \circ \mu'(x_p))\right)}{x_p} \Bigg| i_q \in I, x_p \in X \right\}$$

$$= \left\{ \frac{\left(F_{\hat{\Phi}_{(i_q)}}(x_p) \wedge F'_{\hat{\Psi}_{(i_q)}}(x_p), 1 - (I_{\hat{\Phi}_{(i_q)}}(x_p) \wedge I'_{\hat{\Psi}_{(i_q)}}(x_p)), T_{\hat{\Phi}_{(i_q)}}(x_p) \vee T'_{\hat{\Psi}_{(i_q)}}(x_p), 1 - (\mu(x_p) \vee \mu'(x_p))\right)}{x_p} \Bigg| i_q \in I, x_p \in X \right\}$$

$$= \left\{ \frac{\left(F_{\hat{\Phi}_{(i_q)}}(x_p) \wedge F'_{\hat{\Psi}_{(i_q)}}(x_p), 1 - I_{\hat{\Phi}_{(i_q)}}(x_p) \vee 1 - I'_{\hat{\Psi}_{(i_q)}}(x_p), T_{\hat{\Phi}_{(i_q)}}(x_p) \vee T'_{\hat{\Psi}_{(i_q)}}(x_p), 1 - \mu(x_p) \wedge 1 - \mu'(x_p)\right)}{x_p} \Bigg| i_q \in I, x_p \in X \right\}$$

$$= \left\{ \frac{\left(F_{\hat{\Phi}_{(i_q)}}(x_p) * F'_{\hat{\Psi}_{(i_q)}}(x_p), 1 - I_{\hat{\Phi}_{(i_q)}}(x_p) \circ 1 - I'_{\hat{\Psi}_{(i_q)}}(x_p), T_{\hat{\Phi}_{(i_q)}}(x_p) \circ T'_{\hat{\Psi}_{(i_q)}}(x_p), 1 - \mu(x_p) * 1 - \mu'(x_p)\right)}{x_p} \Bigg| i_q \in I, x_p \in X \right\}$$

$$= \left\{ \frac{\left(F_{\hat{\Phi}_{(i_q)}}(x_p), 1 - I_{\hat{\Phi}_{(i_q)}}(x_p), T_{\hat{\Phi}_{(i_q)}}(x_p), 1 - \mu(x_p)\right)}{x_p} \Bigg| i_q \in I, x_p \in X \right\} \Cap \left\{ \frac{\left(F'_{\hat{\Psi}_{(i_q)}}(x_p), 1 - I'_{\hat{\Psi}_{(i_q)}}(x_p), T'_{\hat{\Psi}_{(i_q)}}(x_p), 1 - \mu'(x_p)\right)}{x_p} \Bigg| i_q \in I, x_p \in X \right\}$$

$$= \hat{\Phi}^c_{(i_q)}(x_p) \Cap \hat{\Psi}^c_{(i_q)}(x_p).$$

(2) $(\hat{\Phi}_{(i_q)} \Cap \hat{\Psi}_{(i_q)})^c(x_p)$

$$= \left(\left\{ \frac{\left(T_{\hat{\Phi}_{(i_q)}}(x_p) * T'_{\hat{\Psi}_{(i_q)}}(x_p), I_{\hat{\Phi}_{(i_q)}}(x_p) \circ I'_{\hat{\Psi}_{(i_q)}}(x_p), F_{\hat{\Phi}_{(i_q)}}(x_p) \circ F'_{\hat{\Psi}_{(i_q)}}(x_p), \mu(x_p) * \mu'(x_p)\right)}{x_p} \Bigg| i_q \in I, x_p \in X \right\} \right)^c$$

$$= \left\{ \frac{\left(F_{\hat{\Phi}_{(i_q)}}(x_p) \circ F'_{\hat{\Psi}_{(i_q)}}(x_p), 1 - (I_{\hat{\Phi}_{(i_q)}}(x_p) \circ I'_{\hat{\Psi}_{(i_q)}}(x_p)), T_{\hat{\Phi}_{(i_q)}}(x_p) * T'_{\hat{\Psi}_{(i_q)}}(x_p), 1 - (\mu(x_p) * \mu'(x_p))\right)}{x_p} \Bigg| i_q \in I, x_p \in X \right\}$$

$$= \left\{ \frac{\left(F_{\hat{\Phi}_{(i_q)}}(x_p) \vee F'_{\hat{\Psi}_{(i_q)}}(x_p), 1 - (I_{\hat{\Phi}_{(i_q)}}(x_p) \vee I'_{\hat{\Psi}_{(i_q)}}(x_p)), T_{\hat{\Phi}_{(i_q)}}(x_p) \wedge T'_{\hat{\Psi}_{(i_q)}}(x_p), 1 - (\mu(x_p) \wedge \mu'(x_p))\right)}{x_p} \Bigg| i_q \in I, x_p \in X \right\}$$

$$= \left\{ \frac{\left(F_{\hat{\Phi}_{(i_q)}}(x_p) \vee F'_{\hat{\Psi}_{(i_q)}}(x_p), 1 - I_{\hat{\Phi}_{(i_q)}}(x_p) \wedge 1 - I'_{\hat{\Psi}_{(i_q)}}(x_p), T_{\hat{\Phi}_{(i_q)}}(x_p) \wedge T'_{\hat{\Psi}_{(i_q)}}(x_p), 1 - \mu(x_p) \vee 1 - \mu'(x_p)\right)}{x_p} \Bigg| i_q \in I, x_p \in X \right\}$$

$$= \left\{ \frac{\left(F_{\hat{\Phi}_{(i_q)}}(x_p) \circ F'_{\hat{\Psi}_{(i_q)}}(x_p), 1 - I_{\hat{\Phi}_{(i_q)}}(x_p) * 1 - I'_{\hat{\Psi}_{(i_q)}}(x_p), T_{\hat{\Phi}_{(i_q)}}(x_p) * T'_{\hat{\Psi}_{(i_q)}}(x_p), 1 - \mu(x_p) \circ 1 - \mu'(x_p)\right)}{x_p} \Bigg| i_q \in I, x_p \in X \right\}$$

$$= \left\{ \frac{\left(F_{\hat{\Phi}_{(i_q)}}(x_p), 1 - I_{\hat{\Phi}_{(i_q)}}(x_p), T_{\hat{\Phi}_{(i_q)}}(x_p), 1 - \mu(x_p)\right)}{x_p} \Bigg| i_q \in I, x_p \in X \right\} \Cup \left\{ \frac{\left(F'_{\hat{\Psi}_{(i_q)}}(x_p), 1 - I'_{\hat{\Psi}_{(i_q)}}(x_p), T'_{\hat{\Psi}_{(i_q)}}(x_p), 1 - \mu'(x_p)\right)}{x_p} \Bigg| i_q \in I, x_p \in X \right\}$$

$$= \hat{\Phi}^c_{(i_q)}(x_p) \Cup \hat{\Psi}^c_{(i_q)}(x_p).$$

□

4. Two Algorithms of Single-Valued Neutrosophic Fuzzy Soft Sets for Decision-Making

Depending on single-valued neutrosophic fuzzy soft sets, in the following, we introduce two new approaches for fuzzy decision-making problems.

Next, we construct Algorithm 1 as the first type for decision-making (i.e., the first application of a single-valued neutrosophic fuzzy soft set).

Algorithm 1: Determine the optimal decision based on a single-valued neutrosophic fuzzy soft set matrix.

First step: Input the single-valued neutrosophic fuzzy soft set $\hat{\Phi}_{(i_q)} \in (\mathbb{SVNFS})^{XI}$ as follows:

$$\hat{\Phi}_{(i_q)} = \left\{ \frac{(T_{\hat{\Phi}_{(i_q)}}(x_p), I_{\hat{\Phi}_{(i_q)}}(x_p), F_{\hat{\Phi}_{(i_q)}}(x_p), \mu(x_p))}{x_p} \;\middle|\; i_q \in I,\ x_p \in X,\ 0 \le T_{\hat{\Phi}_{(i_q)}}(x_p) + I_{\hat{\Phi}_{(i_q)}}(x_p) + F_{\hat{\Phi}_{(i_q)}}(x_p) \le 3 \right\},$$

to be evaluated by a group of experts n to element x on parameter i, where $T_{\hat{\Phi}_{(i_q)}}(x_p) \in [0,1]$ (i.e., the degree of truth membership), $I_{\hat{\Phi}_{(i_q)}}(x_p)$ (i.e., the degree of indeterminacy membership), $F_{\hat{\Phi}_{(i_q)}}(x_p)$ (i.e., the degree of falsity membership), and $\mu(x_p) \in [0,1]$.

Second step: Input the single-valued neutrosophic fuzzy soft set in matrix form (written as $\mathcal{M}_{q\times p}$, $p, q \in N$):

$$\mathcal{M}_{q\times p} = \begin{pmatrix} (T_{\hat{\Phi}_{(i_1)}}(x_1), I_{\hat{\Phi}_{(i_1)}}(x_1), F_{\hat{\Phi}_{(i_1)}}(x_1), \mu(x_1)) & (T_{\hat{\Phi}_{(i_1)}}(x_2), I_{\hat{\Phi}_{(i_1)}}(x_2), F_{\hat{\Phi}_{(i_1)}}(x_2), \mu(x_2)) & \cdots & (T_{\hat{\Phi}_{(i_2)}}(x_p), I_{\hat{\Phi}_{(i_2)}}(x_p), F_{\hat{\Phi}_{(i_2)}}(x_p), \mu(x_p)) \\ (T_{\hat{\Phi}_{(i_2)}}(x_1), I_{\hat{\Phi}_{(i_2)}}(x_1), F_{\hat{\Phi}_{(i_2)}}(x_1), \mu(x_1)) & (T_{\hat{\Phi}_{(i_2)}}(x_2), I_{\hat{\Phi}_{(i_2)}}(x_2), F_{\hat{\Phi}_{(i_2)}}(x_2), \mu(x_2)) & \cdots & (T_{\hat{\Phi}_{(i_2)}}(x_p), I_{\hat{\Phi}_{(i_2)}}(x_p), F_{\hat{\Phi}_{(i_2)}}(x_p), \mu(x_p)) \\ (T_{\hat{\Phi}_{(i_3)}}(x_1), I_{\hat{\Phi}_{(i_3)}}(x_1), F_{\hat{\Phi}_{(i_3)}}(x_1), \mu(x_1)) & (T_{\hat{\Phi}_{(i_3)}}(x_2), I_{\hat{\Phi}_{(i_3)}}(x_2), F_{\hat{\Phi}_{(i_3)}}(x_2), \mu(x_2)) & \cdots & (T_{\hat{\Phi}_{(i_3)}}(x_p), I_{\hat{\Phi}_{(i_3)}}(x_p), F_{\hat{\Phi}_{(i_3)}}(x_p), \mu(x_p)) \\ \vdots & \vdots & \ddots & \vdots \\ (T_{\hat{\Phi}_{(i_q)}}(x_1), I_{\hat{\Phi}_{(i_q)}}(x_1), F_{\hat{\Phi}_{(i_q)}}(x_1), \mu(x_1)) & (T_{\hat{\Phi}_{(i_q)}}(x_2), I_{\hat{\Phi}_{(i_q)}}(x_2), F_{\hat{\Phi}_{(i_q)}}(x_2), \mu(x_2)) & \cdots & (T_{\hat{\Phi}_{(i_q)}}(x_p), I_{\hat{\Phi}_{(i_q)}}(x_p), F_{\hat{\Phi}_{(i_q)}}(x_p), \mu(x_p)) \end{pmatrix}.$$

Third step: Calculate the center matrix (i.e., $\delta_{\hat{\Phi}_{(i_q)}}(x_p) = (T_{\hat{\Phi}_{(i_q)}}(x_p) + I_{\hat{\Phi}_{(i_q)}}(x_p) + F_{\hat{\Phi}_{(i_q)}}(x_p)) - \mu(x_p)$):

$$C_{q\times p} = \begin{pmatrix} \delta_{\hat{\Phi}_{(i_1)}}(x_1) & \delta_{\hat{\Phi}_{(i_1)}}(x_2) & \cdots, & \delta_{\hat{\Phi}_{(i_1)}}(x_p) \\ \delta_{\hat{\Phi}_{(i_2)}}(x_1) & \delta_{\hat{\Phi}_{(i_2)}}(x_2) & \cdots, & \delta_{\hat{\Phi}_{(i_2)}}(x_p) \\ \vdots & \vdots & \ddots & \vdots \\ \delta_{\hat{\Phi}_{(i_q)}}(x_1) & \delta_{\hat{\Phi}_{(i_q)}}(x_2) & \cdots, & \delta_{\hat{\Phi}_{(i_q)}}(x_p) \end{pmatrix}.$$

Fourth step: Calculate the $d^{max}(x_j)$ (maximum decision), $d^{min}(x_j)$ (minimum decision), and $S(x_j)$ (score) of elements x_j $(j = 1, 2, \cdots, p)$:

$$d^{max}(x_j) = \sum_{i=1}^{q} \left(1 - \delta_{\hat{\Phi}_{(i_q)}}(x_j)\right)^2, \quad d^{min}(x_j) = \sum_{i=1}^{q} (\delta_{\hat{\Phi}_{(i_q)}}(x_j))^2$$
$$S(x_j) = d^{max}(x_j) + d^{min}(x_j).$$

(to understand the motivation behind this method, let ρ be the Euclidean metric on R^q, $\mathbf{0} = (0, \cdots, 0)^T \in R^q$, $\mathbf{1} = (1, \cdots, 1)^T \in R^q$, and $\boldsymbol{\theta}_j = (\theta_{1,x_j}, \theta_{2,x_j}, \cdots, \theta_{q,x_j})^T \in R^q$. Thus $S(x_j) = [\rho(\boldsymbol{\theta}_j, \mathbf{1})]^2 + [\rho(\boldsymbol{\theta}_j, \mathbf{0})]^2$ $(j = 1, 2, \cdots, p)$).

Fifth step: Obtain the decision p satisfying

$$x_p = \max\left\{S(x_1), S(x_2), \cdots, S(x_j)\right\}.$$

Now, we show the principle and steps of the above Algorithm 1 by using the following example.

Example 8. *An investment company wants to choose some investment projects to make full use of idle funds. There are five alternatives $X = \{z_1, z_2, z_3, z_4, z_5\}$ that can be selected: two internet education projects (denoted as z_1 and z_2) and three film studio investments (represented as z_3, z_4, z_5). According to the project investment books, the decision-makers evaluate the five alternatives from the following three parameters $I = \{i_1, i_2, i_3\}$, where i_1 is "human resources", i_2 is "social benefits", and i_3 is "expected benefits". The data of the single-valued neutrosophic fuzzy soft set $\hat{\Phi}_{(i_q)} \in (\mathbb{SVNFS})^{XI}$ is given by*

$$\hat{\Phi} = \left(\begin{array}{c|ccccc} I & z_1 & z_2 & z_3 & z_4 & z_5 \\ \hline i_1 & (0.3,0.7,0.5,0.2) & (0.1,0.8,0.5,0.5) & (0.2,0.6,0.8,0.7) & (0.5,0.6,0.5,0.2) & (0.4,0.7,0.9,0.1) \\ i_2 & (0.9,0.4,0.5,0.7) & (0.3,0.7,0.5,0.4) & (0.8,0.2,0.6,0.8) & (0.3,0.7,0.2,0.5) & (0.7,0.8,0.8,0.3) \\ i_3 & (0.6,0.3,0.5,0.6) & (0.3,0.5,0.6,0.4) & (0.7,0.1,0.6,0.3) & (0.8,0.9,0.6,0.4) & (0.7,0.8,0.9,0.6) \end{array}\right).$$

Now, we will explain the practical meaning of alternatives X by taking the alternative z_1 as an example: the single-valued neutrosophic fuzzy soft set $\hat{\Phi}_{(i_1)}(z_1) = (0.3, 0.7, 0.5, 0.2)$ is the evaluation by four expert groups; the single-valued neutrosophic fuzzy soft value 0.3 (meaning that 30% say yes in the first expert group) in $\hat{\Phi}_{(i_1)}(z_1)$, the single-valued neutrosophic fuzzy soft value 0.7 (meaning 70% say no in the second expert group) in $\hat{\Phi}_{(i_1)}(z_1)$, the single-valued neutrosophic fuzzy soft value 0.5 (meaning 50% say yes in the third expert group) in $\hat{\Phi}_{(i_1)}(z_1)$, and fuzzy value 0.2 (meaning 20% say no in the fourth expert group) in $\hat{\Phi}_{(i_1)}(z_1)$. Then, the single-valued neutrosophic fuzzy soft set in matrix form $\mathcal{M}_{3\times5}$ in the second step of Algorithm 1 is given by

$$\mathcal{M}_{3\times5} = \begin{pmatrix} (0.3,0.7,0.5,0.2) & (0.9,0.4,0.5,0.7) & (0.6,0.3,0.5,0.6) \\ (0.1,0.8,0.5,0.5) & (0.3,0.7,0.5,0.4) & (0.3,0.5,0.6,0.4) \\ (0.2,0.6,0.8,0.7) & (0.8,0.2,0.6,0.8) & (0.7,0.1,0.6,0.3) \\ (0.5,0.6,0.5,0.2) & (0.3,0.7,0.2,0.5) & (0.8,0.9,0.6,0.4) \\ (0.4,0.7,0.9,0.1) & (0.7,0.8,0.8,0.3) & (0.7,0.8,0.9,0.6) \end{pmatrix}.$$

Thus, we obtain the following center matrix $C_{3\times5}$ of $\mathcal{M}_{3\times5}$ in the third step of Algorithm 1:

$$C_{3\times5} = \begin{pmatrix} 1.3 & 1.1 & 0.8 \\ 0.9 & 1.1 & 1 \\ 0.9 & 0.8 & 1.1 \\ 1.4 & 0.7 & 1.9 \\ 1.9 & 2 & 1.8 \end{pmatrix}.$$

By calculating, we get $d^{max}(z_j)$, $d^{min}(z_j)$, and $S(z_j)$ of elements z_j ($j = 1, 2, 3, 4, 5$):

$$d^{max}(z_1) = 0.14, d^{max}(z_2) = 0.02, d^{max}(z_3) = 0.06, d^1(z_4) = 1.06, d^{max}(z_5) = 2.45;$$

$$d^{min}(z_1) = 3.54, d^{min}(z_2) = 3.02, d^{min}(z_3) = 2.66, d^{min}(z_4) = 6.06, d^{min}(z_5) = 10.85;$$

$$S(z_1) = 3.68, S(z_2) = 3.04, S(z_3) = 2.72, S(z_4) = 7.12, S(z_5) = 13.3.$$

Finally, we can see from the fifth step that z_5 is the best decision.

Now, we present Algorithm 2 as a second type for a decision-making problem (i.e., a second application of the single-valued neutrosophic fuzzy soft set) as follows:

Algorithm 2: Determine the optimal decision based on AND operation of two single-valued neutrosophic fuzzy soft sets.

First step: Input the single-valued neutrosophic fuzzy soft sets $\hat{\Phi}_{(i_q)} \in (\mathbb{SVNFS})^{XI}$ and $\hat{\Psi}_{(j_q)} \in (\mathbb{SVNFS})^{XJ}$, defined, respectively, as follows:

$$\hat{\Phi}_{(i_q)} = \left\{ \frac{(T_{\hat{\Phi}_{(i_q)}}(x_p), I_{\hat{\Phi}_{(i_q)}}(x_p), F_{\hat{\Phi}_{(i_q)}}(x_p), \mu(x_p))}{x_p} \,\middle|\, i_q \in I,\ x_p \in X,\ 0 \leq T_{\hat{\Phi}_{(i_q)}}(x_p) + I_{\hat{\Phi}_{(i_q)}}(x_p) + F_{\hat{\Phi}_{(i_q)}}(x_p) \leq 3 \right\},$$

to be evaluated by a group of experts n to element x on parameter i, where $T_{\hat{\Phi}_{(i_q)}}(x_p) \in [0,1]$ (i.e., the degree of truth membership), $I_{\hat{\Phi}_{(i_q)}}(x_p)$ (i.e., the degree of indeterminacy membership), $F_{\hat{\Phi}_{(i_q)}}(x_p)$ (i.e., the degree of falsity membership), and $\mu(x_p) \in [0,1]$,

$$\hat{\Psi}_{(j_q)} = \left\{ \frac{(T'_{\hat{\Psi}_{(j_q)}}(x_p), I'_{\hat{\Psi}_{(j_q)}}(x_p), F'_{\hat{\Psi}_{(j_q)}}(x_p), \mu'(x_p))}{x_p} \,\middle|\, j_q \in J,\ x_p \in X,\ 0 \leq T'_{\hat{\Psi}_{(j_q)}}(x_p) + I'_{\hat{\Psi}_{(j_q)}}(x_p) + F'_{\hat{\Psi}_{(j_q)}}(x_p) \leq 3 \right\}$$

to be evaluated by a group of experts n to element x on parameter j, where $T_{\hat{\Psi}'_{(j_q)}}(x_p) \in [0,1]$ (i.e., the degree of truth membership), $I_{\hat{\Psi}'_{(j_q)}}(x_p)$ (i.e., the degree of indeterminacy membership), $F_{\hat{\Psi}'_{(j_q)}}(x_p)$ (i.e., the degree of falsity membership), and $\mu(x_p) \in [0,1]$.

Second step: Define and calculate the AND operation of two single-valued neutrosophic fuzzy soft sets $\hat{\Phi}_{(i_q)} \in (\mathbb{SVNFS})^{XI}$ and $\hat{\Psi}_{(j_q)} \in (\mathbb{SVNFS})^{XJ}$, denoted by $(\hat{\Phi}\overline{\wedge}\hat{\Psi})_{(i_q,j_q)}$ $(\forall i \in I, j \in J)$, defined as

$$(\hat{\Phi}\overline{\wedge}\hat{\Psi})_{(i_q,j_q)} = \left\{ \frac{(T_{\hat{\Phi}_{(i_q)}}(x_p) \wedge T'_{\hat{\Psi}_{(j_q)}}(x_p), I_{\hat{\Phi}_{(i_q)}}(x_p) \vee I'_{\hat{\Psi}_{(j_q)}}(x_p), F_{\hat{\Phi}_{(i_q)}}(x_p) \vee F'_{\hat{\Psi}_{(j_q)}}(x_p), \mu(x_p) \wedge \mu'(x_p))}{x_p} \middle| i_q \in I, j_q \in J, x_p \in X \right\}.$$

Third step: Define and write the truth membership $(\hat{\Phi}\overline{\wedge}\hat{\Psi})^T_{(i_q,j_q)}$, the indeterminacy membership $(\hat{\Phi}\overline{\wedge}\hat{\Psi})^I_{(i_q,j_q)}$, and the falsity membership $(\hat{\Phi}\overline{\wedge}\hat{\Psi})^F_{(i_q,j_q)}$, respectively, as follows:

$$(\hat{\Phi}\overline{\wedge}\hat{\Psi})^T_{(i_q,j_q)} = \left\{ \frac{(T_{\hat{\Phi}_{(i_q)}}(x_p) \wedge T'_{\hat{\Psi}_{(j_q)}}(x_p), \mu(x_p) \wedge \mu'(x_p))}{x_p} \middle| i_q \in I, j_q \in J, x_p \in X \right\},$$

$$(\hat{\Phi}\overline{\wedge}\hat{\Psi})^I_{(i_q,j_q)} = \left\{ \frac{(I_{\hat{\Phi}_{(i_q)}}(x_p) \vee I'_{\hat{\Psi}_{(j_q)}}(x_p), \mu(x_p) \wedge \mu'(x_p))}{x_p} \middle| i_q \in I, j_q \in J, x_p \in X \right\},$$

and

$$(\hat{\Phi}\overline{\wedge}\hat{\Psi})^F_{(i_q,j_q)} = \left\{ \frac{(F_{\hat{\Phi}_{(i_q)}}(x_p) \vee F'_{\hat{\Psi}_{(j_q)}}(x_p), \mu(x_p) \wedge \mu'(x_p))}{x_p} \middle| i_q \in I, j_q \in J, x_p \in X \right\}.$$

Fourth step: Define and compute the max-matrices of $(\hat{\Phi}\overline{\wedge}\hat{\Psi})^T_{(i_q,j_q)}$, $(\hat{\Phi}\overline{\wedge}\hat{\Psi})^I_{(i_q,j_q)}$, and $(\hat{\Phi}\overline{\wedge}\hat{\Psi})^F_{(i_q,j_q)}$, respectively, for every $x_p \in X$ as follows $(p = 1, 2, \cdots, N)$:

$$(\hat{\Phi}\overline{\wedge}\hat{\Psi})^T_{(i_q,j_q)}(x_p) = \frac{1}{2}\Big((T_{\hat{\Phi}_{(i_q)}}(x_p) \wedge T'_{\hat{\Psi}_{(j_q)}}(x_p)) + (\mu(x_p) \wedge \mu'(x_p)) \Big),$$

$$(\hat{\Phi}\overline{\wedge}\hat{\Psi})^I_{(i_q,j_q)}(x_p) = \Big((I_{\hat{\Phi}_{(i_q)}}(x_p) \vee I'_{\hat{\Psi}_{(j_q)}}(x_p)) \times (\mu(x_p) \wedge \mu'(x_p)) \Big),$$

and

$$(\hat{\Phi}\overline{\wedge}\hat{\Psi})^F_{(i_q,j_q)}(x_p) = \Big((F_{\hat{\Phi}_{(i_q)}}(x_p) \vee F'_{\hat{\Psi}_{(j_q)}}(x_p)) - (\mu(x_p) \wedge \mu'(x_p) \Big)^2.$$

Algorithm 2: *Cont.*

Fifth step: Calculate and write the max-decision τ_T (i.e., $\tau_T : X \to R$), τ_I (i.e., $\tau_I : X \to R$), and τ_F (i.e., $\tau_F : X \to R$) of $(\hat{\Phi}\overline{\wedge}\hat{\Psi})^T_{(i_q,j_q)}$, $(\hat{\Phi}\overline{\wedge}\hat{\Psi})^I_{(i_q,j_q)}$, and $(\hat{\Phi}\overline{\wedge}\hat{\Psi})^F_{(i_q,j_q)}$, respectively, for every $x_p \in X$ as follows ($p = 1, 2, \cdots, N$):

$$\tau_T(x_p) = \sum_{(i,j)\in I\times J} \delta_T(x_p)(i,j), \quad \tau_I(x_p) = \sum_{(i,j)\in I\times J} \delta_F(x_p)(i,j), \quad \text{and} \quad \sum_{(i,j)\in I\times J} \delta_F(x_p)(i,j),$$

where

$$\delta_T(x_p)(i,j) = \begin{cases} (\hat{\Phi}\overline{\wedge}\hat{\Psi})^T_{(i_q,j_q)}(x_p), & (\hat{\Phi}\overline{\wedge}\hat{\Psi})^T_{(i_q,j_q)}(x_p) = \max\{(\hat{\Phi}\overline{\wedge}\hat{\Psi})^T_{(u_q,v_q)}(x_p) : (u,v) \in I \times J\} \\ 0, & \text{otherwise} \end{cases},$$

$$\delta_I(x_p)(i.j) = \begin{cases} (\hat{\Phi}\overline{\wedge}\hat{\Psi})^I_{(i_q,j_q)}(x_p), & (\hat{\Phi}\overline{\wedge}\hat{\Psi})^I_{(i_q,j_q)}(x_p) = \max\{(\hat{\Phi}\overline{\wedge}\hat{\Psi})^I_{(u_q,v_q)}(x_p) : (u,v) \in I \times J\} \\ 0, & \text{otherwise} \end{cases},$$

$$\delta_F(x_p)(i.j) = \begin{cases} (\hat{\Phi}\overline{\wedge}\hat{\Psi})^F_{(i_q,j_q)}(x_p), & (\hat{\Phi}\overline{\wedge}\hat{\Psi})^F_{(i_q,j_q)}(x_p) = \max\{(\hat{\Phi}\overline{\wedge}\hat{\Psi})^F_{(u_q,v_q)}(x_p) : (u,v) \in I \times J\} \\ 0, & \text{otherwise} \end{cases}.$$

Sixth step: Calculate the score $S(x_p)$ of element x_p as follows ($p = 1, 2, \cdots, N$):

$$S(x_p) = \tau_T(x_p) + \tau_I(x_p) + \tau_F(x_p).$$

Seventh step: Obtain the decision p satisfying

$$x_p = \max\left\{S(x_1), S(x_2), \cdots, S(x_j)\right\}.$$

Now, we show the principle and steps of the above Algorithm 2 using the following example.

Example 9. *(Continued from Example 11). Suppose that an investment company also adds three different parameters $J = \{j_1, j_2, j_3\}$, where j_1 is "marketing management", j_2 is "productivity of capital", and j_3 is "interest rates". The data of the single-valued neutrosophic fuzzy soft set $\hat{\Psi}_{(j_q)} \in (\mathbb{SVNFS})^{XJ}$ is given by*

$$\hat{\Psi} = \left(\begin{array}{c|ccccc} J & z_1 & z_2 & z_3 & z_4 & z_5 \\ \hline j_1 & (0.5, 0.6, 0.7, 0.4) & (0.3, 0.2, 0.7, 0.8) & (0.6, 0.9, 0.4, 0.3) & (0.8, 0.8, 0.2, 0.1) & (0.9, 0.5, 0.4, 0.2) \\ j_2 & (0.8, 0.4, 0.5, 0.2) & (0.7, 0.9, 0.2, 0.1) & (0.3, 0.3, 0.9, 0.4) & (0.9, 0.4, 0.5, 0.5) & (0.7, 0.8, 0.7, 0.2) \\ j_3 & (0.9, 0.9, 0.5, 0.3) & (0.5, 0.9, 0.2, 0.1) & (0.6, 0.6, 0.1, 0.5) & (0.5, 0.7, 0.8, 0.8) & (0.6, 0.2, 0.4, 0.7) \end{array}\right).$$

Now, we explain the practical meaning of alternatives X by taking the alternative z_1 as an example: the single-valued neutrosophic fuzzy soft set $\hat{\Psi}_{(j_1)}(z_1) = (0.5, 0.6, 0.7, 0.4)$ is the evaluation by four expert groups; the single-valued neutrosophic fuzzy soft value 0.5 (meaning 50% say yes in the first expert group) in $\hat{\Psi}_{(j_1)}(z_1)$, the single-valued neutrosophic fuzzy soft value 0.6 (meaning 60% say no in the second expert group) in $\hat{\Psi}_{(j_1)}(z_1)$, the single-valued neutrosophic fuzzy soft value 0.7 (meaning 70% say yes in the third expert group) in $\hat{\Psi}_{(j_1)}(z_1)$, and fuzzy value 0.4 (meaning 40% say no in the fourth expert group) in $\hat{\Psi}_{(j_1)}(z_1)$. Then, by computing $(\hat{\Phi}\overline{\wedge}\hat{\Psi})_{(i_q,j_q)}$ ($q = 1, 2, 3$) in the second step of Algorithm 2, we obtain the following:

$\hat{\Phi}\overline{\wedge}\hat{\Psi}$	z_1	z_2	z_3	z_4	z_5
(i_1, j_1)	$(0.3, 0.7, 0.7, 0.2)$	$(0.1, 0.8, 0.7, 0.5)$	$(0.2, 0.9, 0.8, 0.3)$	$(0.5, 0.8, 0.5, 0.1)$	$(0.4, 0.7, 0.9, 0.1)$
(i_1, j_2)	$(0.3, 0.7, 0.5, 0.2)$	$(0.1, 0.9, 0.5, 0.1)$	$(0.2, 0.6, 0.9, 0.4)$	$(0.5, 0.6, 0.5, 0.2)$	$(0.4, 0.8, 0.9, 0.1)$
(i_1, j_3)	$(0.3, 0.9, 0.5, 0.2)$	$(0.1, 0.9, 0.5, 0.1)$	$(0.2, 0.6, 0.8, 0.5)$	$(0.5, 0.7, 0.8, 0.2)$	$(0.4, 0.7, 0.9, 0.1)$
(i_2, j_1)	$(0.5, 0.6, 0.7, 0.4)$	$(0.3, 0.7, 0.7, 0.4)$	$(0.6, 0.9, 0.6, 0.3)$	$(0.3, 0.8, 0.2, 0.1)$	$(0.7, 0.8, 0.8, 0.2)$
(i_2, j_2)	$(0.8, 0.4, 0.5, 0.2)$	$(0.3, 0.9, 0.5, 0.1)$	$(0.3, 0.3, 0.9, 0.4)$	$(0.3, 0.7, 0.5, 0.5)$	$(0.7, 0.8, 0.8, 0.2)$
(i_2, j_3)	$(0.9, 0.9, 0.5, 0.3)$	$(0.3, 0.9, 0.5, 0.1)$	$(0.6, 0.6, 0.6, 0.5)$	$(0.3, 0.7, 0.8, 0.5)$	$(0.6, 0.8, 0.8, 0.3)$
(i_3, j_1)	$(0.5, 0.6, 0.7, 0.4)$	$(0.3, 0.5, 0.6, 0.4)$	$(0.7, 0.8, 0.6, 0.1)$	$(0.8, 0.9, 0.6, 0.1)$	$(0.7, 0.8, 0.9, 0.2)$
(i_3, j_2)	$(0.6, 0.4, 0.5, 0.2)$	$(0.3, 0.9, 0.6, 0.1)$	$(0.3, 0.3, 0.9, 0.3)$	$(0.8, 0.9, 0.6, 0.4)$	$(0.7, 0.8, 0.9, 0.2)$
(i_3, j_3)	$(0.6, 0.9, 0.5, 0.3)$	$(0.3, 0.9, 0.6, 0.1)$	$(0.6, 0.6, 0.6, 0.3)$	$(0.5, 0.9, 0.8, 0.4)$	$(0.6, 0.8, 0.9, 0.6)$

By calculating in the third step of Algorithm 2, we get the truth membership $(\hat{\Phi}\overline{\wedge}\hat{\Psi})^T_{(i_q,j_q)}$, *the indeterminacy membership* $(\hat{\Phi}\overline{\wedge}\hat{\Psi})^I_{(i_q,j_q)}$, *and the falsity membership* $(\hat{\Phi}\overline{\wedge}\hat{\Psi})^F_{(i_q,j_q)}$, *respectively, as follows:* $(q = 1, 2, 3)$*:*

$(\hat{\Phi}\overline{\wedge}\hat{\Psi})^T$	z_1	z_2	z_3	z_4	z_5
(i_1, j_1)	$(0.3, 0.2)$	$(0.1, 0.5)$	$(0.2, 0.3)$	$(0.5, 0.1)$	$(0.4, 0.1)$
(i_1, j_2)	$(0.3, 0.2)$	$(0.1, 0.1)$	$(0.2, 0.4)$	$(0.5, 0.2)$	$(0.4, 0.1)$
(i_1, j_3)	$(0.3, 0.2)$	$(0.1, 0.1)$	$(0.2, 0.5)$	$(0.5, 0.2)$	$(0.4, 0.1)$
(i_2, j_1)	$(0.5, 0.4)$	$(0.3, 0.4)$	$(0.6, 0.3)$	$(0.3, 0.1)$	$(0.7, 0.2)$
(i_2, j_2)	$(0.8, 0.2)$	$(0.3, 0.1)$	$(0.3, 0.4)$	$(0.3, 0.5)$	$(0.7, 0.2)$
(i_2, j_3)	$(0.9, 0.3)$	$(0.3, 0.1)$	$(0.6, 0.5)$	$(0.3, 0.5)$	$(0.6, 0.3)$
(i_3, j_1)	$(0.5, 0.4)$	$(0.3, 0.4)$	$(0.7, 0.1)$	$(0.8, 0.1)$	$(0.7, 0.2)$
(i_3, j_2)	$(0.6, 0.2)$	$(0.3, 0.1)$	$(0.3, 0.3)$	$(0.8, 0.4)$	$(0.7, 0.2)$
(i_3, j_3)	$(0.6, 0.3)$	$(0.3, 0.1)$	$(0.6, 0.3)$	$(0.5, 0.4)$	$(0.6, 0.6)$

$(\hat{\Phi}\overline{\wedge}\hat{\Psi})^I$	z_1	z_2	z_3	z_4	z_5
(i_1, j_1)	$(0.7, 0.2)$	$(0.8, 0.5)$	$(0.9, 0.3)$	$(0.8, 0.1)$	$(0.7, 0.1)$
(i_1, j_2)	$(0.7, 0.2)$	$(0.9, 0.1)$	$(0.6, 0.4)$	$(0.6, 0.2)$	$(0.8, 0.1)$
(i_1, j_3)	$(0.9, 0.2)$	$(0.9, 0.1)$	$(0.6, 0.5)$	$(0.7, 0.2)$	$(0.7, 0.1)$
(i_2, j_1)	$(0.6, 0.4)$	$(0.7, 0.4)$	$(0.9, 0.3)$	$(0.8, 0.1)$	$(0.8, 0.2)$
(i_2, j_2)	$(0.4, 0.2)$	$(0.9, 0.1)$	$(0.3, 0.4)$	$(0.7, 0.5)$	$(0.8, 0.2)$
(i_2, j_3)	$(0.9, 0.3)$	$(0.9, 0.1)$	$(0.6, 0.5)$	$(0.7, 0.5)$	$(0.8, 0.3)$
(i_3, j_1)	$(0.6, 0.4)$	$(0.5, 0.4)$	$(0.8, 0.1)$	$(0.9, 0.1)$	$(0.8, 0.2)$
(i_3, j_2)	$(0.4, 0.2)$	$(0.9, 0.1)$	$(0.3, 0.3)$	$(0.9, 0.4)$	$(0.8, 0.2)$
(i_3, j_3)	$(0.9, 0.3)$	$(0.9, 0.1)$	$(0.6, 0.3)$	$(0.9, 0.4)$	$(0.8, 0.6)$

$(\hat{\Phi}\overline{\wedge}\hat{\Psi})^F$	z_1	z_2	z_3	z_4	z_5
(i_1, j_1)	$(0.7, 0.2)$	$(0.7, 0.5)$	$(0.8, 0.3)$	$(0.5, 0.1)$	$(0.9, 0.1)$
(i_1, j_2)	$(0.5, 0.2)$	$(0.5, 0.1)$	$(0.9, 0.4)$	$(0.5, 0.2)$	$(0.9, 0.1)$
(i_1, j_3)	$(0.5, 0.2)$	$(0.5, 0.1)$	$(0.8, 0.5)$	$(0.8, 0.2)$	$(0.9, 0.1)$
(i_2, j_1)	$(0.7, 0.4)$	$(0.7, 0.4)$	$(0.6, 0.3)$	$(0.2, 0.1)$	$(0.8, 0.2)$
(i_2, j_2)	$(0.5, 0.2)$	$(0.5, 0.1)$	$(0.9, 0.4)$	$(0.5, 0.5)$	$(0.8, 0.2)$
(i_2, j_3)	$(0.5, 0.3)$	$(0.5, 0.1)$	$(0.6, 0.5)$	$(0.8, 0.5)$	$(0.8, 0.3)$
(i_3, j_1)	$(0.7, 0.4)$	$(0.6, 0.4)$	$(0.6, 0.1)$	$(0.6, 0.1)$	$(0.9, 0.2)$
(i_3, j_2)	$(0.5, 0.2)$	$(0.6, 0.1)$	$(0.9, 0.3)$	$(0.6, 0.4)$	$(0.9, 0.2)$
(i_3, j_3)	$(0.5, 0.3)$	$(0.6, 0.1)$	$(0.6, 0.3)$	$(0.8, 0.4)$	$(0.9, 0.6)$

By calculating in the fourth step of Algorithm 2, we obtain the max-matrices of $(\hat{\Phi}\overline{\wedge}\hat{\Psi})^T_{(i_q,j_q)}$, $(\hat{\Phi}\overline{\wedge}\hat{\Psi})^I_{(i_q,j_q)}$, *and* $(\hat{\Phi}\overline{\wedge}\hat{\Psi})^F_{(i_q,j_q)}$ $(p = 1,2,3,4,5;\ q = 1,2,3)$, *respectively, for every* $z_p \in X$ *as follows:*

$$\left(\begin{array}{c|ccccc}
(\hat{\Phi}\overline{\wedge}\hat{\Psi})^T & z_1 & z_2 & z_3 & z_4 & z_5 \\
\hline
(i_1,j_1) & 0.25 & \underline{0.3} & 0.25 & \underline{0.3} & 0.25 \\
(i_1,j_2) & 0.25 & 0.1 & 0.3 & \underline{0.35} & 0.25 \\
(i_1,j_3) & 0.25 & 0.1 & \underline{0.35} & \underline{0.35} & 0.25 \\
(i_2,j_1) & \underline{0.45} & 0.35 & \underline{0.45} & 0.2 & \underline{0.45} \\
(i_2,j_2) & \underline{0.5} & 0.2 & 0.35 & 0.4 & 0.45 \\
(i_2,j_3) & \underline{0.6} & 0.2 & 0.55 & 0.4 & 0.45 \\
(i_3,j_1) & \underline{0.45} & 0.35 & 0.4 & \underline{0.45} & \underline{0.45} \\
(i_3,j_2) & 0.4 & 0.2 & 0.3 & \underline{0.6} & 0.45 \\
(i_3,j_3) & 0.45 & 0.2 & 0.45 & 0.45 & \underline{0.6}
\end{array}\right),$$

$$\left(\begin{array}{c|ccccc}
(\hat{\Phi}\overline{\wedge}\hat{\Psi})^I & z_1 & z_2 & z_3 & z_4 & z_5 \\
\hline
(i_1,j_1) & 0.14 & \underline{0.4} & 0.27 & 0.08 & 0.07 \\
(i_1,j_2) & 0.14 & 0.09 & \underline{0.24} & 0.12 & 0.08 \\
(i_1,j_3) & 0.18 & 0.09 & \underline{0.3} & 0.14 & 0.07 \\
(i_2,j_1) & 0.24 & \underline{0.28} & 0.27 & 0.08 & 0.16 \\
(i_2,j_2) & 0.08 & 0.08 & 0.12 & \underline{0.35} & 0.16 \\
(i_2,j_3) & 0.27 & 0.09 & 0.3 & \underline{0.35} & 0.24 \\
(i_3,j_1) & \underline{0.24} & 0.2 & 0.08 & 0.09 & 0.16 \\
(i_3,j_2) & 0.08 & 0.09 & 0.09 & \underline{0.36} & 0.16 \\
(i_3,j_3) & 0.27 & 0.09 & 0.18 & 0.36 & \underline{0.48}
\end{array}\right),$$

$$\left(\begin{array}{c|ccccc}
(\hat{\Phi}\overline{\wedge}\hat{\Psi})^F & x_1 & z_2 & z_3 & z_4 & z_5 \\
\hline
(i_1,j_1) & 0.25 & 0.04 & 0.25 & 0.16 & \underline{0.64} \\
(i_1,j_2) & 0.09 & 0.16 & 0.25 & 0.09 & \underline{0.64} \\
(i_1,j_3) & 0.09 & 0.16 & 0.09 & 0.36 & \underline{0.64} \\
(i_2,j_1) & 0.09 & 0.09 & 0.09 & 0.01 & \underline{0.36} \\
(i_2,j_2) & 0.09 & 0.16 & 0.25 & 0 & \underline{0.36} \\
(i_2,j_3) & 0.04 & 0.16 & 0.01 & 0.09 & \underline{0.25} \\
(i_3,j_1) & 0.09 & 0.04 & 0.25 & 0.25 & \underline{0.49} \\
(i_3,j_2) & 0.09 & 0.25 & 0.36 & 0.04 & \underline{0.49} \\
(i_3,j_3) & 0.04 & \underline{0.25} & 0.09 & 0.16 & 0.09
\end{array}\right).$$

By calculating in the fifth step of Algorithm 2, we obtain the max-decision τ_T, τ_I, *and* τ_F *of elements* z_p, *respectively, as follows* $(p = 1,2,3,4,5)$*:*

$$\tau_T(z_1) = 2,\ \tau_T(z_2) = 0.3,\ \tau_T(z_3) = 0.8,\ \tau_T(z_4) = 2.05,\ \tau_T(z_5) = 1.5;$$

$$\tau_I(z_1) = 0.24,\ \tau_I(z_2) = 0.68,\ \tau_I(z_3) = 0.54,\ \tau_I(z_4) = 1.06,\ \tau_I(z_5) = 0.48;$$

$$\tau_F(z_1) = 0,\ \tau_F(z_2) = 0.25,\ \tau_F(z_3) = 0,\ \tau_F(z_4) = 0,\ \tau_F(z_5) = 3.87.$$

By calculating in the sixth step of Algorithm 2, the scores $S(z_p)$ *of elements* $z_p(p = 1,2,3,4,5)$, *respectively, are as follows:*

$$S(z_1) = 2.24,\ S(z_2) = 1.23,\ S(z_3) = 1.34,\ S(z_4) = 3.11,\ S(z_5) = 5.85.$$

Finally, we know from the seventh step that z_5 has a high value. Therefore, the experts should select z_5 as the best choice.

Remark 2.

(1) By means of Algorithms 1 and 2, we can see that the final results are in agreement. Thus, x_5 is the most accurate and refinable.

(2) By comparing the steps in Algorithms 1 and 2, we can see that step 4 and step 5 in Algorithm 2 are complicated in their process compared to step 2 and step 3 in Algorithm 1, respectively. So, if we take the complexity of these steps into consideration, Algorithm 2 gives its decision concisely.

(3) Algorithms 1 and 2 that we have elaborated here arrive at their decisions by combining the concept of single-valued neutrosophic fuzzy set theory and soft set theory. As result, we can apply Algorithm 1 to picture fuzzy soft sets [29], generalized picture fuzzy soft sets [13], and interval-valued neutrosophic soft sets [12]. Further, Algorithm 2 can be applied to possibility m-polar fuzzy soft sets [15] and possibility multi-fuzzy soft sets [17].

5. Conclusions

We introduced the notion of the single-valued neutrosophic fuzzy soft set as a novel neutrosophic soft set model. We discussed the five operations of the single-valued neutrosophic fuzzy soft set, such as subset, equal, union, intersection, and complement. The structure properties of the single-valued neutrosophic fuzzy soft set are explained. Then, a novel approach (i.e., Algorithm 1) is presented as a single-valued neutrosophic fuzzy soft set decision method. Lastly, an application (i.e., Algorithm 2) of a single-valued neutrosophic fuzzy soft set for fuzzy decision-making is constructed, and the two approaches (i.e., Algorithms 1 and 2) introduce an important contribution to further research and relevant applications. Therefore, in the future, we will provide a real application with a real dataset or we will apply the two approaches (i.e., Algorithms 1 and 2) to lung cancer disease [30] and coronary artery disease [31]. In addition, we will describe in more detail in order to clarify if the methods (i.e., Algorithms 1 and 2) converge or diverge from standard approaches such as fuzzy sets [1], intuitionistic fuzzy sets [2], picture fuzzy sets [3].

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