



# *Article* **Research on a Control Strategy of the Symmetrical Four-Roller Bending Process Based on Experiment and Numerical Simulation**

**Hongqiang Cao 1,2, Gaochao Yu 1,2,\*, Chunfang Yang 1,2 and Jun Zhao 1,2**

- <sup>1</sup> Key Laboratory of Advanced Forging & Stamping Technology and Science, Yanshan University, Ministry of Education of China, Qinhuangdao 066004, China; frank@ysu.edu.cn (H.C.); cfyang@stumail.ysu.edu.cn (C.Y.); zhaojun@ysu.edu.cn (J.Z.)
- <sup>2</sup> College of Mechanical Engineering, Yanshan University, Qinhuangdao 066004, China
- **\*** Correspondence: gch\_yu@ysu.edu.cn

**Abstract:** The intelligent production of sheet metal is a comprehensive technology involving control science, computer science, and sheet metal production. Intelligent rolling is an important development in the production process of sheet metal. In this paper, a new symmetrical four-roller bending (SFRB) process is introduced, which consists of feeding, pre-bending, reverse roll bending, second bending, and forward roll bending and unloading. A control strategy is proposed for the process, including on-line monitoring of curvature, on-line identification of the springback law, on-line prediction of final reduction, and control strategy. A convenient and reliable on-line curvature monitoring method is proposed. The quantitative relationship between the reduction and the curvature, in the form of a quadratic function, was established by physical experiments and numerical simulation, and the online identification of the springback law was realized. An on-line prediction method of the final reduction is proposed, and the determination principle of the reduction of three pre-bending processes is given. Finally, the control strategy of the SFRB process was verified by physical experiments. The relative error of the curvature radius of the final formed parts can be controlled within 0.8%. This research provides new insights into intelligent rolling.

**Keywords:** thin-wall pipes; symmetrical four-roller bending; springback; control strategy

# **1. Introduction**

The intelligent production of sheet metal is a comprehensive subject that combines control science, computer science, and sheet metal production theory [\[1\]](#page-12-0). Intelligent rolling is an important development in the production process of sheet metal. According to the different number of rollers, the roll bending process can be divided into three forms, namely, two-roller, three-roller, and four-roller. In view of different forms of rolling process, scholars have undertaken a significant amount of research on the relationship between the forming curvature and reduction, defect formation, and structural optimization, using theoretical analysis, numerical simulation, and experimental research methods. In the field of three-roller bending, using different methods, Gandhi et al. [\[2\]](#page-12-1), Zhang et al. [\[3\]](#page-12-2), Hu and Wang [\[4\]](#page-12-3), Gandhi et al. [\[5\]](#page-12-4) and Fu et al. [\[6\]](#page-12-5), Quan et al. [\[7\]](#page-12-6), Chudasama et al. [\[8\]](#page-12-7), and Yu et al. [\[9](#page-12-8)[,10\]](#page-12-9) constructed mathematical models and obtained theoretical results. Zeng et al. [\[11\]](#page-12-10), Ktari et al. [\[12\]](#page-12-11), and Feng et al. [\[13](#page-12-12)[,14\]](#page-12-13) carried out numerical simulations to obtain the deformation characteristics of the three-roller bending process. Huang et al. [\[15\]](#page-13-0) presented a new three-roller setting round process to achieve calibration of the roundness of a whole pipe without changing the pipe circumference. In the field of four-roller bending, Hua et al. [\[16](#page-13-1)[,17\]](#page-13-2), Gao et al. [\[18\]](#page-13-3), and Sun et al. [\[19](#page-13-4)[,20\]](#page-13-5) established mathematical models, and Gu et al. [\[21\]](#page-13-6) developed a finite element model to analyze the four-roll bending process. Yu et al. [\[22\]](#page-13-7) developed the SFRB process and predicted the reduction of the upper roller; results showed the ovality of the formed part was less than 5%. Li et al. [\[23\]](#page-13-8) analyzed the



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two-roller bending process and established a theoretical model. Gong et al. [\[24\]](#page-13-9) analyzed the main influencing factors and the distribution of stress and strain.

Because of the simplification of engineering plastic mechanical and material constitutive models, such as the linear kinematic, linear isotropic, and simple kinematic hardening models [\[9](#page-12-8)[,25](#page-13-10)[,26\]](#page-13-11), it remains a challenge to accurately calculate the reduction according to the target curvature. In production, this mainly depends on the experience of workers and die adjustment, which greatly reduces the production efficiency, and errors are certain to occur. The intelligent control of sheet metal forming is an effective way to address the above problems. Errors caused by theoretical analysis can be compensated for by the monitoring, identification, prediction, and control of the deformation law.

Zhao et al. [\[1\]](#page-12-0) carried out intelligent drawing of both axisymmetric and non-axisymmetric parts. Li [\[27\]](#page-13-12) developed an intelligent control system of the JCO forming process for LASW pipes, including theoretical analysis, real-time monitoring of the forming process, identification of material performance parameters, and prediction of process parameters. Zhao et al. [\[28\]](#page-13-13) developed the control strategy of multi-point bending oneoff straightening and control strategy of three-point bending, multi-step straightening. Zhao et al. [\[29](#page-13-14)[,30\]](#page-13-15) constructed an over-bending setting round intelligent control system for the pipe ends of large pipes, including a pipe-end ovality recognition system, two-step and three-step over-bending setting round control strategy, and real-time prediction of optimum process parameters.

This paper aims to enhance the existing research related to roll bending by proposing the SFRB process. A control strategy is proposed for this process, including on-line monitoring of curvature, on-line identification of the springback law, on-line prediction of the final reduction, and a control strategy process. A convenient and reliable on-line curvature monitoring method is also proposed. The quantitative relationship between the reduction and the curvature was established by physical experiments and numerical simulation, and the online identification of the springback law was realized. An on-line prediction method of the final reduction is also proposed, and the determination principle of the reduction of three pre-bending processes is outlined. Finally, the control strategy of the SFRB process was verified by physical experiments. This research provides new insights into intelligent rolling.

# **2. SFRB Process**

The main working parts of the process are two upper rollers and two lower rollers. The two upper rollers are on the top and active, and the other two rollers are at the bottom and driven. The four rollers are symmetrical along the vertical plane. As shown in Figure [1,](#page-2-0) the main process includes feeding, pre-bending, reverse roll bending, final bending, positive roll bending, unload, and springback [\[22\]](#page-13-7). The process has the advantages of high forming precision, small remaining straight edge, high production efficiency, and simple equipment structure.

<span id="page-2-0"></span>

 $\mathbf{F}(\mathbf{r})$  processing the bending, (**c**) bending completion (**b**) unleed and enginalized (**f**) positive roll bending, (**g**) bending completion, (**h**) unload and springback. (**f**) positive roll bending, (**g**) bending completion, (**h**) unload and springback. **Figure 1.** SFRB process: (**a**) feeding, (**b**) pre-bending, (**c**) reverse roll bending, (**d**) final bending, (**e**) positive roll bending,

# **3. Control Strategy**

# **3. Control Strategy**  *3.1. Online Monitoring of Curvature*

On-line real-time monitoring of the roller bending process can reflect the macroscopic mechanical and geometric parameters of the processed object by adopting effective testing methods. The curvature radius is an important parameter to be monitored online during the roller bending process. At present, in actual production, the method of manual  $\frac{1}{n}$ measurement is mostry adopted, which has low emerging and hage error. To realize the purpose of intelligent detection, in this study a new intelligent measurement method was propose a manger is concerned, which can get a measurement is more and large expected. The measuring principle of the method is as follows: after pre-bending or roll bending, the three positions of the formed part are measured, and the curvature radius of the formed part is obtained using the three-point circle principle. The measurement principle is shown in Figure 2. measurement is mostly adopted, which has low efficiency and large error. To realize the

The measuring point is taken as the coordinate origin, and the direction from the measuring point to position point 1 as the *x* direction to establish the coordinate system. The coordinates of position No.1, position No.2, and position No.3 are(*ρ*1, 0), (*ρ*<sup>2</sup> cos *α*, *ρ*<sup>2</sup> sin *α*), and  $(\rho_3 \cos(\alpha + \beta), \rho_3 \sin(\alpha + \beta))$ . The center coordinate is  $(x_0, y_0)$ . The coordinates of three positions are brought into the equation of circle:

$$
\begin{cases}\n(\rho_1 - x_0)^2 + (0 - y_0)^2 = r^2 \\
(\rho_2 \cos \alpha - x_0)^2 + (\rho_2 \sin \alpha - y_0)^2 = r^2 \\
(\rho_3 \cos(\alpha + \beta) - x_0)^2 + (\rho_3 \sin(\alpha + \beta) - y_0)^2 = r^2\n\end{cases}
$$
\n(1)

From Equation (1), the center( $x_0$ ,  $y_0$ ) and curvature radius *r* can be obtained.

<span id="page-3-0"></span>

**Figure 2.** The principle of measuring the curvature radius.

### *3.2. Online Identification of Springback Law*

The quantitative description of the relationship between the upper roller reduction and curvature is the basis of realizing the intelligent control of roll bending. The accuracy of quantitative description determines the identification accuracy and prediction accuracy of the intelligent control system. Based on the monitoring results of curvature, the nonlinear fitting method is adopted to process the results of experiments and numerical simulations, so as to identify the relationship between the reduction and curvature.

# 3.2.1. Experimental Identification

The SFRB device is shown in Figure [3.](#page-4-0) The main process parameters of the device are shown in Table [1.](#page-3-1) ST12 cold-rolled plate and 304 stainless steel, both with 2 mm thickness, were chosen. The relationship between the reduction and curvature was studied using 158, 162, 166, and 168 mm lower roller spacing under different reduction conditions.

<span id="page-3-1"></span>**Table 1.** Main process parameters.



<span id="page-4-0"></span>

**Figure 3.** The SFRB device.

When the plate and process conditions are determined, the curvature and reduction are described by Equation (2). The parameters were estimated by the user-defined function equation fitting of Origin Software; the results are shown in Figure [4,](#page-4-1) and the fitting parameters are shown in Table [2.](#page-5-0)

<span id="page-4-1"></span>

$$
K(h) = ah^2 + bh + c \tag{2}
$$

**Figure 4.** The relationship between the reduction and curvature from experiments: (**a**) ST12 cold rolled plate, (**b**) 304 stainless steel.



<span id="page-5-0"></span>**Table 2.** Fitting parameters from experiments.

It can be seen from the fitting results that under the condition of keeping the plate and the lower roller spacing unchanged, the relationship between curvature and reduction is a univariate quadratic function. In practical application, this relationship can be used to select the appropriate lower roller spacing and determine the appropriate upper roller reduction.

# 3.2.2. Numerical Simulation Identification

To increase the accuracy of the research results, the numerical simulation method was used to further verify the relationship between the curvature and reduction. The process parameters of numerical simulations were the same as those of the experiments. The mechanical properties of ST12 cold-rolled plate and 304 stainless steel are shown in Figure [5](#page-5-1) and Table [3.](#page-6-0)

<span id="page-5-1"></span>

**Figure 5.** True stress–strain curve.



<span id="page-6-0"></span>**Table 3.** Mechanical properties of pipes.

 $\overline{a}$ 

The finite element model of the SFRB process was developed using ABAQUS software, as shown in Figure [6.](#page-6-1) The two upper rollers and two lower rollers are assumed to be an analytical rigid body. The plate is a deformable body. The master–slave contact condition is set. The frictional coefficient is 0.18. The process was analyzed using the Dynamic/ Implicit module.

<span id="page-6-1"></span>

**Figure 6.** Numerical simulation model.

The relationship between the curvature and reduction and the fitting parameters from the numerical simulation are shown in Figure [7](#page-6-2) and Table [4.](#page-7-0) It can be seen that the relationship between the curvature and reduction still satisfies the one-dimensional quadratic function for ST12 cold rolled plate and 304 stainless steel.

<span id="page-6-2"></span>

**Figure 7.** The relationship between the reduction and the curvature from numerical simulations: (**a**) ST12 cold rolled plate, (**b**) 304 stainless steel.



<span id="page-7-0"></span>**Table 4.** Fitting parameters from numerical simulations.

### 3.2.3. The Least Square Method

According to the experiments and numerical simulations, the relationship between the curvature and reduction was determined as a univariate quadratic function. To determine the three coefficients in Equation (2), at least three pre-bendings are needed; then, the coefficients of *a*, *b*, and *c* are determined by the least square method. Assuming that the *i*th reduction corresponding to the curvature *k<sup>i</sup>* , and the total number of reductions is *n*, then the deviation between the curvature and the function curve can be expressed as:

$$
\sum_{i=1}^{n} \left[ k_i - (ah_i^2 + bh_i + c) \right]
$$
 (3)

The sum of squared residuals is:

$$
S = \sum_{i=1}^{n} \left[ k_i - \left( a h_i^2 + b h_i + c \right) \right]^2
$$
 (4)

In order to solve the minimum value of *S*, let:

$$
\begin{cases}\n\frac{\partial M}{\partial a} = 0\\ \n\frac{\partial M}{\partial b} = 0\\ \n\frac{\partial M}{\partial c} = 0\n\end{cases}
$$
\n(5)

Then:

$$
\begin{cases}\n a \sum_{i=1}^{n} h_i^4 + b \sum_{i=1}^{n} h_i^3 + c \sum_{i=1}^{n} h_i^2 = \sum_{i=1}^{n} h_i^2 k_i \\
 a \sum_{i=1}^{n} h_i^3 + b \sum_{i=1}^{n} h_i^2 + c \sum_{i=1}^{n} h_i = \sum_{i=1}^{n} h_i k_i \\
 a \sum_{i=1}^{n} h_i^2 + b \sum_{i=1}^{n} h_i + nc = \sum_{i=1}^{n} k_i\n\end{cases}
$$
\n(6)

The three coefficients can be solved using Equation (6).

### *3.3. Online Prediction of Final Reduction*

Considering the actual detection and specific operation, the determination principle of the initial three reductions is as follows: The first reduction  $h_1$  is greater than the elastic limit reduction  $h_e$  to ensure plastic deformation. The third reduction  $h_3$  is the reduction when the bending curvature before springback is the target curvature  $h_t$ , to ensure that the bending curvature after the three pre-bendings does not exceed the target curvature. The second reduction  $h_2$  is between the first and the third. According to the above principles, the reduction of the first, second, and third pre-bending are, respectively:

$$
h_1 = (3h_e + h_t)/4
$$
 (7)

$$
h_2 = (3h_e + 5h_t)/8
$$
 (8)

$$
h_3 = h_\mathfrak{t} \tag{9}
$$

In the rolling process, the maximum strain on the plate section occurs on the outer surface, which can be expressed as:

$$
\varepsilon_{\text{out}} = \frac{t}{2\rho} \tag{10}
$$

where,  $\varepsilon_{\text{out}}$  is the strain of outer surface, *t* is the thickness, and  $\rho$  is the radius of curvature of the neutral layer after deformation.

Let

$$
\varepsilon_{\text{out}} = \varepsilon_{\text{e}} = \frac{\sigma_{\text{s}}}{E} \tag{11}
$$

where  $\varepsilon_e$  is the elastic limit strain.

Take Equation (10) into Equation (11), and

$$
\rho_{\rm e} = \frac{Et}{2\sigma_{\rm s}}\tag{12}
$$

where  $\rho_e$  is the radius of curvature of the neutral layer under the elastic limit condition.

Under the condition of four-point bending, the shape of the plate between the two upper rollers is equal to the curvature arc. Due to the short remaining straight edge in the SFRB process, it is assumed that the shape of the slab between the two lower rollers is also an equal curvature arc under the elastic limit condition. The corresponding center angle is  $2\alpha_e$ , as shown in Figure [8;](#page-8-0) then:

$$
\sin \alpha_{\rm e} = \frac{L_1}{2\rho_{\rm e} + t + 2R_1} \tag{13}
$$

where *L* is the plate length between the two lower rollers.

<span id="page-8-0"></span>

**Figure 8.** Geometric relationships in the roller bending process.

According to the geometric relationship, there is:

$$
h_{\rm e} = \rho_{\rm e} (1 - \cos \alpha_{\rm e}) \tag{14}
$$

Substituting Equation (13) into Equation (14), the elastic limit reduction  $h_e$  is given by:

$$
h_e = \frac{Et}{2\sigma_s} \left( 1 - \sqrt{1 - \left( \frac{\sigma_s L_l}{Et + \sigma_s t + 2\sigma_s R_l} \right)^2} \right)
$$
(15)

When the bending curvature is the target curvature, according to the geometric relationship,  $h_t$  can be expressed as:

$$
h_{t} = H - (\rho_{t} + R_{1} + t/2) \cos \alpha_{t} + (\rho_{t} - R_{u} - t/2) \cos \beta_{t}
$$
 (16)

$$
\alpha_{t} = \arcsin \frac{L_1}{2\rho_t + t + 2R_1}
$$
\n(17)

$$
\beta_t = \arcsin \frac{L_u}{2\rho_t - t - 2R_u} \tag{18}
$$

where *H* is the initial vertical distance between upper and lower rollers, *ρ*<sub>t</sub> is the target curvature radius.

Through the on-line monitoring of curvature and the online identification of the springback law, the relationship between the reduction and curvature after rolling *n* times can be obtained, and the final reduction of forming the target curvature can be calculated as:

$$
h_{\rm m} = \frac{\sqrt{b^2 - 4a(c - K_0)} - b}{2a} \tag{19}
$$

In production practice, under the condition of a suitable lower roller span, the relationship between the upper roller reduction and curvature can be determined by pre-bending three times. This can be used to predict the upper roller reduction to roll the shape with an arbitrary curvature, which provides a basis for the control strategy of the four-roller bending process.

#### *3.4. Control Strategy Process*

According to the three steps of on-line monitoring of curvature, on-line identification of springback law, and on-line prediction of final reduction, the flow chart of the control strategy for the four-roller process was designed as shown in Figure [9.](#page-10-0)

There are inevitable errors in the measurement after each pre-bending, and the amount of data obtained from pre-bending three times is not enough to accurately fit the curve of the one variable quadratic function of the reduction and the curvature. Therefore, when the curvature of the nth ( $n \geq 4$ ) rolled part fails to meet the accuracy requirements of the target curvature, the results of this rolling can be fed back to the calculation process, the database of relationship of the reduction and curvature can be expanded, and the three coefficients of the quadratic function can be fitted again using the least square method. This further improves the fitting accuracy of the quadratic function until the rolled part meets the accuracy requirements.

<span id="page-10-0"></span>

**Figure 9.** Flowchart of control strategy.

### **4. Experimental Verification**

According to the above-mentioned control strategy and the SFRB device, the ST12 cold rolled plate and 304 stainless steel plate were selected for the verification experiments. The thickness was 2 mm. The three pre-bending processes and the forming part are shown in Figures [10](#page-10-1) and [11,](#page-11-0) respectively. The experimental results are shown in Tables [5](#page-11-1) and [6.](#page-11-2)

<span id="page-10-1"></span>

**Figure 10.** Three pre-bending processes: (**a**) Pre-bending process, (**b**) Plate after pre-bending, (**c**) Plate after second pre-bending, (**d**) Plate after third pre bending.

<span id="page-11-0"></span>

**Figure 11.** Forming part: (**a**) Before modified rolling, (**b**) After modified rolling.

Procedures	<b>Reduction</b> h/mm	Curvature Radius p/mm	<b>Target Curvature</b> Radius $p_{\rm t}/\text{mm}$	$Error\%$
First pre-bending	5.3	1219.5		
Second pre-bending	8.1	436.7		
Third pre bending	11.2	220.8		
Rolling	13.2	158.7		5.8
Modified rolling	13.4	150.4	150	0.3

<span id="page-11-1"></span>**Table 5.** Experimental results for ST12 cold rolled plate  $(L_1 = 163.5 \text{ mm})$ .

<b>Procedures</b>	Reduction h/mm	Curvature Radius p/mm	<b>Target Curvature</b> Radius $p_{\rm t}/\text{mm}$	$Error\%$
First pre-bending	5.8	1762.3		
Second pre-bending	8.5	663.8		
Third pre bending	11.2	327.2		
Rolling	14.6	161.2	150	7.5
Modified rolling	14.9	151.1		0.7

<span id="page-11-2"></span>**Table 6.** Experimental results for 304 stainless steel ( $L_1$  =163.5 mm).

It can be seen from Tables [5](#page-11-1) and [6](#page-11-2) that the curvature radius of the formed part, which is formed by the reduction predicted from three pre-bending processes, has a large error with the target curvature. However, using the above control strategy, the rolling data is substituted into the quadratic function fitting process to obtain the revised reduction. After the modified rolling, the accuracy of the formed parts is clearly improved. The rolling error of the ST12 cold rolled plate is 0.3%, and that of 304 stainless steel plate is 0.7%. Through many experiments, the relative error of the curvature radius of the formed parts is within 0.8%, which verifies the feasibility of the control strategy.

### **5. Conclusions**

In this paper, a new symmetrical four-roller bending process is introduced, which consists of feeding, pre-bending, reverse roll bending, second bending, and forward roll bending and unloading. A control strategy of the SFRB process is also presented based on experiments and numerical simulation. The main conclusions are as follows:

1. For this process, a control strategy is proposed, including on-line monitoring of curvature, on-line identification of the springback law, on-line prediction of final reduction, and control strategy process.

2. A convenient and reliable on-line curvature monitoring method is proposed. The SFRB device was developed and the finite element model was established. Through physical experiments and numerical simulation, the quantitative relationship between the reduction and the curvature was established, which is a quadratic function relationship, as follows:

$$
K(h) = ah^2 + bh + c \tag{20}
$$

The online identification of the springback law was realized. An on-line prediction method of the final reduction is proposed, and the determination principle of the reduction of three pre-bending processes is given.

3. Through physical experiments, the control strategy of SFRB process was verified. The relative error of curvature radius of the final formed parts can be controlled within 0.8%. This research provides new method for intelligent rolling of steel plate.

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