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Certain Finite Integrals Related to the Products of Special Functions

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Abstract: The aim of this paper is to establish a theorem associated with the product of the Aleph-function, the multivariable Aleph-function, and the general class of polynomials. The results of this theorem are unified in nature and provide a very large number of analogous results (new or known) involving simpler special functions and polynomials (of one or several variables) as special cases. The derived results lead to significant applications in physics and engineering sciences.

Keywords: Aleph-function; multivariable Aleph-function; general class of multivariable polynomials; hypergeometric function

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1. Introduction

Calculus of fractional orders is a field of mathematics study that grows out of the traditional definitions of calculus with integer orders of integral and derivative operators in much the same way that fractional exponents can do as an outgrowth of exponents with integer value. During the last three decades, from the development in computations by mathematical software, fractional calculus has been applied to almost every field of science, in particular mathematics, physics, and engineering. Many applications of fractional calculus can be found in plasma physics and controlled thermonuclear fusion, nonlinear control theory, turbulence and fluid dynamics, stochastic dynamical systems, image processing, nonlinear biological systems, astrophysics, and mathematical biology. The computations of fractional integrals and fractional derivatives involving transcendental functions of one and several variables are important because of the usefulness of their results, e.g., for evaluating differential and integral equations. Motivated by these and other applications, several mathematicians and physicians have made use of the calculus fractional orders in the theory of special functions of one and more variables.

Definition 1. *(Mellin–Barnes integral) Let L be a contour in the complex plane starting at $c - \imath\infty$ and ending at $c + \imath\infty$ with $\Re(s) = c > 0$. We call the Mellin–Barnes integral to any integral in the complex plane whose integrand contemplates at least one gamma function, given by:*

$$I(z) = \frac{1}{2\pi\imath}\int_L f(s)z^{-s}\mathrm{d}s \tag{1}$$

where the density function, $f(s)$, in general, the solution of a differential equation with polynomial coefficients, is given by a quotient of products of gamma functions depending on parameters.

A newly found special function is called the Aleph-function, which occurs as an extension of the I-function, itself a generalization of the well-known and familiar G- and H-functions in one variable. A special case of the Aleph-function has appeared in the investigation of fractional driftless Fokker–Planck equations with power law diffusion coefficients. The Aleph-function was introduced by Südland et al. [1], who named it after the symbol used for its representation. The notation and complete definition in terms of the Mellin–Barnes-type integrals along with the conditions of convergence were presented by Saxena and Pogáany [2]. Later on, several studies were performed that established relationships between Aleph-functions with various fractional integral operators [3–6]. In addition, the multivariable Aleph-function is a generalization of the multivariable I-function defined by Sharma and Ahmad [7], which is itself a generalization of the multivariable H-function defined by Srivastava and Panda [8,9]. The multivariable Aleph-function $\aleph(z_1, \ldots, z_r)$ of complex arguments $z_1, \ldots, z_r$ writes:

$$\aleph(z_1, \cdots, z_r) = \aleph^{0,\mathfrak{n}:m_1,n_1,\cdots,m_r,n_r}_{p_i,q_i,\tau_i;R:p_{i^{(1)}},q_{i^{(1)}},\tau_{i^{(1)}};R^{(1)};\cdots;p_{i^{(r)}},q_{i^{(r)}};\tau_{i^{(r)}};R^{(r)}}\left(\begin{array}{c} z_1 \\ \cdot \\ \cdot \\ z_r \end{array}\middle| \begin{array}{l} \left(a_j;\alpha_j^{(1)},\cdots,\alpha_j^{(r)}\right)_{1,\mathfrak{n}}, \\ \cdots, \\ \left[\tau_i\left(a_{ji};\alpha_{ji}^{(1)},\cdots,\alpha_{ji}^{(r)}\right)\right]_{\mathfrak{n}+1,p_i} : \left(c_j^{(1)},\gamma_j^{(1)}\right)_{1,n_1}, \\ \left[\tau_i\left(b_{ji};\beta_{ji}^{(1)},\cdots,\beta_{ji}^{(r)}\right)\right]_{1,q_i} : \left(d_j^{(1)},\delta_j^{(1)}\right)_{1,m_1}, \\ \left[\tau_{i^{(1)}}\left(c_{ji^{(1)}}^{(1)},\gamma_{ji^{(1)}}^{(1)}\right)\right]_{n_1+1,p_i^{(1)}};\cdots;\left(c_j^{(r)},\gamma_j^{(r)}\right)_{1,n_r},\left[\tau_{i^{(r)}}\left(c_{ji^{(r)}}^{(r)},\gamma_{ji^{(r)}}^{(r)}\right)\right]_{n_r+1,p_i^{(r)}} \\ \left[\tau_{i^{(1)}}\left(d_{ji^{(1)}}^{(1)},\delta_{ji^{(1)}}^{(1)}\right)\right]_{m_1+1,q_i^{(1)}};\cdots;\left(d_j^{(r)},\delta_j^{(r)}\right)_{1,m_r},\left[\tau_{i^{(r)}}\left(d_{ji^{(r)}}^{(r)},\delta_{ji^{(r)}}^{(r)}\right)\right]_{m_r+1,q_i^{(r)}} \end{array}\right)$$

$$= \frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\psi(s_1,\cdots,s_r)\prod_{k=1}^{r}\theta_k(s_k)\,z_k^{s_k}\,\mathrm{d}s_1\cdots\mathrm{d}s_r, \tag{2}$$

with $\omega = \sqrt{-1}$; it is defined by means of the multiple contour integral, given by:

$$\aleph(z_1, \ldots, z_r) := \frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\psi(s_1,\ldots,s_r)\prod_{k=1}^{r}\theta_k(s_k)z_k^{s_k}\,\mathrm{d}s_1\cdots\mathrm{d}s_r. \tag{3}$$

Here,

$$\psi(s_1,\ldots,s_r) := \frac{\prod_{j=1}^{\mathfrak{n}}\Gamma\left(1-a_j+\sum_{k=1}^{r}\alpha_j^{(k)}s_k\right)}{\sum_{i=1}^{R}\tau_i\left\{\prod_{j=\mathfrak{n}+1}^{p_i}\Gamma\left(a_{ji}-\sum_{k=1}^{r}\alpha_{ji}^{(k)}s_k\right)\prod_{j=1}^{q_i}\Gamma\left(1-b_{ji}+\sum_{k=1}^{r}\beta_{ji}^{(k)}s_k\right)\right\}} \tag{4a}$$

and:

$$\theta_k(s_k) := \frac{\prod_{j=1}^{m_k}\Gamma\left(d_j^{(k)}-\delta_j^{(k)}s_k\right)\prod_{j=1}^{n_k}\Gamma\left(1-c_j^{(k)}+\gamma_j^{(k)}s_k\right)}{\sum_{i^{(k)}=1}^{R^{(k)}}\tau_{i^{(k)}}\left\{\prod_{j=m_k+1}^{q_{i^{(k)}}}\Gamma\left(1-d_{ji^{(k)}}^{(k)}+\delta_{ji^{(k)}}^{(k)}s_k\right)\prod_{j=n_k+1}^{p_{i^{(k)}}}\Gamma\left(c_{ji^{(k)}}^{(k)}-\gamma_{ji^{(k)}}^{(k)}s_k\right)\right\}}. \tag{4b}$$

Suppose, as usual, that the parameters $a_j\,(j=1,\cdots,p)$; $b_j\,(j=1,\cdots,q)$; $c_j^{(k)}(j=1,\cdots,n_k)$; $c_{ji^{(k)}}^{(k)}\,(j=n_k+1,\cdots,p_{i^{(k)}})$; $d_j^{(k)}\,(j=1,\cdots,m_k)$; $d_{ji^{(k)}}^{(k)}\,(j=m_k+1,\cdots,q_{i^{(k)}})$ are complex numbers, with $k=1,\cdots,r$, $i=1,\cdots,R$, $i^{(k)}=1,\cdots,R^{(k)}$, and the $\alpha's, \beta's, \gamma's$, and $\delta's$ are assumed to be positive real numbers for standardization purpose such that:

$$U_i^{(k)} = \sum_{j=1}^{n} \alpha_j^{(k)} + \tau_i \sum_{j=n+1}^{p_i} \alpha_{ji}^{(k)} + \sum_{j=1}^{n_k} \gamma_j^{(k)} + \tau_{i^{(k)}} \sum_{j=n_k+1}^{p_{i^{(k)}}} \gamma_{ji^{(k)}}^{(k)} - \tau_i \sum_{j=1}^{q_i} \beta_{ji}^{(k)} - \sum_{j=1}^{m_k} \delta_j^{(k)}$$
$$- \tau_{i^{(k)}} \sum_{j=m_k+1}^{q_{i^{(k)}}} \delta_{ji^{(k)}}^{(k)} \leq 0. \tag{5}$$

The reals numbers τ_i are positives for $i = 1, \cdots, R$; $\tau_{i^{(k)}}$ are positives for $i^{(k)} = 1, \cdots, R^{(k)}$. The contour L_k is in the s_k-plane and runs from $\sigma - i\infty$ to $\sigma + i\infty$ where σ is a real number with a loop, if necessary, to ensure that the poles of $\Gamma\left(d_j^{(k)} - \delta_j^{(k)} s_k\right)$ with $j = 1, \cdots, m_k$ are separated from those of $\Gamma\left(1 - a_j + \sum_{i=1}^{r} \alpha_j^{(k)} s_k\right)$ with $j = 1, \cdots, n$ and $\Gamma\left(1 - c_j^{(k)} + \gamma_j^{(k)} s_k\right)$ with $j = 1, \cdots, n_k$ to the left of contour L_k.

For more details, the reader may refer to the recent works of Ayant [10] and Suthar [6], in particular, as concerns the criteria for the absolute convergence of the above multiple Mellin–Barnes contour integral. They can be obtained by extending the corresponding conditions fulfilled by the multivariable H-function and are given by the conditions:

$$|\arg z_k| < \frac{1}{2} A_i^{(k)} \pi \quad (\text{for } k = 1, 2, \ldots r),$$

in which:

$$A_i^{(k)} := \sum_{j=1}^{n} \alpha_j^{(k)} - \tau_i \sum_{j=n+1}^{p_i} \alpha_{ji}^{(k)} - \tau_i \sum_{j=1}^{q_i} \beta_{ji}^{(k)} + \sum_{j=1}^{n_k} \gamma_j^{(k)} - \tau_{i^{(k)}} \sum_{j=n_k+1}^{p_{i^{(k)}}} \gamma_{ji^{(k)}}^{(k)}$$
$$+ \sum_{j=1}^{m_k} \delta_j^{(k)} - \tau_{i^{(k)}} \sum_{j=m_k+1}^{q_{i^{(k)}}} \delta_{ji^{(k)}}^{(k)} > 0 \tag{6}$$

with $k = 1 \ldots r$, $i = 1, \ldots, R$, and $i^{(k)} = 1, \ldots, R^{(k)}$. The complex numbers z_i are not zero. In the remainder of the paper, we assume that the existence and absolute convergence conditions of the multivariable Aleph-function are satisfied according to the above criteria.

Remark 1. *Its asymptotic behavior when variables become small or large (resp.) is expressed in the following basic forms:*

$$\aleph(z_1, \ldots, z_r) = \mathcal{O}(|z_1|^{\alpha_1}, \ldots, |z_r|^{\alpha_r}), \quad \textit{when } \max(|z_1|, \ldots, |z_r|) \to 0.$$
$$\aleph(z_1, \ldots, z_r) = \mathcal{O}\left(|z_1|^{\beta_1}, \ldots, |z_r|^{\beta_r}\right), \quad \textit{when } \min(|z_1|, \ldots, |z_r|) \to \infty,$$

where, for $k = 1, \ldots, r$,

$$\alpha_k = \min\left(\Re\left(d_j^{(k)} / \delta_j^{(k)}\right)\right), \quad j = 1, \ldots, m_k \tag{7a}$$

and

$$\beta_k = \max\left(\Re\left(\left(c_j^{(k)} - 1\right) / \gamma_j^{(k)}\right)\right), \quad j = 1, \ldots, n_k. \tag{7b}$$

For the sake of simplicity, we use the following notations in what follows,

$$V := m_1, n_1; \ldots; m_r, n_r \qquad \text{and} \tag{8a}$$
$$W := p_{i^{(1)}}, q_{i^{(1)}}, \tau_{i^{(1)}}; R^{(1)}; \ldots; p_{i^{(r)}}, q_{i^{(r)}}, \tau_{i^{(r)}}; R^{(r)}, \tag{8b}$$

and also,

$$A := \left(a_j; \alpha_j^{(1)}, \ldots, \alpha_j^{(r)}\right)_{1,n}, \left[\tau_i\left(a_{ji}; \alpha_{ji}^{(1)}, \ldots, \alpha_{ji}^{(r)}\right)\right]_{n+1,p_i} : \left(c_j^{(1)}, \gamma_j^{(1)}\right)_{1,n_1}, \left[\tau_{i^{(1)}}\left(c_{ji^{(1)}}^{(1)}, \gamma_{ji^{(1)}}^{(1)}\right)\right]_{n_1+1,p_{i^{(1)}}}; \cdots ; \left(c_j^{(r)}, \gamma_j^{(r)}\right)_{1,n_r}, \left[\tau_{i^{(r)}}\left(c_{ji^{(r)}}^{(r)}, \gamma_{ji^{(r)}}^{(r)}\right)\right]_{n_r+1,p_{i^{(r)}}} \tag{9a}$$

$$B := \left[\tau_i\left(b_{ji}; \beta_{ji}^{(1)}, \ldots, \beta_{ji}^{(r)}\right)\right]_{1,q_i} : \left(d_j^{(1)}, \delta_j^{(1)}\right)_{1,m_1}, \left[\tau_{i^{(1)}}\left(d_{ji^{(1)}}^{(1)}, \delta_{ji^{(1)}}^{(1)}\right)\right]_{m_1+1,q_{i^{(1)}}}; \cdots ; \left(d_j^{(r)}, \delta_j^{(r)}\right)_{1,m_r}, \left[\tau_{i^{(r)}}\left(d_{ji^{(r)}}^{(r)}, \delta_{ji^{(r)}}^{(r)}\right)\right]_{m_r+1,q_{i^{(r)}}}. \tag{9b}$$

In other words, the contracted form of the multivariable Aleph-function can be written as:

$$\aleph(z_1, \ldots, z_r) := \aleph^{0,n:V}_{p_i,q_i,\tau_i;R;W}\left(\begin{array}{c|c} z_1 & A \\ \vdots & \vdots \\ z_r & B \end{array}\right). \tag{10}$$

Multivariable H-function: If we set $\tau_i = \tau_{i^{(1)}} = \cdots = \tau_{i^{(k)}} = 1$ and $R = R^{(1)} = \cdots = R^{(k)} = 1$ with $k = 1, \cdots, r$ in (2), then the multivariable Aleph-function reduces to the multivariable H-function, defined by:

$$\begin{aligned} &H(z_1, \cdots, z_r) \\ &= H^{0,n:m_1,n_1;\cdots;m_r,n_r}_{p,q:p_1,q_1;\cdots;p_r,q_r}\left(\begin{array}{c|c} z_1 & \left(a_j; \alpha_j^{(1)}, \cdots, \alpha_j^{(r)}\right)_{1,n} : \left(c_j^{(1)}, \gamma_1^{(1)}\right)_{1,n_1}, \cdots, \left(c_j^{(r)}, \gamma_j^{(r)}\right)_{1,n_r} \\ \cdot & \\ \cdot & \\ z_r & \left(b_j; \beta_j^{(1)}, \cdots, \beta_j^{(r)}\right)_{1,q} : \left(d_j^{(1)}, \delta_j^{(1)}\right)_{1,q_1}, \cdots, \left(d_j^{(r)}, \delta_j^{(r)}\right)_{1,q_r} \end{array}\right) \\ &= \frac{1}{(2\pi\omega)^{r+1}} \int_{L_1} \cdots \int_{L_{r+1}} \phi(s_1, \cdots, s_r) \prod_{i=1}^{r} \theta_i(s_i)\, z_i^{t_i}\, \mathrm{d}s_1 \cdots \mathrm{d}s_r, \end{aligned} \tag{11}$$

where $\phi(s_1, \cdots, s_r)$, $\theta_i(s_i)$ $(i = 1, \cdots, r+1)$ are given by:

$$\phi(s_1, \cdots, s_r) = \frac{\prod_{j=1}^{n} \Gamma\left(1 - a_j + \sum_{i=1}^{r} \alpha_j^{(j)} s_j\right)}{\prod_{j=n+1}^{p} \Gamma\left(a_j - \sum_{i=1}^{r} \alpha_j^{(i)} s_i\right) \prod_{j=1}^{q} \Gamma\left(1 - b_j + \sum_{i=1}^{r} \beta_j^{(i)} s_i\right)}, \tag{12}$$

and:

$$\theta_i(s_i) = \frac{\prod_{j=1}^{m_i} \Gamma\left(d_j^{(i)} - \delta_j^{(i)} s_i\right) \prod_{j=1}^{n_i} \Gamma\left(1 - c_j^{(i)} + \gamma_j^{(i)} s_i\right)}{\prod_{j=n_i+1}^{p_i} \Gamma\left(c_j^{(i)} - \gamma_j^{(i)} s_i\right) \prod_{j=m_i+1}^{q_i} \Gamma\left(1 - d_j^{(i)} + \delta_j^{(i)} s_i\right)}. \tag{13}$$

Aleph-function of one variable: The Aleph-function was first introduced by Südland et al. [1] by means of a Mellin–Barnes-type integral in the following manner for $z \neq 0$ (see, also, [2–5]):

$$\aleph(z) = \aleph^{M,N}_{P_i,Q_i,c_i;r'}\left(z \left| \begin{array}{l} (a_j,A_j)_{1,N},\ldots,\left[c_j\left(a_{ji},A_{ji}\right)\right]_{N+1,P_i;r'} \\ (b_j,B_j)_{1,M},\ldots,\left[c_i\left(b_{ji},B_{ji}\right)\right]_{M+1,Q_i;r'} \end{array}\right.\right) := \frac{1}{2\pi \mathrm{i}} \int_L \Omega^{M,N}_{P_i,Q_i,c_i;r'}(s)\, z^{-s}\, \mathrm{d}s, \tag{14}$$

where:

$$\Omega^{M,N}_{P_i,Q_i,c_i;r'}(s) := \frac{\prod_{j=1}^{M} \Gamma(b_j + B_j s) \prod_{j=1}^{N} \Gamma(1 - a_j - A_j s)}{\sum_{i=1}^{r'} c_i \left\{\prod_{j=M+1}^{Q_i} \Gamma(1 - b_{ji} - B_{ji} s) \prod_{j=N+1}^{P_i} \Gamma(a_{ji} + A_{ji} s)\right\}}. \tag{15}$$

Here and throughout, let $\mathbb{Z}, \mathbb{C}, \mathbb{R}, \mathbb{R}_+$, and $\mathbb{N}$ be the sets of integers, complex numbers, real numbers, positive real numbers, and positive integers, respectively, and let $\mathbb{N}_0 := \mathbb{N} \cup \{0\}$.

The $L = L_{\omega\gamma\infty}$, $(\gamma \in \mathbb{R})$ is a suitable contour of the Mellin–Barnes-type, which runs from $\gamma - \omega\infty$ to $\gamma + \omega\infty$. The poles of $\Gamma(1 - a_j - A_j s)$, $j \in \{1, \cdots, N\}$ are distinct from the poles of $\Gamma(b_j + B_j s)$, $j \in \{1, \cdots, M\}$. The parameters A_j, B_j, A_{ji}, B_{ji} are positive real numbers, and a_j, b_j, a_{ji}, b_{ji} are complex numbers. The parameters P_i, Q_i are nonnegative integers satisfying the conditions $0 \le N \le P_i$, $1 \le M \le Q_i$, $c_i > 0$ for $i \in \{1, \cdots, r'\}$ and $M, N \in \mathbb{Z}$. As usual, the empty product in (15) is interpreted as unity.

The conditions of the existence of the contour integral defined in (14) are given below for $\ell = 1, 2, \ldots, r'$ by:

$$|\arg(z)| < \frac{\pi}{2}\varphi_\ell \quad \text{when } \varphi_\ell > 0 \tag{16a}$$

and:

$$|\arg(z)| < \frac{\pi}{2}\varphi_\ell \text{ and } \Re(\zeta_\ell) + 1 < 0 \quad \text{when } \varphi_\ell \ge 0, \tag{16b}$$

where, for $\ell = 1, 2, \ldots, r'$,

$$\varphi_\ell := \sum_{j=1}^{N} A_j + \sum_{j=1}^{M} B_j - c_\ell \left(\sum_{j=N+1}^{P_\ell} A_{j\ell} + \sum_{j=M+1}^{Q_\ell} B_{j\ell} \right) \quad \text{and} \tag{17a}$$

$$\zeta_\ell := \sum_{j=1}^{M} b_j - \sum_{j=1}^{N} a_j + c_\ell \left(\sum_{j=M+1}^{Q_\ell} b_{j\ell} - \sum_{j=N+1}^{P_\ell} a_{j\ell} \right) + \frac{1}{2}(P_\ell - Q_\ell). \tag{17b}$$

***I*-function of one variable:** If we set $c_i = 1$ for $i \in \{1, \cdots, r'\}$ in (14), then the Aleph-function coincides with the I-function, given as:

$$I^{M,N}_{P_i,Q_i;r'}[z] = \aleph^{M,N}_{P_i,Q_i,1;r'}\left[z \left| \begin{matrix} (a_j, A_j)_{1,N}, \ldots, (a_j, A_j)_{N+1,P_i} \\ (b_j, B_j)_{1,M}, \ldots, (b_j, B_j)_{M+1,Q_i} \end{matrix} \right. \right] = \frac{1}{2\pi\iota} \int_L \Omega^{M,N}_{P_i,Q_i,1;r'}(s) z^{-s}\, \mathrm{d}s, \tag{18}$$

which holds under the conditions (16a)–(17b) (for $c_i = 1$).

***H*-function of one variable:** By taking $c_i \to 1$ and $r' = 1$ in (14), then the Aleph-function reduces to Fox's H-function [11] as follows:

$$H^{M,N}_{P,Q}[z] = \aleph^{M,N}_{P,Q,1;1}\left[z \left| \begin{matrix} (a_j, A_j)_{1,P} \\ (b_j, B_j)_{1,Q} \end{matrix} \right. \right] = \frac{1}{2\pi\iota} \int_L \Omega^{M,N}_{P,Q}(s) z^{-s}\, \mathrm{d}s, \tag{19}$$

where the kernel $\Omega^{M,N}_{P,Q}(s)$ is given by:

$$\Omega^{M,N}_{P,Q}(s) = \frac{\prod_{j=1}^{M} \Gamma(b_j + B_j s) \prod_{j=1}^{N} \Gamma(1 - a_j - A_j s)}{\prod_{j=N+1}^{P} \Gamma(a_j + A_j s) \prod_{j=M+1}^{Q} \Gamma(1 - b_j - B_j s)}. \tag{20}$$

Furthermore, the series representation of the Aleph-function was given by Chaurasia and Singh [12] in the form:

$$\aleph^{M,N}_{P_i,Q_i,c_i;r'}(z) = \sum_{G=1}^{M} \sum_{g=0}^{\infty} \frac{(-1)^g\, \Omega^{M,N}_{P_i,Q_i,c_i;r'}(\eta_{G,g})}{B_G\, g!}\, z^{-\eta_{G,g}}, \tag{21}$$

with $s = \eta_{G,g} = \frac{b_G + g}{B_G}$, $P_i \le Q_i$, $|z| < 1$, as well as $\Omega^{M,N}_{P_i,Q_i,c_i;r'}$ as defined in relation (15).

These preliminaries ultimately come to an end with the definition of Srivastava's generalized polynomials, which was given as follows in [13],

$$S_{N_1,\ldots,N_s}^{M_1,\ldots,M_s}[y_1,\ldots,y_s] := \sum_{K_1=0}^{[N_1/M_1]} \cdots \sum_{K_s=0}^{[N_s/M_s]} \frac{(-N_1)_{M_1K_1}}{K_1!} \cdots \frac{(-N_s)_{M_sK_s}}{K_s!} \times A(N_1,K_1;\cdots;N_s,K_s)\, y_1^{K_1}\cdots y_s^{K_s}. \tag{22}$$

Here, $M_1,\ldots,M_s$ are arbitrary positive integers and the coefficients $A[N_1,K_1;\cdots;N_s,K_s]$ are arbitrary (real or complex) constants. By suitably specializing these coefficients, the general polynomials $S_{N_1,\ldots,N_s}^{M_1,\ldots,M_s}(y_1,\ldots,y_s)$ provide a number of known polynomials that arise as their special cases—these include the Hermite polynomials, the Jacobi polynomials, and the Laguerre polynomials, among others.

Orthogonal polynomials: We consider the class of polynomials $S_n^m(z) = \sum_{k=0}^{[n/m]} \frac{(-n)_{mk}}{k!} A_{n,k}\, z^k$ for $n = 0,1,2,3,\cdots$ (for more details, see, Srivastava and Singh [14]). If $m = 2$, $A_{n,k} = (-1)^k$, then $S_n^2(x) \to x^{n/2} H_n\left[\frac{1}{2\sqrt{x}}\right]$ (Hermite polynomials). If $m = 1$, $A_{n,k} = \binom{n+\alpha}{n} \frac{1}{(\alpha+1)_k}$, then we have the Laguerre polynomials defined by $S_n^1(x) = L_n^\alpha(x)$. If $m = 1$, $A_{n,k} = \binom{n+\alpha}{n} \frac{(\alpha+\beta+n+1)_k}{(\alpha+1)_k}$, then $S_n^1(x) = P_n^{(\alpha,\beta)}(1-2x)$ (Jacobi polynomial).

2. Main Results

Lemma 1. *Let $\alpha,\beta,\gamma \in \mathbb{R}$; the following formulas hold true (see, Slater [15], p. 75).*

If $(1-x)^{\alpha+\beta-\gamma-\frac{1}{2}}\, {}_2F_1[2\alpha,2\beta;2\gamma;x] = \sum_{u=0}^{\infty} \beta_u x^u$, then:

$${}_2F_1[\alpha,\beta;\gamma;x]\, {}_2F_1\left[\gamma-\alpha+\frac{1}{2},\gamma-\beta+\frac{1}{2};\gamma;x\right] = \sum_{u=0}^{\infty} \frac{\left(\gamma+\frac{1}{2}\right)_u}{(\gamma+1)_u} \beta_u x^u. \tag{23}$$

Theorem 1. *Let $\alpha,\beta,\gamma,\rho,u_i,\rho_i,h_j,\ell_j \in \mathbb{R}$ with $i = 1,\ldots,s$, $j = 1,\ldots,r$. If:*

$$(1-x)^{\alpha+\beta-\gamma-\frac{1}{2}}\, {}_2F_1[2\alpha,2\beta;2\gamma;x] = \sum_{u=0}^{\infty} \beta_u\, x^u,$$

then the formula holds:

$$\int_0^1 x^\lambda \left(x^k+c\right)^{-\rho} {}_2F_1[\alpha,\beta;\gamma;x]\, {}_2F_1\left[\gamma-\alpha+\frac{1}{2},\gamma-\beta+\frac{1}{2};\gamma;x\right] \times S_{N_1,\ldots,N_s}^{M_1,\ldots,M_s}\begin{pmatrix} c_1 x^{u_1}\left(x^k+c\right)^{-\rho_1} \\ \cdots \\ \cdots \\ c_s x^{u_s}\left(x^k+c\right)^{-\rho_s} \end{pmatrix} \aleph_{P_i,Q_i,c_i;r'}^{M,N}\left(z x^h\left(x^k+c\right)^{-\psi}\right) \aleph \begin{pmatrix} z_1 x^{h_1}\left(x^k+c\right)^{-\ell_1} \\ \cdots \\ \cdots \\ z_r x^{h_r}\left(x^k+c\right)^{-\ell_r} \end{pmatrix} \mathrm{d}x$$

$$= \sum_{K_1=0}^{[N_1/M_1]} \cdots \sum_{K_s=0}^{[N_s/M_s]} \sum_{G=1}^{M} \sum_{g=0}^{\infty} \sum_{u=0}^{\infty} \frac{(-N_1)_{M_1K_1}}{K_1!} \cdots \frac{(-N_s)_{M_sK_s}}{K_s!} A[N_1,K_1;\cdots;N_s,K_s]\, c_1^{K_1}\cdots c_s^{K_s} \times \frac{(-1)^g\, \Omega_{P_i,Q_i,c_i,r'}^{M,N}\left(\eta_{G,g}\right)}{B_G\, g!} z^{\eta_{G,g}} \frac{\left(\gamma+\frac{1}{2}\right)_u}{(\gamma+1)_u} \beta_u\, c^{-\left(\rho-\psi\eta_{G,g}+\sum_{i=1}^{s} K_i\rho_i\right)}$$

$$\times\ \aleph^{0,n+2:V;1,0}_{p_i+2,q_i+2,\tau_i;R:W;0,1}\left(\begin{matrix} c^{\ell_1}z_1\\ \vdots \\ c^{\ell_r}z_r \\ \frac{x^k}{c}\end{matrix}\ \middle|\ \begin{matrix}\left(1-\rho+\psi\eta_{G,g}-\sum_{i=1}^{s}K_i\rho_i:\ell_1,\ldots,\ell_r,1\right),\\ \cdots \\ \cdots \\ \left(1-\rho+\psi\eta_{G,g}-\sum_{i=1}^{s}K_i\rho_i:\ell_1,\ldots,\ell_r,0\right),\\ \left(-\lambda-u+h\eta_{G,g}-\sum_{i=1}^{s}K_iu_i:h_1,\ldots,h_r,0\right),\ A_1\\ \cdots\\ \cdots\\ \left(-1-\lambda-u+h\eta_{G,g}-\sum_{i=1}^{s}K_iu_i:h_1,\ldots,h_r,0\right),\ B_1\end{matrix}\right), \tag{24}$$

where:

$$A_1 := \left(a_j;\alpha_j^{(1)},\ldots,\alpha_j^{(r)},0\right)_{1,n},\ \left[\tau_i\left(a_{ji};\alpha_{ji}^{(1)},\ldots,\alpha_{ji}^{(r)},0\right)\right]_{n+1,p_i}:\ \left(c_j^{(1)},\gamma_j^{(1)}\right)_{1,n_1},\ \left[\tau_{i^{(1)}}\left(c^{(1)}_{ji^{(1)}},\gamma^{(1)}_{ji^{(1)}}\right)\right]_{n_1+1,p_{i^{(1)}}};\cdots;\left(c_j^{(r)},\gamma_j^{(r)}\right)_{1,n_r},\left[\tau_{i^{(r)}}\left(c^{(r)}_{ji^{(r)}},\gamma^{(r)}_{ji^{(r)}}\right)\right]_{n_r+1,p_{i^{(r)}}}. \tag{25a}$$

$$B_1 := \left[\tau_i\left(b_{ji};\beta_{ji}^{(1)},\ldots,\beta_{ji}^{(r)},0\right)\right]_{1,q_i}:\ \left(d_j^{(1)},\delta_j^{(1)}\right)_{1,m_1},\ \left[\tau_{i^{(1)}}\left(d^{(1)}_{ji^{(1)}},\delta^{(1)}_{ji^{(1)}}\right)\right]_{m_1+1,q_{i^{(1)}}};\cdots;\left(d_j^{(r)},\delta_j^{(r)}\right)_{1,m_r},\left[\tau_{i^{(r)}}\left(d^{(r)}_{ji^{(r)}},\delta^{(r)}_{ji^{(r)}}\right)\right]_{m_r+1,q_{i^{(r)}}};(0,1). \tag{25b}$$

This is provided that the following constraints are satisfied:

$$\min\{\rho_i,u_i,K_i,\ell_j,h_j\}>0 \quad \text{for } i=1,\ldots,s,\ j=1,\ldots,r,$$

$$h>0,\ \psi>0,\ |\gamma-\alpha-\beta|<\frac{1}{2},$$

$$\Re\left(\rho-h\eta_{G,g}+\sum_{i=1}^{s}K_is_i\right)+\sum_{i=1}^{r}h_i\min_{1\le j\le m_i}\Re\left(\frac{d_j^{(i)}}{\delta_j^{(i)}}\right)>0,$$

and in addition,

$|\arg z_k| < \frac{1}{2}A_i^{(k)}\pi$, *for* $k=1,\ldots,r$, *where* $A_i^{(k)}$ *is defined in Equation (6);*
$|\arg z| < \frac{1}{2}\pi\Omega$ *with:*

$$\Omega := \sum_{j=1}^{N}A_j+\sum_{j=1}^{M}B_j-c_i\left(\sum_{j=M+1}^{Q_i}B_{ji}+\sum_{j=N+1}^{P_i}A_{ji}\right).$$

Proof. Using Slater's Lemma 1, we can multiply both sides of Equation (23) by:

$$x^{\lambda}\left(x^k+c\right)^{-\rho}S^{M_1,\ldots,M_s}_{N_1,\ldots,N_s}\begin{pmatrix} c_1\,x^{u_1}\left(x^k+c\right)^{-\rho_1}\\ \cdots\\ \cdots\\ c_s\,x^{u_s}\left(x^k+c\right)^{-\rho_s}\end{pmatrix}\aleph^{M,N}_{P_i,Q_i,c_i;r'}\left(z\,x^h\left(x^k+c\right)^{-\psi}\right)$$
$$\times\ \aleph\begin{pmatrix} z_1\,x^{h_1}\left(x^k+c\right)^{-\ell_1}\\ \cdots\\ \cdots\\ z_r\,x^{h_r}\left(x^k+c\right)^{-\ell_r}\end{pmatrix}, \tag{26}$$

Now, integrating with respect to x between zero and one leads to the equation:

$$\int_0^1 x^{\lambda}\left(x^k+c\right)^{-\rho} {}_2F_1[\alpha,\beta;\gamma;x]\ {}_2F_1\left[\gamma-\alpha+\frac{1}{2},\gamma-\beta+\frac{1}{2};\gamma;x\right]$$
$$\times\, S^{M_1,\ldots,M_s}_{N_1,\ldots,N_s}\begin{pmatrix} c_1\, x^{u_1}\left(x^k+c\right)^{-\rho_1} \\ \cdots \\ \cdots \\ c_s\, x^{u_s}\left(x^k+c\right)^{-\rho_s}\end{pmatrix} \aleph^{M,N}_{P_i,Q_i,c_i;r'}\left(z\, x^h\left(x^k+c\right)^{-\psi}\right)$$
$$\times\, \aleph\begin{pmatrix} z_1\, x^{h_1}\left(x^k+c\right)^{-\ell_1} \\ \cdots \\ \cdots \\ z_r\, x^{h_r}\left(x^k+c\right)^{-\ell_r}\end{pmatrix} \mathrm{d}x$$
$$= \int_0^1 x^{\lambda}\left(x^k+c\right)^{-\rho} \sum_{u=0}^{\infty}\frac{\left(\gamma+\frac{1}{2}\right)_u}{(\gamma+1)_u}\beta_u\, x^u\, S^{M_1,\ldots,M_s}_{N_1,\ldots,N_s}\begin{pmatrix} c_1\, x^{u_1}\left(x^k+c\right)^{-\rho_1} \\ \cdots \\ \cdots \\ c_s\, x^{u_s}\left(x^k+c\right)^{-\rho_s}\end{pmatrix}$$
$$\times\, \aleph^{M,N}_{P_i,Q_i,c_i;r'}\left(zx^h\left(x^k+c\right)^{-\psi}\right) \aleph\begin{pmatrix} z_1x^{h_1}\left(x^k+c\right)^{-\ell_1} \\ \cdots \\ \cdots \\ z_rx^{h_r}\left(x^k+c\right)^{-\ell_r}\end{pmatrix} \mathrm{d}x. \tag{27}$$

Finally, interchanging the order of integration and summations (which is permissible under the conditions stated in Equation (24)) and after a few simplifications, we obtain the final result (denoted here by I):

$$I = \sum_{u=0}^{\infty}\frac{\left(\gamma+\frac{1}{2}\right)_u}{(\gamma+1)_u}\beta_u \int_0^1 x^{\lambda+u}\left(x^k+c\right)^{-\rho} S^{M_1,\ldots,M_s}_{N_1,\ldots,N_s}\begin{pmatrix} c_1\, x^{u_1}\left(x^k+c\right)^{-\rho_1} \\ \cdots \\ \cdots \\ c_s\, x^{u_s}\left(x^k+c\right)^{-\rho_s}\end{pmatrix}$$
$$\times\, \aleph^{M,N}_{P_i,Q_i,c_i;r'}\left(zx^h\left(x^k+c\right)^{-\psi}\right) \aleph\begin{pmatrix} z_1x^{h_1}\left(x^k+c\right)^{-\ell_1} \\ \cdots \\ \cdots \\ z_r\, x^{h_r}\left(x^k+c\right)^{-\ell_r}\end{pmatrix} \mathrm{d}x. \tag{28}$$

From the definitions of the general class of multivariable polynomials defined in (22) formulated in terms of the series representation of the Aleph-function of one variable given in Equation (21) extended to the multivariable Aleph-function in the Mellin–Barnes multiple contour integral defined in Equation (3), the proof proceeds in the following lines.

The quantity:

$$\left(x^k+c\right)^{-\left(\rho-h\eta_{G,g}+\sum_{i=1}^{s}K_i\rho_i+\sum_{j=1}^{r}\ell_j\xi_j\right)}$$

can be expressed as a contour integral by use of the result of Srivastava et al. in [16] and, next, by interchanging the order of summation and integration (which is permissible under the conditions of the theorem). Interpreting the resulting Mellin–Barnes contour integral as an Aleph-function of $r+1$ variables, the desired result is obtained. □

3. Particular Cases

Due to the quite general nature of the multivariable Aleph-function and of the class of multivariable polynomials, several integrals involving simpler functions can be readily evaluated as special cases of the main theorem.

Corollary 1. *If we set $\gamma = \alpha$ in Theorem 1, then the value of β_u in Equation (23) is equal to $\frac{(\beta+\frac{1}{2})_u}{u!}$ and the result in Equation (24) yields the following interesting integral.*

$$\int_0^1 x^{\lambda}\left(x^k+c\right)^{-\rho} {}_2F_1\left[\alpha+\frac{1}{2},\beta+\frac{1}{2};\alpha+1;x\right] S_{N_1,\dots,N_s}^{M_1,\dots,M_s}\begin{pmatrix} c_1 x^{u_1}\left(x^k+c\right)^{-\rho_1} \\ \cdots \\ \cdots \\ c_s x^{u_s}\left(x^k+c\right)^{-\rho_s}\end{pmatrix}$$

$$\times \aleph_{P_i,Q_i,c_i;r'}^{M,N}\left(zx^h\left(x^k+c\right)^{-\psi}\right)\aleph\begin{pmatrix} z_1 x^{h_1}\left(x^k+c\right)^{-\ell_1} \\ \cdots \\ \cdots \\ z_r x^{h_r}\left(x^k+c\right)^{-\ell_r}\end{pmatrix}\mathrm{d}x$$

$$= \sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_s=0}^{[N_s/M_s]}\sum_{G=1}^{M}\sum_{g=0}^{\infty}\sum_{u=0}^{\infty}\frac{(-N_1)_{M_1K_1}}{K_1!}\cdots\frac{(-N_s)_{M_sK_s}}{K_s!}A[N_1,K_1;\cdots;N_s,K_s]\,c_1^{K_1}\dots c_s^{K_s}$$

$$\times\frac{(-1)^g\Omega_{P_i,Q_i,c_i,r'}^{M,N}\left(\eta_{G,g}\right)}{B_G g!}z^{\eta_{G,g}}\frac{\left(\gamma+\frac{1}{2}\right)_u}{(\gamma+1)_u}\frac{\left(\beta+\frac{1}{2}\right)_u}{u!}c^{-(\rho-\psi\eta_{G,g}+\sum_{i=1}^{s}K_i\rho_i)}$$

$$\times\aleph_{p_i+2,q_i+2,\tau_i;R:W;0,1}^{0,n+2:V;1,0}\left(\begin{matrix} c^{\ell_1}z_1 \\ \vdots \\ c^{\ell_r}z_r \\ \frac{x^k}{c}\end{matrix}\,\middle|\,\begin{matrix}\left(1-\rho+\psi\eta_{G,g}-\sum_{i=1}^{s}K_i\rho_i\ :\ \ell_1,\dots,\ell_r,1\right), \\ \cdots \\ \cdots \\ \left(1-\rho+\psi\eta_{G,g}-\sum_{i=1}^{s}K_i\rho_i:\ell_1,\dots,\ell_r,0\right), \\ \left(-\lambda-u+h\eta_{G,g}-\sum_{i=1}^{s}K_iu_i:h_1,\dots,h_r,0\right),\ A_1 \\ \cdots \\ \cdots \\ \left(-1-\lambda-u+h\eta_{G,g}-\sum_{i=1}^{s}K_iu_i:h_1,\dots,h_r,0\right),\ B_1\end{matrix}\right), \quad (29)$$

where A_1 and B_1 are given by Equations (25a) and (25b), respectively. The above relation satisfies the same conditions of validity that are stated for the multiple Aleph-function in Theorem 1 for Equation (24).

Corollary 2. *If we set $\beta = \alpha + \frac{1}{2}$ and $-\nu = \alpha + \frac{1}{2}$ (where ν is a nonnegative integer) in (29), we obtain:*

$$\int_0^1 x^{\lambda}\left(x^k+c\right)^{-\rho}(1-x)^{\nu} S_{N_1,\dots,N_s}^{M_1,\dots,M_s}\begin{pmatrix} c_1 x^{u_1}\left(x^k+c\right)^{-\rho_1} \\ \cdots \\ \cdots \\ c_s x^{u_s}\left(x^k+c\right)^{-\rho_s}\end{pmatrix}\aleph_{P_i,Q_i,c_i;r'}^{M,N}\left()zx^h\left(x^k+c\right)^{-\psi}\right)$$

$$\times\aleph\begin{pmatrix} z_1 x^{h_1}\left(x^k+c\right)^{-\ell_1} \\ \cdots \\ \cdots \\ z_r x^{h_r}(x^k+c)^{-\ell_r}\end{pmatrix}\mathrm{d}x$$

$$= \sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_s=0}^{[N_s/M_s]}\sum_{G=1}^{M}\sum_{g=0}^{\infty}\sum_{u=0}^{\infty}\frac{(-N_1)_{M_1K_1}}{K_1!}\cdots\frac{(-N_s)_{M_sK_s}}{K_s!}A[N_1,K_1;\cdots;N_s,K_s]\,c_1^{K_1}\cdots c_s^{K_s}$$

$$\times \frac{(-1)^g \Omega^{M,N}_{P_i,Q_i,c_i,r'}\left(\eta_{G,g}\right)}{B_G g!} z^{\eta_{G,g}} \frac{(-\nu)_u}{u!} c^{-(\rho-\psi\eta_{G,g}+\sum_{i=1}^{s} K_i \rho_i)}$$

$$\times \aleph^{0,n+2:V;1,0}_{p_i+2,q_i+2,\tau_i;R:W;0,1}\left(\begin{array}{c|c} c^{\ell_1} z_1 & \left(1-\rho+\psi\eta_{G,g}-\sum_{i=1}^{s} K_i\rho_i : \ell_1,\ldots,\ell_r,1\right), \\ \vdots & \cdots \\ c^{\ell_r} z_r & \cdots \\ \frac{x^k}{c} & \left(1-\rho+\psi\eta_{G,g}-\sum_{i=1}^{s} K_i\rho_i : \ell_1,\ldots,\ell_r,0\right), \\ & \left(-\lambda-u+h\eta_{G,g}-\sum_{i=1}^{s} K_i u_i : h_1,\ldots,h_r,0\right), A_1 \\ & \cdots \\ & \cdots \\ & \left(-1-\lambda-u+h\eta_{G,g}-\sum_{i=1}^{s} K_i u_i : h_1,\ldots,h_r,0\right), B_1 \end{array}\right), \quad (30)$$

where (as in Corollary (2)) A_1 and B_1 are both defined in Equations (25a) and (25b), respectively, and satisfy the same conditions of validity that are stated for Formula (24) (see Theorem 1).

Remark 2. *If the Aleph-function and the multivariable Aleph-function reduce to the Fox H-function [16] (see Equation (19)) and to the multivariable H-function (11), respectively [8,9], then the recent results of Ghiya et al. [17] follow as a consequence.*

4. Concluding Remarks

In this paper, we studied and presented results associated with the product of Aleph-functions, multivariable Aleph-functions, and the general class of polynomials defined in (22). The one-variable and multivariable Aleph-functions expressed herein are relatively basic and quite general in nature. Therefore, some suitable adjustments of the parameters of multivariable Aleph-functions and the general class of polynomials make it possible to obtain various other special functions (such as the I-function, the Fox H-function, Meijer's G-function, etc.; see, e.g., [6]) involving a large variety of polynomials. Some of the issues of the main theorem have been already discussed here as special cases in the form of corollaries; they lead to significant applications in physics and engineering sciences.

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