

Article

Concept System and Application of Point Group Symmetry in Mechanical Structure Design

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Abstract: Symmetry has been widely and deeply researched in basic science, and many mature results have been obtained so far. However, the widespread existence of symmetry in applied science is not in direct proportion to the attention it has received. Through a large number of examples studies, almost all mechanical structures are found to have symmetry, and most of them have the characteristics of point group symmetry. Therefore, the concept of point group symmetry in crystallography was extended to the field of machinery and adjusted according to the mechanical structures. First of all, the classification of mechanical point group symmetry is proposed, and how point group symmetry is applied in machinery is illustrated with examples. Then, the requirements of symmetry are analyzed and compared. Furthermore, the data mining software RapidMiner is used to mine the association rules between requirements and symmetry. Based on the mining results, the four selection principles of point group symmetry are summarized to provide ideas for structure design. Finally, a new type of gear pump with radial force balancing is invented by comprehensively using the mining results and selection principles.

Keywords: point group symmetry; mechanical structure design; requirements; data mining; selection principles

1. Introduction

Symmetry initially referred to balance, harmony, and coordination. Until modern times, generalized symmetry was defined as invariance to change [1]. Physics interprets symmetry as regularity, while aesthetics emphasizes that symmetry is the characteristic of beauty and art [2]. Studies have been conducted on symmetry in the natural science [3–5], social science [6], and engineering science [7]. Landscapes, buildings, and mechanical equipment all show symmetrical attributes.

The study of symmetry is usually inseparable from group theory. In the 1930s, as soon as the group theory came out, the concept of symmetry began to merge with it. After that, group theory has become a mathematical language to describe symmetry in various disciplines [8–10]. But different disciplines could adjust the definition and scope of symmetry group according to their own characteristics. In chemistry, point group and space group have been widely recognized and applied, but the third group between the two groups can still be defined according to specific research object, such as unit cell in solid [11]. In biology, combined with the term biological symmetry, group theory can be used to describe biological objects, such as point group symmetry to describe the shape of petals [12]. In architecture, symmetry of each level, such as point symmetry, rotational symmetry, and translational symmetry, is considered as a mathematical symmetry group. Symmetry group or hierarchical symmetry classification of nested control is helpful to further analyze the function of symmetry or asymmetry [13]. In mechanics, group theory can also be used as a reference for classification [14].

Engineering design points out that symmetrical structures satisfy the design criteria of simplicity [15]. Axiomatic design gives a reasoning: as long as FRs (functional requirements) can be met, symmetrical shapes or components should be used as far as possible [16]. The principle of increasing asymmetry in TRIZ (invention problem solving method) emphasizes the innovative idea of symmetry breaking [17]. The earliest systematic research on mechanical symmetry is the applied knowledge of mechanical structure symmetry summarized by Barendsen in his doctoral dissertation [18]. Other common research studies on mechanical symmetry include symmetry recognition or detection, the influence of symmetry features on DFA (design for assembly), the use of symmetry ideas to reduce the calculation time or modeling difficulty [19–21], etc. There are also some scattered research studies on mechanical symmetry, such as using symmetry to restore man-made objects layer by layer, using symmetry as a dynamic feature to solve problems in artificial vision, taking symmetry as one of the criteria for structural optimization [22–25], etc. In a word, mechanical symmetry, like other fields, is usually a method or tool to guide the application.

The relationship between symmetry and requirement is the key point of symmetrical structure design. For example, the point group symmetry is applied to the classification of mechanical parts to study the relationship between the symmetry and assembly efficiency [26], and the idea of symmetry breaking can be used in error proofing design [27].

In view of the advantages of group theory to symmetry classification and the previous achievements, this paper starts with point group symmetry and focuses on its concept and application in mechanical structure.

2. Method

Association rules mining can discover the association between items from the data set. Through the analysis of a large number of examples, we can get the data set which takes point group symmetry and requirements as the items. Then, we can use the data analysis software RapidMiner to mine the association rules between point group symmetry and requirements.

2.1. Data Preparation

2.1.1. Classification of Point Group Symmetry

In crystallography, the various combinations of all symmetry elements converging at one point is called the point group, or symmetry type. The mathematical analysis shows that there are 32 kinds of spatial point groups in crystal [28]. Based on the crystallographic definition of point group symmetry and the Schoenflies symbol, the point group symmetry of mechanical structures can be classified as uniaxial group symmetry, nonaxial group symmetry, and multiaxial group symmetry, according to the number of main axes. According to the types and combinations of symmetry elements, uniaxial group symmetry is divided into 7 kinds, uniaxial group symmetry is divided into 2 kinds, and multiaxial group symmetry is not divided in detail. The concept system of point group symmetry of mechanical structures can be obtained as shown in Figure 1.

The symbol and symmetry elements of point group symmetry are shown in Table 1. The value of n is no less than 2 in all symbols.

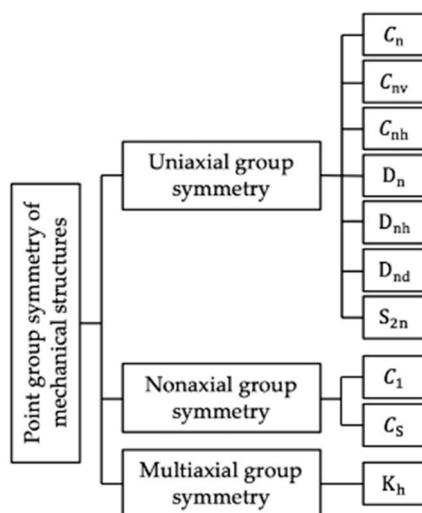


Figure 1. Classification of point group symmetry.

Table 1. Symbols and symmetry elements of point group symmetry.

Symbols	Symmetry Elements	The Relationship between Symmetry Elements
C_1	C_1	/
C_s	σ	/
C_n	C_n	/
C_{nv}	C_n, σ_v	σ_v passes through C_n , the number of σ_v is n
C_{nh}	C_n, σ_h	σ_h is perpendicular to C_n
D_n	C_n, C_2	C_2 is perpendicular to C_n , the number of C_2 is n
D_{nh}	C_n, C_2, σ_h	/
D_{nd}	C_n, C_2, σ_d	σ_d passes through C_n and bisects the angle between C_2
S_{2n}	$C_n, C_{2n}\sigma_h$	$C_{2n}\sigma_h$ is the combination of C_{2n} and σ_h
K_h	C_n, C_n, \dots, C_n	multiple C_n intersect at one point

(1) Uniaxial group symmetry

Uniaxial group symmetry means that there is only one main axis in the structure, and main axis refers to the rotational symmetry element with the highest order. The curve ribs shown in Figure 2a are rotationally symmetrical around the main axis, belonging to C_n (in this case, $n = 5$). In addition to rotational symmetry around the main axis, there are also mirror symmetry elements passing through the main axis, so the straight ribs shown in Figure 2b belong to C_{nv} , and the number of mirror symmetry elements is equal to the order of rotational symmetry, that is, the number of ribs. The curve shape is more difficult to process than the straight shape, while the even number of straight ribs are deformed by internal stress during the casting process and are also prone to vibration during driving [29]. Therefore, considering the machining difficulty and driving stability, ribs of $C_{(2n+1)v}$ are generally used in a wheel.

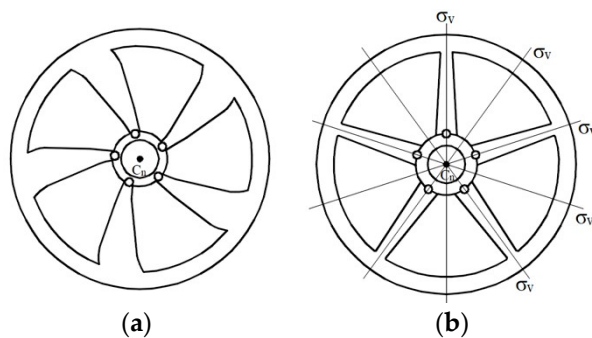


Figure 2. Wheel ribs. (a) Curved ribs; (b) straight ribs.

Structure is the carrier of function, and the requirement is the generalized function, including function, performance and constraint. Multiple symmetry elements structure is more widely used in practice than single symmetry element structure because it can not only achieve functions but also improve performance and meet constraints. Impellers in Figure 3 can properly prove this conclusion. The symmetry element of single impeller shown in Figure 3a is only C_n , which makes the centrifugal pump produce axial unbalanced force during operation, resulting in vibration and noise. In order to solve this problem, as shown in Figure 3b, σ_h is added to change single impeller into bilateral impeller, and then self-balance is realized [30].

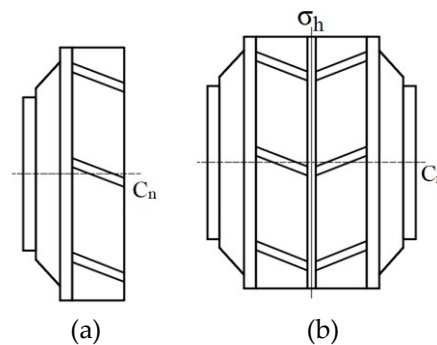


Figure 3. Impellers. (a) Single impeller (C_n); (b) bilateral impeller (C_{nh}).

(2) Nonaxial group symmetry

Nonaxial group symmetry includes C_1 and C_s . In C_1 , symmetry elements only have identity operation, so both asymmetrical structures and symmetrical broken structures belong to C_1 . C_s is also mirror symmetry, with only one mirror surface or line as symmetry element.

There is a principle of increasing asymmetry in TRIZ theory, that is, increasing the asymmetry of asymmetrical structure or changing the symmetrical structure to asymmetrical structure. However, in the comparative analysis of uniaxial group symmetry, it is found that the structures with higher symmetry tend to have better performance, so the principle of increasing asymmetry is not always effective, but it is effective in the process of changing uniaxial group symmetry into nonaxial group symmetry.

In Figure 4, the bearing housing is planned to adopt C_n and C_s structures, respectively. C_n shown in Figure 4a is easy to be confused in front and back positions, so that the deviation between the axis of the housing hole and the axis of the shaft increases due to the reversal of the front and back positions. However, C_s shown in Figure 4b can eliminate the deviation, and the bearing housing changes from uniaxial group symmetry to nonaxial group symmetry, which conforms to the principle of increasing asymmetry.

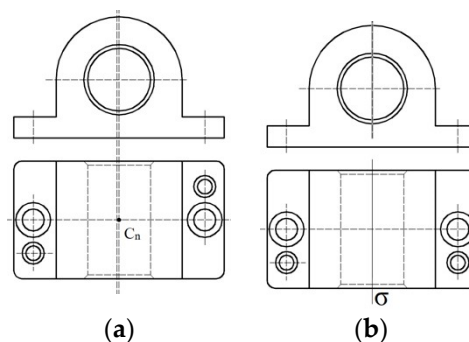


Figure 4. Bearing housing. (a) Unreasonable design (C_n); (b) improved design (C_s).

(3) Multiaxial group symmetry

Multiaxial group symmetry is a general designation for all the point group symmetry with more than one main axis, denoted as K_h . The most common requirements of multiaxial group symmetry are to expand the scope of function and safety redundancy. The device shown in Figure 5 has three main axes, which can collect vibration energy in multiple directions, that is, expand the scope of function. When the energy collection module is damaged, it can be easily disassembled and replaced. The waste of resources caused by the overall scrap of the energy collector can be reduced due to safety redundancy.

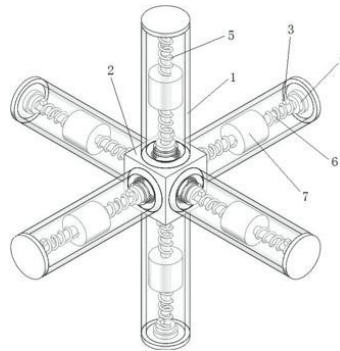


Figure 5. A vibration energy collecting device [31].

2.1.2. The Analysis of Requirements

In addition to the examples listed above, the realization of functions and the improvement of performance by point group symmetry is reflected in many aspects. In the process of analyzing the relationship between point group symmetry and requirements, requirements are divided into functions and performance in this paper. Structure design should first meet functions and subsequently consider specific performance. Symmetry element C_n is usually used to achieve main functions, and other symmetry elements besides C_n are usually used to improve performance. This shows that increasing symmetry is a way to improve structural performance. For example, different types of gears all have symmetry element C_n to meet the transfer of power and motion, but symmetry element σ_h in spur gear and herringbone gear counteracts side effect, and symmetry element σ_v in spur gear and bevel gear puts emphasis on reducing the difficulty of processing.

Based on the analysis of mechanical structures, the requirements of symmetry elements of point group symmetry are summarized, as shown in Table 2.

Combining the contents of Tables 1 and 2, the requirements corresponding to the point group symmetry can be found. Take K_h as an example, K_h contains multiple symmetry elements C_n , so K_h can not only meet all the requirements C_n meets but also repeat these requirements many times. The repetition of requirements can be expressed as expanding the scope of functions, increasing the efficiency of functions, as well as increasing the intensity of functions and safety redundancy. K_h and its requirements are shown in Table 3. The requirements satisfied by all the other types of point group symmetry can be analogized in turn, which will not be described in detail here. As for nonaxial group symmetry C_5 , its symmetry element is only σ , and the corresponding requirements include all the requirements that σ_v , σ_h , and σ_d can meet.

Table 2. Symmetry elements and their requirements.

Requirements	Symmetry Elements			
	C_n		σ_v	
Functions	Transform energy types Control power and motion Transfer power and motion Join or separate material Change the shape of material Store material Fixate or move the position of material		/	
Performance	Expand the scope of function Counteract side effects		Reduce the difficulty of processing Reduce friction coefficient Reduce volume Disassemble easily	
Requirements	Symmetry Elements			
Functions	σ_h	C_2	/	S_{2n}
Performance	Expand the scope of function Reduce volume Counteract side effects	Reduce area Extend service life Reduce vibration and shock Increase stability Reduce deformation	Eliminate interference	Counteract side effects Reduce vibration and noise

Table 3. Requirements that K_h can meet.

	Requirements	
	Functions	Performance
K_h	Transform energy types Control power and motion Transfer power and motion Join or separate material Change the shape of material Store material Fixate or move the position of material	Expand the scope of function Improve the efficiency of function Increase the intensity of function Counteract side effects Safety redundancy

It can be seen from Tables 2 and 3 that a requirement may be realized by several symmetry types. Through association rules mining, it is possible to compare the association strength between point group symmetry and requirements and provide a reference for structural schemes selection.

2.2. Association Rules Mining

This paper used RapidMiner to mine association rules. RapidMiner is an open source data analysis tool that connects the data mining process by dragging operators. Each operator is a process or an algorithm. In order to mine the association rules between point group symmetry and requirements, we only need to drag operators and connect them to establish the model.

2.2.1. Data Preprocessing

After analyzing the symmetry type and requirement of mechanical structures (such as the examples in Figures 2–4, etc.), the symmetry type and requirement are regarded as the items and are organized into a file format that can be recognized by RapidMiner. The sample before and after the file conversion is shown in Figure 6.

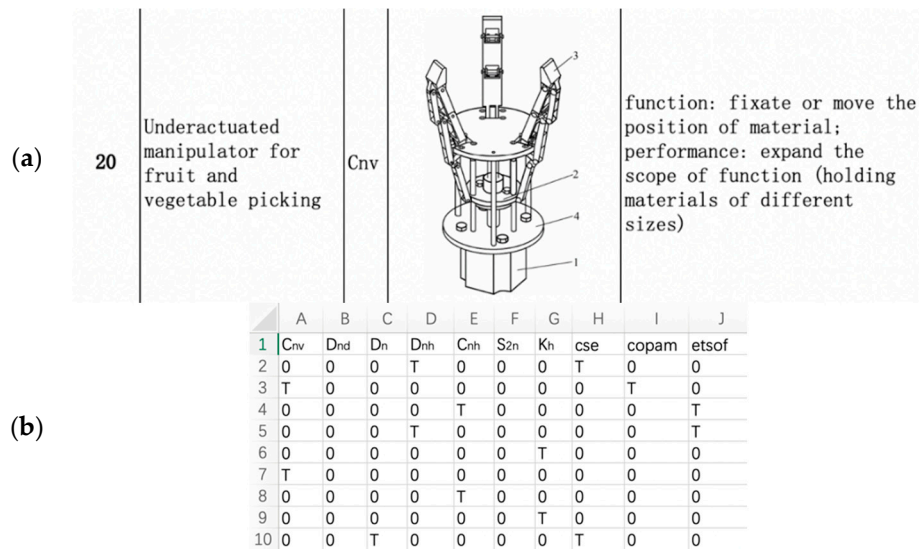


Figure 6. File sample. (a) Sample of structure analysis; (b) sample of files read by RapidMiner.

2.2.2. Data Mining Modeling

- (1) Drag the “Read Excel” operator to read the data.
- (2) Drag the “Nominal to Binominal” and “Select Attributes” operators to solve the problem of semantic deviation and retain the items with semantic existence.
- (3) Drag the “FP-Growth” and “Create Association Rules” operators and connect all operators to complete the modeling. The final association rules mining model is shown in Figure 7.

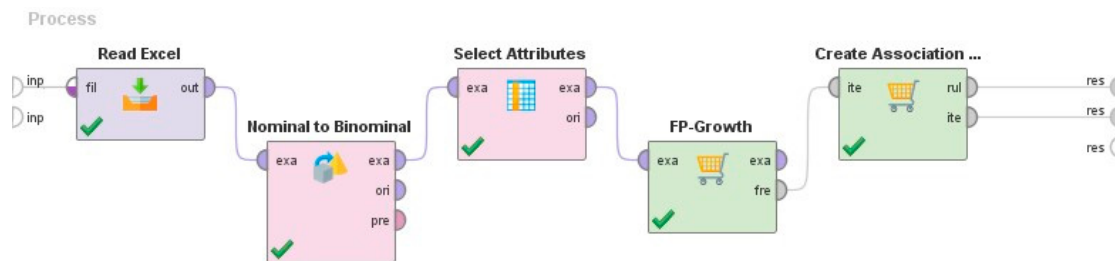


Figure 7. Association rules mining model.

3. Results

3.1. Mining Results

Run the model shown in Figure 7 to get the results of association rules mining. The rule strength is mainly measured by support and confidence. Support means the proportion of an item in the data set. Confidence is a kind of conditional probability, which reflects the proportion of conclusion (symmetry type) when premise (requirement) appears [32]. The following is an analysis of association rules with high support and high confidence.

Figure 8 shows single items with support more than 8%. In point group symmetry, D_{nh} has the highest support (34.7%), followed by C_{nv} (19%). In the requirements, “transfer power and motion (topam)” has the highest support (10.9%), followed by “expand the scope of function (etsof)” (8.4%).

Set the minimum confidence to 25%. Taking D_{nh} as an example, there are 12 premises to get D_{nh} , as shown in Figure 9. The requirements with top three support are “topam”, “cse”, and “etsof”, as shown in Figure 8. All these three requirements can be taken as the premise to get D_{nh} . Among them, the support and confidence obtained by associating “topam” with D_{nh} are the highest, which are 6.8 and 62.2%, respectively. As for “cse”, the support and confidence associated with D_{nh} are higher than

those associated with C_{nv} , as shown by No. 47 in Figure 9 and No. 30 in Figure 10. This comparison also provides a reference for later selection principles and structure design.

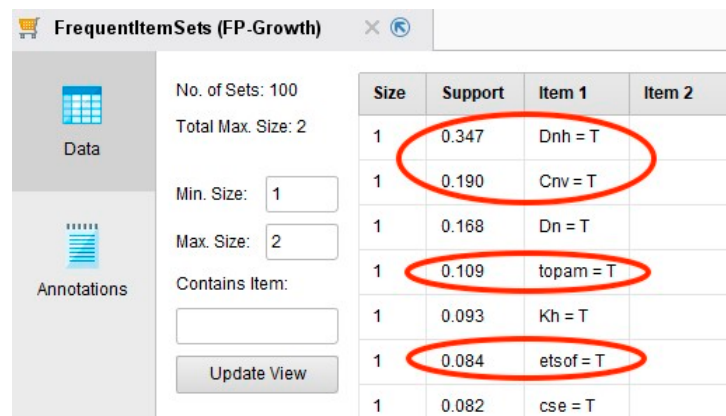


Figure 8. Support of high frequency items.

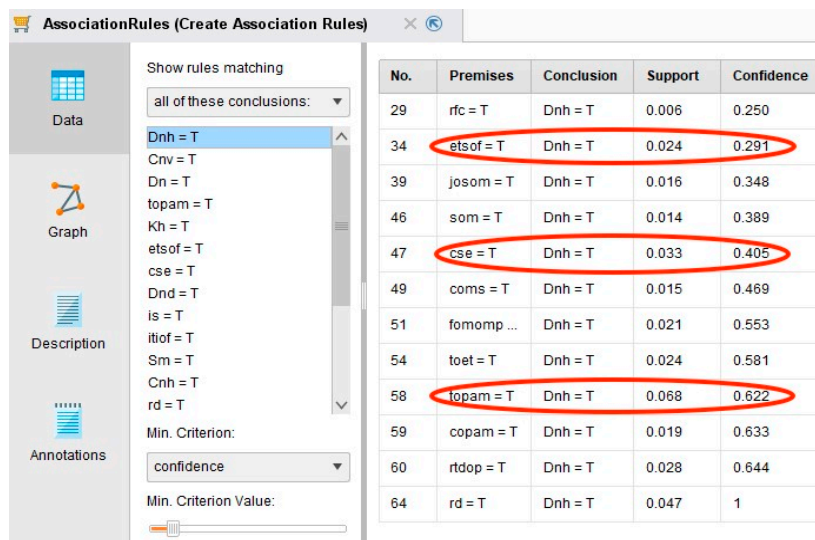


Figure 9. Association rules about D_{nh} .

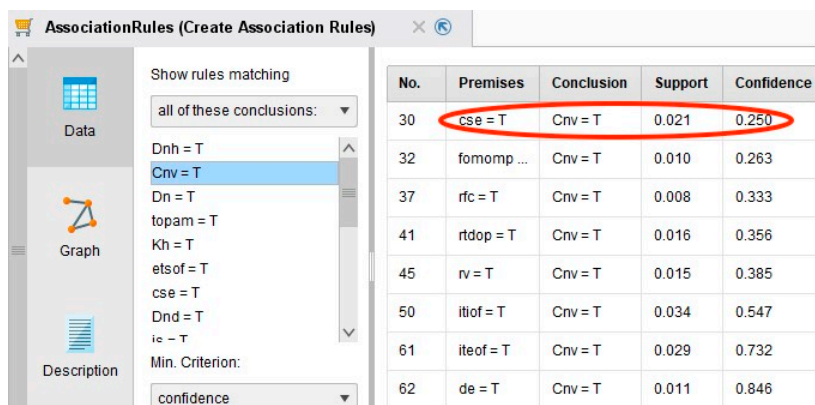


Figure 10. Association rules about C_{nv} .

By analyzing the results mined by association rules, several general selection principles can be summarized. In this way, it is possible to use requirements as the premise and refer to the support and confidence to obtain the appropriate symmetrical structure solution.

3.2. Selection Principles

In the design process, if the requirements that match the selection principles are met, there is no need to find possible solutions from a large number of mining results, thereby saving time and reducing errors.

3.2.1. Principle 1: Increase Symmetry

Usually, we have consensus that a circle is more symmetrical than a square and that a square is more symmetrical than a rectangle. The influencing factors of symmetry are various. Thus far, no quantitative comparison is available of symmetry between two different structures. Figure 11 shows the qualitative comparison of point group symmetry. It can be seen that the highest symmetry is the multiaxial group, and the lowest is the nonaxial group. In uniaxial group symmetry, C_n has the lowest symmetry, and D_{nh} has the highest symmetry.

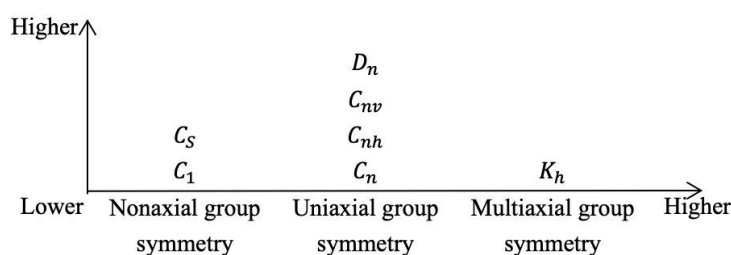


Figure 11. Qualitative comparison of symmetry.

Simplicity, safety, and clarity are the design criteria [15]. Structures with higher symmetry satisfy the simplicity criterion of design. For example, the spur gear of D_{nh} is simpler to manufacture than other gears. Compared with other uniaxial group symmetry, D_{nh} increases symmetry by increasing the spatial dimensions of symmetry elements, thereby improving the performance of the structures. As long as there is no reason for asymmetry, the structural scheme with higher symmetry shall be adopted as far as possible.

From the data mining results shown in Figure 8, we can also see that the support of D_{nh} , C_{nv} , D_n , and K_h are 34.7, 19, 16.8, and 9.3%, respectively, ranking in the top four of all point group symmetry, indicating that in practical applications, point group structures with high symmetry are more applied, which conforms to the principle of increasing symmetry.

Figure 12 is a case of applying selection principle 1 to improve the structure. The traditional electric fan belongs to C_n as shown in Figure 12a, and the order of rotational symmetry is three, which is the same as the number of fan blades, but it can usually be five or other odd numbers. The odd order can reduce vibration and noise caused by fan operation. The structure of C_n can only produce wind on the front side of the blades. To meet the requirement of one fan for continuous use by additional people, a Korean design company has introduced a double-sided fan, which known as the “2-way” fan. As shown in Figure 12b, the new fan belongs to D_n . The order of main axis is even, which reduces the difficulty of assembling blades. Moreover, because of the symmetry element C_2 , the noise problem caused by even order is solved. The structural design of D_n can produce wind from front and back, while the swing angle of the traditional electric fan is approximately 45° to the left or right. Compared with C_n , D_n breaks the limitation of wind outlet from front side, which means expanding the scope of the function.

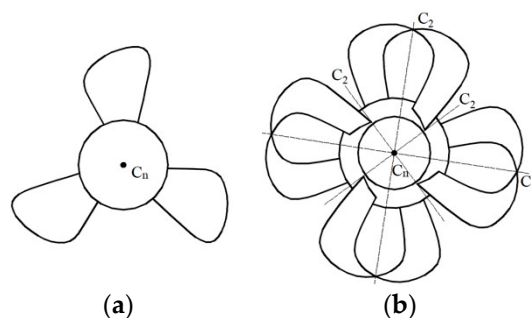


Figure 12. Electric fan. (a) Traditional fan; (b) “2-way” fan.

3.2.2. Principle 2: Increase Asymmetry

For a specific design requirement, if there are multiple schemes of point group symmetry that can meet it, the scheme with higher symmetry should be preferred, no matter from the perspective of simplicity, safety or aesthetics. Therefore, increasing symmetry is the normal design thinking, and increasing asymmetry only occurs under the condition of restrictions.

Changing a multi-axial group symmetry into a uniaxial group symmetry, especially a uniaxial group symmetry into a non-axial group symmetry, is a way to increase asymmetry, which is often used in error proofing design. The improvement of the bearing housing shown in Figure 4 is a typical case.

The result of asymmetry often has the cause of asymmetry, so another important factor of increasing asymmetry is the constraint of asymmetry.

Lifting building components needs to ensure the stability of the process. For building components with asymmetrical internal structures, asymmetrical lifting traverse is needed. For asymmetrical building components, if the mirror-symmetrical lifting traverse shown in Figure 13a is used for lifting, the geometrical center line of the lifting appliance cannot coincide with the geometrical center line of the lifted object. During the lifting process, the object to be lifted rocking and tilting will occur. Under the constraints of the asymmetry of the building components, it is necessary to increase asymmetry of the lifting traverse. For the symmetry breaking lifting traverse shown in Figure 13b, the bracket can be added, and the position of the lifting lugs can be changed, according to the actual structure of the lifted object to keep asymmetrical objects in balance during lifting.

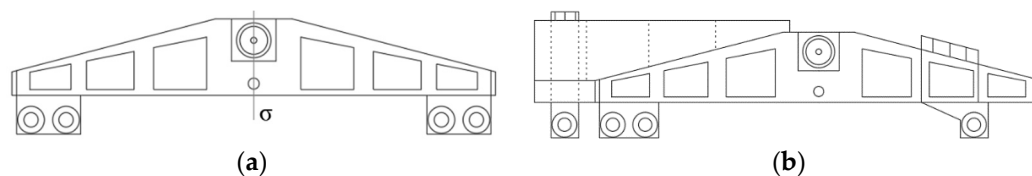


Figure 13. Lifting traverse [33]. (a) Unreasonable design (C_s); (b) improved design (C_1).

3.2.3. Principle 3: Priority

As shown in the results of data mining in Figure 8, the number of practical applications of D_{nh} and C_{nv} is more than that of the other types, combined with selection principle 1, when the requirements can be satisfied by all point group symmetry, D_{nh} is the first priority, and C_{nv} is the next.

D_{nh} cannot appear in two-dimensional graphs. As shown in Figure 14, by increasing the number of keyways, the shaft section can be changed from C_5 to C_{nv} , which can reduce the deformation of the shaft caused by stress concentration, thus reducing the rejection rate. Similar common application example is H-shaped long beam, as shown in Figure 15, the C_{nv} section avoids bending deformation of long beam.

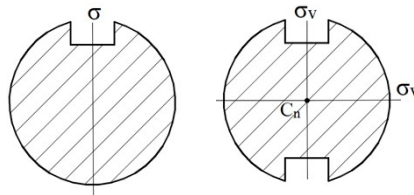


Figure 14. Shaft sections.

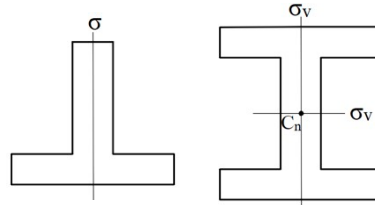


Figure 15. Beam sections.

Although the support of K_h is also ranked the top four, its structure is more complicated than the uniaxial group symmetry, so K_h is a priority only when the requirement is to expand the scope of the function.

The support of D_{nd} and S_{2n} is the smallest as shown in Figure 16, indicating that D_{nd} and S_{2n} have the least priority in practice, usually as an improvement plan of D_{nh} and C_{nh} . For example, a self-aligning roller bearing of D_{nd} can adjust to the angular deviation between the inner and outer raceways. Herringbone tires of S_{2n} have better anti-skid performance and pollution discharge performance.

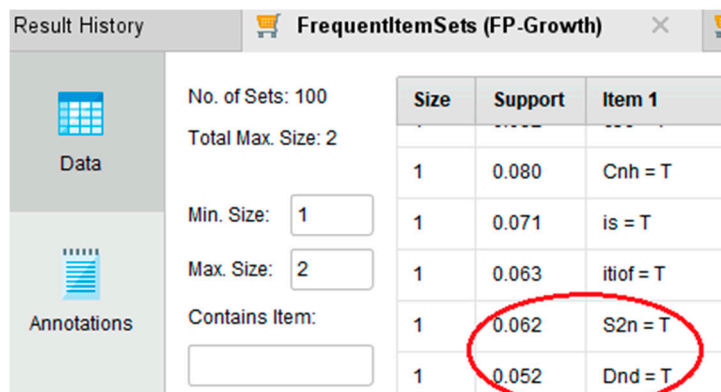


Figure 16. Support of items.

3.2.4. Principle 4: Force Balance

The first impression of symmetry to most people is balance and beauty. In fact, balance is indeed an important function of symmetry. Almost all the symmetry elements in Table 1 have the requirement of counteracting side effects, which includes the balance of force. There are many cases in machinery that use symmetrical structures to achieve self-balancing.

The balance of radial force mainly depends on rotational symmetry. Except for C_1 , all the other point group symmetry can achieve the balance of radial force. The balance of axial force mainly depends on mirror symmetry, and point group symmetry that realize the force self-balance are C_S , C_{nh} , D_{nh} , D_{nd} , S_{2n} , and K_h . As shown in Figure 17, regardless of whether the cylinder arrangement is linear or V-shaped, the engine crankshaft is mirror-symmetrical and belongs to C_S . The symmetry element σ is located in the middle of the shaft and is perpendicular to the axis to counteract the axial forces and moments generated during the movement of the crankshaft.

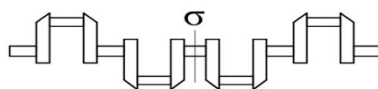


Figure 17. Crankshaft.

The improvement of air compressor is the comprehensive application of selection principle 1, selection principle 3 and principle 4. The working principle of a common piston air compressor is shown in Figure 18a. During operation, the motor drives the crank linkage, which drive its piston to create a linear reciprocating motion. The piston force acts on the crankshaft through the connecting rod, and the overturning moment M_N and resisting moment M_y are generated. The overturning moment M_N acts on the air compressor, causing the air compressor to obviously shake. The resistance moment M_y acts on the spindle at the center of the crank, which accelerates or decelerates the spindle, thereby causing vibration and noise in the air compressor. The lateral force N of the piston pressing on the cylinder block increases their friction and accelerates the wear of the contact surface, thus affecting the sealing performance and reducing the service life of the air compressor [34]. The double crank linkage shown in Figure 18b has only mirror symmetry and belongs to C_S , which means that the harmful forces cancel each other out. Therefore, the vibration and noise of the air compressor caused by the periodic change in resistance moment M_y can be eliminated, and the friction force between the piston and cylinder block caused by the lateral force N can be reduced, thus reducing the wear and prolonging the service life of the air compressor. However, that type of air compressor is single-acting, the output gas with pressure is discontinuous, and the pressure pulsation is large. Figure 18c shows an air compressor of C_{nv} . This layout not only counteracts harmful forces but also outputs continuous high-pressure gas and reduces pressure pulsation.

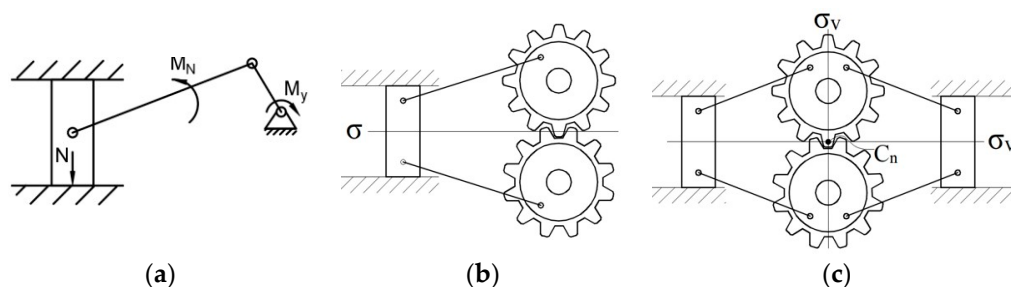


Figure 18. Crank linkage. (a) Single crank linkage; (b) double crank linkage; (c) four crank linkage.

4. Patent

Gear pumps are common hydraulic pumps. Generally, the inlet chamber of a pump is directly connected to the outside world, and its pressure is equal to the ambient pressure. The outlet chamber is generally connected to the actuator, and its pressure is higher than the ambient pressure. The pressure difference between inlet chamber and outlet chamber makes the gear deflect to one side, which easily leads to low efficiency and even jamming of the pump, thus reducing the service life of the gear pump. To improve the efficiency and service life of the pump, it is necessary to redesign the structure to improve the force state of the gear.

4.1. Gear Pump of C_{2v}

In Figure 19, F is the equivalent radial force acting on the gear, pointing from the high-pressure chamber to the low-pressure chamber. Because the symmetry elements of the gears in the pump are C_n and σ_v , thus, the equivalent radial forces of hydraulic pressure acting on the driving and driven gears are the same.

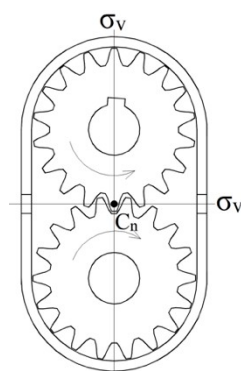


Figure 19. Gear pump of C_{2v} .

4.2. Gear Pump of C_{4v}

Figure 20 shows a four-wheel linkage gear pump [35]. After adding a pair of gears, the structure has σ_v in four dimensions and C_n as the main axis. Like the conventional gear pump, it is of C_{nv} , but the value of n is increased. The equivalent radial force can be counteracted by this symmetrical layout, thus effectively reducing the hydraulic pressure on the gears, extending the life of the gear pump, and improving the efficiency by double suction and double output.

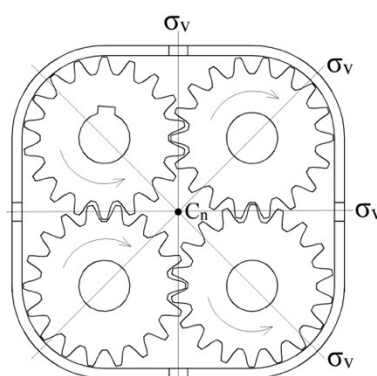


Figure 20. Gear pump of C_{4v} .

The gear pump transfers oil via the gear meshing motion. In the four-wheel linkage gear pump, four gears form four pairs of gears. The transportation route of oil at the four meshing points is not clear; thus, the hydraulic pressure issue is not effectively solved. At the same time, the larger size on the plane perpendicular to the main axis will make the pump body occupy too much space.

The purpose of force balance has been defined. Firstly, the application of point group symmetry in force balance is explained through an improved piston air compressor shown in Figure 18. Then, combined with the selection principles, a new type of gear pump with radial force balancing is invented, and the authorization announcement has been obtained [36].

4.3. Gear Pump of D_{2h}

4.3.1. Determination of Scheme

The gear pump of C_{nv} needs to be improved to solve the problem of force imbalance, and there is no reason for asymmetry, so, according to the selection principle 1, selection principle 3, and selection principle 4, the symmetry type of improved structure is determined to be D_{nh} . Since the structure of the original gear pump belongs to C_{nv} , only σ_h is added to the symmetry elements of C_{nv} , and the final symmetry elements forming D_{nh} are shown in Figure 21.

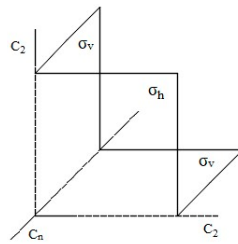


Figure 21. The spatial position of symmetry elements of D_{nh} ($n = 2$).

The structure of the gear pump improved to D_{nh} ($n = 2$) is shown in Figure 22. The size along the main axis is obviously shorter than the size perpendicular to the main axis. Two gear pairs are added along the main axis of the conventional gear pump, and the increased mirror element is σ_h , which can counteract additional side-effect forces. Compared with the four-wheel gear pump, the new gear pump adopts the symmetrical layout of D_{nh} , which has a smaller volume and a clearer oil transmission route.

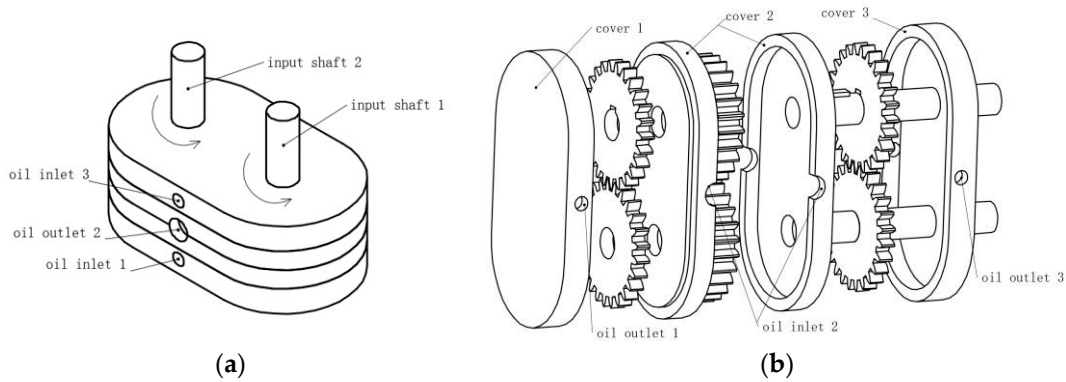


Figure 22. New gear pump. (a) Assembly; (b) exploded view.

4.3.2. Balance of Force

The new gear pump has three pairs of gears along the main axis, and each pair of gears has symmetry elements σ_v and C_n . The radial force at each shaft is equivalent to F_1 , F_2 , and F_3 , as shown in Figure 23, which is linearly related to the width of the gear. When other variables are controlled, the following conditions are established:

$$2B_{first\ gear\ pair} = B_{second\ gear\ pair} = 2B_{third\ gear\ pair} \quad (B_x \text{ is the width of the gear})$$

Oil inlet 1 and oil inlet 3 are located on the same side and on a different side from oil inlet 2. The radial force is directed from the outlet chamber to the inlet chamber such that F_1 and F_3 point to the same direction, opposite to F_2 .

Thus, the following equation can be obtained:

$$2F_1 = F_2 = 2F_3$$

The forces and moments on the input shaft 1 or input shaft 2 are shown in Figure 23, and M_1 and M_2 are bending couples generated by radial forces. Because of σ_h , M_1 and M_2 are of the same size and opposite directions and can cancel each other out.

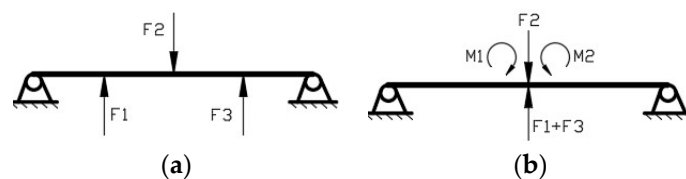


Figure 23. Force analysis. (a) Radial force on the input shafts; (b) force addition.

According to the selection principles, the symmetry type of gear pump is changed from C_{nv} to D_{nh} . Through the force analysis, it is proved that the layout of D_{nh} can balance the side effects generated during the gear operation. The high-pressure oil delivered out of three oil outlets can be applied to one working mechanism or to three working mechanisms at the same time. When the oil is applied to three working mechanisms, in addition to counteracting the radial force, the design of D_{nh} can also meet the needs of synchronous operation of different mechanisms.

5. Conclusions

In this paper, the idea of group theory was introduced into mechanical structure design, and the research started from the point group symmetry, and the following progress was made:

1. Based on the definition and symbol of crystal point group symmetry and the characteristics of machinery, the concept system of the point group symmetry of mechanical structures was proposed.
2. The requirements of symmetry elements were summarized, and the corresponding requirements of point group symmetry could be obtained through the combination of symmetry elements.
3. By using the data mining software RapidMiner, the association rules between point group symmetry and requirements were mined. The results of mining could be used as a reference for structural improvement or innovation.
4. Four selection principles for point group symmetry were proposed. Compared with a large number of mining results, the selection principles could provide a faster design idea.
5. Based on the selection principles and the structural characteristics, a new gear pump with radial force balancing was invented.

This paper only discusses the point group symmetry of mechanical structures. The application of space group symmetry and group theory in micromechanical systems, such as dynamic symmetry, is the subject of future research. In addition, it is more important to refine the regular knowledge related to symmetry and to use symmetry as a method of design.

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