Supplementary Materials: Comparison of Computer Extended Descriptive Geometry (CeDG) with CAD in the modeling of sheet metal patterns

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Methods

Four - way cylindric hopper with conical coupling

CEDG approach.

The next figures and text clarify the descriptive geometry procedures used to build this hopper model and its development (pattern) under Geogebra Dynamic Geometry Software (DGS), succinctly presented in the Methods Section of the main text.

The Figure S1 shows the intersection of the main cylinder with the horizontal plane, obtained trough the main axes. The conic is calculated by means of the ellipse foci. The model is prepared for the reference value `rCil` = 3 m that may be modified using the presented slider control [1,2].

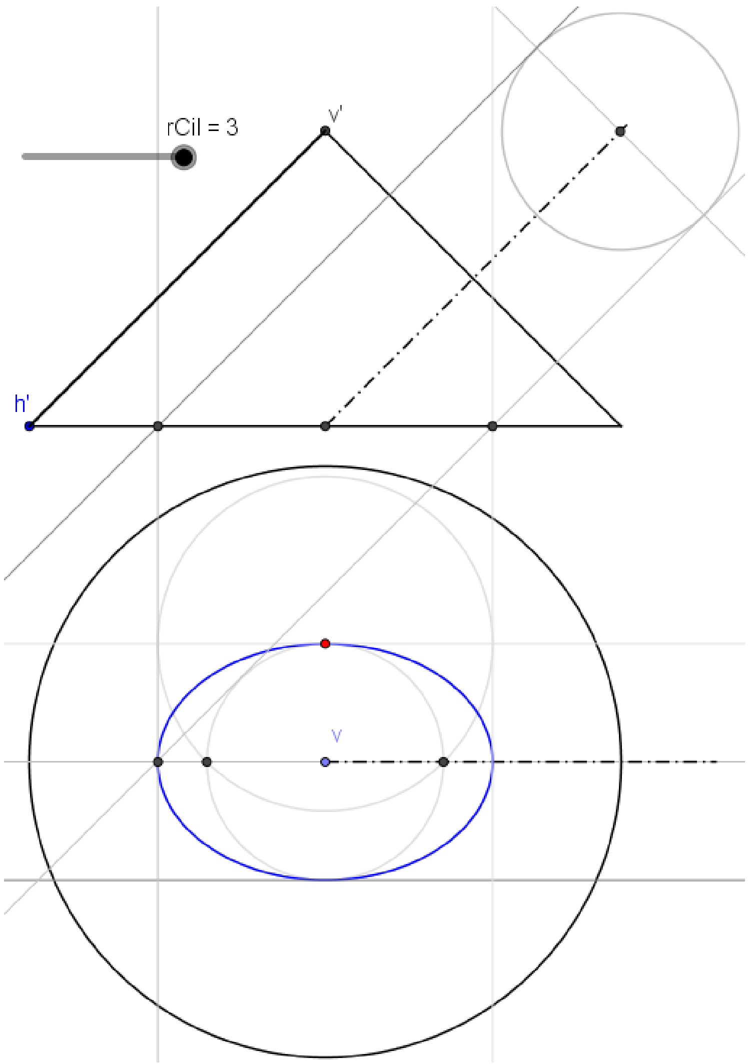


Figure S1. Elliptic section of the cylindric hopper.

The upper junction is calculated by the intersection cylinder - cone, using the technique of auxiliary planes containing generatrix lines [3]. The figure S2 shows an auxiliary plane between limit planes L_1 and L_2 that cuts the ellipse (cylinder's directrix) in points 1 and 2 and the circular cone base in point `a`. The associated generatrix lines included in the same auxiliary plane intersect themselves in points `1a'-1a` and `2a'-2a` (vertical - horizontal projections of points `1A` and `2B`).

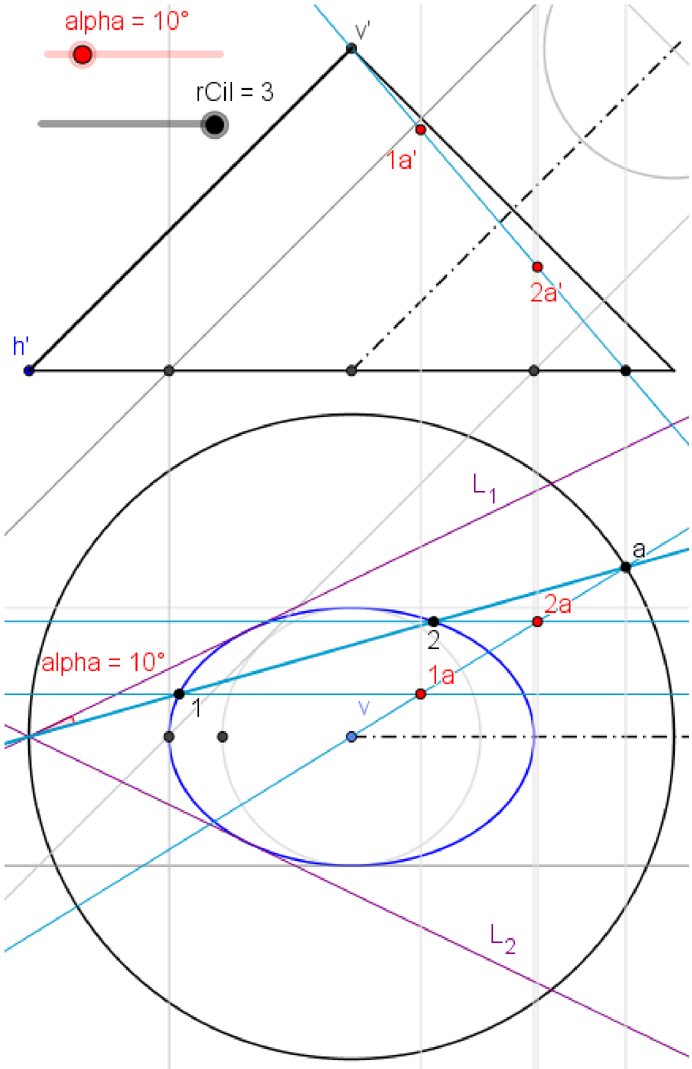


Figure S2. Cylindric hopper with two points of the conical border loci (`1a-1a'` and `2a-2a'`).

After the calculation of the points of the conical junction for an auxiliary plane, the same descriptive geometry procedure may be automatically extended to the full range of auxiliary planes between the limit planes to give the whole border.

This is achieved through the following `locus` mathematical objects:

```
loc1 = Locus(Q,alpha)
loc2 = Locus(S,alpha)
loc3 = Locus(U,alpha)
loc4 = Locus(W,alpha)
```

CEDG Code 1: Conical border loci generation

The horizontal projection of the conical border is given by the sum of `loc1` and `loc2` loci, associated with the points `Q=2a` and `S=1a`, whereas the sum of `loc3` and `loc4` loci provides the vertical projection of the conical border. The parameter `alpha` is the angle between the general auxiliary plane and L_1 limit plane (their lines in the horizontal plane), in such a way that the auxiliary plane is L_1 for `alpha`=0 and L_2 for `alpha` = 51.75°. The orthographic views of the conical junction of the hopper's model is presented in Figure S3.

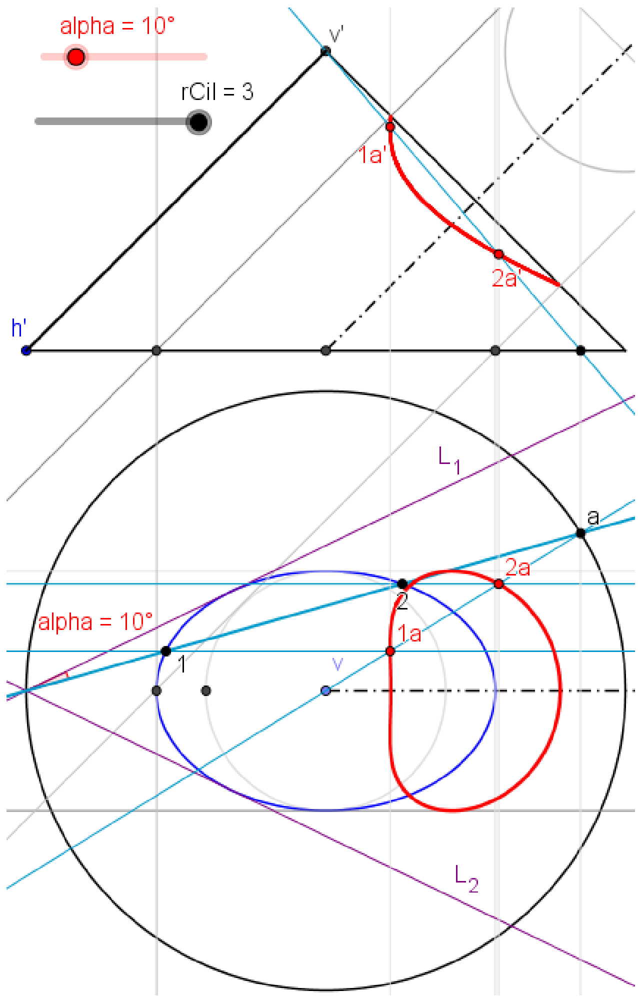


Figure S3. Cylindric hopper with conical junction.

The lateral holes performed by the revolution cylindric surface with axis perpendicular to that of hopper are calculated in a similar manner. A generic point `p'` of the vertical projection locus is carried out to the generatrix lines of the main cylindric hopper, to give the horizontal projections p_1 and p_2, of each hole (Figure S4).

The loci are generated through the movement of the point `p'` along the perimeter of the vertical projection circle. This is achieved with the following `locus` mathematical objects:

```
loc9 = Locus(B,V)
loc10 = Locus(C,V)
```

CEDG Code 2: Lateral holes loci generation

Loci `loc9` and `loc10` are associated to the points `B`=p_1 and `C` = p_2, whereas `V` = `p'`.

An extension of the general method of inscribed prism [4] is used to compute the flat pattern after the modeling of the 4-way cylindric hopper, as shown in Figure S5.

The cylinder is sectioned by a plane perpendicular to the axis and a true-length projection of that section is obtained (see Figure S5a). The perimeter of this section is straighten up to give a straight line perpendicular to generatrix lines in the pattern. The points `1A` and `2B` of the conical border are carried out to their generatrix lines in the pattern, using the shortest generatrix to start the process (Figure S5b). The same procedure applies to the points of the bottom flat border to give the transformed ellipse in the pattern. The distances between generatrix lines in the development and the shortest generatrix (left side) are equal to the true arc lengths of the cylindric section. The same technique is achieved to place the point `p'` of the lateral hole into the flat development.

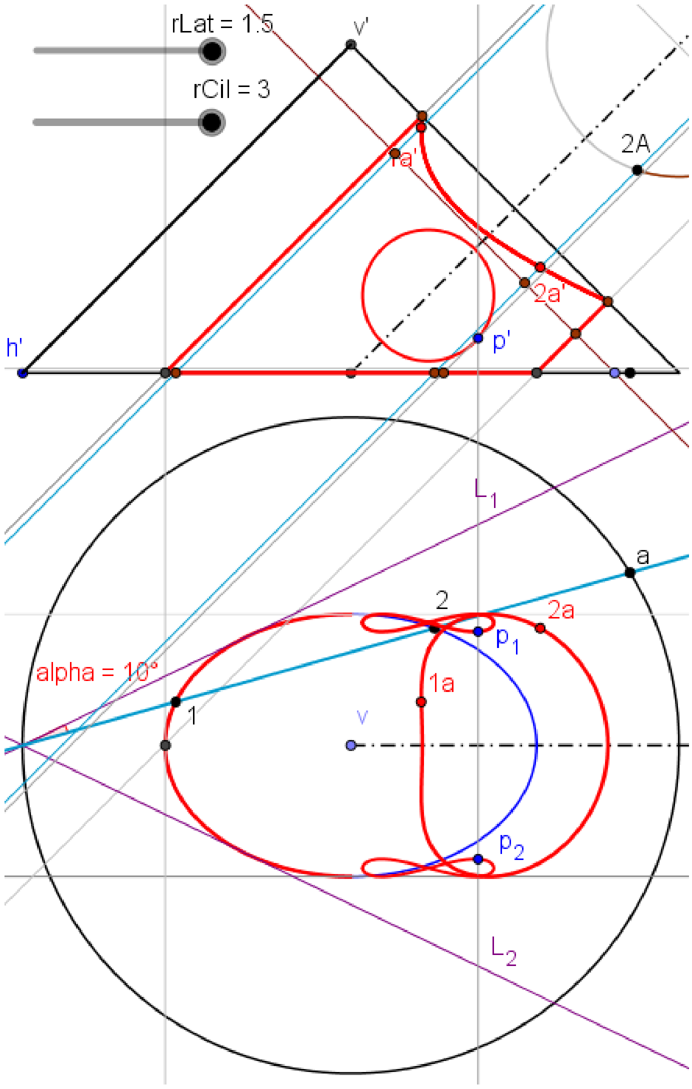


Figure S4. 4-way cylindric hopper showing the points generating the lateral loci.

The full loci of transformed holes and borders are computed through `locus` mathematical objects to give the pattern of the cylindric hopper. The half-pattern drawing obtained with this procedure is shown in Figure S5b, taking advantage of its symmetry. The additional green circle that contains three points of the lateral hole pattern is placed on top of the later to emphasize that the lateral hole is not circular.

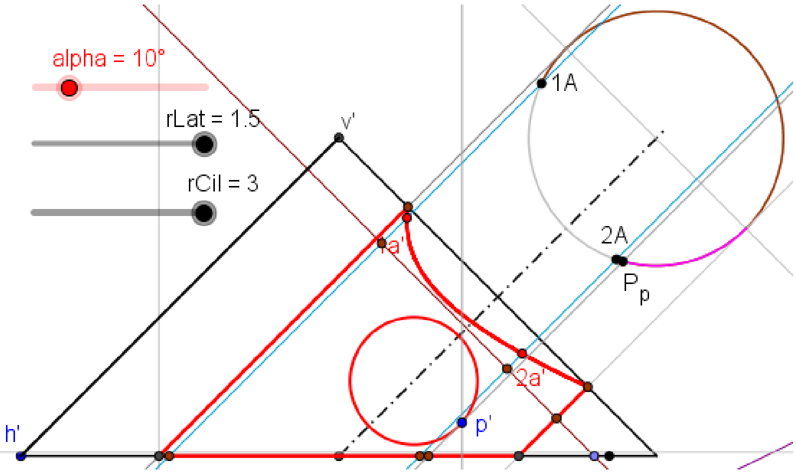
Cylindric connection of tronco-conic hopper outlet with round head

CEDG method.

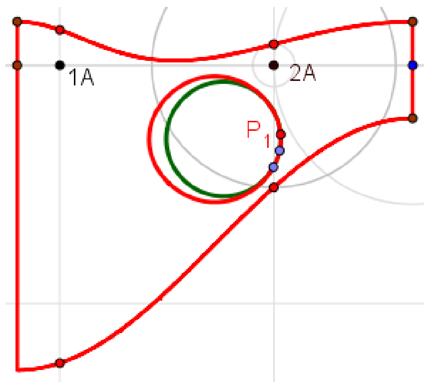
The following subsection clarifies the descriptive geometry procedures used to build a computer parametric model of the required cylindric connection and its flat pattern under Geogebra based CEDG.

The round outlet (center o′$_2$) is translated along the axis until the ends of the projected segment are placed in the same circle (center e′) that those of the horizontal round inlet (center o′$_1$), to obtain the position of the circular section in a non-revolution cone [4], as shows in Figure S6 for a projection plane parallel to the main symmetry plane of the cone.

After the tronco-conic surface is calculated, the direction of the round head centered in o′$_3$ of a vertical pipe that connects it to the non-parallel round outlet of the tronco-conic surface (center o′$_2$) is obtained. The projected segment of the round head is parallel to



(a) Vertical projection of the cylindric hopper, including the true-length view of the section perpendicular to the axis.



(b) Half-pattern obtained by an accurate extension of the inscribed prism method.

Figure S5. Vertical model and half-pattern of the 4-way cylindric hopper.

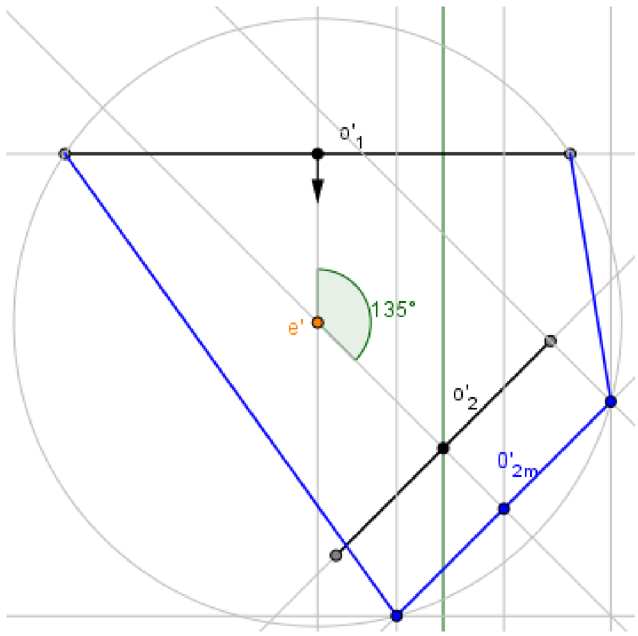


Figure S6. Tronco-conic surface determination with two non-parallel round borders.

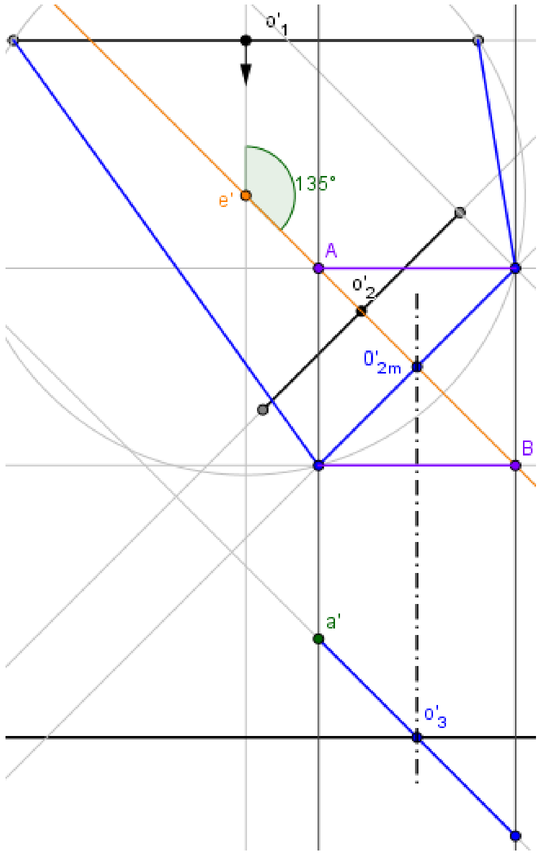


Figure S7. Determination of the bottom round head of the connecting vertical pipe.

segment A-B, which is generated by perpendiculars to the pipe axis through the ends of the o'_2 center round border (Figure S7).

The completed computer parametric model of this system is presented through the orthographic views in Figure S8a. It must be noted that the graphical drawing does not add any rendering and it keeps many auxiliary lines, points and legends to clarify the descriptive geometry foundation.

The flat pattern of the connecting vertical pipe is calculated by the same extension of the general method of inscribed prism [4] used in the first study case. The flattening process starts with the shortest generatrix line, which contains the point A at the plane section perpendicular to the pipe axis (Figure S8b).

A second generic generatrix line with the points b'_u (upper) and b'_l (lower) in vertical projection (Figure S8a) and the associated point B at the plane section perpendicular to the pipe axis are carried out to the flat pattern (Figure S8b).

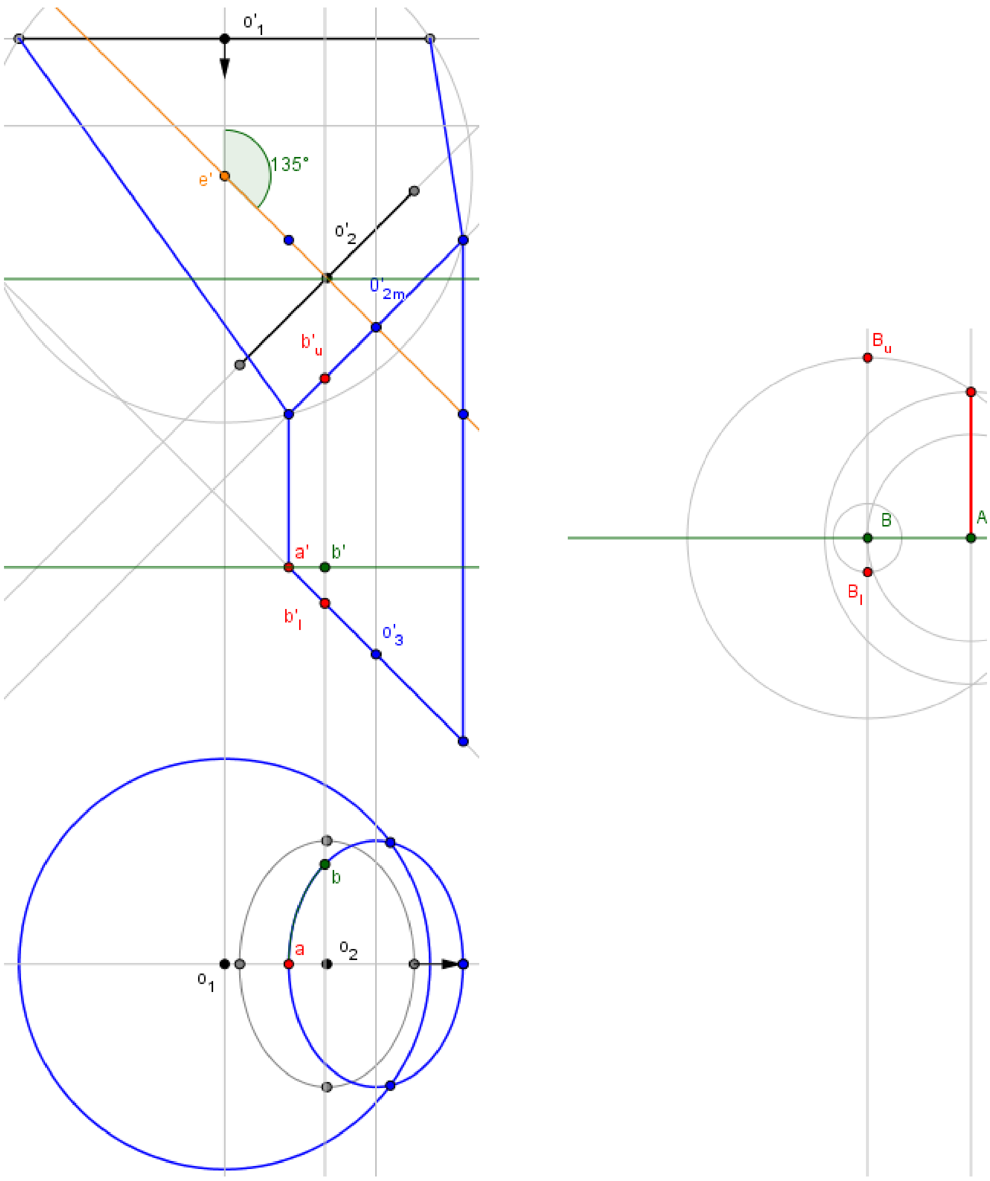
The points b'_u and b'_l in vertical projection are used to generate the loci of the transformed round borders of the pipe at the flat pattern, thanks to the `locus` mathematical object referred in the first study case. The full flat pattern associated with the computer parametric model of the system is presented in Figure S9.

Round - poligonal section transition with circular branch

CEDG method.

This subsection describes the descriptive geometry procedures and techniques used to build the computer parametric model of the transition surface and the associated flat pattern.

The required transformer is presented in Figure S10 by means of the orthographic views. The vertices of the triangles in the round section are obtained by assuring that triangles are tangent to conical surfaces at the adjacent generatrix lines [5].



(a) Required system model by two orthographic views.

(b) Starting of the pipe flattening using the generatrix line that contains the point B.

Figure S8. Tronco-conic hopper connected with vertical pipe (left) and building structure of the flat pattern (right).

This is accomplished by means of the intersection of round and poligonal section planes, which gives the line `i` (horizontal projection in Figure S10). The edges of the poligonal section are extended to cut the `i` line, generating points from which lines tangent to the round section are finally executed. This is shown in Figure S10 for points `a` and `b`. The complete descriptive geometry technique is reported in [5].

The lateral round pipe does not intersect with conical surfaces of the transformer for the whole range of the system's parameters. As a consequence the lateral hole will be an elliptic curve (Figure S10).

The accurate flattening of the round - poligonal transformer may be performed using an extension of the general technique of inscribed pyramid for conical surfaces. However, there is not a straight line in the development of oblique cones associated with a secant section of the cone, in opposition to cylindric surfaces. The computation of any point of the

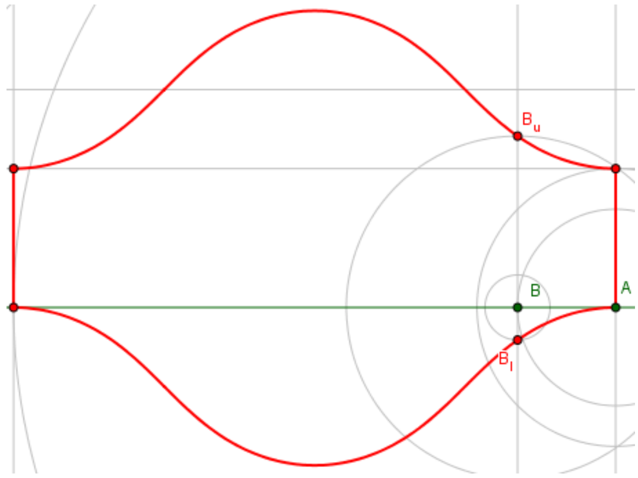


Figure S9. Full pattern of the vertical pipe, showing the generating points at each transformed border.

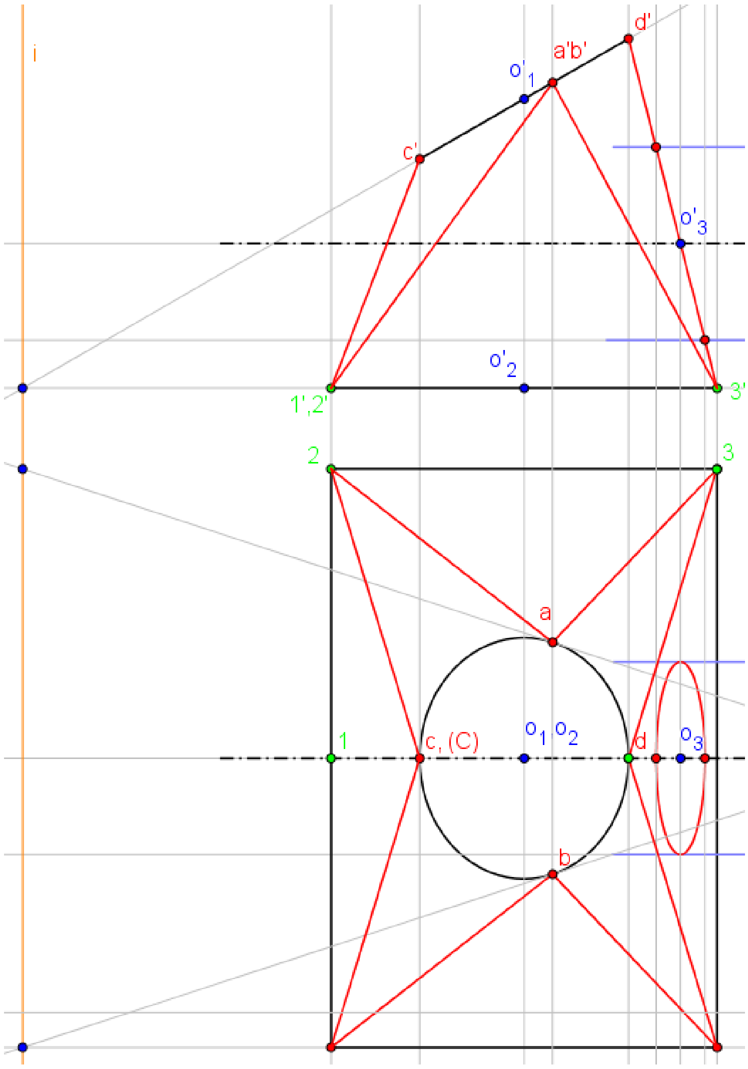


Figure S10. Round-poligonal transformer with additional lateral junction for round pipe, given by orthographic views (see text).

transformed curves from the cones's borders is associated with two degrees of freedom, instead of one.

It is thus needed a recursive `locus` mathematical object that generates each new point using just the previous generated point. As this type of `locus` object does not exist in Geogebra, we have designed a Geogebra Scripting code that executes an iterative loop with the required functional scope. The GGB code that computes the flat pattern of the cone with vertex 2 of Figure S10 is presented below.

```
1  tpathi = PathParameter(S_3);
2  tpathf = PathParameter(T_3);
3  inctpath = (tpathf - tpathi)/(NG-1);
4  LISTFLATINIT = {"ListFlat = {}","SetValue(ListFlat, 1, A_3)","SetValue(tpath,
      tpathi)"};
5  DEFFLAG = {"If(tpathi<tpathf, SetValue(FLAG," + "Text(tpath<tpathf,false)" +
     "),
6      SetValue(FLAG," +"Text(tpath>tpathf,false)"+"))"};
7  DO = {"SetValue(ListFlat, Length(ListFlat)+1, GenDevPointOblicuo1(Escala,
     LengthA,
8      LengthB, Height, HeadAng, DiamC, Z_2, c, A, C, d_2, l_1, Point(d_2,tpath)
       ,
9      Point(d_2,tpath+inctpath), B_3, ListFlat(Length(ListFlat)), d_5, n_1, B_2
       , F_3))",
10     "SetValue(tpath, tpath+inctpath)"};
11 LOOP = {"If("+FLAG+",Execute(Join(DO,LOOP)))"};
12 Execute(Join(LISTFLATINIT,DEFFLAG,LOOP));
```

CEDG Code 3: GGB code that computes the flat pattern of the cone with vertex 2 of Figure S10.

The first three code lines divides the border of the cone to be flattened in `NG` - 1 pieces, where `NG` is the number of generatrix lines. The command `GenDevPointOblicuo1`, included in the list of command codes of line 6 (DO list of strings) is executed for each border's point, from the beginning point (`tpathi`) to the end point (`tpathf`), to calculate the position of the associated flat transformed border's point. This is achieved by means of a conditional `LOOP` defined in the line 7, and executed in the last line of code. All the points that define the transformed curve are stored in the `ListFlat` variable, which is a list of points initialized at `LISTFLATINIT` commands sequence.

The command `GenDevPointOblicuo1` is a custom tool function of Geogebra created in a graphic way, during the calculation of a generic flat transformed border's point by means of descriptive geometry techniques [4]. Therefore, the inputs to this command are objects and variables of this model. We have used the true length between border's points instead of the chord length approximation.

A parametric pattern solution of the round-poligonal transformer with `NG` = 10 is shown in Figure S11. We take advantage of the symmetry of the transformer with respect to the vertical plane that contains the round - poligonal sections to compute the half of the pattern. Parameters's values are controlled with sliders. Buttons `FlattenV2` and `FlattenV3` execute the scripting code to flatten conical surfaces with vertex 2 and vertex 3. The points that define the transformed border of the cone are subsequently joined through spline interpolation to give the pattern solution in the Results Section (main text).

Maximum value of lateral round pipe diameter.

The maximum value of the lateral round pipe diameter (DiamCyl) that keeps the intersection limited to the triangle with vertex `d` (Figure S10) must be calculated to obtain the pattern associated to the third set of parameters' values (main text).

A first and direct procedure projects the cited triangle using the axis of pipe as direction, what gives the circular section of the pipe that can be adjusted to make it tangent to the triangle edges. This technique can be applied easily in CEDG and CAD systems. The Figure S12b shows the implementation into Solid Edge 2020, with a solution `DiamCyl` = 27.09 cm.

A second technique accomplishes the maximum value of `DiamCyl` through the rotation of the triangle of vertex `B` to place it parallel to a projective plane. We have then access to the true length planar forms. An homothetic transformation of the elliptic pipe section at

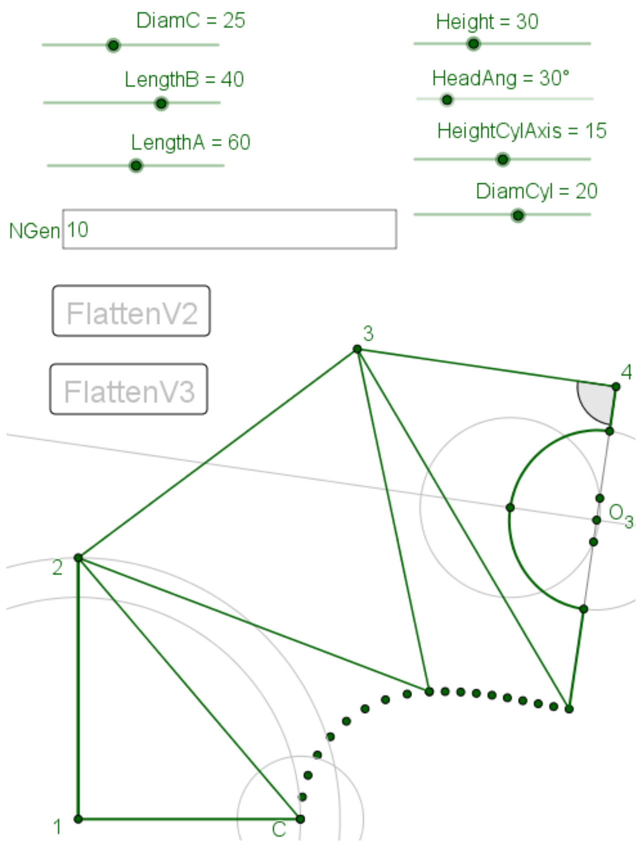
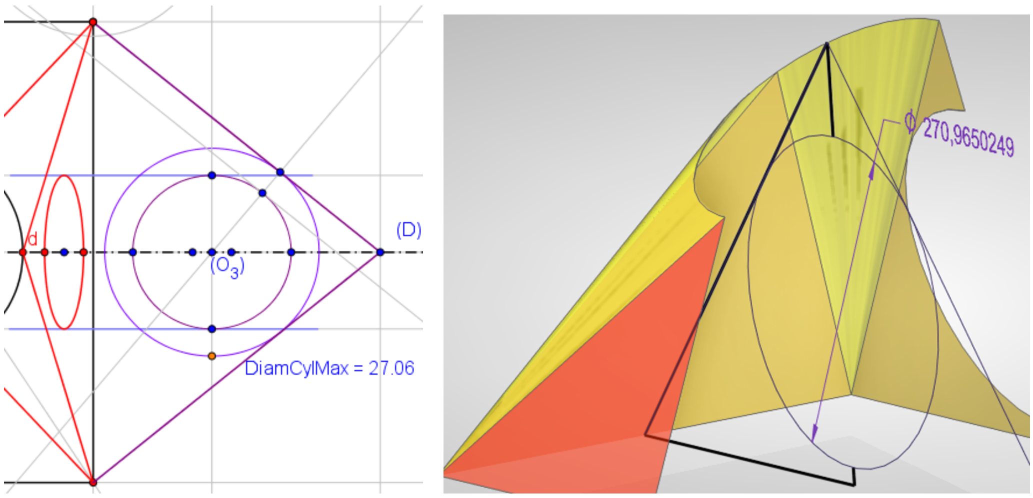


Figure S11. Flat half pattern of the round-poligonal transformer for `NG` = 10 and the reference set of parameters' values (main text).

this plane can make this curve tangent to the triangle edges, what gives maximum `DiamCyl` = 27.06 cm (Figure S12a). This method requires a 3D modeling approach that facilitates the execution of descriptive geometry procedures. Therefore it was implemented in CEDG.



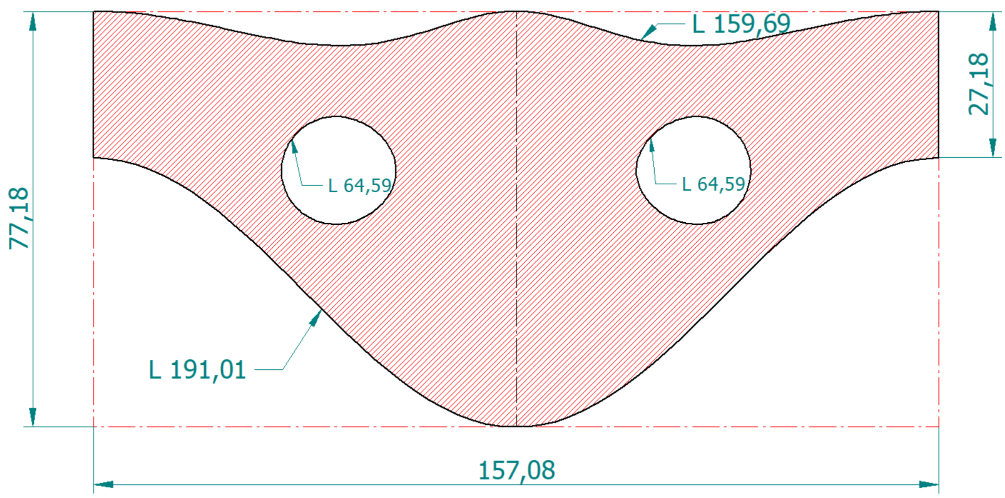
(a) CEDG. **(b)** Solid Edge 2020.

Figure S12. Maximum value of `DiamCyl` (cm) obtained by two methods and modeling approaches.

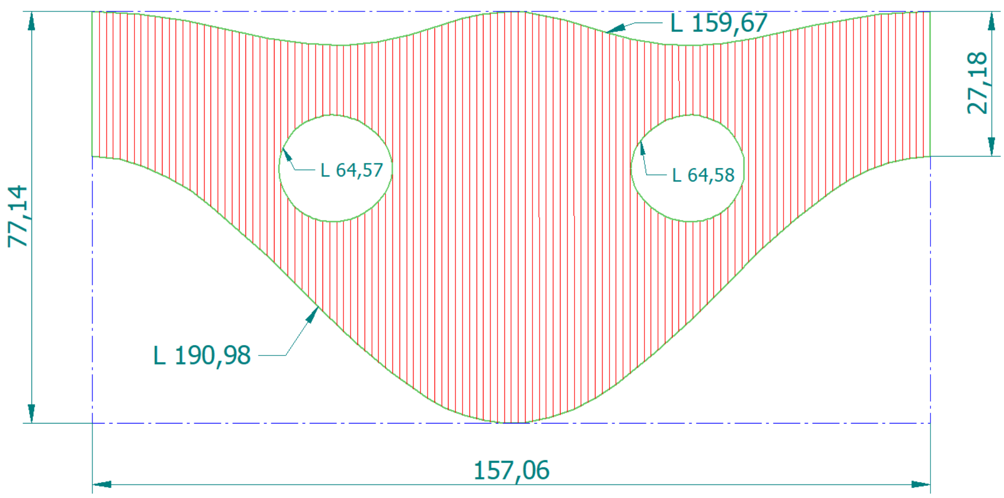
Although both methods must provide exact solutions (limited by the numerical precision of the computer implementation), they are limited by the accuracy of the transformer's model. This is addressed in the discussion Section of main text.

Results

Four - way cylindric hopper with conical coupling



(a) Solid Edge based.



(b) LogiTRACE based (120 generatrix lines).

Figure S13. Patterns of the 4-way cylindric hopper obtained by CAD approaches for the modified parameters' values. Dimensions are in cm.

The figure S13 shows the dimensioned flat patterns' drawings calculated through CAD approaches for `rCil`=2.5 m and `rLat`= 1 m (modified values).

The dimensioned pattern drawing calculated through the CEDG approach is presented in the figure S14.

The Table 1 summarizes the dimensions of the calculated flat patterns, together with the relative errors obtained according to the Methods Section of main text. The dimensions names are also presented in the main text.

The dimensions obtained by the CEDG were equivalent to the exact values, as explained in the main text.

The transformed curves associated to the hopper junctions are built as a set of interpolating curves in both CAD approaches. Therefore, their lengths, L_c, L_e and L_h were computed as the sum of these curves.

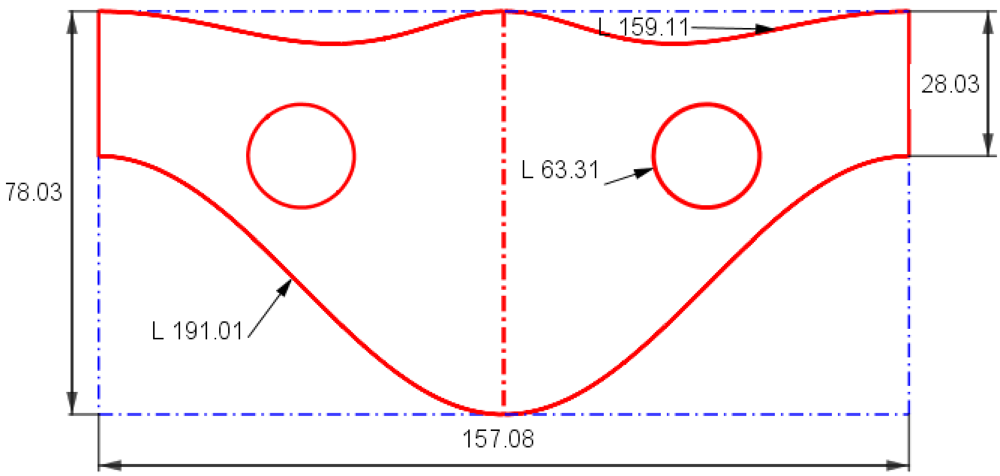


Figure S14. Dimensioned (cm) pattern of the 4-way cylindric hopper obtained by CEDG approach for the modified parameters' values.

Table 1: Calculated dimensions (cm) and associated relative errors (%) in the flat pattern of the cylindric hopper for the modified parameters' values, as a function of the technological approach.

Approach	L_e (*RE*%)	L_c (*RE*%)	L_h (*RE*%)	L_G (*RE*%)	L_g (*RE*%)
CEDG	191.01 (0%)	159.11 (0%)	63.31 (0%)	78.03 (0%)	28.03 (0%)
LogiTRACE†	190.98 (0.016%)	159.67 (0.352%)	64.57 (1.990%)	77.14 (1.154%)	27.18 (3.032%)
Solid Edge	191.01 (0%)	159.69 (0.365%)	64.59 (2.022%)	77.18 (1.089%)	27.18 (3.032%)

† 120 generatrix lines.

The transformed curves are associated to `locus` mathematical objects in Geogebra based CEDG. However, the `locus` lengths were not calculated using the in-series `Perimeter` command, but with a custom method that covers the locus with the number of points needed to reach the required accuracy in the length measure (two decimals in cm units).

Cylindric connection of tronco-conic hopper outlet with round head

Figures S15 and S16 show the dimensioned flat patterns' drawings calculated through the CAD approaches for alpha = 120° (see main text).

The dimensioned pattern drawing calculated through the CEDG approach is presented in the figure S17.

The Table 2 summarizes the main dimensions and relative errors of the calculated flat patterns. The dimensions names are defined in the main text. The dimensions obtained with the CEDG approach were equivalent to the exact values, as described in the main text.

Table 2: Calculated dimensions (cm) and associated relative errors (%) in the flat pattern of the cylindric connection between hopper and round head for the modified parameters' values, as a function of the technological approach.

Approach	L_c (*RE*%)	L_G (*RE*%)	L_g (*RE*%)	L_e (*RE*%)
CEDG	188.50 (0%)	139.56 (0%)	35.63 (0%)	145.33 (0%)
LogiTRACE†	188.49 (0.005%)	138.99 (0.408%)	35.13 (1.403%)	145.31 (0.014%)
Solid Edge	188.50 (0%)	139.05 (0.365%)	35.13 (1.403%)	145.35 (0.028%)

† 120 generatrix lines.

The lengths of transformed curves associated to the hopper's junctions were obtained using the same methods pointed in the first study case.

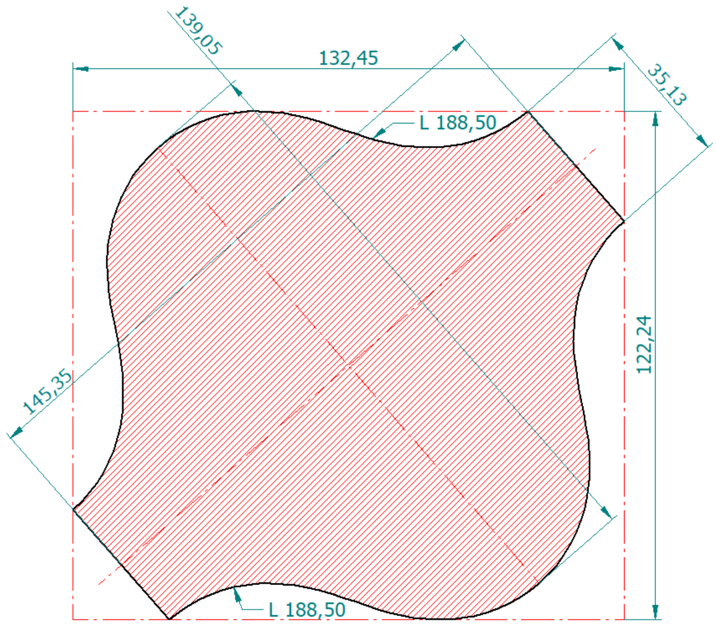


Figure S15. Dimensioned (cm) pattern of the cylindric connection hopper - round head obtained by Solid Edge for the modified parameters' values.

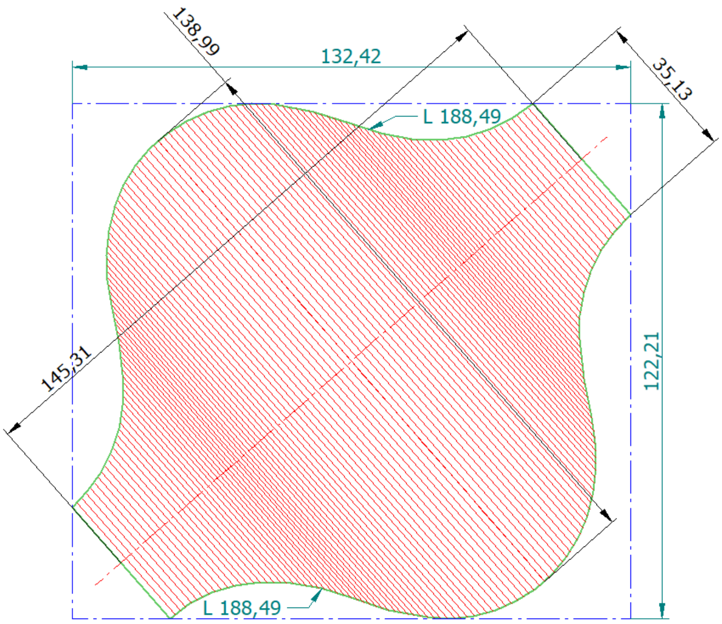


Figure S16. Dimensioned (cm) pattern of the cylindric connection hopper - round head obtained by LogiTRACE with 120 generatrix lines for the modified parameters' values.

The value of dimension L_c for alpha = 120° is kept equal to the value calculated for alpha = 135°, as expected, since the perimeter length of the round head does not depend on alpha angle. Calculation errors for L_c and L_e are much smaller than 0.1 %, in agreement to the errors obtained for alpha = 135° (main text).

The values of L_g dimensions calculated with Solid Edge and LogiTRACE are the same, what confirms that this pattern's dimension is propagated from the Solid Edge 3D model (main text).

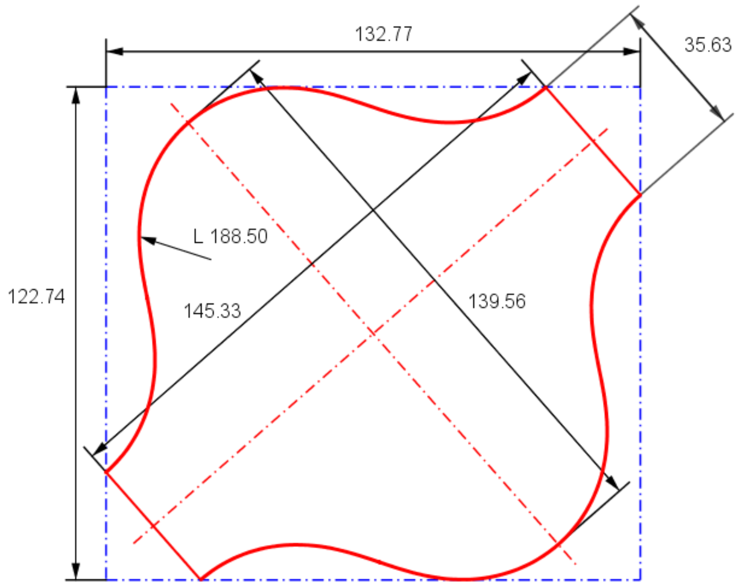


Figure S17. Dimensioned (cm) pattern of the cylindric connection between hopper and round head obtained by CEDG approach for the modified parameters' values.

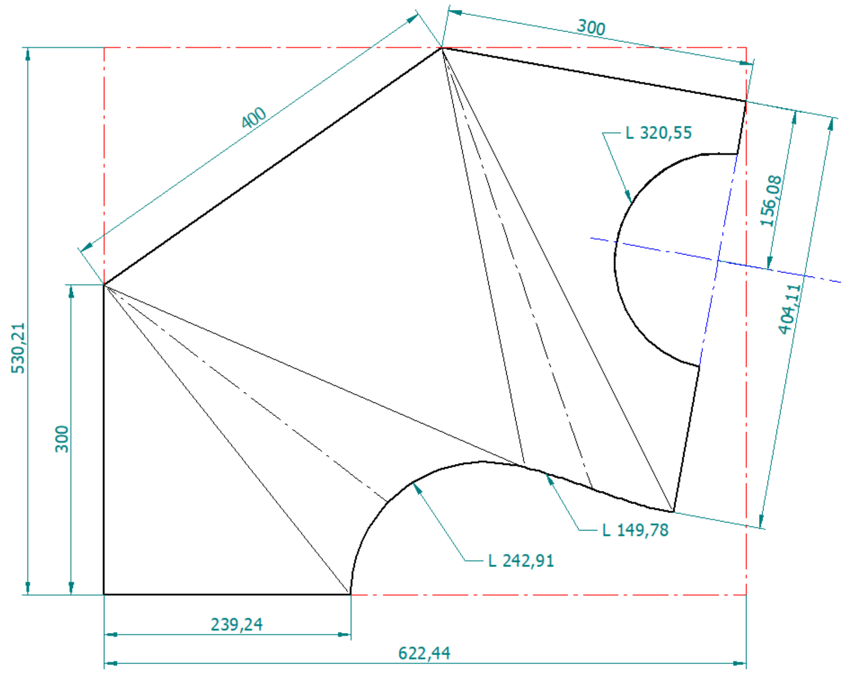


Figure S18. Dimensioned (mm) half- pattern of the round-poligonal transformer with circular branch obtained by Solid Edge for `HeadAng` equal to 45°.

Round - poligonal section transition with circular branch

Figures S18 and S19 show the dimensioned flat patterns' drawings of the round-poligonal section transition with round branch calculated through the CAD approaches for the second set of parameters', defined from the set of reference values with `HeadAng` modified to 45°. The related dimensioned pattern drawing computed through the CEDG approach is shown in the figure S20.

The Table 3 summarizes the main dimensions and relative errors of the flat patterns of figures S18, S19 and S17. The dimensions names are defined in the main text.

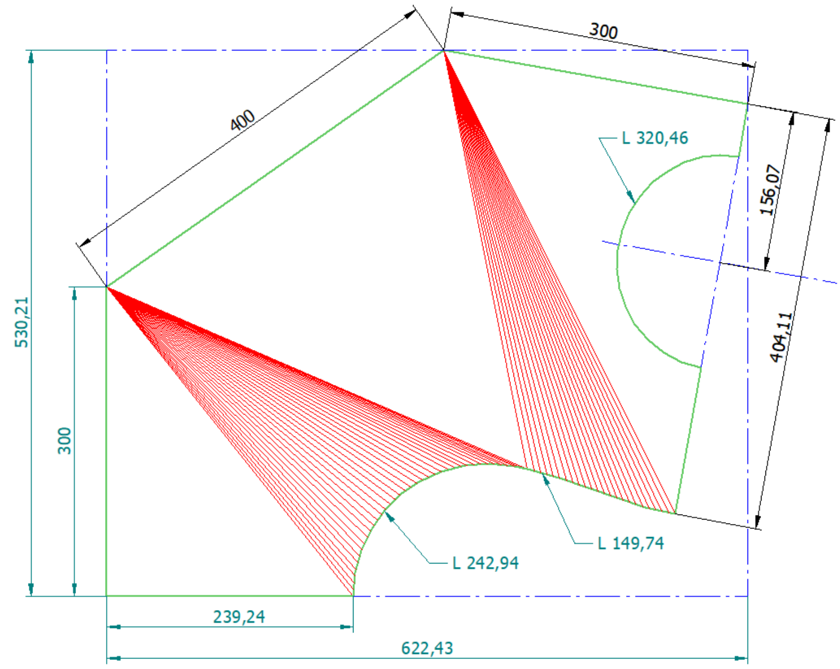


Figure S19. Dimensioned (mm) half- pattern of the round-poligonal transformer with circular branch obtained by LogiTRACE with 120 generatrix lines for `HeadAng` equal to 45°.

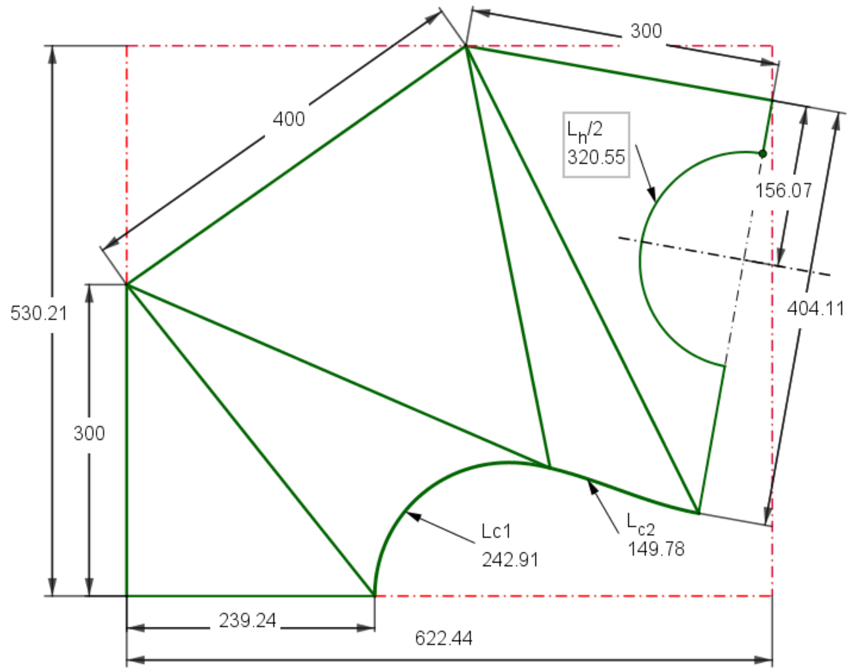


Figure S20. Dimensioned (mm) half- pattern of the round-poligonal transformer with circular branch obtained by CEDG with 120 generatrix lines for `HeadAng` equal to 45°.

Figure S21 shows the dimensioned flat patterns' drawings of the round-poligonal section transition with round branch obtained through CAD approaches for the third set of parameters, defined from the set of reference values with the maximum value of the lateral round pipe diameter, `DiamCyl` = 270.9 mm, calculated with Solid Edge (Methods section).

The related dimensioned pattern drawing computed through the CEDG approach is presented in the figure S22. This was obtained for a maximum value of `DiamCyl` = 270.6 mm, calculated with CEDG (Methods section). Differences between the maximum `DiamCyl` values are addressed in the Discussion section.

Table 3: Calculated dimensions (mm) and associated relative errors (%) of the flat pattern of the round - poligonal transformer with circular branch, for `HeadAng` = 45°, as a function of the technological approach.

Approach	L_{c1} **(*RE*%)**	L_{c2} **(*RE*%)**	L_h **(*RE*%)**	L_{t1} **(*RE*%)**	L_{t2} **(*RE*%)**
CEDG†	242.91 (0%)	149.78 (0%)	641.10 (0%)	239.24 (0%)	404.11 (0%)
LogiTRACE†	242.94 (0.012%)	149.74 (0.027%)	640.92 (0.028%)	239.24 (0%)	404.11 (0%)
Solid Edge	242.91 (0%)	149.78 (0%)	641.10 (0%)	239.24 (0%)	404.11 (0%)

† 120 generatrix lines.

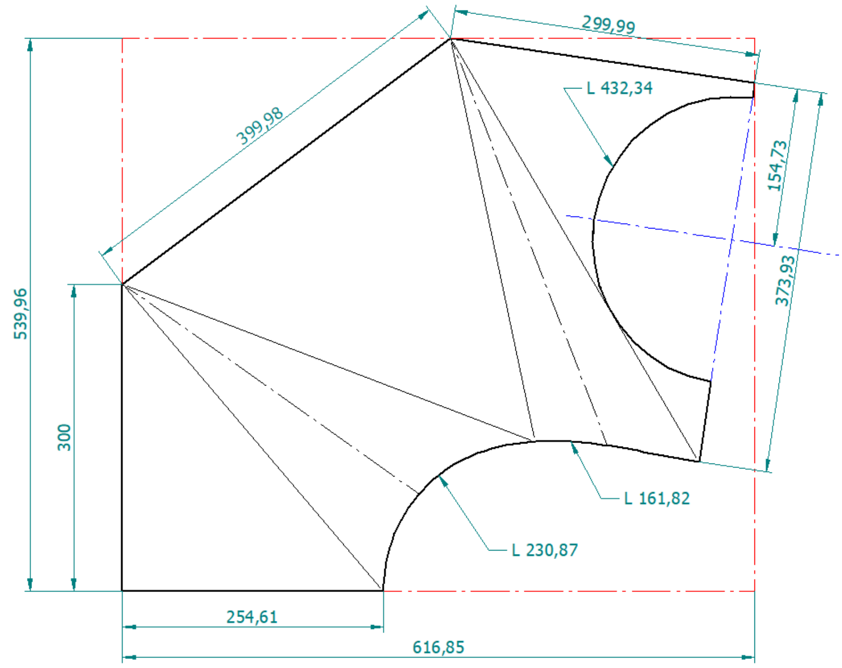
The Table 4 summarizes the main dimensions and relative errors of the flat patterns of figures S21a, S21b and S22. The dimensions names are defined in the main text.

Table 4: Calculated dimensions (mm) and associated relative errors (%) of the round - poligonal transformer with circular branch flat pattern, for the maximum value of `DiamCyl`, as a function of the technological approach.

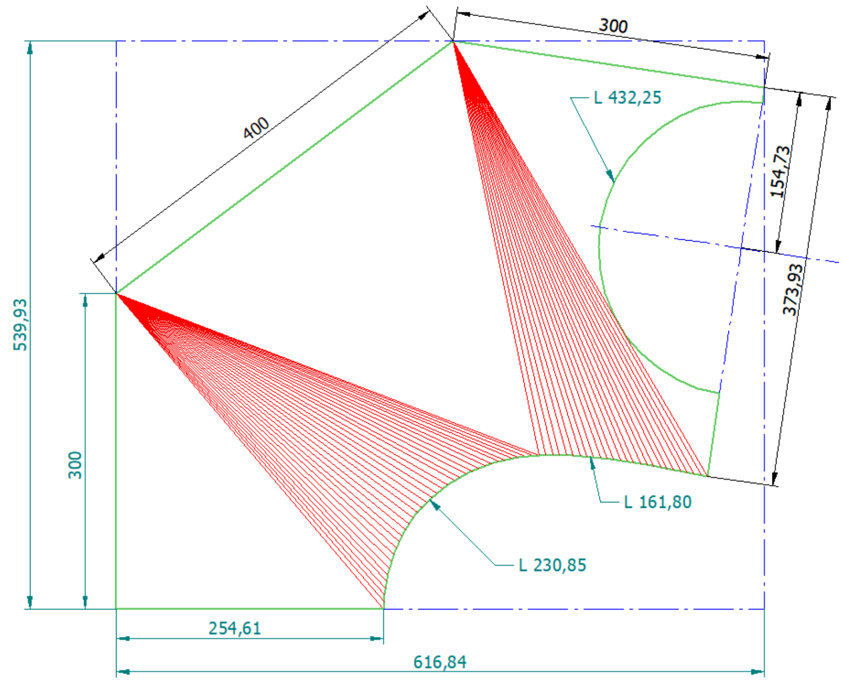
Approach	L_{c1} **(*RE*%)**	L_{c2} **(*RE*%)**	L_h **(*RE*%)**	L_{t1} **(*RE*%)**	L_{t2} **(*RE*%)**
CEDG†	230.87 (0%)	161.82 (0%)	863.56 (0%)	254.60 (0%)	373.93 (0%)
LogiTRACE†	230.85 (0.009%)	161.80 (0.012%)	864.5 (0.109%)	254.61 (0.004%)	373.93 (0%)
Solid Edge	230.87 (0%)	161.82 (0%)	864.68 (0.130%)	254.61 (0.004%)	373.93 (0%)

† 120 generatrix lines.

The lengths of transformed curves were obtained using the same methods pointed in the previous study cases. With the exception of L_h, all of the patterns' dimensions of Table 4 are equal to those ones obtained for the reference parameters' values (main text), what confirms the stability of the computer models, since only L_h depends on `DiamCyl`. Relative errors of L_h in Table 4 include inaccuracies from the flattened process and the calculation of maximum `DiamCyl`.



(a) Solid Edge based.



(b) LogiTRACE based (120 generatrix lines).

Figure S21. Dimensioned (mm) half- pattern of the round-poligonal transformer with circular branch from CAD approaches for the maximum lateral round.

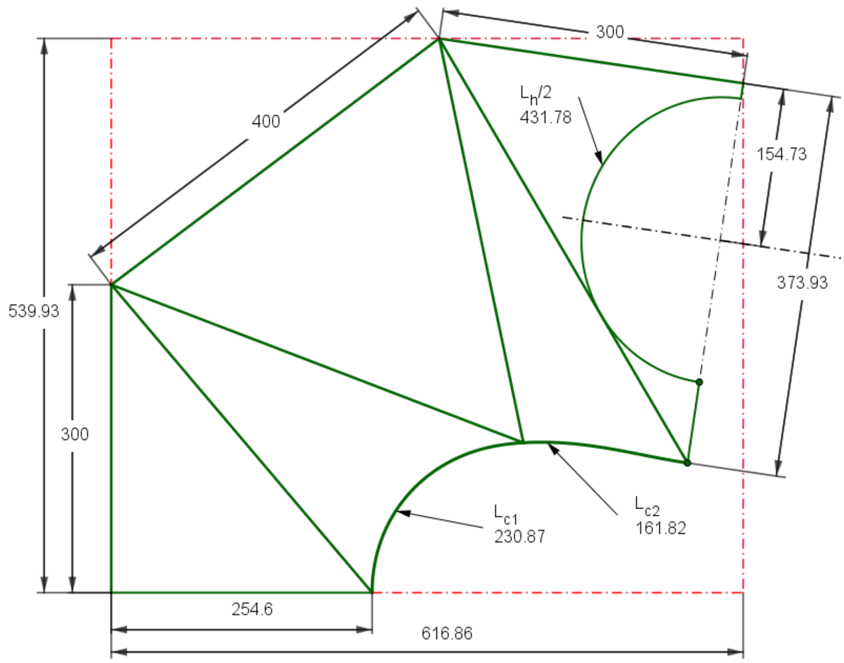


Figure S22. Dimensioned (mm) half- pattern of the round-poligonal transformer with circular branch from CEDG with 120 generatrix lines for maximum lateral round.

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