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Analysis and Prediction for Confirmed COVID-19 Cases in Czech Republic with Uncertain Logistic Growth Model

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Abstract: This paper presents an uncertain logistic growth model to analyse and predict the evolution of the cumulative number of COVID-19 infection in Czech Republic. Some fundamental knowledge about the uncertain regression analysis are reviewed firstly. Stochastic regression analysis is invalid to model cumulative number of confirmed COVID-19 cases in Czech Republic, by considering the disturbance term as random variables, because that the normality test and the identical distribution test of residuals are not passed, and the residual plot does not look like a null plot in the sense of probability theory. In this case, the uncertain logistic growth model is applied by characterizing the disturbance term as uncertain variables. Then parameter estimation, residual analysis, the forecast value and confidence interval are studied. Additionally, the uncertain hypothesis test is proposed to evaluate the appropriateness of the fitted logistic growth model and estimated disturbance term. The analysis and prediction for the cumulative number of COVID-19 infection in Czech Republic can propose theoretical support for the disease control and prevention. Due to the symmetry and similarity of epidemic transmission, other regions of COVID-19 infections, or other diseases can be disposed in a similar theory and method.

Keywords: uncertainty theory; uncertain statistics; uncertain hypothesis test; uncertain regression model; COVID-19

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1. Introduction

The pandemic COVID-19 remains a challenge globally, which gives rise to seriously threatens of human health, economic losses and social panic in different degree. As of 16 August 2021, a total of 1,676,222 confirmed cases and 30,737 death has been reported in Czech Republic. In addition, the confirmed case is the individual whose blood sample is confirmed to be positive and has the ability to transmit the virus, and the death case means the individual who infects the COVID-19 and dies from the virus. How do we model its evolution based on the cumulative confirmed cases from 6 June 2021 to 16 August 2021 in Czech Republic?

Regression analysis is a set of statistical techniques exploring the relationship between explanatory variables and response variables. Although stochastic regression analysis, under the framework of probability theory, has got lots of attention and researches. However, it can only be applied to the case that the estimated distribution being close enough to the true frequency, which can not be satisfied in many cases. Motivated by this, the uncertain regression analysis was firstly proposed in 2018 by Yao-Liu [1] based on the knowledge framework of uncertainty theory [2].

In this paper, we will adopt the uncertain regression analysis to interpret and analyse the cumulative confirmed of COVID-19 cases in Czech Republic. As an important branch of uncertain statistics, uncertain regression analysis [1] is a set of statistical techniques that use uncertainty theory to explore the relationship between explanatory variables and response variables.

After the least squares estimation was proposed to estimate the unknown parameters in uncertain regression models [1], some other estimation methods were presented, such as the absolute deviations estimation [3], Tukey's biweight estimations [4], the maximum likelihood estimation [5], and so on. In order to make interval estimation for predicting the response variables, Lio-Liu [6] suggested a method to determine the expected value and variance of uncertain disturbance term. The uncertain hypothesis test was initialized by Ye-Liu [7] as a statistical tool that uses uncertainty theory to judge whether some hypotheses are correct or not based on observed data. Then the uncertain significance test [8] was introduced. Uncertain regression analysis has also been successfully extended in many directions, including uncertain multivariable regression model [9], multivariate regression analysis [10,11], nonparametric regression analysis [12], and so on. In addition, some other uncertain regression models were analysed, such as the uncertain Chapman-Richards growth model [13], the uncertain Verhulst-Pearl model [14], the uncertain Gompertz regression model [15], the uncertain revised regression model [16], and so on.

It is worth mentioning that the uncertainty theory has been successfully applied to the cumulative number of COVID-19 infections in China, for instance, uncertain SIR model [17], uncertain SEIAR model [18], uncertain logistic growth model [19], initial value estimation [20], uncertain time series [21], and so on. All above researches have studied and analysed the cumulative number of COVID-19 infections in China by using the uncertainty theory, from the angle of uncertain differential equation, uncertain regression analysis and uncertain time series analysis. In this manuscript, the cumulative number of COVID-19 infections in Czech Republic is dealt with initially by using an uncertain logistic growth model. The upper limit number of infection, and an accurate expected forecast value are obtained.

The rest frame is organized as follows. In next section, some fundamental knowledge about the uncertain regression analysis are introduced. Uncertain logistic growth model is given for COVID-19 cases in Czech Republic in Section 3 by characterizing the disturbance term as an uncertain variable. The data for the cumulative number of COVID-19 infection in Czech Republic is given firstly. Then parameter estimation, the forecast value, confidence interval, and the uncertain hypothesis test are used to analyse and predict the evolution of the cumulative number of confirmed COVID-19 infections in Czech Republic. In addition, stochastic regression analysis is invalid to model cumulative number of confirmed COVID-19 cases in Czech Republic, because that the normality test (Lilliefors test) and the identical distribution test (Kolmogorov–Smirnov test) of residuals are not passed, and the residual plot does not look like a null plot in the sense of probability theory. A brief summary and discussion are given in the last. So other regions of COVID-19 infections, or other diseases can be analysed and forecasted by using the uncertain logistic growth model due to the symmetry and similarity.

2. Uncertain Regression Analysis

In this section, we introduce some fundamental knowledge about the uncertain regression analysis, including parameter estimation, residual analysis, forecast value and confidence interval, and hypothesis test for the uncertain logistic growth model.

2.1. Uncertain Logistic Growth Model

The uncertain logistic growth model, as an uncertain regression model, was proposed in order to explore the functional relationship between t and y,

$$y = \beta_0/(1 + \beta_1 \exp(-\beta_2 t)) + \epsilon,\ \beta_0 > 0, \beta_1 > 0, \beta_2 > 0, \quad (1)$$

where $(\beta_0, \beta_1, \beta_2)$ is a vector of parameters, and ϵ is an uncertain disturbance term (uncertain variable).

2.2. Parameter Estimation and Residual Analysis

When $(t, y_t), t = 1, 2, \cdots, n$, are observed, Yao and Liu [1] defined the uncertain least squares estimations $\hat{\beta}_0, \hat{\beta}_1, \hat{\beta}_2$ of $\beta_0, \beta_1, \beta_2$ in the uncertain logistic growth model (1) is the solution of the minimization problem:

$$\min_{\beta_0>0,\beta_1>0,\beta_2>0} \sum_{t=1}^{n} (y_t - \beta_0/(1+\beta_1 \exp(-\beta_2 t)))^2. \tag{2}$$

which is a traditional minimization problem, solved by the least square method (lsqnonlin in Matlab). Thus, the fitted logistic growth model is:

$$y = \hat{\beta}_0/(1+\hat{\beta}_1 \exp(-\hat{\beta}_2 t)). \tag{3}$$

Then for each index t ($t = 1, 2, \cdots, n$), the t-th residual is:

$$\hat{\epsilon}_t = y_t - \hat{\beta}_0/(1+\hat{\beta}_1 \exp(-\hat{\beta}_2 t)), \tag{4}$$

which will be regarded as the samples of the uncertain disturbance term ϵ in the uncertain logistic growth model. Therefore, the expected value of the uncertain disturbance term can be estimated as the average of residuals, i.e.,

$$\hat{e} = \frac{1}{n} \sum_{t=1}^{n} \hat{\epsilon}_t, \tag{5}$$

and the variance of the disturbance term can be estimated as:

$$\hat{\sigma}^2 = \frac{1}{n} \sum_{t=1}^{n} (\hat{\epsilon}_t - \hat{e})^2. \tag{6}$$

2.3. Forecast

As the hypothesis, the disturbance term ϵ is a normal uncertain variable with expected value $\hat{e}$ and variance $\hat{\sigma}^2$. Thus, the forecast uncertain variable of response variable y with respect to a new explanatory variable t is:

$$\hat{y} = \hat{\beta}_0(1+\hat{\beta}_1 \exp(-\hat{\beta}_2 t)) + \hat{\epsilon},\ \hat{\epsilon} \sim \mathcal{N}(\hat{e}, \hat{\sigma}), \tag{7}$$

in which $\hat{e}, \hat{\sigma}^2$ are the expected value and variance of the disturbance term shown in (5) and (6), respectively.

The forecast value is defined as the expected value of the forecast uncertain variable y

$$\hat{\mu} = \hat{\beta}_0(1+\hat{\beta}_1 \exp(-\hat{\beta}_2 t)) + \hat{e}. \tag{8}$$

According to the operational law for calculating the inverse uncertainty distributions [2], $\hat{y}$ has a normal uncertainty distribution $\mathcal{N}(\hat{\mu}, \hat{\sigma})$, i.e.,

$$\hat{\Psi}(x) = \left(1 + \exp\left(\frac{\pi(\hat{\mu} - x)}{\sqrt{3}\hat{\sigma}}\right)\right)^{-1}, \tag{9}$$

and the inverse uncertainty distribution of $\hat{y}$ is equivalent to:

$$\hat{\Psi}^{-1}(\alpha) = \hat{\beta}_0(1+\hat{\beta}_1 \exp(-\hat{\beta}_2 x)) + \Phi^{-1}(\alpha), \tag{10}$$

where $\Phi^{-1}(\alpha)$ is the inverse uncertainty distribution,

$$\Phi^{-1}(\alpha) = e + \frac{\sigma\sqrt{3}}{\pi} \ln \frac{\alpha}{1-\alpha}. \tag{11}$$

Taking α as a confidence level (e.g., 95%), Lio-Liu [5] suggested the α confidence interval of response variable y is:

$$\left[\hat{\mu} - \hat{b}, \hat{\mu} + \hat{b}\right],$$

where

$$\hat{b} = \frac{\hat{\sigma}\sqrt{3}}{\pi} \ln \frac{1+\alpha}{1-\alpha}$$

is the minimum b such that:

$$\hat{\Psi}(\mu + b) - \hat{\Psi}(\mu - b) \geq \alpha. \quad (12)$$

2.4. Uncertain Hypothesis Test

Uncertain hypothesis test was initialized by Ye-Liu [7] as a statistical tool that uses uncertainty theory to judge whether some hypotheses are correct or not. Let ϵ be an uncertain variable with a normal uncertainty distribution $\mathcal{N}(e, \sigma)$ where e and σ^2 are unknown parameters. Then the hypotheses are:

$$H_0 : e = \hat{e} \text{ and } \sigma = \hat{\sigma} \text{ versus } H_1 : e \neq \hat{e} \text{ or } \sigma \neq \hat{\sigma}.$$

The statement H_0 is called a null hypothesis, and H_1 is called an alternative hypothesis. Given a significance level α, the hypotheses are

$$W = \Big\{(\epsilon_1, \epsilon_2, \cdots, \epsilon_n) : \text{ there are at least } \alpha \text{ of indexes } t\text{'s with } 1 \leq t \leq n \text{ such that } \epsilon_t < \Phi^{-1}\left(\frac{\alpha}{2}\right) \text{ or } \epsilon_t > \Phi^{-1}\left(1 - \frac{\alpha}{2}\right)\Big\} \quad (13)$$

with Φ^{-1} is the inverse uncertainty distribution of $\mathcal{N}(\hat{e}, \hat{\sigma})$, i.e.,

$$\Phi^{-1}(\alpha) = \hat{e} + \frac{\hat{\sigma}\sqrt{3}}{\pi} \ln \frac{\alpha}{1-\alpha}. \quad (14)$$

If the vector of observed data belongs to the rejection region W, i.e.,

$$(\epsilon_1, \epsilon_2, \cdots, \epsilon_n) \in W,$$

then we reject H_0. Otherwise, we accept H_0.

3. Uncertain Logistic Growth Model for COVID-19 Cases in Czech Republic

In this section, we use the uncertain regression analysis to modeling the cumulative data of confirmed COVID-19 infections in Czech Republic from 6 June 2021 to 16 August 2021 (see Table 1).

3.1. Data and Model

Table 1 shows the cumulative numbers of COVID-19 infections in Czech Republic from 6 June 2021 to 16 August 2021, via the website of the (MedSci https://www.medsci.cn/ (accessed on 16 August 2021)). Let $t = 1, 2, \cdots, 72$ denotes the data from 6 June 2021 to 16 August 2021. For instance, $t = 1$ and 72 represent 6 June 2021 and 16 August 2021, respectively. In order to find the functional relationship between t (the date) and y (the cumulative number of COVID-19 infections in Czech Republic), we may use the observed data:

$$(t, y_t), \; t = 1, 2, \cdots, 72,$$

in which y_t are the cumulative numbers shown in Table 1 on days t, $t = 1, 2, \cdots, 72$, respectively. For example,

$$y_1 = 1663363, \ \ y_{72} = 1676222,$$

which is shown in Figure 1.

Table 1. Cumulative numbers of confirmed COVID-19 infections in Czech Republic from 6 June 2021 to 16 August 2021.

1,663,363	1,663,517	1,663,607	1,663,998	1,664,382	1,664,649	1,664,839	1,665,022
1,665,097	1,665,139	1,665,327	1,665,526	1,665,660	1,665,818	1,665,961	1,666,025
1,666,082	1,666,192	1,666,325	1,666,521	1,666,686	1,666,821	1,666,890	1,666,947
1,667,115	1,667,287	1,667,435	1,667,608	1,667,796	1,667,935	1,668,040	1,668,170
1,668,277	1,668,891	1,668,891	1,669,182	1,669,351	1,669,496	1,669,745	1,670,073
1,670,348	1,670,583	1,670,823	1,671,027	1,671,145	1,671,372	1,671,685	1,671,933
1,672,140	1,672,340	1,672,409	1,672,547	1,672,764	1,673,017	1,673,219	1,673,429
1,673,576	1,673,694	1,673,769	1,673,926	1,674,183	1,674,410	1,674,577	1,674,726
1,674,906	1,675,010	1,675,179	1,675,450	1,675,675	1,675,868	1,676,080	1,676,222

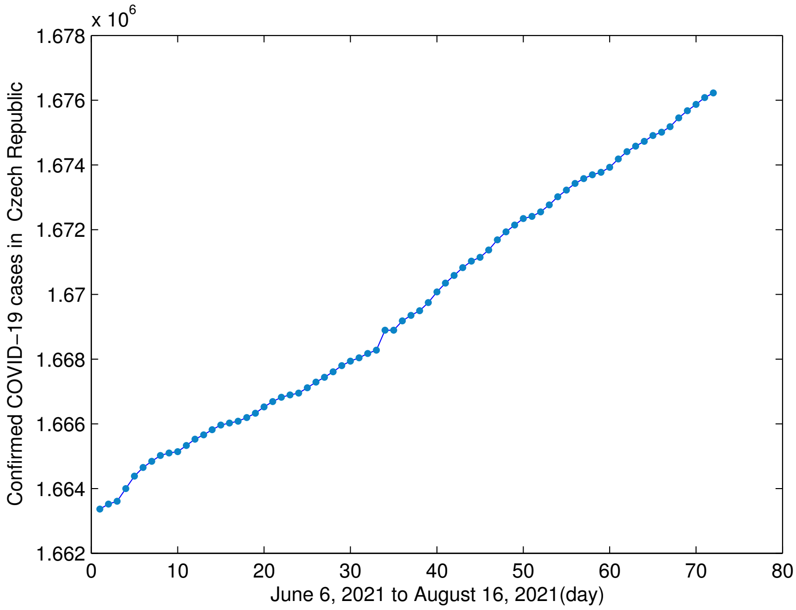

Figure 1. Cumulative numbers of confirmed COVID-19 infections in Czech Republic from 6 June 2021 to 16 August 2021.

We use the uncertain logistic growth model to invest the relationship between the time t and the cumulative number of confirmed COVID-19 cases in Czech Republic y_t. The uncertain logistic growth model is shown in (1),

$$y = \beta_0 / (1 + \beta_1 \exp(-\beta_2 t)) + \epsilon, \ \beta_0 > 0, \beta_1 > 0, \beta_2 > 0,$$

where $(\beta_0, \beta_1, \beta_2)$ is a vector of parameters, and ϵ is an uncertain disturbance term (uncertain variable).

3.2. *Parameter Estimation and Residual Analysis*

With the observed data of confirmed COVID-19 cases in Czech Republic from 6 June 2021 to 16 August 2021, $(t, y_t),\ t = 1, 2, \cdots, 72$, solving the following minimization problem:

$$\min_{\beta_0>0,\beta_1>0,\beta_2>0} \sum_{t=1}^{72} (y_t - (\beta_0/(1+\beta_1 \exp(-\beta_2 t))))^2,$$

we can obtain the parameter values $\hat{\beta}_0, \hat{\beta}_1, \hat{\beta}_2$ for $\beta_0, \beta_1, \beta_2$,

$$\hat{\beta}_0 = 1.5743 \times 10^7,\ \ \hat{\beta}_1 = 8.4666,\ \ \hat{\beta}_2 = 1.213 \times 10^{-4}.$$

Thus, the fitted logistic growth model is:

$$y = 1.5743 \times 10^7 / \left(1 + 8.4666 \exp\left(1.213 \times 10^{-4} t\right)\right), \tag{15}$$

which matching the data excellently.

Then, for each index t $(t = 1, 2, \cdots, 72)$, the t-th residual is:

$$\epsilon_t = y_t - 1.5743 \times 10^7 / \left(1 + 8.4666 \exp\left(1.213 \times 10^{-4} t\right)\right), \tag{16}$$

which can be regarded as the samples of the uncertain disturbance term ϵ (shown in Figure 2) in the uncertain regression model

$$y = 1.5743 \times 10^7 / \left(1 + 8.4666 \exp\left(1.213 \times 10^{-4} t\right)\right) + \epsilon, \tag{17}$$

Then, the average of the expected values of residuals $\hat{e}_t$ is obtained, i.e.,

$$\hat{e} = \frac{1}{72} \sum_{t=1}^{72} \hat{e}_t = 0.0172, \tag{18}$$

and the variance is:

$$\hat{\sigma}^2 = \frac{1}{72} \sum_{t=1}^{72} (\hat{e}_t - \hat{e})^2 = 299.7350^2. \tag{19}$$

Stochastic regression analysis is invalid to model cumulative number of confirmed COVID-19 cases in Czech Republic. Firstly, based on the Lilliefors test for the normality of residuals in Figure 2, we have the p-values is less than 10^{-3}, which suggests that the normality of residuals has been disproved by the test. The QQplot shown in Figure 3 can also support this point. Then, by using the Kolmogorov–Smirnov test, the residuals in Figure 2 are not from the same population in the sense of probability theory. For example, by using the test to check whether the first 10 residuals and the rest residuals in Figure 2 are identically distributed in the sense of probability theory, we have that the p-value is 0.0019, which means it does not pass the identical distribution test. Finally, stochastic regression analysis requires that a residual plot should look like a null plot which has constant mean, constant variance, and no separated points. It seems that the residual plot shown in Figure 2 is not a null plot in the sense of probability theory. Under these situations, the distribution function we obtained is not close enough to the real frequency for the disturbance term. Therefore, the disturbance term can not be characterized as a random variable, that is to see, stochastic regression analysis is invalid in this case.Consequently, the disturbance term can be characterized as an uncertain variable, and we use the uncertain regression analysis and uncertain hypothesis to exam the observed data in Table 1.

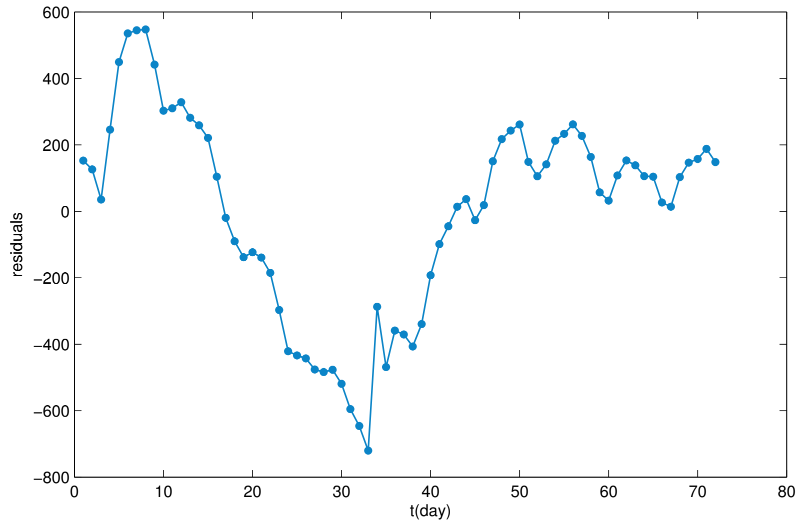

Figure 2. Residual plots for the uncertain logistic growth model.

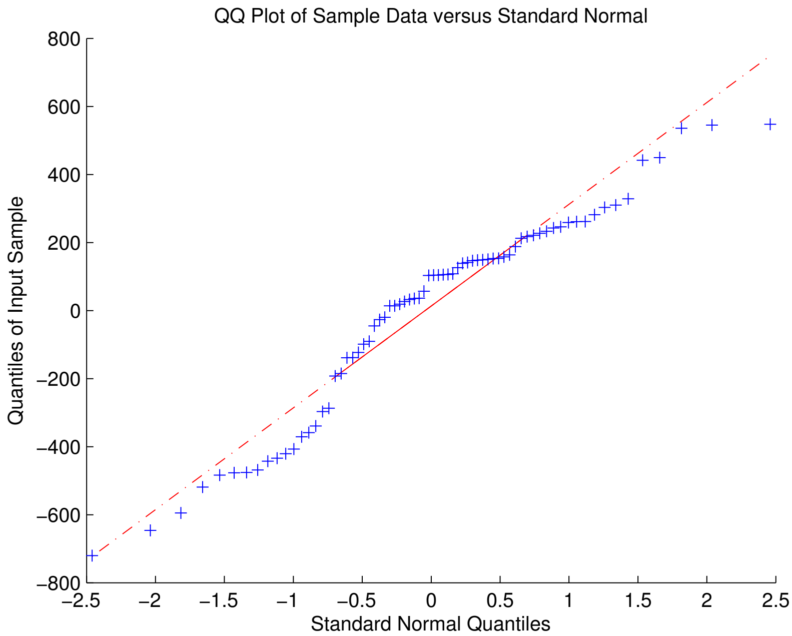

Figure 3. QQplot for the residuals.

3.3. Forecast

The forecast uncertain variable of cumulative number of confirmed COVID-19 infections in Czech Republic on day t is:

$$\hat{y} = 1.5743 \times 10^7 / \left(1 + 8.4666 \exp\left(1.213 \times 10^{-4} t\right)\right) + \hat{\varepsilon},\ \hat{\varepsilon} \sim \mathcal{N}\left(0.0172, 299.7350^2\right). \quad (20)$$

For instance, the forecast uncertain variable of cumulative number of confirmed COVID-19 infections in Czech Republic on day 73 (17 August 2021) follows the normal uncertainty distribution:

$$\hat{y}(73) = 1.5743 \times 10^7 / \left(1 + 8.4666 \exp\left(1.213 \times 10^{-4} \times 73\right)\right) + \hat{\varepsilon}.$$

We can see that $\hat{y}(73) \sim \mathcal{N}(1676256, 299.7350^2)$ due to $\hat{\varepsilon} \sim \mathcal{N}(0.0172, 299.7350^2)$. Thus, the excepted forecast value of cumulative number of confirmed COVID-19 infections in Czech Republic on 17 August 2021 is 1,676,256. Furthermore, the 95% confidence interval is:

$$1676256 \pm \frac{1730.8\sqrt{3}}{\pi} \ln \frac{1+0.95}{1-0.95},$$

i.e., [1,675,650.30, 1,676,861.13].

3.4. Uncertain Hypothesis Test

As assumption, ϵ is a normal uncertain variable $\mathcal{N}(e, \sigma)$ with unknown expected value e and unknown variance σ^2. $(\epsilon_1, \epsilon_2, \cdots, \epsilon_n)$ obtained in Equation (16) are 72 observed data of the uncertain variable ϵ. For a certain significance level $\alpha = 0.05$, we obtain:

$$\Phi^{-1}\left(1 - \frac{\alpha}{2}\right) = 605.4129, \quad \Phi^{-1}\left(\frac{\alpha}{2}\right) = -605.4129.$$

where Φ^{-1} is the inverse uncertainty distribution of $\mathcal{N}(0, 299.7350)$, i.e.,

$$\Phi^{-1}(\alpha) = 0 + \frac{299.7350\sqrt{3}}{\pi} \ln \frac{\alpha}{1-\alpha}. \tag{21}$$

It follows from $\alpha \times 72 = 3.6$ and uncertain hypothesis test that the test for the hypotheses:

$$H_0 : e = 0 \text{ and } \sigma = 299.7350 \quad \text{versus} \quad H_1 : e \neq 0 \text{ or } \sigma \neq 299.7350$$

is

$$W = \Big\{(\epsilon_1, \epsilon_2, \cdots, \epsilon_{72}) : \text{ there are at least 4 of indexes } t'\text{s with } 1 \leq t \leq 72 \text{ such that } \epsilon_t < -605.4129 \text{ or } \epsilon_t > 605.4129\Big\} \tag{22}$$

with Φ^{-1} is the inverse uncertainty distribution of $\mathcal{N}(\hat{e}, \hat{\sigma})$. Since there only exist 2 outliers $\epsilon_{32} : -646.0414 \notin [-605.4129, 605.4129]$ and $\epsilon_{33} : -720.1483 \notin [-605.4129, 605.4129]$ (shown in Figure 4), we have $(\epsilon_1, \epsilon_2, \cdots, \epsilon_{72}) \notin W$. Thus, we accept H_0. In other words, the uncertain regression model (20) is appropriate for fitting the the cumulative number of confirmed COVID-19 data in Czech Republic. As the forecast of uncertain logistic growth model, the total number of confirmed COVID-19 infections in Czech Republic continues to increase, but does not exceed 1.5743×10^7 as expected.

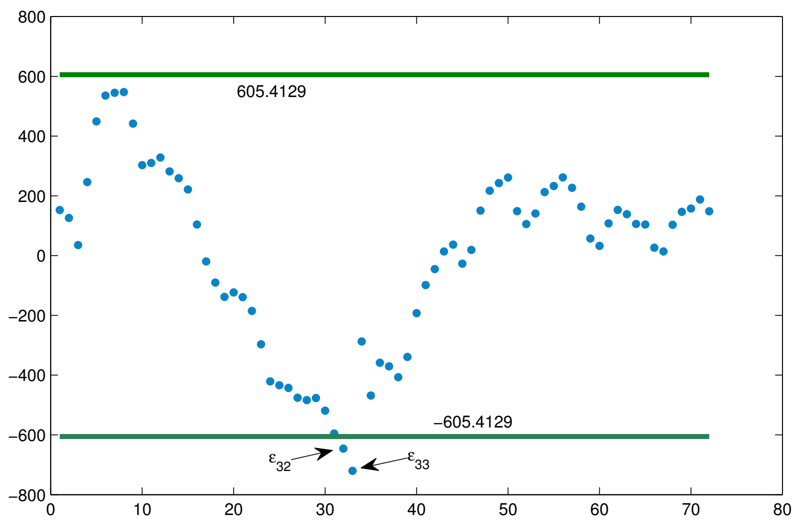

Figure 4. Uncertain hypothesis test: two outliers ($\epsilon_{32} = -646.0414$ and $\epsilon_{33} = -720.1483$).

4. Discussion

In this manuscript, an uncertain logistic growth model was applied to formulated the cumulative number of COVID-19 data in Czech Republic. Stochastic regression analysis is invalid to model cumulative number of confirmed COVID-19 cases in Czech Republic, by considering the disturbance term as random variables, because that the normality test and the identical distribution test of residuals are not passed, and the residual plot does not look like a null plot in the sense of probability theory. In this case, the uncertain logistic growth model was employed by considering the perturbation as an uncertain distribution term. Then parameter estimation, residual analysis, forecast value, and confidence interval were proposed, also the cumulative number of confirmed COVID-19 on August 17, 2021 in Czech Republic was predicted. Moreover the uncertain hypothesis test was employed to illustrate the uncertain logistic growth model is a good fit to the observed data. In the future, uncertain logistic growth model can also be applied to modeling the prediction and precaution for other epidemics due to the symmetry and similarity of disease. If there is a very little data reported or the limitations of data time period choosing, the prediction will be not as accurate as expected.

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