Quantum Coherence of Atoms with Dipole–Dipole Interaction and Collective Damping in the Presence of an Optical Field

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Abstract: We investigate the effect of the interatomic distances and thermal reservoir on the coherence dynamics of the atoms considering the dipole–dipole interaction (DDI) and collective damping effect (CDE). We show that the control and protection of the coherence are very sensitive to the interatomic distances and reservoir temperature. Furthermore, we explore the distance effect between atoms and reservoir temperature on the time evolution of the total quantum correlation between the two atoms. The obtained results could be useful to execute these quantum phenomena and also considered as a good indication to implement realistic experiments with optimal conditions.

1. Introduction

Numerous quantum phenomena have been regarded as resources for performing various tasks in quantum optics and information during the last few decades, both theoretically and experimentally (QOI). In general, quantum correlations require the coherent superposition of quantum states [1–7]. It all started with Einstein, Podolsky, and Rosen's idea of the "EPR paradox" [8]. They discussed how quantum mechanics theory might give rise to "spooky action at a distance". The matter has been described by E. Schrödinger as the capability of local measurements to impact a quantum system without having access to it [9]. Subsequently, Bell introduced an inequality to demonstrate that this action could create a quantum correlation that defies any classical description [10]. Quantum coherence underlies various quantum effects in nanomaterials [11,12], quantum measurements and quantum metrology [13–17], applications of quantum mechanics to biological objects [18–20], connection between quantum coherence with entanglement [21], operational interpretation of coherence in quantum key distribution [22], application of coherence to the thermodynamics [23,24], conversion coherence to quantum correlations [25], extraction the work from coherence [26], coherence in a nuclear magnetic resonance quantum simulator [27], relationship between the coherence and quantum speed limit [28], non-Markovianity measure based on the amount of coherence [29] and connection between

coherence and EPR steering [30]. In keeping with the fundamental significance of quantum coherence, only lately has a rigorous theory of coherence been established, along with some essential restrictions, to ensure that the quantum coherence is considered as a physical resource [31]. Thus, certain quantum measures have been suggested to verify these constraints, in particular those based on the relative entropy and the l_1 norm [31]. Moreover, the convex-roof construction or nonlocal correlation can be used to detect the amount of coherence [21,32] and an operational theory of quantum coherence has been proposed [33].

Quantum coherence has recently attracted much attention to the development of experimental technique with the control and observation of quantum phenomena in different quantum systems. Realistic quantum systems are always unavoidably interacting with their environment, which results in decoherence during the dynamics [34]. During the past few decades, the relaxation and dephasing in open quantum systems have been largely studied in the literature considering Markovian and non-Markovian dynamics. Investigations of the decoherence phenomenon in open systems with Markov approximation has generated great interest in various domains ranging from grasping the fundamental features of quantum mechanics to the advanced experimental applications in QOI [35]. Coherence, along with many other quantum properties, is often studied without considering the influence of the environment on the quantum systems. Recently, many works have been focused on the dynamics of the quantum coherence under the effect of the external environment [36–45]. Based the above considerations, the purpose of this paper is to investigate the effect of atom distance and reservoir temperature on the evolution of quantum coherence and total correlation considering DDI and CDE. We find that the measure of the coherence decreases from its maximum initial value and then experiences oscillations that dampen with time, and it presents a constant behaviour when the time takes large values. On the other hand, we display the quantum correlation dynamics with respect to the distance between the atoms and bath temperature when the atoms are initially started from a separable state. We find that the oscillations of the measure of total correlation also depend on the atoms' distance and provides rapid oscillations that are damped with the time.

The following is a breakdown of the current paper's structure. The physical model and its dynamics are introduced in Section 2. The several measures of quantumness that are utilized in this study are described in Section 3. In Section 4, we present and discuss the findings. The final part contains the conclusions.

2. The Hamiltonian Model and Dynamics

Let us consider a system of two atoms in interaction with an electromagnetic field [46,47]. The Hamiltonian of the physical system is formulated as

$$
\begin{aligned}
H = \quad & - \ i \sum_{i=1}^{2} \sum_{\vec{k}s} \left[\vec{d}_i \cdot \vec{g}_{\vec{k}s}(\vec{r}_i) a_{\vec{k}s} \left(\sigma_i^+ e^{-i(\omega_k - \omega_i)t} \right) \right] \\
& - \ i \sum_{i=1}^{2} \sum_{\vec{k}s} \left[\vec{d}_i \cdot \vec{g}_{\vec{k}s}(\vec{r}_i) a_{\vec{k}s} \left(\sigma_i^- e^{-i(\omega_k + \omega_i)t} \right) \right] + H.c.
\end{aligned}
\tag{1}
$$

where, the atoms are supposed to be close in order to take into account the DDI and CDE. The distance between the atoms are used to prescribe the separation between the two atoms. The quantity $\vec{d}_i$ defines the transition dipole moment, σ_i^+ (σ_i^-) describes the dipole raising (lowering operator), ω_i is the transition frequency, the vector $\vec{r}_i$ denotes the position of the ith atom, $a_{\vec{k}s}$ defines the annihilation operator of the field with the mode $\vec{k}s$ of a frequency ω_k and a wave vector $\vec{k}$. The expression of the coupling constant $\vec{g}_{\vec{k}s}$ is

$$
\vec{g}_{\vec{k}s}(\vec{r}_i) = \sqrt{\frac{\omega_k}{2\epsilon_0 \hbar V}} \hat{e}_{\vec{k}s} \exp i\vec{k} \cdot \vec{r}_i,
\tag{2}
$$

where $\hat{e}_{\vec{k}s}$ is the polarization vector of the electric field and V is the quantization volume. The master equation of two-atom density operator can be written as

$$
\begin{aligned}
\frac{\partial \rho}{\partial t} = & - i\omega_0 \sum_{i=1}^{2}[\sigma_i^z, \rho] - i\sum_{i\neq j}\Omega_{ij}\left[\sigma_i^+\sigma_j^-, \rho\right] \\
& - \frac{1}{2}(\bar{n}+1)\sum_{i,j=1}^{2}\gamma_{ij}\left(\sigma_i^+\sigma_j^-\rho - 2\sigma_j^-\rho\sigma_i^+ + \rho\sigma_i^+\sigma_j^-\right) \\
& - \frac{1}{2}\bar{n}\sum_{i,j=1}^{2}\gamma_{ij}\left(\sigma_i^-\sigma_j^+\rho - 2\sigma_j^+\rho\sigma_i^- + \rho\sigma_i^-\sigma_j^+\right).
\end{aligned}
\tag{3}
$$

Here, the operator σ_i^z indicates the Pauli operator with the component Z for the ith atom, Ω_{ij} and γ_{ij} describe the DDI and CDE, respectively, $\bar{n}$ represents the thermal number and γ_{ii} describes the spontaneous decay rate. They are introduced by

$$
\begin{aligned}
\Omega_{ij} = & - \frac{3}{4}\gamma\left[1 - \left(\vec{d}.\vec{r}_{ij}\right)^2\right]\frac{\cos\left[k_0 r_{ij}\right]}{k_0 r_{ij}} \\
& + \frac{3}{4}\gamma\left[1 - 3\left(\vec{d}.\vec{r}_{ij}\right)^2\right]\left[\frac{\sin\left[k_0 r_{ij}\right]}{(k_0 r_{ij})^2} + \frac{\cos\left[k_0 r_{ij}\right]}{(k_0 r_{ij})^3}\right],
\end{aligned}
\tag{4}
$$

and

$$
\begin{aligned}
\gamma_{ij} = & \frac{3}{2}\gamma\left[1 - \left(\vec{d}.\vec{r}_{ij}\right)^2\right]\frac{\sin\left[k_0 r_{ij}\right]}{k_0 r_{ij}} \\
& + \frac{3}{2}\gamma\left[1 - 3\left(\vec{d}.\vec{r}_{ij}\right)^2\right]\left[\frac{\cos\left[k_0 r_{ij}\right]}{(k_0 r_{ij})^2} - \frac{\sin\left[k_0 r_{ij}\right]}{(k_0 r_{ij})^3}\right]
\end{aligned}
\tag{5}
$$

where $r_{ij} = |r_i - r_j|$ prescribes the interatomic distance with the unit vector $\hat{r}_{ij}$, $k_0 = \omega_0/c$, and $\vec{d}$ is the unit vector of the atomic transition dipole moment. We consider that two atoms are initially prepared in the density matrix of the form

$$
\rho(0) = \frac{1}{4}\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{pmatrix}
\tag{6}
$$

with maximal value of the quantum coherence for the reduced atomic density matrix.

3. Coherence and Total Correlation

The measures of quantumness, such as quantum coherence and total quantum correlation, are reviewed in this section.

The diagonal components of the system density operator determine the essential characteristics of coherence. The l_1 norm of quantum coherence exactly detects the amount of coherence by taking into account the non-diagonal elements' absolute values. The distance between the interested state and the nearest incoherent state is the measure of coherence. The l_1 norm of coherence is introduced as

$$
C_l = \min_{\delta\in\mathcal{I}}\|\rho - \delta\|_{l_1} = \sum_{i\neq j}|\rho_{ij}|.
\tag{7}
$$

Here, $\mathcal{I}$ describes the set of incoherent states and i (j) is the row (column) index. The relative entropy coherence is defined by

$$C_r = S\left(\rho || \rho_{\text{diag}}\right) = S\left(\rho_{\text{diag}}\right) - S(\rho) \tag{8}$$

where the function $S(\rho) = -Tr(\rho \log_2 \rho)$ denotes the von Neumann entropy and the operator ρ_{diag} describes the quantum incoherent state. C_{l_1} and C_r both achieve monotony for all system states. It has been proven that the function C_{l_1} characterizes the upper bound of the function C_r for a pure state.

Currently, the total quantum correlation is a useful physical resource and its concept is associated to the local quantum uncertainty (LQU) and can be considered as a measure of quantum correlations [48]. It is defined by [49]

$$U_{1D} = \min_{K_1^\Gamma} I\left(\rho, K_1^\Gamma\right), \tag{9}$$

where the parameter Γ describes the spectrum of K_1^Γ, I indicates the amount of the Skew information, and the minimization over a chosen spectrum of observables results in a specific measurement of the family. The LQU is defined by

$$U_{1D}(\rho_{12}) = 1 - \lambda_{\max}\{W_{12}\}, \tag{10}$$

where $\lambda_{\max}$ is the maximum eigenvalue of the 3×3 symmetric W_{12} with the matrix elements

$$(W_{12})_{ij} = \text{Tr}\{\sqrt{\rho_{12}}(\sigma_{i1} \otimes I)\sqrt{\rho_{12}}(\sigma_{j1} \otimes I)\}. \tag{11}$$

For pure states, the LQU is normalized to 1 and coincides with the value of the linear entropy.

4. Result and Discussions

The numerical results associated to the amount of coherence and quantum correlation are presented in Figures 1–3. These figures display the dynamical behaviour of measures of quantumness according to the main physical parameters of the model and the initial state form of the two-atom system.

To show the influence of distance between the atoms and reservoir temperature on the time evolution of the coherence, in Figures 1 and 2 we plot, respectively, C_{l_1} and C_r versus the time γt for various values of r considering $\bar{n} = 0$ and $\bar{n} = 0.5$. The black and blue lines are for $\bar{n} = 0$ and $\bar{n} = 0.5$, respectively. In general, the figures show some important dynamical features of the quantum coherence with an oscillatory behaviour. We can observe that the amount of the quantum coherence decreases from its maximal value and then shows oscillations that dampen with time and it reaches a constant value when the time becomes large. The rise in the value of the coherence depends on the DDI and CDE. This phenomenon can be comprehended as the reversed flow of the information between atoms and environment, where the DDI is responsible for the oscillations of the functions C_l and C_r via the exchange of the energy between the atoms. Furthermore, the exchange of the energy is damped out and then with time the oscillations of C_l and C_r are reduced. We can see that the delay in the coherence loss can be caused by the control of the distance parameter r. For small values of r, the measures C_l and C_r exhibit rapid oscillations during the dynamics. This behaviour can be explained by the increase of the emitted energy by one of the atoms that result an enhancement in the oscillations of the quantum coherence. As the parameter r increases, the energy exchange between the atoms decreases, so the measures C_l and C_r decrease exponentially, which agrees with the results of the case of two atoms each locally interacting its own environment. On the other side, it is observed that the temperature effect on the quantum coherence is similar to the different values of the parameter r. The presence of the thermal reservoir, $\bar{n} \neq 0$, leads to destroying the amount of the quantum coherence during the time evolution. The obtained results demonstrate a good understanding of the influence of the physical parameters on the quantum coherence considering the distance between atoms for open systems.
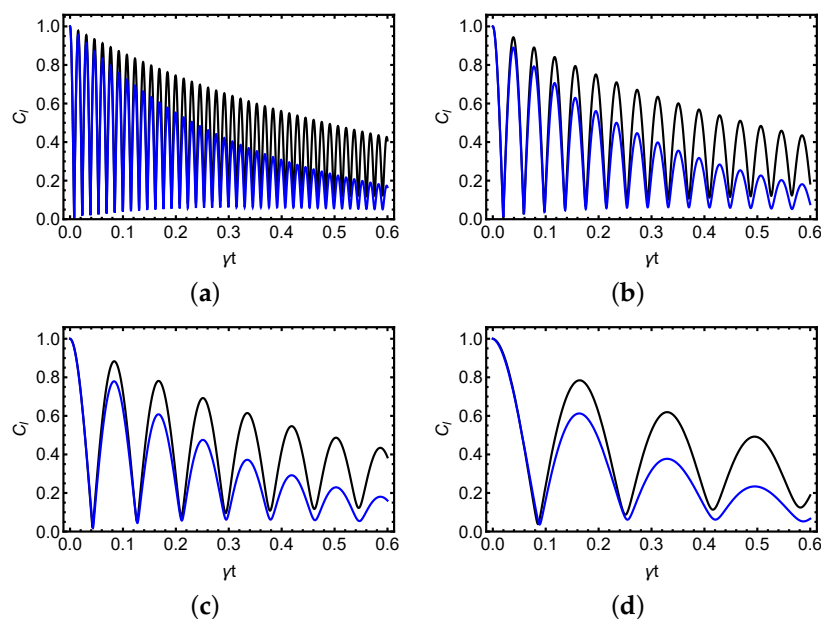
Figure 1. Quantum measure of the coherence, C_l, against the time γt for various values of r. Panel (**a**) is for $k_0 r = 0.15$, panel (**b**) is for $k_0 r = 0.2$, panel (**c**) is for $k_0 r = 0.25$ and panel (**d**) is for $k_0 r = 0.3$. The black line corresponds to $\bar{n} = 0$ and blue line corresponds to $\bar{n} = 0.5$. In general, the figure demonstrates some interesting dynamic characteristics of the quantum coherence with regard to the values of r and $\bar{n}$, where the measure of the coherence depends largely on the distance of the two atoms, displaying oscillations during the temporal evolution. The dynamical behaviour of the coherence measure in thermal environment is similar to that of the vacuum case with a decrease in the amount of measure.
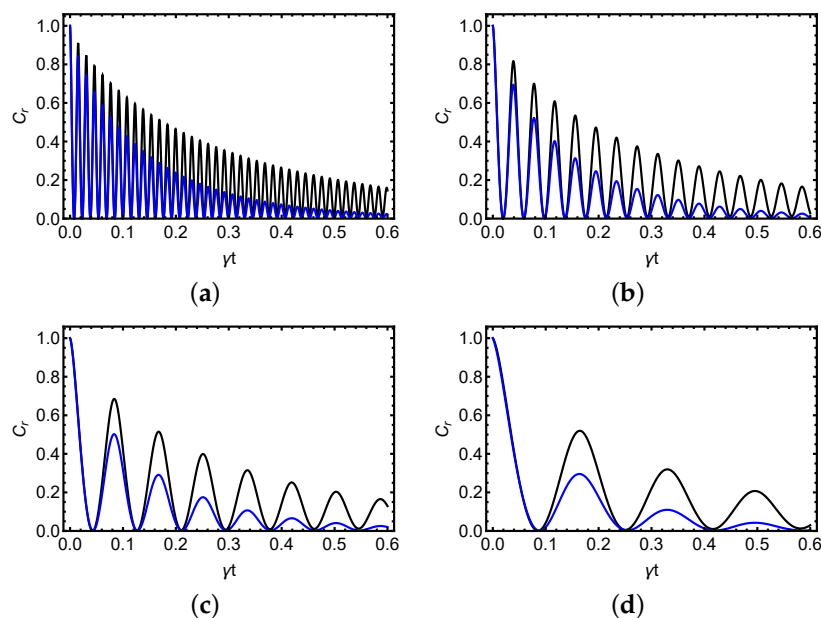

Figure 2. Quantum measure of the coherence, C_r, against the time γt for various values of r. Panel (**a**) is for $k_0 r = 0.15$, panel (**b**) is for $k_0 r = 0.2$, panel (**c**) is for $k_0 r = 0.25$ and panel (**d**) is for $k_0 r = 0.3$. The black line corresponds to $\bar{n} = 0$ and blue line corresponds to $\bar{n} = 0.5$. In general, the figure demonstrates some interesting dynamic characteristics of the quantum coherence with regard to the values of r and $\bar{n}$, where the measure of the coherence depends largely on the distance of the two atoms, displaying oscillations during the temporal evolution. The dynamical behaviour of the coherence measure in thermal environment is similar to that of the vacuum case with a decrease in the amount of measure.

Figure 3 displays the dynamics of the total quantum correlation versus the dimensionless time γt for various values of r with $\bar{n} = 0$ and $\bar{n} = 0.5$. The black and blue lines are for $\bar{n} = 0$ and $\bar{n} = 0.5$, respectively. We can see that the dynamical behaviours of the total correlation and coherence are similar according to the values of the parameters r and $\bar{n}$. These features make coherence a good candidate for describing the quantum correlation in the present model, and therefore its implementation in various fields of QOI. It is well known that in some circumstances, the environments do not have to destroy the correlation and coherence among quantum systems, but can instead be produced over time. We may deduce that this possibility is reliant on the dynamic generator selected and the initial system state. Beginning with the initially separable state of the two atoms, in which each atom is specified in an optimum state, coherence and correlation can maintain throughout time. This is possible in the current physical model and is enabled by the reduced dynamics master equation. In a nutshell, the results demonstrate that the control and preservation of the coherence and correlation can occur by controlling the DDI and CDE in terms of the distance of atoms as well as exploiting the temperature effect to describe realistic experimental scenarios with optimal conditions.
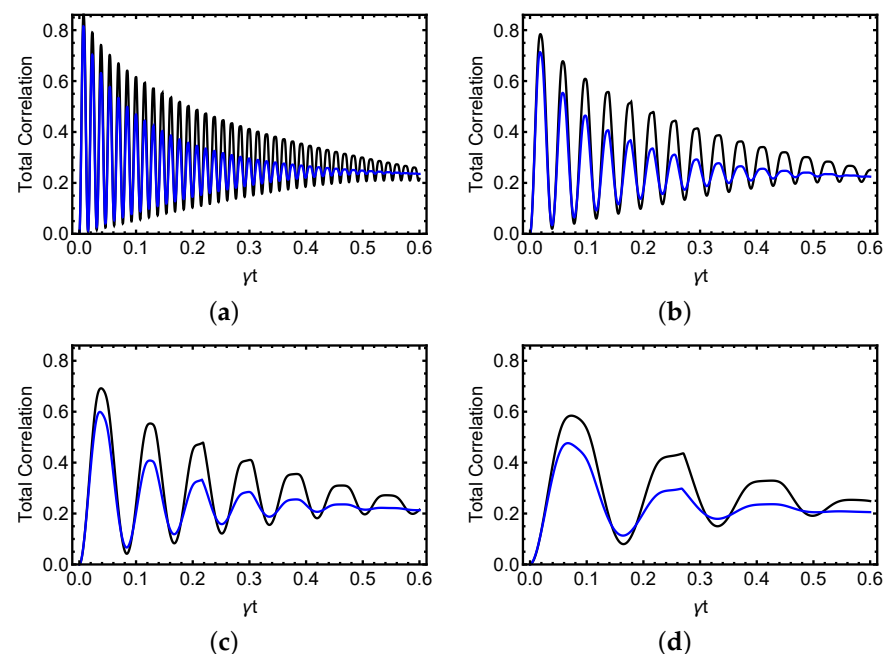

Figure 3. Quantum measure of the total quantum correlation, U_{1D}, against the time γt for various values of r. Panel (**a**) is for $k_0 r = 0.15$, panel (**b**) is for $k_0 r = 0.2$, panel (**c**) is for $k_0 r = 0.25$ and panel (**d**) is for $k_0 r = 0.3$. The black line corresponds to $\bar{n} = 0$ and blue line corresponds to $\bar{n} = 0.5$. In general, the figure demonstrates some interesting dynamic characteristics of the total correlation with regard to the values of r and $\bar{n}$, where the measure of the correlation depends largely on the distance of the two atoms, displaying oscillations during the temporal evolution. The dynamical behaviour of the correlation measure in thermal environment is similar to that of the vacuum case with a decrease in the amount of measure.

5. Conclusions

We have investigated the dynamical behaviour of the coherence and total quantum correlation for atoms in the presence of the thermal reservoir. We have explored the effect of atoms' distance and temperature on the time variation of coherence by taking into account the DDI and CDE. We have found that the measure of the coherence decreases from its maximum initial value and then experiences oscillations that dampen with time and it presents a constant behaviour when the time takes large values. Moreover, we have shown that the control and preservation control of the quantum coherence can be adjusted by a considerable choice of the atoms' distance and temperature of the bath. The decrease in

the value of the distance leads to rapid oscillations of the measure function of coherence and its amount decreases with an increasing temperature. On the other hand, we have also studied the quantum correlation dynamics with respect to the distance between the atoms and bath temperature when the atoms are initially started from a separable state. We have shown that the oscillations of the measure of total correlation also depend on the atoms' distance and provides rapid oscillations that are damped with the time. The correlation dynamics in the thermal reservoir is similar to that of the vacuum reservoir with a decrease in the amount of measure. Finally, the results demonstrate that the control and preservation of the coherence and correlation can occur by controlling the DDI and CDE in terms of the distance of atoms as well as exploiting the temperature effect to describe realistic experimental scenarios with optimal conditions.

Finally, we mention that we investigate here only the coherence dynamics for the case of diatomic systems. Certainly a study of polyatomic systems will make a useful contribution to understanding the dynamics of coherence. Another important line of research is to consider the dynamical behaviour of the coherence in terms of interatomic distances under the influence of non-Markovian environments.

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