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# On the Trace Anomaly of the Chaudhuri–Choi–Rabinovici Model

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**Abstract:** Recently a non-supersymmetric conformal field theory with an exactly marginal deformation in the large $N$ limit was constructed by Chaudhuri–Choi–Rabinovici. On a non-supersymmetric conformal manifold, the $c$ coefficient of the trace anomaly in four dimensions would generically change. In this model, we, however, find that it does not change in the first non-trivial order given by three-loop diagrams.

**Keywords:** conformal field theory; trace anomaly

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Conformal field theories are no longer conformally invariant in curved space-time due to the trace anomaly in even space-time dimensions. They do, however, play a fundamental role in understanding the structure of the energy–momentum tensor and the renormalization group flow.

In four-dimensional conformal field theories, the trace anomaly has the form

$$T^{\mu}_{\mu} = c\text{Weyl}^2 - a\text{Euler} \tag{1}$$

and it is known that coefficient $a$ cannot change under exactly marginal deformations, but coefficient $c$ may [1–7]. However, there has been no explicit field theory example where $c$ changes (except for the effective holographic constructions in [2]). The main obstruction has been that we have no good examples of non-supersymmetric conformal field theories with exactly marginal deformations; in superconformal field theories, while it is easier to realize exactly marginal deformations, $c$ does not change [8].

Recently, Chaudhuri, Choi and Rabinovici have constructed a non-supersymmetric conformal field theory with an exactly marginal deformation in the large $N$ limit [9] (see also [10,11] for other recently constructed examples of non-supersymmetric field theories with exactly marginal deformations in different dimensions than four). This theory may serve as a first non-trivial check if $c$ can really change under exactly marginal deformations. In this short note, we, however, show that it does not change at the first non-trivial order given by three-loop diagrams.

The model (called complex bifundamental model in [9]) is given by four $SU(N_c)$ gauge theories with names $1, 1', 2$ and $2'$, each of which has $N_f$ Dirac fermions in the fundamental representation. We have two complex scalars in the bifundamental representations $\Phi_1$ (under gauge group 1 and $1'$) and $\Phi_2$ (under gauge group 2 and $2'$). The gauge coupling constant for each gauge group is $g_i$. It has no Yukawa interaction, the absence of which is protected by chiral symmetry, but it has a scalar potential

$$\begin{aligned} V = {}& \widetilde{h}_1 \text{Tr}\left[\Phi_1^\dagger \Phi_1 \Phi_1^\dagger \Phi_1\right] + \widetilde{h}_2 \text{Tr}\left[\Phi_2^\dagger \Phi_2 \Phi_2^\dagger \Phi_2\right] \\ & + \widetilde{f}_1 \text{Tr}\left[\Phi_1^\dagger \Phi_1\right] \text{Tr}\left[\Phi_1^\dagger \Phi_1\right] + \widetilde{f}_2 \text{Tr}\left[\Phi_2^\dagger \Phi_2\right] \text{Tr}\left[\Phi_2^\dagger \Phi_2\right] + 2\widetilde{\zeta} \text{Tr}\left[\Phi_1^\dagger \Phi_1\right] \text{Tr}\left[\Phi_2^\dagger \Phi_2\right]. \end{aligned} \tag{2}$$

We take the Veneziano limit of $N_c, N_f \to \infty$ with fixed $x = \frac{N_f}{N_c}$ and consider the limit $x \to \frac{21}{4}$ to make the theory weakly coupled.

In terms of rescaled coupling constants ($i = 1, 2$)

$$\lambda_i = \frac{N_c g_i^2}{16\pi^2} \tag{3}$$

the renormalization group $\beta$ functions in the Veneziano limit are expressed as (no sum over $i$ unless explicitly shown)

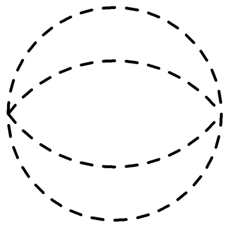
$$\begin{aligned} \beta_{\lambda_i} &= -\frac{21-4x}{3}\lambda_i^2 + \frac{-54+26x}{3}\lambda_i^3 \\ \beta_{h_i} &= 8h_i^2 - 12\lambda_i h_i + \frac{3}{2}\lambda_i^2 \\ \beta_{f_i} &= 4f_i^2 + 16 f_i h_i + 12 h_i^2 + 4\zeta^2 - 12\lambda_i f_i + \frac{9}{2}\lambda_i^2 \\ \beta_\zeta &= \zeta \sum_{i=1}^{2} (4f_i + 8h_i - 6\lambda_i). \end{aligned} \tag{4}$$

The zero of the $\beta$ functions was studied in [9] and they found that there exists a conformal manifold given by

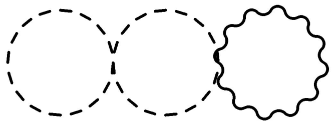
$$\begin{aligned} \lambda_1 &= \lambda_2 = \lambda = \frac{21-4x}{-54+26x} \\ h_1 &= h_2 = \frac{3-\sqrt{6}}{4}\lambda \\ f_p &\equiv \frac{f_1+f_2}{2} = \sqrt{\frac{3}{2}}\lambda \\ \zeta^2 + f_m^2 &= \frac{18\sqrt{6}-39}{16}\lambda^2 \end{aligned} \tag{5}$$

where $f_m \equiv \frac{f_1 - f_2}{2}$. From the last line of Equation (**??**), we see that it has the topology of a circle. As long as $\lambda$ is small, we may neglect higher order corrections.

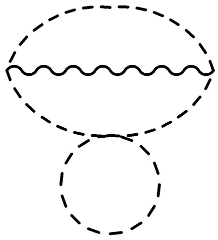
We now ask if the coefficient $c$ in the trace anomaly can change on this conformal manifold. In addition to the coupling constant-independent contributions from the one-loop diagrams (that count a number of fields), the coupling constant-dependent contributions to the trace anomaly that are relevant for us come from the three-loop diagrams shown in Figure 1. The detailed computation for Figure 1A (as well as other two-loop diagrams) can be found in [12–14], but we only need the relative coefficient, so we can simply work on combinatorics.



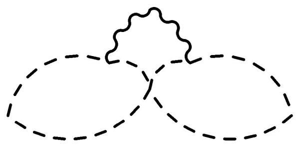
(**A**)



(**B**)



(**C**)



(**D**)

**Figure 1.** Three-loop Feynman diagrams that could contribute to $c$. Wavy lines correspond to gauge fields and dotted lines correspond to scalar fields.

The three-loop Figure 1B–D are not evaluated in the literature, but we see that Figure 1B,C do not contribute to $c$. This is because the divergence can be simply removed by adding the "mass counter-term". Figure 1D may contribute in general, but the contributions to $c$ in our theory do not depend on $\zeta$ or $f_m$ from the symmetry of the diagrams (It cannot be proportional to $\zeta$ because the gauge fields cannot connect $\Phi_1$ and $\Phi_2$. The relevant diagrams are all symmetric with respect to the exchange of $f_1$ and $f_2$).

As for Figure 1A, since the overall contribution to $c$ is known, we can just enumerate diagrams appearing in the Wick contractions of

$$\langle \widetilde{f}_1 \mathrm{Tr}[\Phi_1^\dagger \Phi_1]\mathrm{Tr}[\Phi_1^\dagger \Phi_1](x) \widetilde{f}_1 \mathrm{Tr}[\Phi_1^\dagger \Phi_1]\mathrm{Tr}[\Phi_1^\dagger \Phi_1](y)\rangle_{\mathrm{free}}$$
$$or$$
$$\langle 2\widetilde{\zeta} \mathrm{Tr}[\Phi_1^\dagger \Phi_1]\mathrm{Tr}[\Phi_2^\dagger \Phi_2](x) 2\widetilde{\zeta} \mathrm{Tr}[\Phi_1^\dagger \Phi_1]\mathrm{Tr}[\Phi_2^\dagger \Phi_2](y)\rangle_{\mathrm{free}}$$

We see that only planar diagrams will contribute in the Veneziano limit.

Up to an overall proportionality factor, the result in the Veneziano limit is summarized as

$$c_{2,3-\text{loop}} = -4f_m^2 - 4\zeta^2 + c_\lambda \lambda^2 \quad (6)$$

on the conformal manifold, where $c_\lambda$ is some numerical constant, which is unimportant for our discussions (A typo in the two-loop gauge contribution [14] that could affect $c_\lambda$ has been corrected in [15]). Since the relative coefficient appearing here coincides with what appears in the last line of Equation (**??**), we conclude that $c$ does not change on the conformal manifold, although the value itself is perturbatively corrected. We also note that these two- and three-loop diagrams do not change the value of $a$ as anticipated [1,16] (rather trivially without cancellation, unlike $c$).

The result is surprising in the sense that we generically expect that $c$ would change on a non-supersymmetric conformal manifold. It is an interesting question to see whether the higher loop corrections modify our conclusion. It may be possible to relate the all-loop argument for the existence of the exactly marginal deformation in [9] with the computation of $c$ by closing all the external lines in beta functions to make vacuum diagrams.

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