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Domination in Join of Fuzzy Incidence Graphs Using Strong Pairs with Application in Trading System of Different Countries

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Abstract: Fuzzy graphs (FGs), broadly known as fuzzy incidence graphs (FIGs), are an applicable and well-organized tool to epitomize and resolve multiple real-world problems in which ambiguous data and information are essential. In this article, we extend the idea of domination of FGs to the FIG using strong pairs. An idea of strong pair dominating set and a strong pair domination number (SPDN) is explained with various examples. A theorem to compute SPDN for a complete fuzzy incidence graph (CFIG) is also provided. It is also proved that in any fuzzy incidence cycle (FIC) with l vertices the minimum number of elements in a strong pair dominating set are $M[\gamma_s(C_l(\sigma, \phi, \eta))] = \lceil \frac{l}{3} \rceil$. We define the joining of two FIGs and present a way to compute SPDN in the join of FIGs. A theorem to calculate SPDN in the joining of two strong fuzzy incidence graphs is also provided. An innovative idea of accurate domination of FIGs is also proposed. Some instrumental and useful results of accurate domination for FIC are also obtained. In the end, a real-life application of SPDN to find which country/countries has/have the best trade policies among different countries is examined. Our proposed method is symmetrical to the optimization.

Keywords: strong pair domination; accurate domination; fuzzy incidence graph; complete fuzzy incidence graph

1. Introduction

A graph is an easy way to express information, together with the relationship between various kinds of entities. The entities are indicated by vertices and relations are illustrated by edges. In this paper, we use finite and simple graphs. A mathematical framework to explain inconclusiveness and ambiguity in real-life situations was first recommended by Zadeh. He initiated the idea of Fuzzy set (FS) [1,2]. When there is an inadequacy in the explanation of the objects and their relationship, we need to draw a FG model. Zadeh's FS prepares a way for the theory of FGs which has been contemplated by Rosenfeld [3]. The study of FGs opens a new door for many researchers to participate in this field. For instance, Gani and Ahamed [4] presented an idea to find order and size in FGs. Bhutani et al. [5] discussed degrees for fuzzy end nodes and studied some properties of fuzzy end nodes and fuzzy cut nodes further. The notion of domination in FGs was first suggested by Somasundaram and Somasundaram [6]. Later, Somasundaram [7] discussed domination in products of FGs. Gani and Ahamed [8] initiated the notion of strong and weak domination in FGs along with their different properties. Natarajan and Ayyaswamy [9] initiated the idea of strong (weak) domination in FGs and obtained some interesting results for this new parameter in FGs. Various types of degrees of a node in FGs were introduced by Manjusha and Sunitha [10]. They obtained the lower and upper bound

of domination number for the FGs and also proved that in a fuzzy tree each node of a dominating set is incident on a fuzzy bridge. Manjusha and Sunitha [11] introduced the idea of total domination in FGs using strong arcs and discussed a lower bound and an upper bound for the strong total domination number in FGs. They also presented the necessary and sufficient condition for the set of fuzzy cut nodes to be a strong total dominating set. The strong domination number of complete FG and complete bipartite FG was introduced by Manjusha and Sunitha [12]. After them, the equitable dominating set, minimal fuzzy equitable dominating set, strong (weak) fuzzy equitable dominating set, and fuzzy equitable independent set in FGs were discussed by Dharmalingam and Rani [13]. Then, 2-domination in FGs was introduced by Gani and Devi [14]. Gani and Rahman [15] proposed an innovative idea of an accurate 2-dominating set and accurate 2-domination number of a FG. They also elaborated some properties of accurate 2-dominating sets and also some bounds of accurate 2-domination numbers. Ponnappan et al. [16] brought the notion of edge domination and total edge domination in FGs. They determined the edge domination number and the total edge domination number for several classes of FG and obtained bounds for the same. The concept of very excellent domination in FGs along with their various properties was proposed by Nithya and Dharmalingam [17].

Bhutani and Rosenfeld [18] came up with an idea of strong arcs in FGs. Mathew and Mordeson [19] presented the novel idea of fuzzy influence graph and characterized influence cutpairs in fuzzy influence graphs since their removal increases the number of connected components of a fuzzy network and thus weakens the potential flow in the network. They also discussed some of the properties of influence pairs and influence cut-nodes. Rashmanlou and Jun [20] defined three new operations on interval-valued FGs; namely, direct product, semi-strong product, and strong product. Likewise, they gave sufficient conditions for each one of them to be complete and show that if any of these products is complete, then at least one factor is a complete interval-valued FG. The idea of a complement of an FG was initiated by Sunitha and Vijayakumar [21]. Yin et. al [22] introduced the degree of a vertex the degree and total degree of a vertex in q-rung orthopair FGs. They also found some product operations on q-rung orthopair FGs, including direct product, Cartesian product, semi-strong product, strong product, and lexicographic product. For some other useful and constructive work on FGs, we suggest to the reader [23–27].

FGs are incapable of intimating the impact of a vertex on an edge.This deficiency in FGs became the main reason to introduce the idea of FIGs. Dinesh [28] was the first researcher who introduced the concept of FIGs. For example, if vertices represent various hospitals and edges serve as roads connecting these hospitals, we can have an FG indicating the extent of traffic from one hospital to another hospital. The hospital having the highest number of patients will have the most ramps in the hospital. So, if L and M are two hospitals and LM is a road connecting them then (L, LM) can present the ramp system from the road LM to the hospital L. In unweighted graphs, both L and M will have an impact of 1 on LM. In a directed graph, the influence of L on LM given by (L, LM) is 1 whereas (M, LM) is 0. This idea can be generalized by FIGs. Malik et al. [29] provided initial results to the development a final area of $(s, t]$-fuzzy type incidence graphs, namely $[\widehat{t}, \widehat{s})$-fuzzy incidence for complementary FIGs. They applied the results to problems involving human trafficking. They were particularly interested in the role played by countries' vulnerability and their government's response to human trafficking. Mathew and Mordeson [30] presented various connectivity concepts in FIGs. They established the existence of a strong path between any node arc pair of a FIG. They also discussed several other structural properties of FIGs. Fang et al. [31] presented the idea of connectivity index and Wiener index of FIGs. They also introduced three different types of nodes including fuzzy incidence connectivity enhancing node, fuzzy incidence connectivity reducing node, and fuzzy incidence connectivity neutral node. After them, Mathew et al. [32] presented the notion of node connectivity and arc connectivity in FIGs. They also discussed that the incidence is used to model flows in human trafficking networks. The conception of fuzzy end nodes and relationships among weak incidence pairs, fuzzy incidence end nodes,

and fuzzy incidence cutvertices was developed by Mordeson and Mathew [33]. Later, Nazeer et al. [34] introduced intuitionistic fuzzy incidence graphs (IFIGs) as a generalization of FIGs and explained various types of products including Cartesian product (CP), composition, tensor product, and normal product in IFIGs. They also provided an application of CP and composition of two IFIGs in the textile industry to find the best combinations of departments indicating the highest percentage of progress and the lowest percentage of non-progress. Nazeer et al. [35] came up with an innovative idea of domination in FIGs along with an algorithm and an application for the selection of the best medical lab among a variety of labs. Mordeson et al. [36] introduced the notion of a vague incidence graph and its eccentricity. They applied the results to problems involving human trafficking and illegal immigration. For more detailed and comprehensive work on FIGs, we may refer to the reader [28,37].

There are multiple reasons to initiate the idea of domination in the joining of FIGs using strong pairs (SPs). Firstly, we are unable to apply FGs to the real-life application provided in Section 6 of SPDN in matters of trade of different countries because FGs do not have a feature to give the impact of a vertex on an edge but FIGs have this property. This deficiency of FGs motivates us to introduce these ideas for FIGs. Secondly, the purpose to propose these perceptions to FIGs is that Selvam and Ponnappan [38] proposed the idea of domination in the join of FGs using strong edges (SEs), and Ponnappan and Selvam [39] came up with the idea of accurate domination in FGs using SEs. We broaden their ideas for FIGs by using SPs. Thirdly, in FGs if an edge uv is a SE then vu will also be a SE but in FIG if a pair (u,uv) is a SP then the pair (v,uv) don't need to be a SP. It may or may not be a SP. Fourthly, in FGs if a vertex u dominates vertex v then vertex v will also dominate a vertex u but in FIGs, if a vertex u dominates a vertex v then it is not necessary that a vertex v will also dominate a vertex u because in FIGs if a pair (u,uv) is a SP then a pair (v,uv) may or may not be a SP.

The remainder of this article is organized as follows. In Section 2, some fundamental definitions and preliminary results which are helpful to comprehend the remaining part of the article are explained with various examples. The domination in FIGs using SPs and some significant results are expressed in Section 3. In Section 4, domination in the joining of FIGs is elaborated. Section 5 provides the information of accurate domination in FIGs. A real-life application of domination in matters of trading among different countries is explained in Section 6. A comparative analysis of our study with the previous study is provided in Section 7. Conclusions and future directions are explained in Section 8.

2. Preliminaries

We recall some fundamental definitions in FIGs and propose some novel definitions and ideas. We present some preliminary definitions from [1,30,38] in the following sections. In this article, the minimum and maximum operators are indicated by $\wedge$ and $\vee$, respectively.

Definition 1 ([38])**.** *A FG $G(\sigma, \phi)$ is a pair of function $\sigma : V \to [0,1]$ and $\phi : V \times V \to [0,1]$ where for each $u, v \in V$, we have $\phi(u,v) \le \sigma(u) \wedge \sigma(v)$.*

Definition 2 ([38])**.** *A path ϖ of length n in a FG $G(\sigma, \phi)$ is a sequence of different vertices $s_0, s_1, s_2, ... s_n$ of the type $\phi(s_{i-1}, s_i) > 0$ where $1 \le i \le n$. Vertices s and t in a FG are called to be connected if $\exists s - t$ path in $G(\sigma, \phi)$. In a FG if s and t are connected by means of length i, then $\phi^i(s,t) = \vee\{\phi(s,t_1) \wedge \phi(t_1, t_2)... \wedge \phi(t_{i-1}, t) | s, t_1, t_2, ... t \in \varpi\}$.*

Definition 3 ([38])**.** *The strongest path (SGP) joining any two vertices s and t is a path corresponding to greatest strength between s and t. The strength of the SGP is expressed by $\phi^\infty(s,t)$. Where $\phi^\infty(s,t) = \vee\{\phi^i(s,t) | i = 1, 2, 3, \dots, \infty\}$.*

Example 1. *Figure 1 indicates a FG $G(\sigma, \phi)$ with $\sigma(p) = 0.3, \sigma(q) = 0.4, \sigma(r) = 0.3, \sigma(s) = 0.2, \sigma(t) = 0.5, \sigma(u) = 0.5, \sigma(v) = 0.7, ; \phi(pq) = 0.2, \phi(ps) = 0.07, \phi(pu) = 0.3, \phi(qr) = 0.2, \phi(qu) = 0.4, \phi(qv) = 0.3, \phi(rs) = 0.05, \phi(st) = 0.2$. In this FG there are four paths*

from u to r namely, $P_1 : u - q - r$ path of length 2 and strength 0.2, $P_2 : u - q - p - s - r$ path of length 4 and strength 0.05, $P_3 : u - p - q - r$ path of length 3 and strength 0.2 and $P_4 : u - p - s - r$ path of length 3 and strength 0.05. The strength of the SGP joining u and r is $\phi^\infty(u, r) = \vee\{0.2, 0.05, 0.2, 0.05\} = 0.2$.
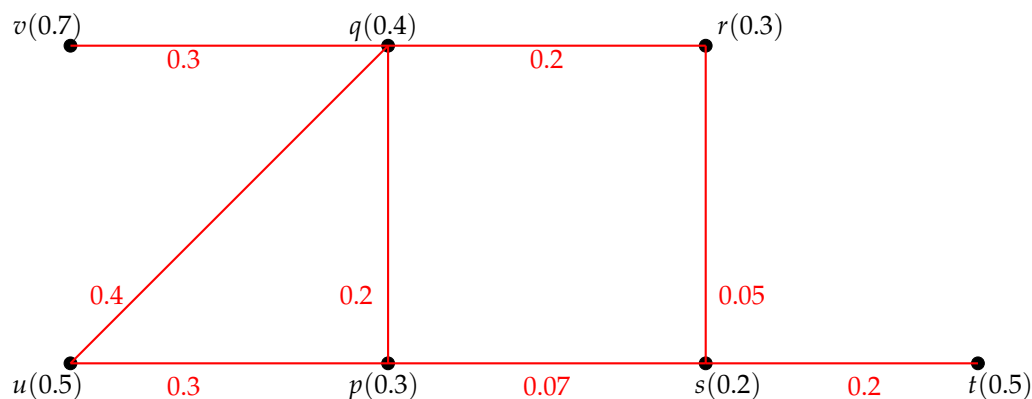

Figure 1. FG with strongest path.

Definition 4 ([38]). *An edge of a FG $G(\sigma, \phi)$ is said be SE if $\phi^\infty(s, t) = \phi(s, t)$.*

Definition 5 ([38]). *Let $G(\sigma, \phi)$ be a FG. Assume s and t are any two vertices of a FG. A vertex s dominates t if the edge (s, t) is SE. A subset Z of V is said to be a strong edge dominating set (SEDS) of FG if for each $t \in V - Z \exists s \in Z$ such that s dominates t. A SEDS Z is named as a minimal SEDS if no proper subset of Z is a SEDS. The minimum fuzzy cardinality (MFC) taken over all minimal SEDS is said to be strong edge domination number (SEDN), indicated by $\gamma_s(G)$.*

Definition 6 ([30]). *Let G be a simple graph with vertex set $V(G)$ and edge set $E(G)$. $G = (V, E, I)$ is called an incidence graph (IG) where $I \subseteq V \times E$. Figure 2 shows an IG.*

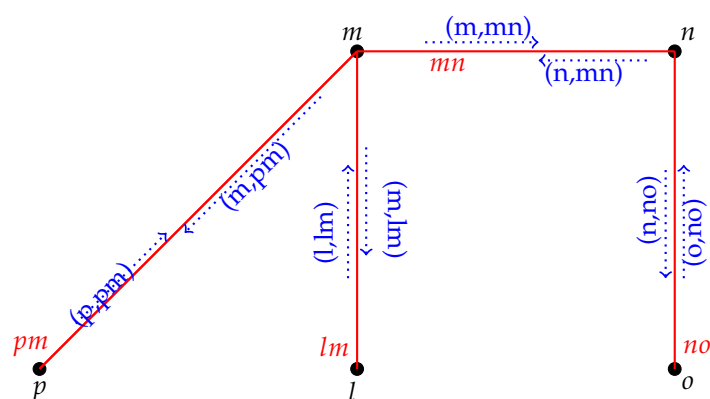If (l, lm) is in IG, then (l, lm) is said to be an incidence pair or pair of an IG.

Figure 2. Incidence graph.

Definition 7 ([1]). *If T is any set, a mapping $\phi : T \to [0, 1]$ is known as fuzzy subset (FSS) of S.*

Definition 8 ([30]). *Assume $G = (V, E)$ is a graph with σ is FSS of V, ϕ is FSS of E and η is FSS of $V \times E$. If $\eta(v, e) \leq \sigma(v) \wedge \phi(e) \, \forall \, v \in V$ and $e \in E$, then η is named as fuzzy incidence (FI) of G.*

Definition 9 ([30])**.** *Consider $G = (V, E)$ is a graph and (σ, ϕ) be a fuzzy subgraph of G. If η is a FI of G, then $\widehat{G} = (\sigma, \phi, \eta)$ is named as FIG of G.*

Definition 10 ([30])**.** *Any $s \in V$ is said to be in the support of σ ($Supp(\sigma)$) if $\sigma(s) > 0$, $st \in E$ is said to be in the support of ϕ ($Supp(\phi)$) if $\phi(st) > 0$ and $(s, tu) \in V \times E$ is said to be in the support of η ($Supp(\eta)$) if $\eta(s, tu) > 0$. The support of σ, ϕ and η are expressed by σ^*, ϕ^*, and η^*, respectively. Let $st \in \phi^*$. Then st is an edge of the FIG $\widehat{G} = (\sigma, \phi, \eta)$ and if $(s, st) \in \eta^*$ then (s, st) is called pair of FIG. In FIG $m = m_0, (m_0, m_0m_1), m_0m_1$ is an incidence path (IP) of length one, $m = m_0, (m_0, m_0m_1), m_0m_1, (m_1, m_0m_1), m_1$ is an IP of length two. In a FIG two vertices s_i and s_j are said to be connected if they are joined by an IP. $s - t = s, (s, st), st, (t, st), t$ and $s - st = s, (s, st), st$ are two different types of IPs in FIG. The incidence strength (IS) of a FIG $\widehat{G} = (\sigma, \phi, \eta)$ is defined as $\wedge \{\eta(v, e) | (v, e) \in \eta^*\}$.*

Definition 11 ([30])**.** *The FIG $\widehat{G} = (\sigma, \phi, \eta)$ is a cycle if $(\sigma^*, \phi^*, \eta^*)$ is a cycle. The FIG $\widehat{G} = (\sigma, \phi, \eta)$ is a fuzzy cycle if $(\sigma^*, \phi^*, \eta^*)$ is a cycle and there exists no unique $st \in \phi^*$ such that $\phi(st) = \wedge\{\phi(uv) | uv \in \phi^*\}$. The FIG $\widehat{G} = (\sigma, \phi, \eta)$ is a fuzzy incidence cycle (FIC), and there exists no unique $(s, st) \in \eta^*$ such that $\eta(s, st) = \wedge\{\eta(u, uv) | (u, uv) \in \eta^*\}$.*

A FIC is shown in Figure 3.
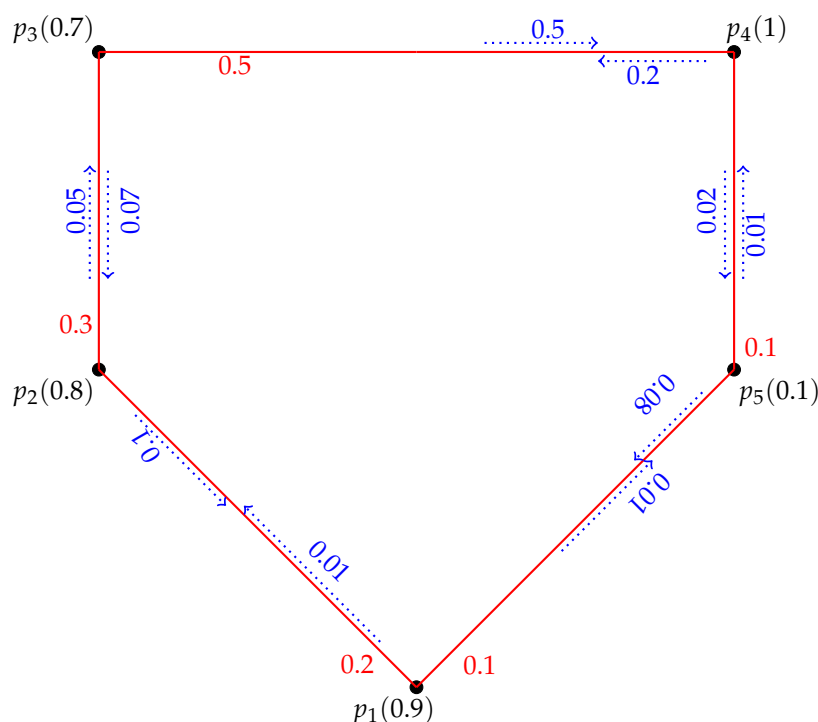
Figure 3. Fuzzy incidence cycle.

Proposition 1 ([30])**.** *Every FIC is a strong cycle.*

Definition 12 ([30])**.** *A FIG $\widehat{G} = (\sigma, \phi, \eta)$ is said to be complete FIG if $\eta(s, st) = \sigma(s) \wedge \phi(st)$ for every $(s, st) \in \eta^*$.*

3. Domination in Fuzzy Incidence Graph Using Strong Pair

Domination in graphs has plenty of uses in various fields. The idea of domination using SPs with various examples is explained in this section. An idea of strong pair dominating set (SPDS) and SPDN is discussed. In the future these concepts will be helpful to find SPDS and SPDN in various types of products of FIGs.

Definition 13. *Let $\widehat{G} = (\sigma, \phi, \eta)$ be a FIG. An edge in FIG $\widehat{G} = (\sigma, \phi, \eta)$ is called an effective edge if $\phi(s, t) = \sigma(s) \wedge \sigma(t)$.*

Definition 14. *A pair in FIG $\widehat{G} = (\sigma, \phi, \eta)$ is called an effective pair (EP) if $\eta(s, st) = \sigma(s) \wedge \phi(st)$.*

Definition 15. *Let $\widehat{G} = (\sigma, \phi, \eta)$ be a FIG. The IS of an IP between the vertex s and an edge st is defined as $IS(s, st) = \wedge\{\eta(u, uv)|(u, uv) \in \eta^*\}$.*

Definition 16. *Assume $\widehat{G} = (\sigma, \phi, \eta)$ is a FIG. If a vertex s and an edge st in a FIG are connected by means of length w, then $\eta^w(s, st) = \vee\{\eta(s, s_1 t_1) \wedge \eta(t_1, s_2 t_2) \wedge ... \wedge \eta(t_{w-1}, st)\}$.*

Definition 17. *Consider $\widehat{G} = (\sigma, \phi, \eta)$ is a FIG. The greatest incidence strength (GIS) between s and st is the largest value of all ISs of all IPs between s and st and it is represented by $\eta^\circ(s, st) = \vee\{\eta^w(s, st)|w = 1, 2, 3, \ldots, \infty\}$.*

Example 2. *Figure 4 indicates a FIG $\widehat{G} = (\sigma, \phi, \eta)$ with $\sigma(p) = 0.3, \sigma(q) = 0.4, \sigma(r) = 0.3, \sigma(s) = 0.2, \sigma(t) = 0.5; \phi(pq) = 0.3, \phi(pr) = 0.3, \phi(qr) = 0.2, \phi(rt) = 0.3, \phi(rs) = 0.2, \phi(st) = 0.1; \eta(p, pq) = 0.3, \eta(q, pq) = 0.1, \eta(p, pr) = 0.1, \eta(r, pr) = 0.1, \eta(q, qr) = 0.2, \eta(r, qr) = 0.2, \eta(r, rt) = 0.1, \eta(t, rt) = 0.3, \eta(r, rs) = 0.1, \eta(s, rs) = 0.1, \eta(s, st) = 0.07, \eta(t, st) = 0.05$. In this FIG there are two IPs from p to pq of length 1 and length 4, respectively, namely, $\eta^1 = p, (p, pq), pq$ with $IS(p, pq) = 0.3$ and $\eta^4 = p, (p, pr), pr, (r, pr), r, (r, rq),$ $rq, (q, rq), q, (q, pq), qp = pq$ having $IS(p, pq) = \wedge\{0.1, 0.1, 0.2, 0.2, 0.1\} = 0.1$. The GIS between p and pq is $\eta^\circ(p, pq) = \vee\{0.3, 0.1\} = 0.3$.*
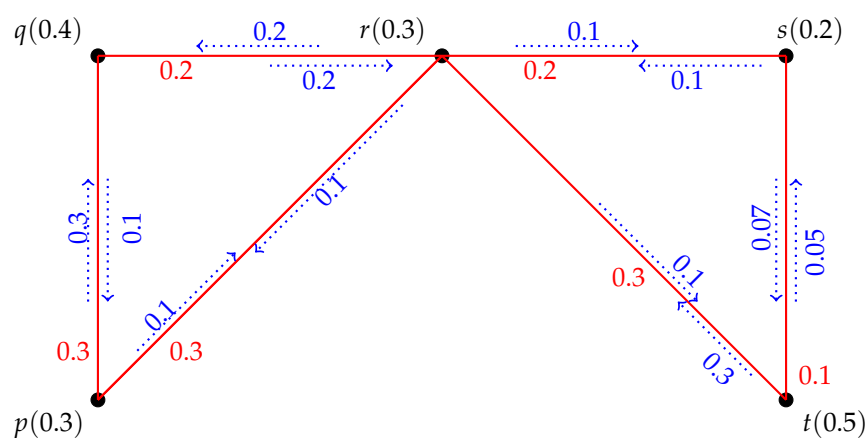
Figure 4. FIG with strong and non strong pairs.

Definition 18. *Assume a FIG $\widehat{G} = (\sigma, \phi, \eta)$. A pair (s, st) is called SP if $\eta^\circ(s, st) = \eta(s, st)$ otherwise it is not strong.*

Example 3. *Figure 4 represents a FIG $\widehat{G} = (\sigma, \phi, \eta)$ with $\sigma(p) = 0.3, \sigma(q) = 0.4, \sigma(r) = 0.3, \sigma(s) = 0.2, \sigma(t) = 0.5; \phi(pq) = 0.3, \phi(pr) = 0.3, \phi(qr) = 0.2, \phi(rt) = 0.3, \phi(rs) = 0.2, \phi(st) = 0.1; \eta(p, pq) = 0.3, \eta(q, pq) = 0.1, \eta(p, pr) = 0.1, \eta(r, pr) = 0.1, \eta(q, qr) = 0.2, \eta(r, qr) = 0.2, \eta(r, rt) = 0.1, \eta(t, rt) = 0.3, \eta(r, rs) = 0.1, \eta(s, rs) = 0.1, \eta(s, st) = 0.07, \eta(t, st) = 0.05$. In this FIG there are two IPs from p to pq namely, $p, (p, pq), pq$ with $IS(p, pq) = 0.3$ and $p, (p, pr), pr, (r, pr), r, (r, rq), rq, (q, rq), q, (q, pq), qp = pq$ having $IS(p, pq) = \wedge\{0.1, 0.1, 0.2, 0.2, 0.1\} = 0.1$. Therefore, (p, pq) is a SP because $\eta^\circ(p, pq) = \vee\{0.3, 0.1\} = 0.3 = \eta(p, pq)$.*

Definition 19. *Let $\widehat{G} = (\sigma, \phi, \eta)$ be a FIG. Let s and t be any two vertices of $\widehat{G} = (\sigma, \phi, \eta)$. A vertex s dominates t if (s, st) is a SP. A subset Z of V is named as a SPDS of $\widehat{G}$ if for each $t \in V - Z$ $\exists s \in Z$ such that s dominates t.*

Definition 20. *Let $\widehat{G} = (\sigma, \phi, \eta)$ be a FIG. Then a SPDS Z is named as minimal SPDS if no proper subset of Z is a SPDS.*

Definition 21. *Consider $\widehat{G} = (\sigma, \phi, \eta)$ is a FIG. Then the MFC taken over all minimal SPDS is said to be SPDN, represented by $\gamma_s(G)$ or γ_s.*

Definition 22. *Let $\widehat{G} = (\sigma, \phi, \eta)$ be a FIG. Then the lowest number of elements in the minimal SPDS is expressed by $M[\gamma_s]$ where $M[\gamma_s] = \wedge\{|Z_k|/Z_k\}$ is a minimal SPDS.*

Example 4. *A FIG $\widehat{G} = (\sigma, \phi, \eta)$ is provided in Figure 5 with $\sigma(p) = 0.3, \sigma(q) = 0.2, \sigma(r) = 0.5; \phi(pq) = 0.1, \phi(pr) = 0.2, \phi(qr) = 0.2; \eta(p, pq) = 0.1, \eta(q, pq) = 0.1, \eta(p, pr) = 0.1, \eta(r, pr) = 0.05, \eta(q, qr) = 0.2, \eta(r, qr) = 0.1$. In this FIG $\eta(p, pq), (p, pr), (q, qp)$ and (q, qr) are all SPs. Therefore, there are two minimal SPDSs which are $Z_1 = \{p\}$ because p dominates q and r and $Z_2 = \{q\}$ because q dominates p and r. This implies $\gamma_s = 0.2$ because $\sigma(q)$ has the lowest fuzzy cardinality.*
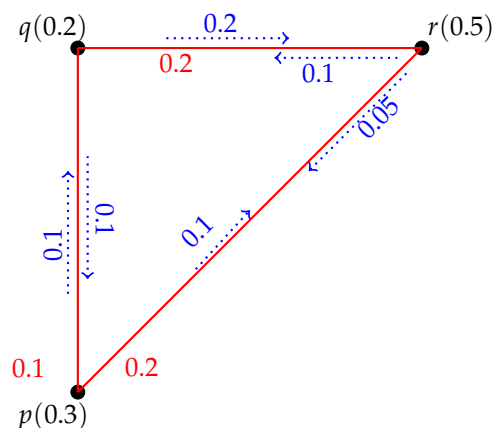

Figure 5. FIG with $\gamma_s = 0.2$.

Theorem 1. *A pair (s,st) in FIG $\widehat{G} = (\sigma, \phi, \eta)$ is a SP if and only if there does not exist an IP $s = m_0, (m_0, m_0 m_1), m_0 m_1, (m_1, m_0 m_1), m_1, \ldots, m_0 m_n = st$ of the kind $\eta(m_j, m_j m_{j+1}) > \eta(s, st)$ for each $j = 0, 1, 2, \ldots, n-1$.*

Proof. Let (s, st) be a SP. Assume $\exists$ an IP $\Delta : s = m_0, (m_0, m_0 m_1), m_0 m_1, (m_1, m_0 m_1), m_1, \ldots, m_0 m_n = st$ of the type $\eta(m_j, m_j m_{j+1}) > \eta(s, st)$ for each $j = 0, 1, 2, \ldots, n-1$. Then GIS of $\Delta > \eta(s, st)$. This means $\eta°(s, st) > \eta(s, st)$. Therefore, (s, st) is not SP which is a contradiction. Consider there does not exist an IP $s = m_0, (m_0, m_0 m_1), m_0 m_1, (m_1, m_0 m_1), m_1, \ldots, m_0 m_n = st$ such that $\eta(m_j, m_j m_{j+1}) > \eta(s, st)$ for every $j = 0, 1, 2, \ldots, n-1$. This implies that for each IP Δ of $s - st$ $\exists$ a pair $(m_j, m_j m_{j+1})$ of the type $\eta(m_j, m_j m_{j+1}) \leq \eta(s, st)$ for some j. This implies for each IP Δ of $s - st$, GIS of $\Delta \leq \eta(s, st)$. This implies $\eta°(s, st) = \eta(s, st)$. Hence (s, st) is a SP. $\square$

Example 5. *Let $\widehat{G} = (\sigma, \phi, \eta)$ be a FIG shown in Figure 4 with $\sigma(p) = 0.3, \sigma(q) = 0.4, \sigma(r) = 0.3, \sigma(s) = 0.2, \sigma(t) = 0.5; \phi(pq) = 0.3, \phi(pr) = 0.3, \phi(qr) = 0.2, \phi(rt) = 0.3, \phi(rs) = 0.2, \phi(st) = 0.1; \eta(p, pq) = 0.3, \eta(q, pq) = 0.1, \eta(p, pr) = 0.1, \eta(r, pr) = 0.1, \eta(q, qr) = 0.2, \eta(r, qr) = 0.2, \eta(r, rt) = 0.1, \eta(t, rt) = 0.3, \eta(r, rs) = 0.1, \eta(s, rs) = 0.1, \eta(s, st) = 0.07, \eta(t, st) = 0.05$. In this FIG there are two IPs from p to pq namely, $p, (p, pq), pq$ with $\eta(p, pq) = 0.3$ and $p, (p, pr), pr, (r, pr), r, (r, rq), rq, (q, rq), q, (q, pq), qp = pq$ having $\eta(p, pr) = 0.1, \eta(r, pr) = 0.1, \eta(r, rq) = 0.2, \eta(q, rq) = 0.2$ and $\eta(q, qp) = 0.1$. Therefore, according to Theorem 1 (p, pq) is a SP because there does not exist any pair of the type such that $\eta(p, pr) = 0.1 \not> \eta(p, pq) = 0.3, \eta(r, pr) = 0.1 \not> \eta(p, pq) = 0.3, \eta(r, rq) = 0.2 \not> \eta(p, pq) = 0.3, \eta(q, rq) = 0.2 \not> \eta(p, pq) = 0.3$ and $\eta(q, qp) = 0.1 \not> \eta(p, pq) = 0.3$.*

Definition 23. *Consider s is a vertex in a FIG $\widehat{G} = (\sigma, \phi, \eta)$. We define $\sigma^{\star}(s) = \vee\{\eta(s, st)|s, st \in \sigma^* \cup \phi^*\}$ where $\sigma^{\star}(s)$ is the highest weights of the pairs incidence at s.*

Theorem 2. *Let $\widehat{G} = (\sigma, \phi, \eta)$ be a FIG and consider (s, st) is a pair in FIG such that $\sigma^{\star}(s) = \eta(s, st)$ then (s, st) is a SP.*

Proof. Assume (s, st) is not a SP then there exists an IP $\Delta : s = s_0, (s_0, s_0 s_1), s_0 s_1, (s_1, s_0 s_1), s_1 , \ldots, s_0 s_m = st$ such that GIS of $\Delta > (s, st)$. This means $\eta(s_j, s_j t_{j+1}) > \eta(s, st)$ for every $j = 0, 1, 2, \ldots, m - 1$. This implies $\sigma^{\star}(s) > \eta(s, st)$ which is a contradiction. □

Example 6. *Consider a FIG $\widehat{G} = (\sigma, \phi, \eta)$ provided in Figure 5 with $\sigma(p) = 0.3, \sigma(q) = 0.2, \sigma(r) = 0.5; \phi(pq) = 0.1, \phi(pr) = 0.2, \phi(qr) = 0.2; \eta(p, pq) = 0.1, \eta(q, pq) = 0.1, \eta(p, pr) = 0.1, \eta(r, pr) = 0.05, \eta(q, qr) = 0.2, \eta(r, qr) = 0.1$. In this FIG (q, qr) is a SP because $\sigma^{\star}(q) = \eta(q, qr) = 0.2$.*

Theorem 3. *Let $\widehat{G} = (\sigma, \phi, \eta)$ be a FIG. If (s, st) is an EP then it must be a SP.*

Proof. Assume (s, st) is an EP. Then $\eta(s, st) = \sigma(s) \wedge \phi(st)$. Suppose $\sigma(s) < \phi(st)$. Then $\eta(s, st) = \sigma(s)$. Therefore, for any IP $\Delta : s = s_0, (s_0, s_0 s_1), s_0 s_1, (s_1, s_0 s_1), s_1, \ldots, s_0 s_m = st$, GIS of $\Delta \leq \sigma(s)$ and $\eta(s, st) = \sigma(s)$. This implies $\eta^{\circ}(s, st) = \sigma(s) = \eta(s, st)$. Therefore, (s, st) is a SP. □

Example 7. *Let $\widehat{G} = (\sigma, \phi, \eta)$ be a FIG given in Figure 4 with $\sigma(p) = 0.3, \sigma(q) = 0.4, \sigma(r) = 0.3, \sigma(s) = 0.2, \sigma(t) = 0.5; \phi(pq) = 0.3, \phi(pr) = 0.3, \phi(qr) = 0.2, \phi(rt) = 0.3, \phi(rs) = 0.2, \phi(st) = 0.1; \eta(p, pq) = 0.3, \eta(q, pq) = 0.1, \eta(p, pr) = 0.1, \eta(r, pr) = 0.1, \eta(q, qr) = 0.2, \eta(r, qr) = 0.2, \eta(r, rt) = 0.1, \eta(t, rt) = 0.3, \eta(r, rs) = 0.1, \eta(s, rs) = 0.1, \eta(s, st) = 0.07, \eta(t, st) = 0.05$. From Figure 4 it is clear that the pairs $(p, pq), (q, qr), (r, qr)$ and (t, tr) are all EPs because $\eta(p, pq) = 0.3 = \sigma(p) \wedge \phi(pq) = 0.3 \wedge 0.3 = 0.3, \eta(q, qr) = 0.2 = \sigma(q) \wedge \phi(qr) = 0.4 \wedge 0.2 = 0.2, \eta(r, qr) = 0.2 = \sigma(r) \wedge \phi(qr) = 0.3 \wedge 0.2 = 0.2$ and $\eta(t, tr) = 0.3 = \sigma(t) \wedge \phi(tr) = 0.5 \wedge 0.3 = 0.3$. Figure 4 shows that there are two IPs from p to pq such that $p, (p, pq), pq$ with IS$= \eta(p, pq) = 0.3$ and $p, (p, pr), pr, (r, pr), r, (r, rq), rq, (q, rq), q, (q, pq), qp = pq$ with IS$= \wedge\{\eta(p, pr) = 0.1, \eta(r, pr) = 0.1, \eta(r, rq) = 0.2, \eta(q, rq) = 0.2, \eta(q, qp) = 0.1\} = 0.1$. Now $\eta^{\circ}(p, pq) = \vee\{0.3, 0.1\} = 0.3$. Since $\eta^{\circ}(p, pq) = 0.3 = \eta(p, pq)$. This shows that (p, pq) is a SP. Similarly, $(q, qr), (r, qr)$ and (t, tr) are all SPs. Hence Theorem 3 is verified.*

Theorem 4. *Let $\widehat{G} = (\sigma, \phi, \eta)$ be a CFIG then $\gamma_s = \wedge\{\sigma(s)|s \in V\}$.*

Proof. Assume Z is a dominating set. Consider $s \in Z$ and $t \in V - Z$. Since $\widehat{G} = (\sigma, \phi, \eta)$ is a *CFIG* therefore every pair is an EP. Then by Theorem 4 each pair is also a SP. Therefore, s dominates every $t \in V - Z$. This means $Z = \{s\}$ is a SPDS. Hence $\gamma_s = \wedge\{\sigma(s)|s \in V\}$. □

Example 8. *Let $\widehat{G} = (\sigma, \phi, \eta)$ be a CFIG with $\sigma(p) = 0.3, \sigma(q) = 0.4, \sigma(r) = 0.5; \phi(pq) = 0.2, \phi(pr) = 0.1, \phi(qr) = 0.3; \eta(p, pq) = 0.2, \eta(q, pq) = 0.2, \eta(p, pr) = 0.1, \eta(r, pr) = 0.1, \eta(q, qr) = 0.3, \eta(r, qr) = 0.3$. Since $\widehat{G}$ is a CFIG therefore each pair is an EP. Therefore, by Theorem 3 every pair is also a SP. This means that each vertex dominates to another vertex i.e. p dominates q and r, q dominates p and r, and r dominates p and q. Now $Z_1 = \{p\}$, $Z_2 = \{q\}$ and $Z_3 = \{r\}$ are all SPDSs. Therefore, $\gamma_s = \wedge\{\sigma(p) = 0.3, \sigma(q) = 0.4, \sigma(r) = 0.5\} = 0.3$. Hence Theorem 4 is verified.*

Theorem 5. *Let $C_l(\sigma, \phi, \eta)$ be a FIC with l vertices then $M[\gamma_s(C_l(\sigma, \phi, \eta))] = \lceil \frac{l}{3} \rceil$.*

Proof. Let $C_l(\sigma, \phi, \eta)$ be a FIC then by Proposition 1 every FIC is a strong cycle therefore each pair in C_l is SP. Therefore, every vertex of C_l can dominate up to two other vertices and itself. Taking each third vertex beginning from the second and utilizing the end when l

is indivisible by three yields a minimal SPDS of size $\lceil \frac{l}{3} \rceil$. This implies $M[\gamma_s(C_l(\sigma, \phi, \eta))] = \lceil \frac{l}{3} \rceil$. $\square$

Example 9. *Let $\widehat{G} = (\sigma, \phi, \eta)$ be a FIC with five vertices (C_5) given in Figure 3 with $\sigma(p_1) = 0.9, \sigma(p_2) = 0.8, \sigma(p_3) = 0.7, \sigma(p_4) = 1, \sigma(p_5) = 0.1; \phi(p_1 p_2) = 0.2, \phi(p_1 p_5) = 0.1, \phi(p_2 p_3) = 0.3, \phi(p_3 p_4) = 0.5, \phi(p_4 p_5) = 0.1; \eta(p_1, p_1 p_2) = 0.01, \eta(p_2, p_1 p_2) = 0.1, \eta(p_1, p_1 p_5) = 0.01, \eta(p_5, p_1 p_5) = 0.08, \eta(p_2, p_2 p_3) = 0.05, \eta(p_3, p_2 p_3) = 0.07, \eta(p_3, p_3 p_4) = 0.5, \eta(p_4, p_3 p_4) = 0.2, \eta(p_4, p_4 p_5) = 0.02, \eta(p_5, p_4 p_5) = 0.01$. Since C_5 is a FIC and by Proposition 1 each pair in a FIC is a SP. Therefore, according to Example 6 $M[\gamma_s(C_5(\sigma, \phi, \eta))] = \lceil \frac{5}{3} = 1.66 \rceil = 2$ which are $\{p_1, p_3\}, \{p_1, p_4\}, \{p_2, p_4\}, \{p_2, p_5\}$ and $\{p_3, p_5\}$.*

4. Domination in the Join of Fuzzy Incidence Graphs

In this section, some new ideas of domination along with various examples are explained. The method to find join of two FIGs is provided in this section. The definition of strong fuzzy incidence graph ($SFIG$) is also obtained in this section. The join of two SFIGs is always SFIG is also elaborated in this section.

Definition 24. *Let $\widehat{G}_1 = (\sigma_1, \phi_1, \eta_1)$ and $\widehat{G}_2 = (\sigma_2, \phi_2, \eta_2)$ be two FIGs. The join of $\widehat{G}_1 = (\sigma_1, \phi_1, \eta_1)$ and $\widehat{G}_2 = (\sigma_2, \phi_2, \eta_2)$ is expressed by $\widehat{G}_1 \oplus \widehat{G}_2$ and is defined as the FIG $\widehat{G}(\sigma_1 \oplus \sigma_2, \phi_1 \oplus \phi_2, \eta_1 \oplus \eta_2)$ where*

$$(\sigma_1 \oplus \sigma_2)s = \begin{cases} \sigma_1(s) \; if \; s \in \widehat{G}_1 \\ \sigma_2(s) \; if \; s \in \widehat{G}_2 \end{cases}$$

$$(\phi_1 \oplus \phi_2)(st) = \begin{cases} \phi_1(st) \; if \; s, t \in \widehat{G}_1 \\ \phi_2(st) \; if \; s, t \in \widehat{G}_2 \\ \sigma_1(s) \wedge \sigma_2(s) \; if \; s \in \widehat{G}_1 \; and \; t \in \widehat{G}_2 \end{cases}$$

and

$$(\eta_1 \oplus \eta_2)(s, st) = \begin{cases} \eta_1(s, st) \; if \; (s, st) \in \widehat{G}_1 \\ \eta_2(s, st) \; if \; (s, st) \in \widehat{G}_2 \\ \sigma(s) \wedge \sigma(t) \wedge \eta(s, st_i) \; if \; s, t_i \in \widehat{G}_1 \; and \; t \in \widehat{G}_2, where \; \eta = \eta_1 \oplus \eta_2 \end{cases}$$

Definition 25. *A FIG $\widehat{G} = (\sigma, \phi, \eta)$ is called a $SFIG$ if every pair in $\widehat{G} = (\sigma, \phi, \eta)$ is strong.*

Theorem 6. *Let $\widehat{G}_1 = (\sigma_1, \phi_1, \eta_1)$ and $\widehat{G}_2 = (\sigma_2, \phi_2, \eta_2)$ be two SFIGs. Then $\gamma_s(\widehat{G}_1 \oplus \widehat{G}_2) = \wedge\{\sigma_1(s), \sigma_2(t)\}$ where $s \in \widehat{G}_1$ and $t \in \widehat{G}_2$.*

Proof. Assume $\widehat{G}_1 = (\sigma_1, \phi_1, \eta_1)$ and $\widehat{G}_2 = (\sigma_2, \phi_2, \eta_2)$ are two SFIGs. Since $\widehat{G}_1$ and $\widehat{G}_2$ are two SFIGs then $\widehat{G}_1 \oplus \widehat{G}_2$ will also a SFIG. So, each vertex in $\widehat{G}_1 \oplus \widehat{G}_2$ will dominates other vertices. Then by definition of SPDN, $\gamma_s(\widehat{G}_1 \oplus \widehat{G}_2) = \wedge\{\sigma_1(s), \sigma_2(t)\}$. $\square$

Theorem 7. *Let $\widehat{G}_e = (\sigma_1, \phi_1, \eta_1)$ and $\widehat{G}_f = (\sigma_2, \phi_2, \eta_2)$ be two SFIGs with $e \geq 2$ and $f \geq 2$, where e and f are representing the number of vertices in $\widehat{G}_e$ and $\widehat{G}_f$, respectively. Then $M[\gamma_s(\widehat{G}_e \oplus \widehat{G}_f)] = 1$.*

Proof. Consider $\widehat{G}_e = (\sigma_1, \phi_1, \eta_1)$ and $\widehat{G}_f = (\sigma_2, \phi_2, \eta_2)$ are two SFIGs. Let Z_1 and Z_2 be SPDSs of $\widehat{G}_e$ and $\widehat{G}_f$, respectively. Since $\widehat{G}_e$ and $\widehat{G}_f$ are SFIGs. Then $\widehat{G}_e \oplus \widehat{G}_f$ will also a SFIG with $M[\gamma_s(\widehat{G}_e \oplus \widehat{G}_f)] = 1$ because one vertex in $\widehat{G}_e \oplus \widehat{G}_f$ dominates to all remaining vertices. $\square$

Example 10. *Assume $\widehat{G}_e = (\sigma_1, \phi_1, \eta_1)$ is a SFIG as shown in Figure 6 with $\sigma_1(p_1) = 0.2$, $\sigma_1(p_2) = 0.5$; $\phi_1(p_1p_2) = 0.2$; $\eta_1(p_1, p_1p_2) = 0.2$, $\eta_1(p_2, p_1p_2) = 0.1$ and $\widehat{G}_f = (\sigma_2, \phi_2, \eta_2)$ is a SFIG as shown in Figure 7 with $\sigma_2(q_1) = 0.3$, $\sigma_2(q_2) = 0.4$; $\phi_2(q_1q_2) = 0.3$; $\eta_2(q_1, q_1q_2) = 0.2$, $\eta_2(q_2, q_1q_2) = 0.2$. From Figure 8 it can be seen that $M[\gamma_s(\widehat{G}_e \oplus \widehat{G}_f)] = 1$.*
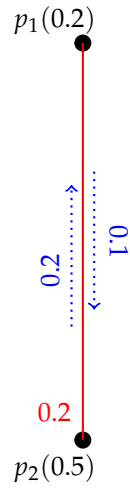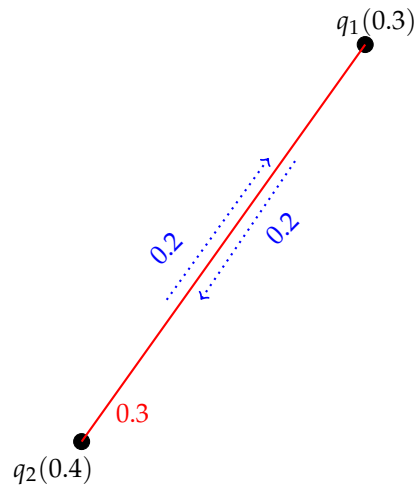

Figure 6. SFIG $\widehat{G}_e$.

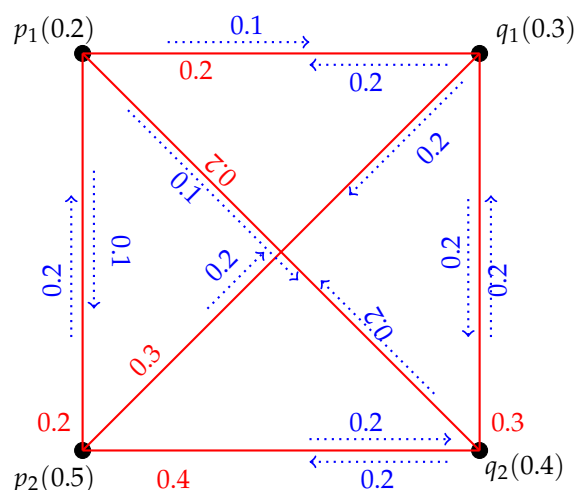

Figure 7. SFIG $\widehat{G}_f$.

Figure 8. SFIG $\widehat{G}_e \oplus \widehat{G}_f$ with $M[\gamma_s(\widehat{G}_e \oplus \widehat{G}_f)] = 1$.

5. Accurate Domination in Fuzzy Incidence Graphs

An innovative idea of accurate domination in FIG is proposed in this section. Some meaningful results related to accurate domination are also examined. Some useful results of accurate domination for FIC are elaborated.

Definition 26. *A SPDS Z of a FIG $\widehat{G} = (\sigma, \phi, \eta)$ is named as accurate dominating set (ADS) if V-Z has no SPDS of cardinality $|Z|$.*

Definition 27. *Let $\widehat{G} = (\sigma, \phi, \eta)$ be a FIG. Then the accurate domination number (ADN) of $\widehat{G} = (\sigma, \phi, \eta)$ is the lowest cardinality of an ADS and it is represented by $\gamma_{\ddot{a}}$.*

Example 11. *Figure 9 shows a FIG $\widehat{G} = (\sigma, \phi, \eta)$ with $\sigma(p) = 0.5, \sigma(q) = 0.3, \sigma(r) = 0.2, \sigma(s) = 0.7; \phi(pq) = 0.3, \phi(pr) = 0.2, \phi(qr) = 0.2, \phi(rs) = 0.2; \eta(p, pq) = 0.2, \eta(p, pr) = 0.01, \eta(q, qr) = 0.1, \eta(q, qp) = 0.2, \eta(r, rq) = 0.05, \eta(r, rp) = 0.02, \eta(r, rs) = 0.2, \eta(s, sr) = 0.2$. In $\widehat{G} = (\sigma, \phi, \eta)$ all pairs are strong except (p, pr) and an ADS $Z = \{r\}$ with $\gamma_{\ddot{a}} = 1$.*
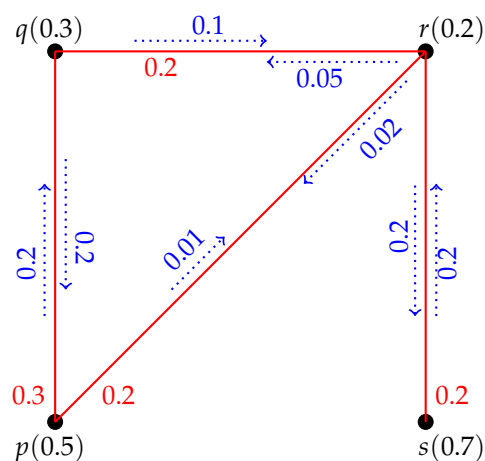

Figure 9. FIG with $\gamma_{\ddot{a}} = 1$.

Theorem 8. *Let $C_l(\sigma, \phi, \eta)$ be a FIC, then $\gamma_{\ddot{a}}(C_l) = \frac{l}{3}$, where $l = 3h$ and $h \geq 1$.*

Proof. Assume $m_1, m_2, m_3, \ldots, m_n$ are the vertices of $C_l(\sigma, \phi, \eta)$. By definition of FIC every pair in FIC is SP. Let Z be the minimum $SPDS$ of $C_l(\sigma, \phi, \eta)$. Therefore, every vertex

$m_i \in Z$ absolutely dominates two vertices in $V - Z$. By Definition 23 $M[\gamma_s C_l(\sigma, \phi, \eta)] = \frac{l}{3}$. Therefore, Z is an ADS with $\gamma_{\ddot{a}}(C_l) = \frac{l}{3}$. □

Theorem 9. *Let $C_l(\sigma, \phi, \eta)$ be a FIC, then $\gamma_{\ddot{a}}(C_l) = \lceil \frac{l}{3} \rceil$, where $l = 3h + 1$ and $h \geq 1$.*

Proof. Consider $m_1, m_2, m_3, \ldots, m_n$ are the vertices of $C_l(\sigma, \phi, \eta)$. Since $C_l(\sigma, \phi, \eta)$ is a FIC then by definition of FIC each pair is SP. So, each vertex m_i dominates to vertices m_{i-1} and m_{i+1}. Assume Z is the minimum SPDS of $C_l(\sigma, \phi, \eta)$. Then by Definition 23 $M[\gamma_s(C_l)(\sigma, \phi, \eta)] = \lceil \frac{l}{3} \rceil$. This implies Z is an ADS with $\gamma_{\ddot{a}}(C_l) = \lceil \frac{l}{3} \rceil$. □

Theorem 10. *Let $C_l(\sigma, \phi, \eta)$ be a FIC, then $\gamma_{\ddot{a}}(C_l) = \lceil \frac{l}{3} \rceil$, where $l = 3h + 2$ and $h \geq 1$.*

Proof. Consider $m_1, m_2, m_3, \ldots, m_n$ are the vertices of $C_l(\sigma, \phi, \eta)$. Since $C_l(\sigma, \phi, \eta)$ is a FIC then by definition of FIC each pair is SP. So, each vertex m_i dominates to vertices m_{i-1} and m_{i+1}. Let Z be the minimum SPDS of $C_l(\sigma, \phi, \eta)$. Then by Definition 23 $M[\gamma_s(C_l)(\sigma, \phi, \eta)] = \lceil \frac{l}{3} \rceil$. This implies Z is an ADS with $\gamma_{\ddot{a}}(C_l) = \lceil \frac{l}{3} \rceil$. □

6. Real-Life Application of Strong Pair Domination Number

Here we include a real-life application of SPDN in matters of trade among different countries. As an illustrative case, consider a network (FIG) $\widehat{G}$ of six vertices representing six different countries $C_1, C_2, C_3, C_4, C_5,$ and C_6 as shown in Figure 10. The membership value (MSV) of the vertices shows the population of the country. The MSV of the edges represents the relationship among these countries and the MSV of the pairs denotes the best trade policies among these countries. The investors are interested in investing their money in these countries but, before they invest, they want to know which country/countries has/have the best trade policies. With the help of SPDN, we will be able to find which country/countries has/have the best trade policies. In turn, this knowledge would help them tackle any risk in their investment.

In Figure 10, $(C_1, C_1 C_6)$ is SP because $\eta(C_1, C_1 C_6) = 0.09 = \eta^{\circ}(C_1, C_1 C_6)$. Similarly, $(C_2, C_2 C_1), (C_2, C_2 C_4), (C_3, C_3 C_2), (C_3, C_3 C_4), (C_4, C_4 C_2), (C_4, C_4 C_3), (C_4, C_4 C_5), (C_4, C_4 C_6),$ $(C_5, C_5 C_4), (C_6, C_6 C_1), (C_6, C_6 C_4)$ and $(C_6, C_6 C_5)$ are all SPs. Also, $\eta(C_1, C_1 C_2) = 0.01 \neq 0.03 = \eta^{\circ}(C_1, C_1 C_2)$. Therefore, $(C_1, C_1 C_6)$ is not SP. In similar way, $(C_2, C_2 C_3)$ and $(C_5, C_5 C_6)$ are not SPs. The SPDSs are $Z_1 = \{C_4, C_6\}, Z_2 = \{C_3, C_6\}, Z_3 = \{C_1, C_4\}, Z_4 = \{C_1, C_3, C_5\}$ with $\gamma_s(Z_1) = \sigma(C_4) + \sigma(C_6) = 0.8 + 0.1 = 0.9, \gamma_s(Z_2) = 0.7 + 0.1 = 0.8, \gamma_s(Z_3) = 0.9 + 0.8 = 1.7$ and $\gamma_s(Z_4) = 0.9 + 0.7 + 1 = 2.6$. So, investors can invest in country C_3 and C_6 because these two countries have best trade policies among all other countries.
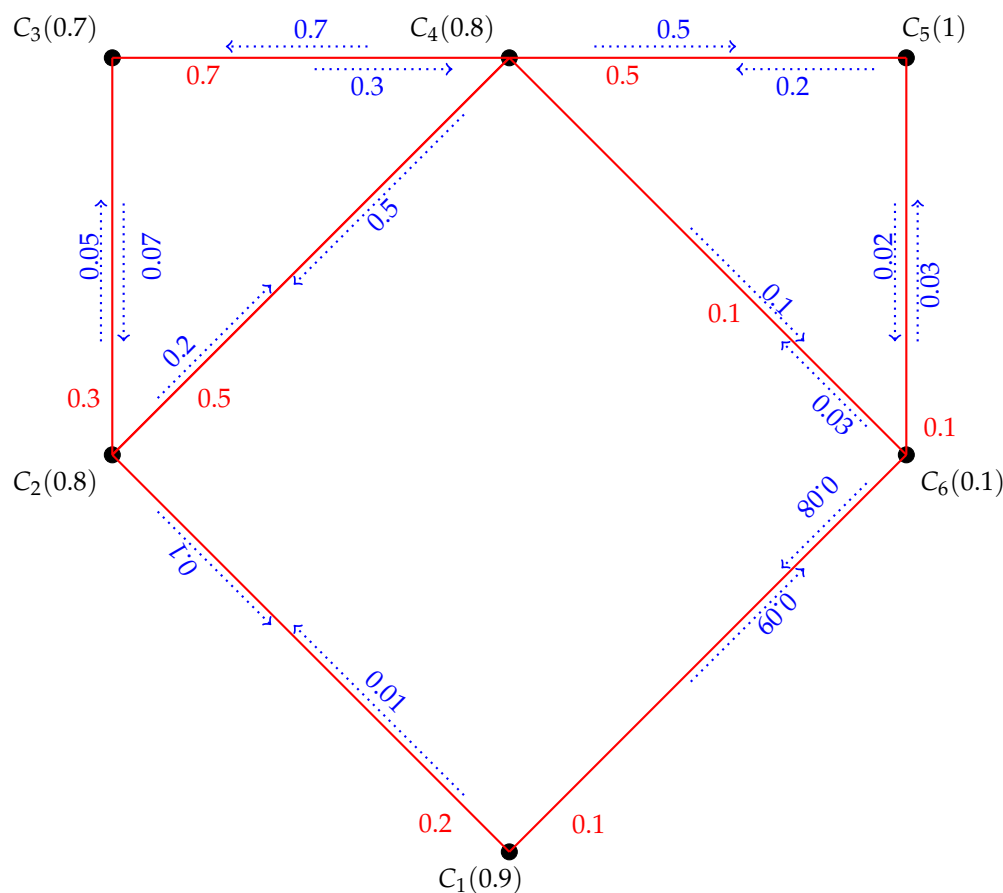
Figure 10. A graphical model representing trade among different countries.

7. Comparative Analysis

In Figure 10, a FIG indicating six different countries C_1, C_2, C_3, C_4, C_5, and C_6. The MSV of the vertices representing the inhabitants of the country, the MSV of the edges expressing the relationship among these countries and the MSV of the pairs represent the best trade policies among these countries. In the case of FIG we get SPDS $Z_2 = \{C_3, C_6\}$ with $\gamma_s(Z_2) = 0.8$. But in Figure 10, if we remove all the pairs we get a FG. In the case of FG, we find all strong and non SEs. First we check $C_1 C_2$ is a SE or not. In Figure 10 it can be seen that there are five possible paths from C_1 to C_2 which are $P_1 : C_1 - C_2$ is a path of length 1 with strength $= \phi(C_1 C_2) = 0.2$. $P_2 : C_1 - C_6 - C_5 - C_4 - C_2$ is a path of length 4 with strength $= \wedge\{\phi(C_1 C_6), \phi(C_6 C_5), \phi(C_5 C_4), \phi(C_4 C_2) = 0.1, 0.1, 0.5, 0.5\} = 0.1$. $P_3 : C_1 - C_6 - C_5 - C_4 - C_3 - C_2$ is a path of length 5 with strength $= \wedge\{\phi(C_1 C_6), \phi(C_6 C_5), \phi(C_5 C_4), \phi(C_4 C_3), \phi(C_3 C_2) = 0.1, 0.1, 0.7, 0.3\} = 0.1$. $P_4 : C_1 - C_6 - C_4 - C_2$ is a path of length 3 with strength $= \wedge\{\phi(C_1 C_6), \phi(C_6 C_4), \phi(C_4 C_2) = 0.1, 0.1, 0.5\} = 0.1$. $P_5 : C_1 - C_6 - C_4 - C_3 - C_2$ is a path of length 4 with strength $= \wedge\{\phi(C_1 C_6), \phi(C_6 C_4), \phi(C_4 C_3), \phi(C_3 C_2) = 0.1, 0.1, 0.7, 0.3\} = 0.1$. The strength of the SGP joining C_1 and C_2 is $\phi^\infty(C_1, C_2) = \vee\{0.2, 0.1, 0.1, 0.1, 0.1\} = 0.2$ is the largest value of all the strengths between C_1 and C_2. Since $\phi^\infty(C_1, C_2) = \phi(C_1, C_2) = 0.2$ therefore $C_1 C_2$ is a SE. In the same way, $C_1 C_6, C_2 C_4, C_3 C_4, C_4 C_5, C_4 C_6$ and $C_5 C_6$ are all SEs. But $C_2 C_3$ is not a SE because $\phi^\infty(C_2, C_3) = 0.5 \neq \phi(C_2, C_3) = 0.3$. All possible SEDSs of the FG are $Z_1 = \{C_1, C_4\}, Z_2 = \{C_1, C_3, C_5\}, Z_3 = \{C_2, C_3, C_6\}$ and $Z_4 = \{C_2, C_3, C_5\}$ with $\gamma_s(Z_1) = \sigma(C_1) + \sigma(C_4) = 0.9 + 0.8 = 1.7$. Similarly, $\gamma_s(Z_2) = 2.6, \gamma_s(Z_3) = 1.6$ and $\gamma_s(Z_4) = 2.5$. Now, it can be seen in Figure 10 that in case of FIG the set $Z_2 = \{C_3, C_6\}$ with $\gamma_s(Z_2) = 0.8$ has two countries which have best trade policies among all other countries but in the case of FG we are unable to talk about best trade policies due to non-availability

of pairs. FGs can show the relationship among different countries but silent to talk about trade policies among different countries. Therefore, FIGs are more beneficial and effective than FGs.

8. Conclusions

The concept of domination in graphs is very rich both from a theoretical as well as applications point of view. Domination ideas in graphs are crucial from a different perspective. It has multiple uses in various fields of life. Different authors have investigated more than thirty domination parameters. In this article, we propose the novel idea of domination in FIGs using SPs. The notion of SPDS and SPDN for various types of FIGs with different examples is also explained. An idea of domination in the join of FIGs and accurate domination using SPs with a variety of examples in FIGs is presented in this article. Some significant results related to accurate domination are also investigated. Several handy and key results of accurate domination for FIC with different examples are also discussed. SPDN and ADN are introduced for different FIGs. SPDN for CFIGs is also explored with certain examples. Some beneficial and instrumental theorems of domination in the join of FIGs are also explained. An application of SPDN in matters of trade among different countries is accomplished too. In the future, we aim to extend our research work to Hamiltonian FIGs, the coloring of FIGs, and the chromaticity of FIGs. Further work on these ideas will be reported in upcoming papers.

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References

1. Zadeh, L.A. Fuzzy sets. *Inf. Control* **1965**, *8*, 338–353. [CrossRef]
2. Zadeh, L.A. Is there a need for fuzzy logic? *Inf. Sci.* **2008**, *178*, 2751–2779. [CrossRef]
3. Rosenfeld, A. *Fuzzy Graphs, Fuzzy Sets and Their Applications to Cognitive and Decision Processes*; Elsevier: Amsterdam, The Netherlands, 1975; pp. 77–95.
4. Gani, A.N.; Ahamed, M.B. Order and size in fuzzy graphs. *Bull. Pure Appl. Sci.* **2003**, *22*, 145–148.
5. Bhutani, K.R.; Mordeson, J.; Rosenfeld, A. On degrees of end nodes and cut nodes in fuzzy graphs. *Iran. J. Fuzzy Syst.* **2004**, *1*, 57–64.
6. Somasundaram, A.; Somasundaram, S. Domination in fuzzy graphs-I. *Pattern Recognit. Lett.* **1998**, *19*, 787–791. [CrossRef]
7. Somasundaram, A. Domination in products of fuzzy graphs. *Int. J. Uncertain. Fuzziness Knowl. Based Syst.* **2005**, *13*, 195–204. [CrossRef]
8. Gani, A.N.; Ahamed, M.B. Strong and weak domination in fuzzy graphs. *East Asian Math. J.* **2007**, *23*, 1–8.
9. Natarajan, C.; Ayyaswamy, S. On strong (weak) domination in fuzzy graphs. *Int. J. Math. Comput. Sci.* **2010**, *4*, 1035–1037.
10. Manjusha, O.; Sunitha, M. Notes on domination in fuzzy graphs. *J. Intell. Fuzzy Syst.* **2014**, *27*, 3205–3212. [CrossRef]
11. Manjusha, O.; Sunitha, M. Total domination in fuzzy graphs using strong arcs. *Ann. Pure Appl. Math.* **2014**, *9*, 23–33.
12. Manjusha, O.; Sunitha, M. Strong domination in fuzzy graphs. *Fuzzy Inf. Eng.* **2015**, *7*, 369–377. [CrossRef]
13. Dharmalingam, K.; Rani, M. Equitable domination in fuzzy graphs. *Int. J. Pure Appl. Math.* **2014**, *94*, 661-667. [CrossRef]
14. Gani, A.N.; Devi, K.P. 2-domination in fuzzy graphs. *Int. J. Fuzzy Math. Arch.* **2015**, *9*, 119–124.
15. Gani, A.N.; Rahman, A.A. Accurate 2-domination in fuzzy graphs using strong arcs. *Int. J. Pure Appl. Math.* **2018**, *118*, 279–286.
16. Ponnappan, C.; Ahamed, S.B.; Surulinathan, P. Edge domination in fuzzy graphs new approach. *Int. J. It Eng. Appl. Sci. Res.* **2015**, *4*, 14–17.
17. Nithya, P.; Dharmalingam, K. Very excellent domination in fuzzy grpahs. *Int. J. Comput. Appl. Math.* **2017**, *12*, 313–326.
18. Bhutani, K.R.; Rosenfeld, A. Strong arcs in fuzzy graphs. *Inf. Sci.* **2003**, *152*, 319–322. [CrossRef]
19. Mathew, S.; Mordeson, J.N. Fuzzy influence graphs. *New Math. Nat. Comput.* **2017**, *13*, 311–325. [CrossRef]
20. Rashmanlou, H.; Jun, Y.B. Complete interval-valued fuzzy graphs. *Ann. Fuzzy Math. Inform.* **2013**, *6*, 677–687.

21. Sunitha, M.; Vijayakumar, A. Complement of a fuzzy graph. *Indian J. Pure Appl. Math.***2002**, *33*, 1451–1464.
22. Yin, S.; Li, H.; Yang, Y. Product operations on q-rung orthopair fuzzy graphs. *Symmetry* **2019**, *11*, 1–23. [CrossRef]
23. Mordeson, J.N.; Nair, P.S. Fuzzy graphs and fuzzy hypergraphs. *Physica* **2012**, *46*. [CrossRef]
24. Bershtein, L.S.; Bozhenyuk, A.V.; Fuzzy graphs and fuzzy hypergraphs. In *Encyclopedia of Artificial Intelligence*; IGI Global: Hershey, PA, USA, 2009; pp. 704–709. [CrossRef]
25. Mathew, S.; Mordeson, J.N.; Malik, D.S. *Fuzzy Graph Theory*; Springer: Berlin/Heidelberg, Germany, 2018.
26. Mordeson, J.N.; Chang-Shyh, P. Operations on fuzzy graphs. *Inf. Sci.* **1994**, *79*, 159–170. [CrossRef]
27. Yeh, R.T.; Bang, S.Y. Fuzzy relations, fuzzy graphs, and their applications to culstering analysis. In *Fuzzy Sets and Their Applications to Cognitive and Decision Processes*; Elsevier: Amsterdam, The Netherlands, 1975; pp. 125–149.
28. Dinesh, T. Fuzzy incidence graph-an introduction. *Adv. Fuzzy Sets Syst.* **2016**, *21*, 33–48. [CrossRef]
29. Malik, D.S.; Mathew, S.; Mordeson, J.N. Fuzzy incidence graphs: Applications to human trafficking. *Inf. Sci.* **2018**, *447*, 244–255. [CrossRef]
30. Mathew, S.; Mordeson, J.N. Connectivity concepts in fuzzy incidence graphs. *Inf. Sci.* **2017**, *382–383*, 326–333. [CrossRef]
31. Fang, J.; Nazeer, I.; Rashid, T.; Liu, J.B. Connectivity and Wiener index of fuzzy incidence graphs. *Math. Probl. Eng.* **2021**, *2021*, 1–7. [CrossRef]
32. Mathew, S.; Mordeson, J.N.; Yang, H.L. Incidence cuts and connectivity in fuzzy incidence graphs. *Iran. J. Fuzzy Syst.* **2019**, *16*, 31–43.
33. Mordeson, J.N.; Mathew, S. Fuzzy end nodes in fuzzy incidence graphs. *New Math. Nat. Comput.* **2017**, *13*, 13–20. [CrossRef]
34. Nazeer, I.; Rashid, T.; Keikha, A. An application of product of intuitionistic fuzzy incidence graphs in textile industry. *Complexity* **2021**. [CrossRef]
35. Nazeer, I.; Rashid, T.; Guirao, J.L.G. Domination of fuzzy incidence graphs with the algorithm and application for the selection of a medical lab. *Math. Probl. Eng.* **2021**, *2021*, 1–11. [CrossRef]
36. Mordeson, J.N.; Mathew, S.; Borzooei, R.A. Vulnerability and government response to human trafficking: Vague fuzzy incidence graphs. *New Math. Nat. Comput.* **2018**, *14*, 203–219. [CrossRef]
37. Mordeson, J.N.; Mathew, S.; Malik, D.S. *Fuzzy Incidence Graphs, Fuzzy Graph Theory with Applications to Human Trafficking*; Springer: Berlin/Heidelberg, Germany, 2018; pp. 87–137.
38. Selvam, A.; Ponnappan, C. Domination in join of fuzzy graphs using strong arcs. *Mater. Today Proc.* **2021**, *37*, 67–70. [CrossRef]
39. Ponnappan, C.; Selvam, A. Accurate domination in fuzzy graphs using strong arcs. *Mater. Today Proc.* **2021**, *37*, 115–118. [CrossRef]