

Essay

The Λ and the CDM as Integration Constants

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Abstract: Notoriously, the two main problems of the standard Λ CDM model of cosmology are the cosmological constant Λ and the cold dark matter, CDM. This essay shows that both the Λ and the CDM arise as integration constants in a careful derivation of Einstein's equations from first principles in a Lorentz gauge theory. The dark sector of the universe might only reflect the geometry of a spontaneous symmetry breaking that is necessary for the existence of spacetime and an observer therein.

Keywords: cosmology; dark matter and dark energy; gauge theory of gravity

1. Introduction

General relativity is the local version of special relativity. Gravity is thus understood to be a gauge theory of the Lorentz group. The basic variable is then a Lorentz connection 1-form ω^a_b , which defines the covariant derivative \mathbf{D} , and thereby the curvature 2-form $\mathbf{R}^a_b = \mathbf{d}\omega^a_b + \omega^a_c \wedge \omega^c_b$ is subject to the 3-form Bianchi identity $\mathbf{D}\mathbf{R}^a_b = 0$ inherited from the Jacobi identity of the Lorentz algebra.

Since the beginning [1], the role of translations in the inhomogeneous Lorentz group has been elusive. What has been clear is that in order to recover the dynamics of general relativity, some extra structure is required besides the connection ω^a_b . The standard approach since Kibble's work [2] has been to introduce the coframe field \mathbf{e}^a , another 1-form valued in the Lorentz algebra. Not long ago, the more economical possibility of introducing solely a scalar field τ^a was put forward by Złóśnik et al. [3]. Only then was gravity described by variables that are fully analogous to the fields of the standard Yang–Mills theory.

The symmetry-breaking scalar τ^a has been called the (Cartan) *Khronon* because it encodes the foliation of spacetime. The theory of Złóśnik et al. is pre-geometric in the sense that there exist symmetric solutions (say $\tau^a = 0$) where there is no spacetime. Only in a spontaneously broken phase $\tau^2 < 0$, there emerges a coframe field $\mathbf{e}^a = \mathbf{D}\tau^a$. Furthermore, if the coframe field is non-degenerate, a metric tensor $g_{\mu\nu} \mathbf{d}x^\mu \otimes \mathbf{d}x^\nu = \eta_{ab} \mathbf{e}^a \otimes \mathbf{e}^b$. In terms of the two fundamental fields, the Lorentz connection and the *Khronon* scalar, the theory realises the idea of observer space [4]. As the field τ picks a time-like value, thus specifying the foliation of spacetime, the symmetry of the (complexified) Lorentz group is reduced to the little group of rotations.

A serendipitous discovery was the fact that in the broken phase, the theory does not quite reduce to general relativity, but to general relativity with dust [3]. The presence of this “dust of time” could explain the cosmological observations without dark matter. In this essay, we shall elucidate how this geometrical dark matter appears as an integration constant at the level of field equations. In addition, we consider the next-to-simplest model by introducing the cosmological Λ -term. This will require another symmetry-breaking field, the (Weyl) *Kairon* σ^a , which happens to impose unimodularity.



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The conclusion we wish to present is that a minimalistic gauge theory of gravity includes both the Λ and CDM, and they both enter into the field equations as integration constants in the broken phase.

2. Dark Matter

Let us first make the case for dark matter. In the original, quite dense article [3], the result was derived by a Hamiltonian analysis that may not be easy to follow in detail. Therefore, we believe the simple derivation below could be useful.

The $SO(4,C)$ action can be written in the quadratic form [3] (in units $c = \hbar = 8\pi G = 1$),

$$I_G = \frac{1}{2} \int \left(\frac{1}{8} \tau^2 \epsilon^{bd}{}_{ac} + i \tau^b \tau^d \eta_{ac} \right) \mathbf{R}^a{}_b \wedge \mathbf{R}^c{}_d, \quad (1)$$

and the variations with respect to the two fields yield (what have been called “the infernal equations”),

$$\mathbf{D}(^+ \mathbf{R}^a{}_b \wedge \mathbf{D}\tau^b) = 0, \quad (2a)$$

$$\frac{1}{2} \mathbf{D}^+ (\mathbf{D}\tau^{[a} \wedge \mathbf{D}\tau^{b]}) = \tau^{[a} + \mathbf{R}^b]{}_c \wedge \mathbf{D}\tau^c. \quad (2b)$$

The anti/self-dual projections of a field $X^a{}_b$ in the adjoint representation are denoted as ${}^\pm X^a{}_b$, and defined by the property $\epsilon^{ad}{}_{bc} {}^\pm X^c{}_d = \pm 2i {}^\pm X^a{}_b$. There emerges a formal solution to (2a),

$$^+ \mathbf{R}^a{}_b \wedge \mathbf{D}\tau^b = \mathbf{M}^a \quad \text{where} \quad \mathbf{D}\mathbf{M}^a = 0. \quad (3)$$

To make further progress, we will assume that $\tau^2 < 0$, so that we can call $\mathbf{D}\tau^a = \mathbf{e}^a$ and have the coframe field at hand. Then, since $\epsilon^{ad}{}_{bc} {}^+ \mathbf{R}^c{}_d = 2i {}^+ \mathbf{R}^a{}_b$, we can write the following:

$$-2\mathbf{M}^a = i \epsilon^{ac}{}_{bd} {}^+ \mathbf{R}^b{}_c \wedge \mathbf{e}^d = i \left({}^+ \mathbf{R}^a{}_b - \frac{1}{2} \delta_b^a + R \right) * \mathbf{e}^b. \quad (4)$$

We have thus recovered the Einstein field equations for the self-dual curvature, sourced by a yet unknown 3-form \mathbf{M}^a .

It remains to be shown that this source term behaves as idealised dust. By combining (3) with (2b), we see that ${}^- (\tau^{[a} \mathbf{M}^{b]}) = 0$. At this point, we can pick the simplifying gauge $\tau^a = \tau \delta_0^a$, wherein it becomes apparent that the spatial 3-forms $\mathbf{M}^I = 0$ vanish. By construction (3), we have $\mathbf{D}\mathbf{M}^a = 0$, which yields two further constraints, $\omega^I{}_0 \wedge \mathbf{M}^0 = 0$ and $\mathbf{d}\mathbf{M}^0 = 0$. The former implies that \mathbf{M}^0 is a spatial 3-form, $\mathbf{M}^0 = -(i\rho/2) * \mathbf{e}^0$ for some function ρ , and the latter implies that this function ρ dilutes with the spatial volume. Thus, ρ indeed effectively describes the energy density of dust.

Though the derivation was particularly transparent with the gauge choice $\tau^a = \tau \delta_0^a$, the conclusion naturally holds in any other gauge. We have also checked that coupling matter with (1) would not change the form of \mathbf{M}^a .

Thus, the dust component is not put in by adding an energy-momentum tensor into the field equations or a matter Lagrangian into the action, but it is an effective term that arises in the generic solutions of the theory. Any physical solution imposes the distinction between time and space, and the implied spontaneous breaking of the Lorentz symmetry introduces a background energy density with an exact vanishing pressure. This is fundamentally different from baryonic matter, whose finite pressure is crucial to take into account in precision cosmology. Moreover, any hypothetical particle dark matter would only be described ideal dust in a coarse-grained approximation at cosmological scales.

3. Cosmological Constant

Simply adding the constant Λ ad hoc would not be compatible with first principles, but do require the extension of the Lorentz to the de Sitter gauge group [5]. Instead, we now more frugally supplement the action (1) with two terms,

$$I = I_G - \frac{1}{24} \int \Lambda \epsilon_{abcd} \mathbf{D}\tau^a \wedge \mathbf{D}\tau^b \wedge \mathbf{D}\tau^c \wedge \mathbf{D}\tau^d - \mathbf{D}\Lambda \wedge \mathbf{B}. \quad (5)$$

Had we added only the first new term, the variation with respect to the Λ would prohibit a viable spacetime by imposing that $\sqrt{-g} = 0$. Therefore, we also had to include the second new term, such that it rather imposes the constancy of Λ with the 3-form Lagrange multiplier \mathbf{B} . It is not difficult to see that the two new equations of motion are:

$$\mathbf{d}\Lambda = 0, \quad (6a)$$

$$\mathbf{d}\mathbf{B} = \frac{1}{24} \epsilon_{abcd} \mathbf{D}\tau^a \wedge \mathbf{D}\tau^b \wedge \mathbf{D}\tau^c \wedge \mathbf{D}\tau^d, \quad (6b)$$

and that field Equation (4) is only modified by the addition of the appropriate Λ -term. Whilst (6a) ensures the constancy of the Λ , the consequence of (6b) is unimodularity. In the broken phase, it becomes $\partial_\mu * B^\mu = \sqrt{-g}$, and since the vector density $*B^\mu$ is not fixed, we are free to set $\sqrt{-g} = 1$. We note that the action in (5) is nothing but the polynomial realisation of the well-known unimodular method [6] now embedded into the minimal Lorentz gauge theory (1). Unimodularity solves the “old” problem of vacuum energy [7].

The new field \mathbf{B} is a measure of global time. Consider a spacetime volume V bounded by a spatial hypersurface ∂V . Now, due to the Stokes theorem,

$$\oint_{\partial V} \mathbf{B} = \int_V \sqrt{-g} \mathbf{d}^4x = \text{vol}(V). \quad (7)$$

The spontaneous breaking of unimodular invariance, given by the normalisation chosen for $\partial_\mu * B^\mu$, determines the unit of global time. In the broken phase, we could identify the *Kairon* scalar field $\sigma^a = \mathbf{e}^a \cdot * \mathbf{B}$ in the fundamental representation of the Lorentz group, on the same footing as the scalar field τ^a . Such a dual pair of fields arises from the embedding of the Lorentz gauge theory (1) into the conformal gauge theory [8], and intuition from previous studies [9] also suggests the natural gauge choice of $\mathbf{D}\tau^a \sim \mathbf{D}\sigma^a$.

We have already reached the conclusion of this essay: the similar magnitude of the observed energy densities due to the Λ and the CDM [10] could be the result of their common origin in the conformal geometry of the observer space.

4. Conclusions

Rather than unknown particles or modified gravity, the dark sector of the universe could be the manifestation of a spontaneous symmetry breaking that underpins the emergence of a metric spacetime. In a rigorous derivation of the Einstein’s field equations from a more fundamental, pre-geometric theory, both the Λ and the CDM appear as integration constants. Though spontaneous symmetry breaking has been considered the origin of the difference between time and space [11,12], results similar to ours have not, to our knowledge, been arrived at in less minimalist settings. Though a wide variety of different proposals share some of the features of our theory [13–15], they are based on the reduction of gravitational variables in contrast to the introduction of additional fields.

Of the 12 real components of the complexified Lorentz connection ω^a_b , the six self-dual pieces ${}^+ \omega^a_b + {}^+ \omega^{a*}_b$ account for the usual spin connection, whereas three independent degrees of freedom encoded in the boosts ${}^- \omega^I_0$ give rise to the spatial triad through $\mathbf{e}^I = \mathbf{D}\tau^I$ in the broken phase. Perhaps, along the lines of [16], the three remaining anti-self-dual rotations ${}^- \omega^I_J$ could be related to the $SU(2)_L$ connection in the particle sector, and the scalars τ^a and σ^a could be related to the Higgs field. Another speculation is that

our formulation might provide an improved starting point for loop quantum gravity that is currently suffering from a “covariance crisis” [17].

The cosmology we propose could be falsified by either a direct or indirect detection of a dark matter particle, which could account for the observed ρ . The structure formation of such particle dark matter cannot explain the formation of supermassive black holes at the center of galaxies, but perhaps they could have their origins in the caustics that could theoretically form in the evolution of an idealised dust. In laboratory experiments, at least the naive expectation is that the cosmological density field is negligible.

To conclude, we propose a possible theory behind the two main parameters of the standard Λ CDM model of cosmology [10]. The theory predicts that the CDM is ideal dust, and suggests a new approach to avoid fine-tunings of the Λ .

- The 1st Λ problem, the sensitivity of gravity to vacuum energy, is resolved [7].
- The 2nd Λ problem, the observed value of Λ , could be related to the age of the Universe (The question of why the ρ is small seems like the same question as why the universe is so old. The association of large numbers in physics with the age of the universe goes back, via Dirac, to Weyl. Recently, it was pointed out that Dirac’s large number hypothesis might be realised in a model with two “dilaton” [5]. However, in this essay, we only discussed the cosmological numbers related to the Λ and the CDM.).
- The 3rd Λ problem, the coincidence that $\Lambda \sim \rho$, may have a rationale in their dual origin.

In particular, from the construction of the 3-form M^a , we could infer that $\partial\tau \sim 1/\sqrt{\rho}$, and from (7), we may read that $\partial_\mu\sigma^\mu \sim 1/\sqrt{\Lambda}$. It remains to be investigated whether the duality $\tau \sim \sigma$ could indeed explain the cosmic coincidence.

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