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Complex Intuitionistic Fuzzy Aczel-Alsina Aggregation Operators and Their Application in Multi-Attribute Decision-Making

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Abstract: To handle complex, risk-illustrating, and asymmetric information, the theory discussed in this analysis is much more suitable for evaluating the above dilemmas. To manage ambiguity and inconsistency in real-life problems, the principle of Aczel–Alsina (AA) t-norm and t-conorm was initiated in 1980. These norms are massively modified and different from prevailing norms due to parameter p, where $0 < p < +\infty$. The major contribution of this analysis is to analyze the AA operational laws (addition, multiplication, score value, accuracy value) under the complex intuitionistic fuzzy (CIF) settings. Furthermore, we initiated the principle of CIFAA weighted averaging (CIFAAWA), CIFAA ordered weighted averaging (CIFAAOWA), CIFAA hybrid averaging (CIFAAHA), CIFAA weighted geometric (CIFAAWG), CIFAA ordered weighted geometric (CIFAAOWG), CIFAA hybrid geometric (CIFAAHG), as well as their beneficial results. Additionally, to consider the elaborated works, a multi-attribute decision-making (MADM) technique was explored to investigate the supremacy and feasibility of the developed works. The main influence of this manuscript is how to choose the best decision under the availability of asymmetric types of information given by different experts. Finally, we performed the sensitivity analysis and graphically show the presented work with the help of several examples.

Keywords: complex intuitionistic fuzzy sets; Aczel–Alsina aggregation operators; decision-making strategy

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1. Introduction

The MADM technique has attained a lot of attention from different intellectuals due to its beneficial ability to investigate the best decisions out of a group, and several individuals have employed it in the region of problematic dilemmas. Some cases are very difficult if an expert employs fuzzy numbers instead of the crisp number. For this, the principle of the fuzzy set (FS) [1] was initiated by increasing the range of crips set in the unit interval. Keeping the benefits of the FS, numerous people have utilized it in the region of different circumstances. For illustration, see studies on ordered aggregation operators [2], the idea of immediate probabilities [3], decision-making modeling based on immediate probabilities [4], ordered weighted aggregation (OWA) operators for FSs [5], OWA operators of different criteria with mixed uncertain satisfaction [6], super migrative based on aggregation function [7], deviation under aggregation mapping [8], generalized ordered modular averaging operators [9], OWA operators in the weighted averaging and their application in decision-making [10], analysis of fuzzy research under biometric indicators [11], and on averaging operators for intuitionistic fuzzy sets [12]. The fundamental theory of FS has gotten massive attention from the side of very well-known academics. However, there may be a rule in the decision-making processes whereby the decision-maker may be of truth grade (TG) and falsity grade (FG) for the attribute values, then the FS fails to tackle this case. To survive this challenge, Atanassov [13] diagnosed an intuitionistic FS (IFS). IFS has been developed with two different mappings in the shape of TG and FG, which hold the following axiom: that is, the total of two mappings is controlled to the unit interval. To fill this gap in the research, several people have employed it in the region of bipolar soft sets [14], discussing the quality of the image by using measures [15], Aczel-Alsina aggregation operators [16], decision-making frameworks for determining the large-scale rooftop [17], different sorts of measures under right angle triangle [18], analysis of weight-based hybrid approach [19], three-ways decision-making techniques [20], uncertain database measures under belief function [21], how to determine the average of knowledge under IFSs [22], time-series approaches by using the IFSs [23], and AHP techniques based on IFSs and their application [24].

The above-referred studies uncovering those various decision-making strategies have been created under IFS and interval-valued IFS speculations. In any case, it is dissected that these hypotheses can manage only one-dimensional decision-making issues. Nonetheless, some truly overwhelming issues incorporate two-dimensional information, that is, data identified with the traits and periodicity of the boundaries worried about the issue. To depict such two-dimensional data utilizing these hypotheses, the chief should think about at least two FSs/IFSs, which might expand the execution time and the number of calculations required while tackling the issue. Thus, to depict occasional data in the judgment esteems, the fundamentals of complex FS (CFS) were diagnosed by Ramot et al. [25]. CFS covers only the TG in the shape of these sorts of structures, which is deepened on the real part and imaginary part, whose values belong to the unit interval. Additionally, many operational laws and their inequalities based on CFS were diagnosed by Zhang et al. [26], the principle of complex fuzzy logic was elaborated by Ramot et al. [27], neuro-fuzzy systems were initiated by Chen et al. [28], and Liu et al. [29] investigated the cross-entropy measures for CFSs. The fundamental theory of CFS has gotten massive attention from the side of very well-known academics. However, there may be a rule in the decision-making processes whereby the decision-maker may be TG and FG for the attribute values, and the CFS fails to tackle this case. To survive this challenge, Alkouri and Salleh [30] diagnosed the complex IFS (CIFS). IFS has developed with two different mappings in the shape of TG and FG in the shape of complex numbers, which hold the following axiom: that is, the total of the real part (also for the imaginary part) of the two mappings is controlled to the unit interval. To fill the gap in the research works, several people have employed it in the region of aggregation operators [31], generalized geometric aggregation operators [32], robust averaging/geometric aggregation operators [33], generalized complex intuitionistic fuzzy aggregation operators [34], and prioritized aggregation operators for complex intuitionistic fuzzy soft sets [35]. To investigate the addition and multiplication of any two fuzzy numbers, one needs to use t-norm and t-conorm, because using crisp addition and multiplication exceed the value of fuzzy numbers from the unit interval. The concept of the triangular norm was initiated by Menger [36] in 1942. Further, Garg and Rani [37] developed the distance measures to fill the gap in the research work. Xia et al. [38] initiated the probabilistic t-norm and t-conorm. Liu [39] explored the Hamacher t-norm and t-conorm. Deschrijver et al. [40] elaborated the Lukasiewicz t-norm and t-conorm. Klement and Mesiar [41] presented the idea of the t-norm.

The main theory of Aczel–Alsina t-norm and t-conorm based on classical information was derived by Aczel and Alsina [42], which is a very dominant and valuable idea used for evaluating awkward and unreliable information in genuine life problems. The principle of the Aczel–Alsina t-norm and t-conorm is that they are massively modified when compared to the prevailing t-norms (Hamacher). The mathematical approach of diagnosed information and related operators is very closely connected with the theory of symmetry. Considering the above examination, we understand that decision-making issues are getting progressively more perplexing generally. To have the option to pick the unrivaled alternative(s) for the MADM issues and impart the questionable information in an undeniably more worthwhile way is an important problem. Moreover, it is important to oversee how to consider the association between input disputes. The major contribution of this analysis is illustrated below.

1. To analyze the AA operational laws (addition, multiplication, score value, accuracy value) under the CIF settings.
2. To initiate the principle of CIFAAWA, CIFAAOWA, CIFAAHA, CIFAAWG, CIFAAOWG, CIFAAHG, and their beneficial results.
3. To consider the elaborated works, a MADM technique was explored to investigate the supremacy and feasibility of the developed works.
4. To demonstrate the sensitivity analysis and graphically show the presented works with the help of several examples, where the main theme of this analysis is also described in the form of Figure 1.

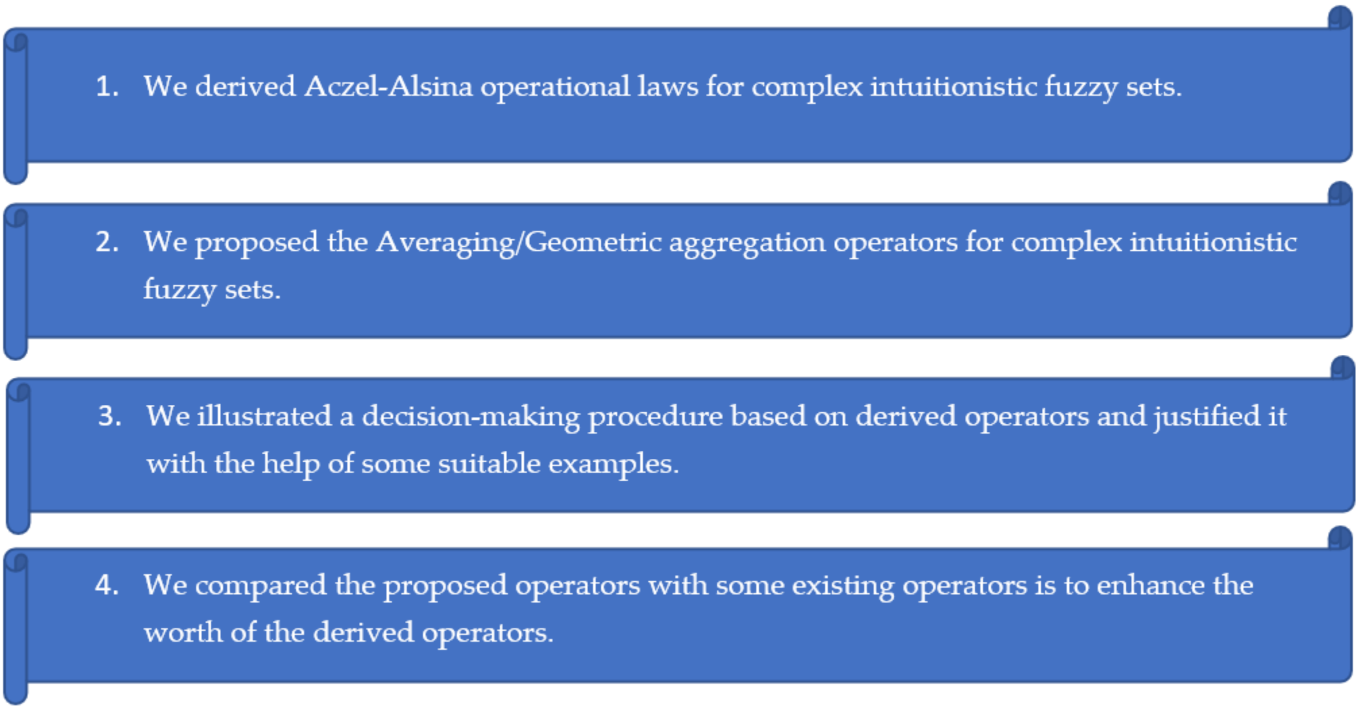

Figure 1. The main theme of the proposed analysis is given above.

The major key of this analysis is elaborated as follows: in Section 2, we revise several prevailing principles such as CPFSs, SSs, Aczel–Alsina t-norm and t-conorm, and their algebraic laws. In Section 3, we analyze the AA operational laws (addition, multiplication, score value, accuracy value) under the CIF settings and their use results. In Section 4, we initiate the principle of CIFAAWA, CIFAAOWA, CIFAAHA, CIFAAWG, CIFAAOWG, CIFAAHG, and their beneficial results. In Section 5, we consider the elaborated works, and an MADM technique is explored to investigate the supremacy and feasibility of the developed works. Finally, we demonstrate the sensitivity analysis and graphically show the presented works with the help of several examples. The conclusion of this script is deliberated in Section 6.

2. Preliminaries

This theory aims to revise several prevailing principles such as CPFSs, Aczel–Alsina t-norm and t-conorm, SSs, and their algebraic laws. The object $\widetilde{\mathbb{X}_U}$ statess the universal sets.

Definition 1. *[30] A CIFS* $\overline{\overline{\mathfrak{C}_C}}$ *is invented by:*

$$\overline{\overline{\mathfrak{C}_C}} = \left\{ \left(\overline{\overline{\mathcal{M}_{\overline{\overline{\mathfrak{C}_C}}}}}(\widetilde{\chi_E}), \overline{\overline{\mathcal{N}_{\overline{\overline{\mathfrak{C}_C}}}}}(\widetilde{\chi_E}) \right) : \widetilde{\chi_E} \in \widetilde{\mathbb{X}_U} \right\} \tag{1}$$

where $\overline{\overline{\mathcal{M}_{\overline{\overline{\mathfrak{C}_C}}}}}(\widetilde{\chi_E}) = \overline{\overline{\mathcal{M}_{\overline{\overline{R}}}}}(\widetilde{\chi_E}) e^{i2\pi(\overline{\overline{\mathcal{M}_{\overline{\overline{I}}}}}(\widetilde{\chi_E}))}$ *and* $\overline{\overline{\mathcal{N}_{\overline{\overline{\mathfrak{C}_C}}}}}(\widetilde{\chi_E}) = \overline{\overline{\mathcal{N}_{\overline{\overline{R}}}}}(\widetilde{\chi_E}) e^{i2\pi(\overline{\overline{\mathcal{N}_{\overline{\overline{I}}}}}(\widetilde{\chi_E}))}$, *stated the TD and FD with* $0 \leq \overline{\overline{\mathcal{M}_{\overline{\overline{R}}}}}(\widetilde{\chi_E}) + \overline{\overline{\mathcal{N}_{\overline{\overline{R}}}}}(\widetilde{\chi_E}) \leq 1$ *and* $0 \leq \overline{\overline{\mathcal{M}_{\overline{\overline{I}}}}}(\widetilde{\chi_E}) + \overline{\overline{\mathcal{N}_{\overline{\overline{I}}}}}(\widetilde{\chi_E}) \leq 1$. *The expression* $\overline{\overline{\Re_{\overline{\overline{\mathfrak{C}_C}}}}}(\widetilde{\chi_E}) = \overline{\overline{\Re_{\overline{\overline{R}}}}}(\widetilde{\chi_E}) e^{i2\pi(\overline{\overline{\Re_{\overline{\overline{I}}}}}(\widetilde{\chi_E}))} = \left(1 - \left(\overline{\overline{\mathcal{M}_{\overline{\overline{R}}}}}(\widetilde{\chi_E}) + \overline{\overline{\mathcal{N}_{\overline{\overline{R}}}}}(\widetilde{\chi_E})\right)\right) e^{i2\pi(1-(\overline{\overline{\mathcal{M}_{\overline{\overline{I}}}}}(\widetilde{\chi_E}) + \overline{\overline{\mathcal{N}_{\overline{\overline{I}}}}}(\widetilde{\chi_E})))}$ *is called neutral grade. The complex intuitionistic fuzzy number (CIFN) is invented by:* $\overline{\overline{\mathfrak{C}_{C_j}}} = \left(\overline{\overline{\mathcal{M}_{\overline{\overline{R_j}}}}} e^{i2\pi(\overline{\overline{\mathcal{M}_{\overline{\overline{I_j}}}}})}, \overline{\overline{\mathcal{N}_{\overline{\overline{R_j}}}}} e^{i2\pi(\overline{\overline{\mathcal{N}_{\overline{\overline{I_j}}}}})} \right), j = 1, 2, \ldots, n.$

Definition 2. *[30] By considering any two CIFNs* $\overline{\overline{\mathfrak{C}_{C_j}}} = \left(\overline{\overline{\mathcal{M}_{\overline{\overline{R_j}}}}} e^{i2\pi(\overline{\overline{\mathcal{M}_{\overline{\overline{I_j}}}}})}, \overline{\overline{\mathcal{N}_{\overline{\overline{R_j}}}}} e^{i2\pi(\overline{\overline{\mathcal{N}_{\overline{\overline{I_j}}}}})} \right), j = 1, 2$, *then*

$$\overline{\overline{\mathfrak{C}_{C_1}}} \oplus \overline{\overline{\mathfrak{C}_{C_2}}} = \left(\overline{\overline{\mathcal{M}_{\overline{\overline{R_1}}}}} + \overline{\overline{\mathcal{M}_{\overline{\overline{R_2}}}}} - \overline{\overline{\mathcal{M}_{\overline{\overline{R_1}}}} \mathcal{M}_{\overline{\overline{R_2}}}} e^{i2\pi(\overline{\overline{\mathcal{M}_{\overline{\overline{I_1}}}}} + \overline{\overline{\mathcal{M}_{\overline{\overline{I_2}}}}} - \overline{\overline{\mathcal{M}_{\overline{\overline{I_1}}} \mathcal{M}_{\overline{\overline{I_2}}}}})}, \left(\overline{\overline{\mathcal{N}_{\overline{\overline{R_1}}} \mathcal{N}_{\overline{\overline{R_2}}}}} \right) e^{i2\pi(\overline{\overline{\mathcal{N}_{\overline{\overline{I_1}}} \mathcal{N}_{\overline{\overline{I_2}}}}})} \right) \tag{2}$$

$$\overline{\overline{\mathfrak{C}_{C_1}}} \otimes \overline{\overline{\mathfrak{C}_{C_2}}} = \left(\left(\overline{\overline{\mathcal{M}_{\overline{\overline{R_1}}} \mathcal{M}_{\overline{\overline{R_2}}}}} \right) e^{i2\pi(\overline{\overline{\mathcal{M}_{\overline{\overline{I_1}}} \mathcal{M}_{\overline{\overline{I_2}}}}})}, \overline{\overline{\mathcal{N}_{\overline{\overline{R_1}}}}} + \overline{\overline{\mathcal{N}_{\overline{\overline{R_2}}}}} - \overline{\overline{\mathcal{N}_{\overline{\overline{R_1}}} \mathcal{N}_{\overline{\overline{R_2}}}}} e^{i2\pi(\overline{\overline{\mathcal{N}_{\overline{\overline{I_1}}}}} + \overline{\overline{\mathcal{N}_{\overline{\overline{I_2}}}}} - \overline{\overline{\mathcal{N}_{\overline{\overline{I_1}}} \mathcal{N}_{\overline{\overline{I_2}}}}})} \right) \tag{3}$$

$$\overline{\overline{\psi_S \mathfrak{C}_{C_1}}} = \left(\left(1 - \left(1 - \overline{\overline{\mathcal{M}_{\overline{\overline{R_1}}}}} \right)^{\overline{\overline{\psi_S}}} \right) e^{i2\pi(1-(1-\overline{\overline{\mathcal{M}_{\overline{\overline{I_1}}}}})^{\overline{\overline{\psi_S}}})}, \overline{\overline{\mathcal{N}_{\overline{\overline{R_1}}}}}^{\overline{\overline{\psi_S}}} e^{i2\pi(\overline{\overline{\mathcal{N}_{\overline{\overline{I_1}}}}}^{\overline{\overline{\psi_S}}})} \right) \tag{4}$$

$$\overline{\overline{\mathfrak{C}_{C_1}}}^{\overline{\overline{\psi_S}}} = \left(\overline{\overline{\mathcal{M}_{\overline{\overline{R_1}}}}}^{\overline{\overline{\psi_S}}} e^{i2\pi(\overline{\overline{\mathcal{M}_{\overline{\overline{I_1}}}}}^{\overline{\overline{\psi_S}}})}, \left(1 - \left(1 - \overline{\overline{\mathcal{N}_{\overline{\overline{R_1}}}}} \right)^{\overline{\overline{\psi_S}}} \right) e^{i2\pi(1-(1-\overline{\overline{\mathcal{N}_{\overline{\overline{I_1}}}}})^{\overline{\overline{\psi_S}}})} \right) \tag{5}$$

Definition 3. *[30] By considering any CIFNs* $\overline{\overline{\mathfrak{C}_{C_j}}} = \left(\overline{\overline{\mathcal{M}_{\overline{\overline{R_j}}}}} e^{i2\pi(\overline{\overline{\mathcal{M}_{\overline{\overline{I_j}}}}})}, \overline{\overline{\mathcal{N}_{\overline{\overline{R_j}}}}} e^{i2\pi(\overline{\overline{\mathcal{N}_{\overline{\overline{I_j}}}}})} \right), j = 1$, *then the score value (SV) and accuracy value (AV) are given by:*

$$\overline{\overline{\mathcal{S}_{SV}}}\left(\overline{\overline{\mathfrak{C}_{C_1}}}\right) = \frac{1}{2}\left(\overline{\overline{\mathcal{M}_{\overline{\overline{R_1}}}}} + \overline{\overline{\mathcal{M}_{\overline{\overline{I_1}}}}} - \overline{\overline{\mathcal{N}_{\overline{\overline{R_1}}}}} - \overline{\overline{\mathcal{N}_{\overline{\overline{I_1}}}}} \right), \overline{\overline{\mathcal{S}_{SV}}}\left(\overline{\overline{\mathfrak{C}_{C_1}}}\right) \in [-1, 1] \tag{6}$$

$$\overline{\overline{\mathcal{H}_{AV}}}\left(\overline{\overline{\mathfrak{C}_{C_1}}}\right) = \frac{1}{2}\left(\overline{\overline{\mathcal{M}_{\overline{\overline{R_1}}}}} + \overline{\overline{\mathcal{M}_{\overline{\overline{I_1}}}}} + \overline{\overline{\mathcal{N}_{\overline{\overline{R_1}}}}} + \overline{\overline{\mathcal{N}_{\overline{\overline{I_1}}}}} \right), \overline{\overline{\mathcal{H}_{AV}}}\left(\overline{\overline{\mathfrak{C}_{C_1}}}\right) \in [0, 1] \tag{7}$$

Definition 4. *[30] By considering any CIFNs* $\overline{\overline{\mathfrak{C}_{C_j}}} = \left(\overline{\overline{\mathcal{M}_{\overline{\overline{R_j}}}}} e^{i2\pi(\overline{\overline{\mathcal{M}_{\overline{\overline{I_j}}}}})}, \overline{\overline{\mathcal{N}_{\overline{\overline{R_j}}}}} e^{i2\pi(\overline{\overline{\mathcal{N}_{\overline{\overline{I_j}}}}})} \right)$, $j = 1, 2$, *then*

1. When $\overline{\overline{\mathcal{S}_{SV}}}\left(\overline{\overline{\mathfrak{C}_{C_1}}}\right) > \overline{\overline{\mathcal{S}_{SV}}}\left(\overline{\overline{\mathfrak{C}_{C_2}}}\right) \Rightarrow \overline{\overline{\mathfrak{C}_{C_1}}} > \overline{\overline{\mathfrak{C}_{C_2}}}$;
2. When $\overline{\overline{\mathcal{S}_{SV}}}\left(\overline{\overline{\mathfrak{C}_{C_1}}}\right) < \overline{\overline{\mathcal{S}_{SV}}}\left(\overline{\overline{\mathfrak{C}_{C_2}}}\right) \Rightarrow \overline{\overline{\mathfrak{C}_{C_1}}} < \overline{\overline{\mathfrak{C}_{C_2}}}$;
3. When $\overline{\overline{\mathcal{S}_{SV}}}\left(\overline{\overline{\mathfrak{C}_{C_1}}}\right) = \overline{\overline{\mathcal{S}_{SV}}}\left(\overline{\overline{\mathfrak{C}_{C_2}}}\right) \Rightarrow$;
   (i) When $\overline{\overline{\mathcal{H}_{AV}}}\left(\overline{\overline{\mathfrak{C}_{C_1}}}\right) > \overline{\overline{\mathcal{H}_{AV}}}\left(\overline{\overline{\mathfrak{C}_{C_2}}}\right) \Rightarrow \overline{\overline{\mathfrak{C}_{C_1}}} > \overline{\overline{\mathfrak{C}_{C_2}}}$;
   (ii) When $\overline{\overline{\mathcal{H}_{AV}}}\left(\overline{\overline{\mathfrak{C}_{C_1}}}\right) < \overline{\overline{\mathcal{H}_{AV}}}\left(\overline{\overline{\mathfrak{C}_{C_2}}}\right) \Rightarrow \overline{\overline{\mathfrak{C}_{C_1}}} < \overline{\overline{\mathfrak{C}_{C_2}}}$;
   (iii) When $\overline{\overline{\mathcal{H}_{AV}}}\left(\overline{\overline{\mathfrak{C}_{C_1}}}\right) = \overline{\overline{\mathcal{H}_{AV}}}\left(\overline{\overline{\mathfrak{C}_{C_2}}}\right) \Rightarrow \overline{\overline{\mathfrak{C}_{C_1}}} = \overline{\overline{\mathfrak{C}_{C_2}}}$.

Definition 5. *[16] Suppose* $\overline{\overline{\mathbb{T}_{TN}}} : [0,1] \times [0,1] \to [0,1]$, *this states a TN if*

1. $\overline{\overline{\mathbb{T}_{TN}}}(\widetilde{\chi_E}, \widetilde{\chi_E}') = \overline{\overline{\mathbb{T}_{TN}}}(\widetilde{\chi_E}', \widetilde{\chi_E}), \widetilde{\chi_E}, \widetilde{\chi_E}' \in [0,1]$;
2. $\overline{\overline{\mathbb{T}_{TN}}}(\widetilde{\chi_E}, \widetilde{\chi_E}') \leq \overline{\overline{\mathbb{T}_{TN}}}(\widetilde{\chi_E}, \widetilde{\chi_E}'')$, if $\widetilde{\chi_E}' \leq \widetilde{\chi_E}''$;
3. $\overline{\overline{\mathbb{T}_{TN}}}\left(\widetilde{\chi_E}, \overline{\overline{\mathbb{T}_{TN}}}(\widetilde{\chi_E}', \widetilde{\chi_E}'')\right) = \overline{\overline{\mathbb{T}_{TN}}}\left(\overline{\overline{\mathbb{T}_{TN}}}(\widetilde{\chi_E}, \widetilde{\chi_E}'), \widetilde{\chi_E}''\right)$;
4. $\overline{\overline{\mathbb{T}_{TN}}}(\widetilde{\chi_E}, 1) = \widetilde{\chi_E}$;

Definition 6. *[16] Suppose* $\overline{\overline{\mathbb{S}_{TN}}} : [0,1] \times [0,1] \to [0,1]$, *this states a TCN if*

1. $\overline{\overline{\mathbb{S}_{TN}}}(\widetilde{\chi_E}, \widetilde{\chi_E}') = \overline{\overline{\mathbb{S}_{TN}}}(\widetilde{\chi_E}', \widetilde{\chi_E}), \widetilde{\chi_E}, \widetilde{\chi_E}' \in [0,1]$;
2. $\overline{\overline{\mathbb{S}_{TN}}}(\widetilde{\chi_E}, \widetilde{\chi_E}') \leq \overline{\overline{\mathbb{S}_{TN}}}(\widetilde{\chi_E}, \widetilde{\chi_E}'')$, if $\widetilde{\chi_E}' \leq \widetilde{\chi_E}''$;
3. $\overline{\overline{\mathbb{S}_{TN}}}\left(\widetilde{\chi_E}, \overline{\overline{\mathbb{S}_{TN}}}(\widetilde{\chi_E}', \widetilde{\chi_E}'')\right) = \overline{\overline{\mathbb{S}_{TN}}}\left(\overline{\overline{\mathbb{S}_{TN}}}(\widetilde{\chi_E}, \widetilde{\chi_E}'), \widetilde{\chi_E}''\right)$;
4. $\overline{\overline{\mathbb{S}_{TN}}}(\widetilde{\chi_E}, 0) = \widetilde{\chi_E}$;

Definition 7. *[16] Suppose* $\left(\overline{\overline{\mathbb{T}_{TNA}}}^{\psi}\right)_{\psi \in [0,\infty]}$, *then the Aczel–Alsina TN is given by:*

$$\overline{\overline{\mathbb{T}_{TNA}}}^{\psi}(\widetilde{\chi_E}, \widetilde{\chi_E}') = \begin{cases} \overline{\overline{\mathbb{T}_{TN}}}(\widetilde{\chi_E}, \widetilde{\chi_E}') & if\ \psi = 0 \\ min(\widetilde{\chi_E}, \widetilde{\chi_E}') & if\ \psi = \infty \\ e^{-((-log\widetilde{\chi_E})^{\psi} + (-log\widetilde{\chi_E}')^{\psi})^{\frac{1}{\psi}}} & otherwise \end{cases} \tag{8}$$

Definition 8. *[16] Suppose* $\left(\overline{\overline{\mathbb{S}_{TNA}}}^{\psi}\right)_{\psi \in [0,\infty]}$, *then the Aczel–Alsina TCN is given by:*

$$\overline{\overline{\mathbb{S}_{TNA}}}^{\psi}(\widetilde{\chi_E}, \widetilde{\chi_E}') = \begin{cases} \overline{\overline{\mathbb{S}_{TN}}}(\widetilde{\chi_E}, \widetilde{\chi_E}') & if\ \psi = 0 \\ max(\widetilde{\chi_E}, \widetilde{\chi_E}') & if\ \psi = \infty \\ 1 - e^{-((-\log(1-\widetilde{\chi_E}))^{\psi} + (-\log(1-\widetilde{\chi_E}'))^{\psi})^{\frac{1}{\psi}}} & otherwise \end{cases} \tag{9}$$

3. Aczel–Alsina Operational Laws for CIFSs

The major contribution of this section is to analyze the AA operational laws (addition, multiplication, score value, accuracy value) under the CIF settings and their related results.

Definition 9. *By considering any two CIFNs* $\overline{\overline{\mathfrak{C}_{C_j}}} = \left(\overline{\overline{\mathcal{M}_{\overline{\overline{R_j}}}}} e^{i2\pi(\overline{\overline{\mathcal{M}_{\overline{\overline{I_j}}}}})}, \overline{\overline{\mathcal{N}_{\overline{\overline{R_j}}}}} e^{i2\pi(\overline{\overline{\mathcal{N}_{\overline{\overline{I_j}}}}})}\right), j = 1, 2,$ *then by using the Aczel–Alsina TN and TCN, we elaborated:*

$$\overline{\overline{\mathfrak{C}_{C_1}}} \oplus \overline{\overline{\mathfrak{C}_{C_2}}} = \left(\overline{\overline{\mathbb{S}_{TNA}}}^{\psi}\left(\overline{\overline{\mathcal{M}_{\overline{\overline{R_1}}}}}, \overline{\overline{\mathcal{M}_{\overline{\overline{R_2}}}}}\right) e^{i2\pi(\overline{\overline{\mathbb{S}_{TNA}}}^{\psi}(\overline{\overline{\mathcal{M}_{\overline{\overline{I_1}}}}}, \overline{\overline{\mathcal{M}_{\overline{\overline{I_2}}}}}))}, \overline{\overline{\mathbb{T}_{TNA}}}^{\psi}\left(\overline{\overline{\mathcal{N}_{\overline{\overline{R_1}}}}}, \overline{\overline{\mathcal{N}_{\overline{\overline{R_2}}}}}\right) e^{i2\pi(\overline{\overline{\mathbb{T}_{TNA}}}^{\psi}(\overline{\overline{\mathcal{N}_{\overline{\overline{I_1}}}}}, \overline{\overline{\mathcal{N}_{\overline{\overline{I_2}}}}}))}\right) \tag{10}$$

$$\overline{\overline{\mathfrak{C}_{C_1}}} \otimes \overline{\overline{\mathfrak{C}_{C_2}}} = \left(\overline{\overline{\mathbb{T}_{TNA}}}^{\psi}\left(\overline{\overline{\mathcal{M}_{\overline{\overline{R_1}}}}}, \overline{\overline{\mathcal{M}_{\overline{\overline{R_2}}}}}\right) e^{i2\pi(\overline{\overline{\mathbb{T}_{TNA}}}^{\psi}(\overline{\overline{\mathcal{M}_{\overline{\overline{I_1}}}}}, \overline{\overline{\mathcal{M}_{\overline{\overline{I_2}}}}}))}, \overline{\overline{\mathbb{S}_{TNA}}}^{\psi}\left(\overline{\overline{\mathcal{N}_{\overline{\overline{R_1}}}}}, \overline{\overline{\mathcal{N}_{\overline{\overline{R_2}}}}}\right) e^{i2\pi(\overline{\overline{\mathbb{S}_{TNA}}}^{\psi}(\overline{\overline{\mathcal{N}_{\overline{\overline{I_1}}}}}, \overline{\overline{\mathcal{N}_{\overline{\overline{I_2}}}}}))}\right) \tag{11}$$

Definition 10. *By considering any two CIFNs* $\overline{\overline{\mathfrak{C}_{C_j}}} = \left(\overline{\overline{\mathcal{M}_{\overline{\overline{R_j}}}}} e^{i2\pi(\overline{\overline{\mathcal{M}_{\overline{\overline{I_j}}}}})}, \overline{\overline{\mathcal{N}_{\overline{\overline{R_j}}}}} e^{i2\pi(\overline{\overline{\mathcal{N}_{\overline{\overline{I_j}}}}})}\right),$ $j = 1, 2$, *then*

$$\overline{\overline{\mathfrak{C}_{C_1}}} \oplus \overline{\overline{\mathfrak{C}_{C_2}}} = \begin{pmatrix} \left(1 - e^{-\left(\left(-\log\left(1-\overline{\overline{\mathcal{M}_{\overline{\overline{R_1}}}}}\right)\right)^{\psi} + \left(-\log\left(1-\overline{\overline{\mathcal{M}_{\overline{\overline{R_2}}}}}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right) e^{i2\pi\left(1-e^{-\left(\left(-\log\left(1-\overline{\overline{\mathcal{M}_{\overline{\overline{I_1}}}}}\right)\right)^{\psi}+\left(-\log\left(1-\overline{\overline{\mathcal{M}_{\overline{\overline{I_2}}}}}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right)}, \\ \left(e^{-\left(\left(-\log\left(\overline{\overline{\mathcal{N}_{\overline{\overline{R_1}}}}}\right)\right)^{\psi} + \left(-\log\left(\overline{\overline{\mathcal{N}_{\overline{\overline{R_2}}}}}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right) e^{i2\pi\left(e^{-\left(\left(-\log\left(\overline{\overline{\mathcal{N}_{\overline{\overline{I_1}}}}}\right)\right)^{\psi}+\left(-\log\left(\overline{\overline{\mathcal{N}_{\overline{\overline{I_2}}}}}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right)} \end{pmatrix} \tag{12}$$

$$\overline{\overline{\mathfrak{C}_{C_1}}} \otimes \overline{\overline{\mathfrak{C}_{C_2}}} = \begin{pmatrix} \left(e^{-\left(\left(-\log\left(\overline{\overline{\mathcal{M}_{\overline{\overline{R_1}}}}}\right)\right)^{\psi} + \left(-\log\left(\overline{\overline{\mathcal{M}_{\overline{\overline{R_2}}}}}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right) e^{i2\pi\left(e^{-\left(\left(-\log\left(\overline{\overline{\mathcal{M}_{\overline{\overline{I_1}}}}}\right)\right)^{\psi}+\left(-\log\left(\overline{\overline{\mathcal{M}_{\overline{\overline{I_2}}}}}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right)}, \\ \left(1 - e^{-\left(\left(-\log\left(1-\overline{\overline{\mathcal{N}_{\overline{\overline{R_1}}}}}\right)\right)^{\psi} + \left(-\log\left(1-\overline{\overline{\mathcal{N}_{\overline{\overline{R_2}}}}}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right) e^{i2\pi\left(1-e^{-\left(\left(-\log\left(1-\overline{\overline{\mathcal{N}_{\overline{\overline{I_1}}}}}\right)\right)^{\psi}+\left(-\log\left(1-\overline{\overline{\mathcal{N}_{\overline{\overline{I_2}}}}}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right)} \end{pmatrix} \tag{13}$$

$$\overline{\overline{\theta_S \mathfrak{C}_{C_1}}} = \begin{pmatrix} \left(1 - e^{-\left(\overline{\overline{\theta_S}}\left(-\log\left(1-\overline{\overline{\mathcal{M}_{\overline{\overline{R_1}}}}}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right) e^{i2\pi\left(1-e^{-\left(\overline{\overline{\theta_S}}\left(-\log\left(1-\overline{\overline{\mathcal{M}_{\overline{\overline{I_1}}}}}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right)}, \\ \left(e^{-\left(\overline{\overline{\theta_S}}\left(-\log\left(\overline{\overline{\mathcal{N}_{\overline{\overline{R_1}}}}}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right) e^{i2\pi\left(e^{-\left(\overline{\overline{\theta_S}}\left(-\log\left(\overline{\overline{\mathcal{N}_{\overline{\overline{I_1}}}}}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right)} \end{pmatrix} \tag{14}$$

$$\overline{\overline{\mathfrak{C}_{C_1}}}^{\overline{\overline{\theta_S}}} = \begin{pmatrix} \left(e^{-\left(\overline{\overline{\theta_S}}\left(-\log\left(\overline{\overline{\mathcal{M}_{\overline{\overline{R_1}}}}}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right) e^{i2\pi\left(e^{-\left(\overline{\overline{\theta_S}}\left(-\log\left(\overline{\overline{\mathcal{M}_{\overline{\overline{I_1}}}}}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right)}, \\ \left(1 - e^{-\left(\overline{\overline{\theta_S}}\left(-\log\left(1-\overline{\overline{\mathcal{N}_{\overline{\overline{R_1}}}}}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right) e^{i2\pi\left(1-e^{-\left(\overline{\overline{\theta_S}}\left(-\log\left(1-\overline{\overline{\mathcal{N}_{\overline{\overline{I_1}}}}}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right)} \end{pmatrix} \tag{15}$$

Theorem 1. *By considering any two CIFNs* $\overline{\overline{\mathfrak{C}_{C_j}}} = \left(\overline{\overline{\mathcal{M}_{\overline{\overline{R_j}}}}} e^{i2\pi\left(\overline{\overline{\mathcal{M}_{\overline{\overline{I_j}}}}}\right)}, \overline{\overline{\mathcal{N}_{\overline{\overline{R_j}}}}} e^{i2\pi\left(\overline{\overline{\mathcal{N}_{\overline{\overline{I_j}}}}}\right)}\right), j = 1, 2,$ *then*

1. $\overline{\overline{\mathfrak{C}_{C_1}}} \oplus \overline{\overline{\mathfrak{C}_{C_2}}} = \overline{\overline{\mathfrak{C}_{C_2}}} \oplus \overline{\overline{\mathfrak{C}_{C_1}}}$;
2. $\overline{\overline{\mathfrak{C}_{C_1}}} \otimes \overline{\overline{\mathfrak{C}_{C_2}}} = \overline{\overline{\mathfrak{C}_{C_2}}} \otimes \overline{\overline{\mathfrak{C}_{C_1}}}$;
3. $\overline{\overline{\theta_S}}\left(\overline{\overline{\mathfrak{C}_{C_1}}} \oplus \overline{\overline{\mathfrak{C}_{C_2}}}\right) = \overline{\overline{\theta_S \mathfrak{C}_{C_1}}} \oplus \overline{\overline{\theta_S \mathfrak{C}_{C_2}}}$;
4. $\left(\overline{\overline{\theta_{S_1}}} + \overline{\overline{\theta_{S_1}}}\right)\overline{\overline{\mathfrak{C}_{C_1}}} = \overline{\overline{\theta_{S_1} \mathfrak{C}_{C_1}}} \oplus \overline{\overline{\theta_{S_2} \mathfrak{C}_{C_1}}}$;
5. $\left(\overline{\overline{\mathfrak{C}_{C_1}}} \otimes \overline{\overline{\mathfrak{C}_{C_2}}}\right)^{\overline{\overline{\theta_S}}} = \overline{\overline{\mathfrak{C}_{C_1}}}^{\overline{\overline{\theta_S}}} \otimes \overline{\overline{\mathfrak{C}_{C_2}}}^{\overline{\overline{\theta_S}}}$;
6. $\overline{\overline{\mathfrak{C}_{C_1}}}^{\overline{\overline{\theta_{S_1}}}} \otimes \overline{\overline{\mathfrak{C}_{C_1}}}^{\overline{\overline{\theta_{S_1}}}} = \overline{\overline{\mathfrak{C}_{C_1}}}^{\overline{\overline{\theta_{S_1}}}+\overline{\overline{\theta_{S_2}}}}$.

The proof of Theorem 1 is given in Appendix A.

4. Complex Intuitionistic Fuzzy Aczel–Alsina Aggregation Operators

The major contribution of this analysis is to analyze some operators by using the AA operational laws under the CIF settings to initiate the principle of CIFAAWA, CIFAAOWA, CIFAAHA, CIFAAWG, CIFAAOWG, CIFAAHG, and their beneficial results.

Definition 11. *By considering any group of CIFNs* $\overline{\overline{\mathfrak{C}_{C_j}}} = \left(\overline{\overline{\mathcal{M}_{\overline{\overline{R_j}}}}} e^{i2\pi\left(\overline{\overline{\mathcal{M}_{\overline{\overline{I_j}}}}}\right)}, \overline{\overline{\mathcal{N}_{\overline{\overline{R_j}}}}} e^{i2\pi\left(\overline{\overline{\mathcal{N}_{\overline{\overline{I_j}}}}}\right)}\right),$ $j = 1, 2, \ldots, n$, *then the CIFAAWA operator is given by:*

$$CIFAAWA\left(\overline{\overline{\mathfrak{C}_{C_1}}}, \overline{\overline{\mathfrak{C}_{C_2}}}, \ldots, \overline{\overline{\mathfrak{C}_{C_n}}}\right) = \overline{\overline{\mathfrak{W}_1}}\mathfrak{C}_{C_1} \oplus \overline{\overline{\mathfrak{W}_2}}\mathfrak{C}_{C_2} \oplus \ldots \oplus \overline{\overline{\mathfrak{W}_n}}\mathfrak{C}_{C_n} = \oplus_{j=1}^{n}\left(\overline{\overline{\mathfrak{W}_j\mathfrak{C}_{C_j}}}\right) \tag{16}$$

where $\overline{\overline{\mathfrak{W}}} = \left(\overline{\overline{\mathfrak{W}}}_1, \overline{\overline{\mathfrak{W}}}_2, \ldots, \overline{\overline{\mathfrak{W}}}_n\right)^T$, *with a rule* $\sum_{j=1}^{n}\overline{\overline{\mathfrak{W}}}_j = 1$.

Theorem 2. *By considering any group of CIFNs* $\overline{\overline{\mathfrak{C}_{C_j}}} = \left(\overline{\overline{\mathcal{M}}}_{\overline{\overline{R_j}}} e^{i2\pi\left(\overline{\overline{\mathcal{M}}}_{\overline{\overline{I_j}}}\right)}, \overline{\overline{\mathcal{N}}}_{\overline{\overline{R_j}}} e^{i2\pi\left(\overline{\overline{\mathcal{N}}}_{\overline{\overline{I_j}}}\right)}\right)$, $j = 1, 2, \ldots, n$, *then by using Equation (16), we elaborated*

$$CIFAAWA\left(\overline{\overline{\mathfrak{C}_{C_1}}}, \overline{\overline{\mathfrak{C}_{C_2}}}, \ldots, \overline{\overline{\mathfrak{C}_{C_n}}}\right) = \left(\begin{array}{c} \left(1 - e^{-\left(\sum_{j=1}^{n}\overline{\overline{\mathfrak{W}}}_j\left(-\log\left(1-\overline{\overline{\mathcal{M}}}_{\overline{\overline{R_j}}}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right) e^{i2\pi\left(1-e^{-\left(\sum_{j=1}^{n}\overline{\overline{\mathfrak{W}}}_j\left(-\log\left(1-\overline{\overline{\mathcal{M}}}_{\overline{\overline{I_j}}}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right)}, \\ \left(e^{-\left(\sum_{j=1}^{n}\overline{\overline{\mathfrak{W}}}_j\left(-\log\left(\overline{\overline{\mathcal{N}}}_{\overline{\overline{R_j}}}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right) e^{i2\pi\left(e^{-\left(\sum_{j=1}^{n}\overline{\overline{\mathfrak{W}}}_j\left(-\log\left(\overline{\overline{\mathcal{N}}}_{\overline{\overline{I_j}}}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right)} \end{array}\right) \tag{17}$$

The proof of Theorem 2 is given in Appendix B.

Additionally, the propositions of idempotency, boundedness, and monotonicity are also employed here.

Proposition 1. *By considering any group of CIFNs* $\overline{\overline{\mathfrak{C}_{C_j}}} = \left(\overline{\overline{\mathcal{M}}}_{\overline{\overline{R_j}}} e^{i2\pi\left(\overline{\overline{\mathcal{M}}}_{\overline{\overline{I_j}}}\right)}, \overline{\overline{\mathcal{N}}}_{\overline{\overline{R_j}}} e^{i2\pi\left(\overline{\overline{\mathcal{N}}}_{\overline{\overline{I_j}}}\right)}\right)$, $j = 1, 2, \ldots, n$, *If* $\overline{\overline{\mathfrak{C}_{C_j}}} = \overline{\overline{\mathfrak{C}}}$, *then*

$$CIFAAWA\left(\overline{\overline{\mathfrak{C}_{C_1}}}, \overline{\overline{\mathfrak{C}_{C_2}}}, \ldots, \overline{\overline{\mathfrak{C}_{C_n}}}\right) = \overline{\overline{\mathfrak{C}}} \tag{18}$$

The proof of Proposition 1 is given in Appendix C.

Proposition 2. *By considering any group of CIFNs* $\overline{\overline{\mathfrak{C}_{C_j}}} = \left(\overline{\overline{\mathcal{M}}}_{\overline{\overline{R_j}}} e^{i2\pi\left(\overline{\overline{\mathcal{M}}}_{\overline{\overline{I_j}}}\right)}, \overline{\overline{\mathcal{N}}}_{\overline{\overline{R_j}}} e^{i2\pi\left(\overline{\overline{\mathcal{N}}}_{\overline{\overline{I_j}}}\right)}\right)$, $j = 1, 2, \ldots, n$, *If* $\overline{\overline{\mathfrak{C}}}^{-} = \left(\min_j \overline{\overline{\mathcal{M}}}_{\overline{\overline{R_j}}} e^{i2\pi\left(\min_j \overline{\overline{\mathcal{M}}}_{\overline{\overline{I_j}}}\right)}, \max_j \overline{\overline{\mathcal{N}}}_{\overline{\overline{R_j}}} e^{i2\pi\left(\max_j \overline{\overline{\mathcal{N}}}_{\overline{\overline{I_j}}}\right)}\right)$ *and* $\overline{\overline{\mathfrak{C}}}^{+} = \left(\max_j \overline{\overline{\mathcal{M}}}_{\overline{\overline{R_j}}} e^{i2\pi\left(\max_j \overline{\overline{\mathcal{M}}}_{\overline{\overline{I_j}}}\right)}, \min_j \overline{\overline{\mathcal{N}}}_{\overline{\overline{R_j}}} e^{i2\pi\left(\min_j \overline{\overline{\mathcal{N}}}_{\overline{\overline{I_j}}}\right)}\right)$, *then*

$$\overline{\overline{\mathfrak{C}}}^{-} \leq CIFAAWA\left(\overline{\overline{\mathfrak{C}_{C_1}}}, \overline{\overline{\mathfrak{C}_{C_2}}}, \ldots, \overline{\overline{\mathfrak{C}_{C_n}}}\right) \leq \overline{\overline{\mathfrak{C}}}^{+} \tag{19}$$

The proof of Proposition 2 is given in Appendix D.

Proposition 3. *By considering any group of CIFNs* $\overline{\overline{\mathfrak{C}_{C_j}}} = \left(\overline{\overline{\mathcal{M}}}_{\overline{\overline{R_j}}} e^{i2\pi\left(\overline{\overline{\mathcal{M}}}_{\overline{\overline{I_j}}}\right)}, \overline{\overline{\mathcal{N}}}_{\overline{\overline{R_j}}} e^{i2\pi\left(\overline{\overline{\mathcal{N}}}_{\overline{\overline{I_j}}}\right)}\right)$, $j = 1, 2, \ldots, n$, *If* $\overline{\overline{\mathfrak{C}_{C_j}}} \leq \overline{\overline{\mathfrak{C}_{C_j}}}' = \left(\overline{\overline{\mathcal{M}}}'_{\overline{\overline{R_j}}} e^{i2\pi\left(\overline{\overline{\mathcal{M}}}'_{\overline{\overline{I_j}}}\right)}, \overline{\overline{\mathcal{N}}}'_{\overline{\overline{R_j}}} e^{i2\pi\left(\overline{\overline{\mathcal{N}}}'_{\overline{\overline{I_j}}}\right)}\right)$, *then*

$$CIFAAWA\left(\overline{\overline{\mathfrak{C}_{C_1}}}, \overline{\overline{\mathfrak{C}_{C_2}}}, \ldots, \overline{\overline{\mathfrak{C}_{C_n}}}\right) \leq CIFAAWA\left(\overline{\overline{\mathfrak{C}_{C_1}}}', \overline{\overline{\mathfrak{C}_{C_2}}}', \ldots, \overline{\overline{\mathfrak{C}_{C_n}}}'\right) \tag{20}$$

Proof. Omitted. □

Definition 12. *By considering any group of CIFNs* $\overline{\overline{\mathfrak{C}_{C_j}}} = \left(\overline{\overline{\mathcal{M}_{\overline{\overline{R_j}}}}} e^{i2\pi(\overline{\overline{\mathcal{M}_{\overline{\overline{I_j}}}}})}, \overline{\overline{\mathcal{N}_{\overline{\overline{R_j}}}}} e^{i2\pi(\overline{\overline{\mathcal{N}_{\overline{\overline{I_j}}}}})}\right)$, $j = 1, 2, \ldots, n$, *then the CIFAAOWA operator is given by:*

$$CIFAAOWA\left(\overline{\overline{\mathfrak{C}_{C_1}}}, \overline{\overline{\mathfrak{C}_{C_2}}}, \ldots, \overline{\overline{\mathfrak{C}_{C_n}}}\right) = \overline{\overline{\mathfrak{W}_1 \mathfrak{C}_{C_{\varphi(1)}}}} \oplus \overline{\overline{\mathfrak{W}_2 \mathfrak{C}_{C_{\varphi(2)}}}} \oplus \ldots \oplus \overline{\overline{\mathfrak{W}_n \mathfrak{C}_{C_{\varphi(n)}}}} = \oplus_{j=1}^{n}\left(\overline{\overline{\mathfrak{W}_j \mathfrak{C}_{C_{\varphi(j)}}}}\right) \quad (21)$$

where $\overline{\overline{\mathfrak{W}}} = \left(\overline{\overline{\mathfrak{W}}}_1, \overline{\overline{\mathfrak{W}}}_2, \ldots, \overline{\overline{\mathfrak{W}}}_n\right)^T$, *with a rule* $\sum_{j=1}^{n} \overline{\overline{\mathfrak{W}}}_j = 1$, *with parameter* $\varphi(1), \varphi(2), \ldots, \varphi(n)$ *based on* $\overline{\overline{\mathfrak{C}_{C_{\varphi(j)}}}} \leq \overline{\overline{\mathfrak{C}_{C_{\varphi(j-1)}}}}$.

Theorem 2. *By considering any group of CIFNs* $\overline{\overline{\mathfrak{C}_{C_j}}} = \left(\overline{\overline{\mathcal{M}_{\overline{\overline{R_j}}}}} e^{i2\pi(\overline{\overline{\mathcal{M}_{\overline{\overline{I_j}}}}})}, \overline{\overline{\mathcal{N}_{\overline{\overline{R_j}}}}} e^{i2\pi(\overline{\overline{\mathcal{N}_{\overline{\overline{I_j}}}}})}\right)$, $j = 1, 2, \ldots, n$, *then by using Equation (21), we elaborated*

$$CIFAAOWA\left(\overline{\overline{\mathfrak{C}_{C_1}}}, \overline{\overline{\mathfrak{C}_{C_2}}}, \ldots, \overline{\overline{\mathfrak{C}_{C_n}}}\right) = \left(\begin{array}{c} \left(1 - e^{-\left(\sum_{j=1}^{n} \overline{\overline{\mathfrak{W}}}_j \left(-\log\left(1 - \overline{\overline{\mathcal{M}_{\overline{\overline{R_{\varphi(j)}}}}}}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right) e^{i2\pi\left(1 - e^{-\left(\sum_{j=1}^{n} \overline{\overline{\mathfrak{W}}}_j \left(-\log\left(1 - \overline{\overline{\mathcal{M}_{\overline{\overline{I_{\varphi(j)}}}}}}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right)}, \\ \left(e^{-\left(\sum_{j=1}^{n} \overline{\overline{\mathfrak{W}}}_j \left(-\log\left(\overline{\overline{\mathcal{N}_{\overline{\overline{R_{\varphi(j)}}}}}}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right) e^{i2\pi\left(e^{-\left(\sum_{j=1}^{n} \overline{\overline{\mathfrak{W}}}_j \left(-\log\left(\overline{\overline{\mathcal{N}_{\overline{\overline{I_{\varphi(j)}}}}}}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right)} \end{array}\right) \quad (22)$$

Proof. Omitted. □

Additionally, the propositions of idempotency, boundedness, and monotonicity are also employed here.

Proposition 4. *By considering any group of CIFNs* $\overline{\overline{\mathfrak{C}_{C_j}}} = \left(\overline{\overline{\mathcal{M}_{\overline{\overline{R_j}}}}} e^{i2\pi(\overline{\overline{\mathcal{M}_{\overline{\overline{I_j}}}}})}, \overline{\overline{\mathcal{N}_{\overline{\overline{R_j}}}}} e^{i2\pi(\overline{\overline{\mathcal{N}_{\overline{\overline{I_j}}}}})}\right)$, $j = 1, 2, \ldots, n$, *If* $\overline{\overline{\mathfrak{C}_{C_j}}} = \overline{\overline{\mathfrak{C}}}$, *then*

$$CIFAAOWA\left(\overline{\overline{\mathfrak{C}_{C_1}}}, \overline{\overline{\mathfrak{C}_{C_2}}}, \ldots, \overline{\overline{\mathfrak{C}_{C_n}}}\right) = \overline{\overline{\mathfrak{C}}} \quad (23)$$

Proof. Omitted. □

Proposition 5. *By considering any group of CIFNs* $\overline{\overline{\mathfrak{C}_{C_j}}} = \left(\overline{\overline{\mathcal{M}_{\overline{\overline{R_j}}}}} e^{i2\pi(\overline{\overline{\mathcal{M}_{\overline{\overline{I_j}}}}})}, \overline{\overline{\mathcal{N}_{\overline{\overline{R_j}}}}} e^{i2\pi(\overline{\overline{\mathcal{N}_{\overline{\overline{I_j}}}}})}\right)$, $j = 1, 2, \ldots, n$, *If* $\overline{\overline{\mathfrak{C}}}^{-} = \left(\min_j \overline{\overline{\mathcal{M}_{\overline{\overline{R_j}}}}} e^{i2\pi(\min_j \overline{\overline{\mathcal{M}_{\overline{\overline{I_j}}}}})}, \max_j \overline{\overline{\mathcal{N}_{\overline{\overline{R_j}}}}} e^{i2\pi(\max_j \overline{\overline{\mathcal{N}_{\overline{\overline{I_j}}}}})}\right)$ *and* $\overline{\overline{\mathfrak{C}}}^{+} = \left(\max_j \overline{\overline{\mathcal{M}_{\overline{\overline{R_j}}}}} e^{i2\pi(\max_j \overline{\overline{\mathcal{M}_{\overline{\overline{I_j}}}}})}, \min_j \overline{\overline{\mathcal{N}_{\overline{\overline{R_j}}}}} e^{i2\pi(\min_j \overline{\overline{\mathcal{N}_{\overline{\overline{I_j}}}}})}\right)$, *then*

$$\overline{\overline{\mathfrak{C}}}^{-} \leq CIFAAOWA\left(\overline{\overline{\mathfrak{C}_{C_1}}}, \overline{\overline{\mathfrak{C}_{C_2}}}, \ldots, \overline{\overline{\mathfrak{C}_{C_n}}}\right) \leq \overline{\overline{\mathfrak{C}}}^{+} \quad (24)$$

Proof : Omitted. □

Proposition 6. *By considering any group of CIFNs* $\overline{\overline{\mathfrak{C}_{C_j}}} = \left(\overline{\overline{\mathcal{M}_{\overline{\overline{R_j}}}}} e^{i2\pi(\overline{\overline{\mathcal{M}_{\overline{\overline{I_j}}}}})}, \overline{\overline{\mathcal{N}_{\overline{\overline{R_j}}}}} e^{i2\pi(\overline{\overline{\mathcal{N}_{\overline{\overline{I_j}}}}})}\right)$, *$j = 1, 2, \ldots, n$, if* $\overline{\overline{\mathfrak{C}_{C_j}}} \leq \overline{\overline{\mathfrak{C}_{C_j}}}' = \left(\overline{\overline{\mathcal{M}_{\overline{\overline{R_j}}}}}' e^{i2\pi(\overline{\overline{\mathcal{M}_{\overline{\overline{I_j}}}}}')}, \overline{\overline{\mathcal{N}_{\overline{\overline{R_j}}}}}' e^{i2\pi(\overline{\overline{\mathcal{N}_{\overline{\overline{I_j}}}}}')}\right)$, *then*

$$CIFAAOWA\left(\overline{\overline{\mathfrak{C}_{C_1}}}, \overline{\overline{\mathfrak{C}_{C_2}}}, \ldots, \overline{\overline{\mathfrak{C}_{C_n}}}\right) \leq CIFAAOWA\left(\overline{\overline{\mathfrak{C}_{C_1}}}', \overline{\overline{\mathfrak{C}_{C_2}}}', \ldots, \overline{\overline{\mathfrak{C}_{C_n}}}'\right) \tag{25}$$

Proof. Omitted. □

Definition 13. *By considering any group of CIFNs* $\overline{\overline{\mathfrak{C}_{C_j}}} = \left(\overline{\overline{\mathcal{M}_{\overline{\overline{R_j}}}}} e^{i2\pi(\overline{\overline{\mathcal{M}_{\overline{\overline{I_j}}}}})}, \overline{\overline{\mathcal{N}_{\overline{\overline{R_j}}}}} e^{i2\pi(\overline{\overline{\mathcal{N}_{\overline{\overline{I_j}}}}})}\right)$, *$j = 1, 2, \ldots, n$, then the CIFAAHA operator is given by:*

$$CIFAAHA\left(\overline{\overline{\mathfrak{C}_{C_1}}}, \overline{\overline{\mathfrak{C}_{C_2}}}, \ldots, \overline{\overline{\mathfrak{C}_{C_n}}}\right) = \overline{\overline{\mathfrak{W}_1}}\overline{\overline{\dot{\mathfrak{C}}_{C_{\varphi(1)}}}} \oplus \overline{\overline{\mathfrak{W}_2}}\overline{\overline{\dot{\mathfrak{C}}_{C_{\varphi(2)}}}} \oplus \ldots \oplus \overline{\overline{\mathfrak{W}_n}}\overline{\overline{\dot{\mathfrak{C}}_{C_{\varphi(n)}}}} = \oplus_{j=1}^{n}\left(\overline{\overline{\mathfrak{W}_j}}\overline{\overline{\dot{\mathfrak{C}}_{C_{\varphi(j)}}}}\right) \tag{26}$$

where $\overline{\overline{\mathfrak{W}}} = \left(\overline{\overline{\mathfrak{W}}}_1, \overline{\overline{\mathfrak{W}}}_2, \ldots, \overline{\overline{\mathfrak{W}}}_n\right)^T$*, with a rule* $\sum_{j=1}^{n}\overline{\overline{\mathfrak{W}}}_j = 1$*, with parameter* $\varphi(1), \varphi(2), \ldots, \varphi(n)$ *based on* $\overline{\overline{\dot{\mathfrak{C}}_{C_{\varphi(j)}}}} \leq \overline{\overline{\dot{\mathfrak{C}}_{C_{\varphi(j-1)}}}}$*. Additionally,* $\overline{\overline{\dot{\mathfrak{C}}_{C_{\varphi(j)}}}} = n\overline{\overline{\mathfrak{W}}}_j''\overline{\overline{\dot{\mathfrak{C}}_{C_{\varphi(j)}}}}$ *with* $\sum_{j=1}^{n}\overline{\overline{\mathfrak{W}}}_j'' = 1$.

Theorem 3. *By considering any group of CIFNs* $\overline{\overline{\mathfrak{C}_{C_j}}} = \left(\overline{\overline{\mathcal{M}_{\overline{\overline{R_j}}}}} e^{i2\pi(\overline{\overline{\mathcal{M}_{\overline{\overline{I_j}}}}})}, \overline{\overline{\mathcal{N}_{\overline{\overline{R_j}}}}} e^{i2\pi(\overline{\overline{\mathcal{N}_{\overline{\overline{I_j}}}}})}\right)$, *$j = 1, 2, \ldots, n$, then by using Equation (26), we elaborated*

$$CIFAAHA\left(\overline{\overline{\mathfrak{C}_{C_1}}}, \overline{\overline{\mathfrak{C}_{C_2}}}, \ldots, \overline{\overline{\mathfrak{C}_{C_n}}}\right) = \left(\begin{array}{c} \left(1 - e^{-\left(\sum_{j=1}^{n}\overline{\overline{\mathfrak{W}_j}}\left(-\log\left(1-\dot{\mathcal{M}}_{\overline{\overline{R_{\varphi(j)}}}}^{=}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right) e^{i2\pi\left(1-e^{-\left(\sum_{j=1}^{n}\overline{\overline{\mathfrak{W}_j}}\left(-\log\left(1-\dot{\mathcal{M}}_{\overline{\overline{I_{\varphi(j)}}}}^{=}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right)}, \\ \left(e^{-\left(\sum_{j=1}^{n}\overline{\overline{\mathfrak{W}_j}}\left(-\log\left(\dot{\mathcal{N}}_{\overline{\overline{R_{\varphi(j)}}}}^{=}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right) e^{i2\pi\left(e^{-\left(\sum_{j=1}^{n}\overline{\overline{\mathfrak{W}_j}}\left(-\log\left(\dot{\mathcal{N}}_{\overline{\overline{I_{\varphi(j)}}}}^{=}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right)} \end{array}\right) \tag{27}$$

Proof. Omitted. □

Additionally, the propositions of idempotency, boundedness, and monotonicity are also employed here.

Proposition 7. *By considering any group of CIFNs* $\overline{\overline{\mathfrak{C}_{C_j}}} = \left(\overline{\overline{\mathcal{M}_{\overline{\overline{R_j}}}}} e^{i2\pi(\overline{\overline{\mathcal{M}_{\overline{\overline{I_j}}}}})}, \overline{\overline{\mathcal{N}_{\overline{\overline{R_j}}}}} e^{i2\pi(\overline{\overline{\mathcal{N}_{\overline{\overline{I_j}}}}})}\right)$, *$j = 1, 2, \ldots, n$, If* $\overline{\overline{\mathfrak{C}_{C_j}}} = \overline{\overline{\mathfrak{C}}}$*, then*

$$CIFAAHA\left(\overline{\overline{\mathfrak{C}_{C_1}}}, \overline{\overline{\mathfrak{C}_{C_2}}}, \ldots, \overline{\overline{\mathfrak{C}_{C_n}}}\right) = \overline{\overline{\mathfrak{C}}} \tag{28}$$

Proof. Omitted. □

Proposition 8. *By considering any group of CIFNs* $\overline{\overline{\mathfrak{C}_{C_j}}} = \left(\overline{\overline{\mathcal{M}}}_{\overline{\overline{R}}_j} e^{i2\pi(\overline{\overline{\mathcal{M}}}_{\overline{\overline{I}}_j})}, \overline{\overline{\mathcal{N}}}_{\overline{\overline{R}}_j} e^{i2\pi(\overline{\overline{\mathcal{N}}}_{\overline{\overline{I}}_j})}\right)$, $j = 1, 2, \ldots, n$, *If* $\overline{\overline{\mathfrak{C}}}^- = \left(\min_j \overline{\overline{\mathcal{M}}}_{\overline{\overline{R}}_j} e^{i2\pi(\min_j \overline{\overline{\mathcal{M}}}_{\overline{\overline{I}}_j})}, \max_j \overline{\overline{\mathcal{N}}}_{\overline{\overline{R}}_j} e^{i2\pi(\max_j \overline{\overline{\mathcal{N}}}_{\overline{\overline{I}}_j})}\right)$ *and* $\overline{\overline{\mathfrak{C}}}^+ = \left(\max_j \overline{\overline{\mathcal{M}}}_{\overline{\overline{R}}_j} e^{i2\pi(\max_j \overline{\overline{\mathcal{M}}}_{\overline{\overline{I}}_j})}, \min_j \overline{\overline{\mathcal{N}}}_{\overline{\overline{R}}_j} e^{i2\pi(\min_j \overline{\overline{\mathcal{N}}}_{\overline{\overline{I}}_j})}\right)$, *then*

$$\overline{\overline{\mathfrak{C}}}^- \leq CIFAAHA\left(\overline{\overline{\mathfrak{C}_{C_1}}}, \overline{\overline{\mathfrak{C}_{C_2}}}, \ldots, \overline{\overline{\mathfrak{C}_{C_n}}}\right) \leq \overline{\overline{\mathfrak{C}}}^+ \tag{29}$$

Proof. Omitted. □

Proposition 9. *By considering any group of CIFNs* $\overline{\overline{\mathfrak{C}_{C_j}}} = \left(\overline{\overline{\mathcal{M}}}_{\overline{\overline{R}}_j} e^{i2\pi(\overline{\overline{\mathcal{M}}}_{\overline{\overline{I}}_j})}, \overline{\overline{\mathcal{N}}}_{\overline{\overline{R}}_j} e^{i2\pi(\overline{\overline{\mathcal{N}}}_{\overline{\overline{I}}_j})}\right)$, $j = 1, 2, \ldots, n$, *If* $\overline{\overline{\mathfrak{C}_{C_j}}} \leq \overline{\overline{\mathfrak{C}_{C_j}}}' = \left(\overline{\overline{\mathcal{M}}}'_{\overline{\overline{R}}_j} e^{i2\pi(\overline{\overline{\mathcal{M}}}'_{\overline{\overline{I}}_j})}, \overline{\overline{\mathcal{N}}}'_{\overline{\overline{R}}_j} e^{i2\pi(\overline{\overline{\mathcal{N}}}'_{\overline{\overline{I}}_j})}\right)$, *then*

$$CIFAAHA\left(\overline{\overline{\mathfrak{C}_{C_1}}}, \overline{\overline{\mathfrak{C}_{C_2}}}, \ldots, \overline{\overline{\mathfrak{C}_{C_n}}}\right) \leq CIFAAHA\left(\overline{\overline{\mathfrak{C}_{C_1}}}', \overline{\overline{\mathfrak{C}_{C_2}}}', \ldots, \overline{\overline{\mathfrak{C}_{C_n}}}'\right) \tag{30}$$

Proof: Omitted. □

Definition 14. *By considering any group of CIFNs* $\overline{\overline{\mathfrak{C}_{C_j}}} = \left(\overline{\overline{\mathcal{M}}}_{\overline{\overline{R}}_j} e^{i2\pi(\overline{\overline{\mathcal{M}}}_{\overline{\overline{I}}_j})}, \overline{\overline{\mathcal{N}}}_{\overline{\overline{R}}_j} e^{i2\pi(\overline{\overline{\mathcal{N}}}_{\overline{\overline{I}}_j})}\right)$, $j = 1, 2, \ldots, n$, *then the CIFAAWG operator is given by:*

$$CIFAAWG\left(\overline{\overline{\mathfrak{C}_{C_1}}}, \overline{\overline{\mathfrak{C}_{C_2}}}, \ldots, \overline{\overline{\mathfrak{C}_{C_n}}}\right) = \overline{\overline{\mathfrak{C}_{C_1}}}^{\overline{\overline{\mathfrak{W}}}_1} \otimes \overline{\overline{\mathfrak{C}_{C_2}}}^{\overline{\overline{\mathfrak{W}}}_2} \otimes \ldots \otimes \overline{\overline{\mathfrak{C}_{C_n}}}^{\overline{\overline{\mathfrak{W}}}_n} = \otimes_{j=1}^{n}\left(\overline{\overline{\mathfrak{C}_{C_j}}}^{\overline{\overline{\mathfrak{W}}}_j}\right) \tag{31}$$

where $\overline{\overline{\mathfrak{W}}} = \left(\overline{\overline{\mathfrak{W}}}_1, \overline{\overline{\mathfrak{W}}}_2, \ldots, \overline{\overline{\mathfrak{W}}}_n\right)^T$, *with a rule* $\sum_{j=1}^{n} \overline{\overline{\mathfrak{W}}}_j = 1$.

Theorem 4. *By considering any group of CIFNs* $\overline{\overline{\mathfrak{C}_{C_j}}} = \left(\overline{\overline{\mathcal{M}}}_{\overline{\overline{R}}_j} e^{i2\pi(\overline{\overline{\mathcal{M}}}_{\overline{\overline{I}}_j})}, \overline{\overline{\mathcal{N}}}_{\overline{\overline{R}}_j} e^{i2\pi(\overline{\overline{\mathcal{N}}}_{\overline{\overline{I}}_j})}\right)$, $j = 1, 2, \ldots, n$, *then by using Equation (31), we elaborated*

$$CIFAAWG\left(\overline{\overline{\mathfrak{C}_{C_1}}}, \overline{\overline{\mathfrak{C}_{C_2}}}, \ldots, \overline{\overline{\mathfrak{C}_{C_n}}}\right) = \left(\begin{array}{c} \left(e^{-\left(\sum_{j=1}^{n}\left(-\log\left(\overline{\overline{\mathcal{M}}}_{\overline{\overline{R}}_j}\right)\right)^{\psi\overline{\overline{\mathfrak{W}}}_j}\right)^{\frac{1}{\psi}}}\right) e^{i2\pi\left(e^{-\left(\sum_{j=1}^{n}\left(-\log\left(\overline{\overline{\mathcal{M}}}_{\overline{\overline{I}}_j}\right)\right)^{\psi\overline{\overline{\mathfrak{W}}}_j}\right)^{\frac{1}{\psi}}}\right)}, \\ \left(1 - e^{-\left(\sum_{j=1}^{n}\left(-\log\left(1-\overline{\overline{\mathcal{N}}}_{\overline{\overline{R}}_j}\right)\right)^{\psi\overline{\overline{\mathfrak{W}}}_j}\right)^{\frac{1}{\psi}}}\right) e^{i2\pi\left(1-e^{-\left(\sum_{j=1}^{n}\left(-\log\left(1-\overline{\overline{\mathcal{N}}}_{\overline{\overline{I}}_j}\right)\right)^{\psi\overline{\overline{\mathfrak{W}}}_j}\right)^{\frac{1}{\psi}}}\right)} \end{array}\right) \tag{32}$$

Definition 15. *By considering any group of CIFNs* $\overline{\overline{\mathfrak{C}_{C_j}}} = \left(\overline{\overline{\mathcal{M}}}_{\overline{\overline{R}}_j} e^{i2\pi(\overline{\overline{\mathcal{M}}}_{\overline{\overline{I}}_j})}, \overline{\overline{\mathcal{N}}}_{\overline{\overline{R}}_j} e^{i2\pi(\overline{\overline{\mathcal{N}}}_{\overline{\overline{I}}_j})}\right)$, $j = 1, 2, \ldots, n$, *then the CIFAAOWG operator is given by:*

$$CIFAAOWG\left(\overline{\overline{\mathfrak{C}_{C_1}}}, \overline{\overline{\mathfrak{C}_{C_2}}}, \ldots, \overline{\overline{\mathfrak{C}_{C_n}}}\right) = \overline{\overline{\mathfrak{C}_{C_{\varphi(1)}}}}^{\overline{\overline{\mathfrak{W}}}_1} \otimes \overline{\overline{\mathfrak{C}_{C_{\varphi(2)}}}}^{\overline{\overline{\mathfrak{W}}}_2} \otimes \ldots \otimes \overline{\overline{\mathfrak{C}_{C_{\varphi(n)}}}}^{\overline{\overline{\mathfrak{W}}}_n} = \otimes_{j=1}^{n}\left(\overline{\overline{\mathfrak{C}_{C_{\varphi(j)}}}}^{\overline{\overline{\mathfrak{W}}}_j}\right) \tag{33}$$

where $\overline{\overline{\mathfrak{W}}} = \left(\overline{\overline{\mathfrak{W}}}_1, \overline{\overline{\mathfrak{W}}}_2, \ldots, \overline{\overline{\mathfrak{W}}}_n\right)^T$*, with a rule* $\sum_{j=1}^{n} \overline{\overline{\mathfrak{W}}}_j = 1$*, with parameter* $\varphi(1), \varphi(2), \ldots, \varphi(n)$ *based on* $\overline{\overline{\mathfrak{C}_{C_{\varphi(j)}}}} \leq \overline{\overline{\mathfrak{C}_{C_{\varphi(j-1)}}}}$.

Theorem 5. *By considering any group of CIFNs* $\overline{\overline{\mathfrak{C}_{C_j}}} = \left(\overline{\overline{\mathcal{M}_{\overline{\overline{R}}_j}}} e^{i2\pi\left(\overline{\overline{\mathcal{M}_{\overline{\overline{I}}_j}}}\right)}, \overline{\overline{\mathcal{N}_{\overline{\overline{R}}_j}}} e^{i2\pi\left(\overline{\overline{\mathcal{N}_{\overline{\overline{I}}_j}}}\right)}\right)$, $j = 1, 2, \ldots, n$*, then by using Equation (33), we elaborated*

$$CIFAAOWG\left(\overline{\overline{\mathfrak{C}_{C_1}}}, \overline{\overline{\mathfrak{C}_{C_2}}}, \ldots, \overline{\overline{\mathfrak{C}_{C_n}}}\right) = \begin{pmatrix} \left(e^{-\left(\sum_{j=1}^{n}\left(-\log\left(\overline{\overline{\mathcal{M}_{\overline{\overline{R}}_{\varphi(j)}}}}\right)\right)^{\psi\overline{\overline{\mathfrak{W}}}_j}\right)^{\frac{1}{\psi}}}\right) e^{i2\pi\left(e^{-\left(\sum_{j=1}^{n}\left(-\log\left(\overline{\overline{\mathcal{M}_{\overline{\overline{I}}_{\varphi(j)}}}}\right)\right)^{\psi\overline{\overline{\mathfrak{W}}}_j}\right)^{\frac{1}{\psi}}}\right)}, \\ \left(1 - e^{-\left(\sum_{j=1}^{n}\left(-\log\left(1-\overline{\overline{\mathcal{N}_{\overline{\overline{R}}_{\varphi(j)}}}}\right)\right)^{\psi\overline{\overline{\mathfrak{W}}}_j}\right)^{\frac{1}{\psi}}}\right) e^{i2\pi\left(1-e^{-\left(\sum_{j=1}^{n}\left(-\log\left(1-\overline{\overline{\mathcal{N}_{\overline{\overline{I}}_{\varphi(j)}}}}\right)\right)^{\psi\overline{\overline{\mathfrak{W}}}_j}\right)^{\frac{1}{\psi}}}\right)} \end{pmatrix} \quad (34)$$

Proof. Omitted. □

Additionally, the propositions of idempotency, boundedness, and monotonicity are also employed here.

Proposition 10. *By considering any group of CIFNs* $\overline{\overline{\mathfrak{C}_{C_j}}} = \left(\overline{\overline{\mathcal{M}_{\overline{\overline{R}}_j}}} e^{i2\pi\left(\overline{\overline{\mathcal{M}_{\overline{\overline{I}}_j}}}\right)}, \overline{\overline{\mathcal{N}_{\overline{\overline{R}}_j}}} e^{i2\pi\left(\overline{\overline{\mathcal{N}_{\overline{\overline{I}}_j}}}\right)}\right)$, $j = 1, 2, \ldots, n$*, If* $\overline{\overline{\mathfrak{C}_{C_j}}} = \overline{\overline{\mathfrak{C}}}$*, then*

$$CIFAAOWG\left(\overline{\overline{\mathfrak{C}_{C_1}}}, \overline{\overline{\mathfrak{C}_{C_2}}}, \ldots, \overline{\overline{\mathfrak{C}_{C_n}}}\right) = \overline{\overline{\mathfrak{C}}} \quad (35)$$

Proof. Omitted. □

Proposition 11. *By considering any group of CIFNs* $\overline{\overline{\mathfrak{C}_{C_j}}} = \left(\overline{\overline{\mathcal{M}_{\overline{\overline{R}}_j}}} e^{i2\pi\left(\overline{\overline{\mathcal{M}_{\overline{\overline{I}}_j}}}\right)}, \overline{\overline{\mathcal{N}_{\overline{\overline{R}}_j}}} e^{i2\pi\left(\overline{\overline{\mathcal{N}_{\overline{\overline{I}}_j}}}\right)}\right)$, $j = 1, 2, \ldots, n$*, If* $\overline{\overline{\mathfrak{C}}}^{-} = \left(\min_j \overline{\overline{\mathcal{M}_{\overline{\overline{R}}_j}}} e^{i2\pi\left(\min_j \overline{\overline{\mathcal{M}_{\overline{\overline{I}}_j}}}\right)}, \max_j \overline{\overline{\mathcal{N}_{\overline{\overline{R}}_j}}} e^{i2\pi\left(\max_j \overline{\overline{\mathcal{N}_{\overline{\overline{I}}_j}}}\right)}\right)$ *and* $\overline{\overline{\mathfrak{C}}}^{+} = \left(\max_j \overline{\overline{\mathcal{M}_{\overline{\overline{R}}_j}}} e^{i2\pi\left(\max_j \overline{\overline{\mathcal{M}_{\overline{\overline{I}}_j}}}\right)}, \min_j \overline{\overline{\mathcal{N}_{\overline{\overline{R}}_j}}} e^{i2\pi\left(\min_j \overline{\overline{\mathcal{N}_{\overline{\overline{I}}_j}}}\right)}\right)$*, then*

$$\overline{\overline{\mathfrak{C}}}^{-} \leq CIFAAOWG\left(\overline{\overline{\mathfrak{C}_{C_1}}}, \overline{\overline{\mathfrak{C}_{C_2}}}, \ldots, \overline{\overline{\mathfrak{C}_{C_n}}}\right) \leq \overline{\overline{\mathfrak{C}}}^{+} \quad (36)$$

Proof. Omitted. □

Proposition 12. *By considering any group of CIFNs* $\overline{\overline{\mathfrak{C}_{C_j}}} = \left(\overline{\overline{\mathcal{M}_{\overline{\overline{R}}_j}}} e^{i2\pi\left(\overline{\overline{\mathcal{M}_{\overline{\overline{I}}_j}}}\right)}, \overline{\overline{\mathcal{N}_{\overline{\overline{R}}_j}}} e^{i2\pi\left(\overline{\overline{\mathcal{N}_{\overline{\overline{I}}_j}}}\right)}\right)$, $j = 1, 2, \ldots, n$*, If* $\overline{\overline{\mathfrak{C}_{C_j}}} \leq \overline{\overline{\mathfrak{C}_{C_j}}}' = \left(\overline{\overline{\mathcal{M}_{\overline{\overline{R}}_j}}}' e^{i2\pi\left(\overline{\overline{\mathcal{M}_{\overline{\overline{I}}_j}}}'\right)}, \overline{\overline{\mathcal{N}_{\overline{\overline{R}}_j}}}' e^{i2\pi\left(\overline{\overline{\mathcal{N}_{\overline{\overline{I}}_j}}}'\right)}\right)$*, then*

$$CIFAAOWG\left(\overline{\overline{\mathfrak{C}_{C_1}}}, \overline{\overline{\mathfrak{C}_{C_2}}}, \ldots, \overline{\overline{\mathfrak{C}_{C_n}}}\right) \leq CIFAAOWG\left(\overline{\overline{\mathfrak{C}_{C_1}}}', \overline{\overline{\mathfrak{C}_{C_2}}}', \ldots, \overline{\overline{\mathfrak{C}_{C_n}}}'\right) \quad (37)$$

Proof. Omitted. □

Definition 16. *By considering any group of CIFNs* $\overline{\overline{\mathfrak{C}_{C_j}}} = \left(\overline{\overline{\mathcal{M}}}_{\overline{\overline{R}}_j} e^{i2\pi\left(\overline{\overline{\mathcal{M}}}_{\overline{\overline{I}}_j}\right)}, \overline{\overline{\mathcal{N}}}_{\overline{\overline{R}}_j} e^{i2\pi\left(\overline{\overline{\mathcal{N}}}_{\overline{\overline{I}}_j}\right)}\right)$, $j = 1, 2, \ldots, n$, *then the CIFAAHG operator is given by:*

$$CIFAAHG\left(\overline{\overline{\mathfrak{C}_{C_1}}}, \overline{\overline{\mathfrak{C}_{C_2}}}, \ldots, \overline{\overline{\mathfrak{C}_{C_n}}}\right) = \overline{\overline{\dot{\mathfrak{C}}_{C_{\varphi(1)}}}}^{\overline{\overline{\mathfrak{W}}}_1} \otimes \overline{\overline{\dot{\mathfrak{C}}_{C_{\varphi(2)}}}}^{\overline{\overline{\mathfrak{W}}}_2} \otimes \ldots \otimes \overline{\overline{\dot{\mathfrak{C}}_{C_{\varphi(n)}}}}^{\overline{\overline{\mathfrak{W}}}_n} = \otimes_{j=1}^{n}\left(\overline{\overline{\dot{\mathfrak{C}}_{C_{\varphi(j)}}}}^{\overline{\overline{\mathfrak{W}}}_j}\right) \tag{38}$$

where $\overline{\overline{\mathfrak{W}}} = \left(\overline{\overline{\mathfrak{W}}}_1, \overline{\overline{\mathfrak{W}}}_2, \ldots, \overline{\overline{\mathfrak{W}}}_n\right)^T$*, with a rule* $\sum_{j=1}^{n} \overline{\overline{\mathfrak{W}}}_j = 1$*, with parameter* $\varphi(1), \varphi(2), \ldots, \varphi(n)$ *based on* $\overline{\overline{\dot{\mathfrak{C}}_{C_{\varphi(j)}}}} \leq \overline{\overline{\dot{\mathfrak{C}}_{C_{\varphi(j-1)}}}}$*. Additionally,* $\overline{\overline{\dot{\mathfrak{C}}_{C_{\varphi(j)}}}} = n\overline{\overline{\mathfrak{W}}}''_j \overline{\overline{\dot{\mathfrak{C}}_{C_{\varphi(j)}}}}$ *with* $\sum_{j=1}^{n} \overline{\overline{\mathfrak{W}}}''_j = 1$.

Theorem 6. *By considering any group of CIFNs* $\overline{\overline{\mathfrak{C}_{C_j}}} = \left(\overline{\overline{\mathcal{M}}}_{\overline{\overline{R}}_j} e^{i2\pi\left(\overline{\overline{\mathcal{M}}}_{\overline{\overline{I}}_j}\right)}, \overline{\overline{\mathcal{N}}}_{\overline{\overline{R}}_j} e^{i2\pi\left(\overline{\overline{\mathcal{N}}}_{\overline{\overline{I}}_j}\right)}\right)$, $j = 1, 2, \ldots, n$, *then by using Equation (38), we elaborated*

$$CIFAAHG\left(\overline{\overline{\mathfrak{C}_{C_1}}}, \overline{\overline{\mathfrak{C}_{C_2}}}, \ldots, \overline{\overline{\mathfrak{C}_{C_n}}}\right) = \begin{pmatrix} \left(e^{-\left(\sum_{j=1}^{n}\left(-\log\left(\dot{\mathcal{M}}_{\overline{\overline{R}}_{\varphi(j)}}\right)\right)^{\psi\overline{\overline{\mathfrak{W}}}_j}\right)^{\frac{1}{\psi}}}\right) e^{i2\pi\left(e^{-\left(\sum_{j=1}^{n}\left(-\log\left(\dot{\mathcal{M}}_{\overline{\overline{I}}_{\varphi(j)}}\right)\right)^{\psi\overline{\overline{\mathfrak{W}}}_j}\right)^{\frac{1}{\psi}}}\right)}, \\ \left(1-e^{-\left(\sum_{j=1}^{n}\left(-\log\left(1-\dot{\mathcal{N}}_{\overline{\overline{R}}_{\varphi(j)}}\right)\right)^{\psi\overline{\overline{\mathfrak{W}}}_j}\right)^{\frac{1}{\psi}}}\right) e^{i2\pi\left(1-e^{-\left(\sum_{j=1}^{n}\left(-\log\left(1-\dot{\mathcal{N}}_{\overline{\overline{I}}_{\varphi(j)}}\right)\right)^{\psi\overline{\overline{\mathfrak{W}}}_j}\right)^{\frac{1}{\psi}}}\right)} \end{pmatrix} \tag{39}$$

Proof. Omitted. □

Additionally, the propositions of idempotency, boundedness, and monotonicity are also employed here.

Proposition 13. *By considering any group of CIFNs* $\overline{\overline{\mathfrak{C}_{C_j}}} = \left(\overline{\overline{\mathcal{M}}}_{\overline{\overline{R}}_j} e^{i2\pi\left(\overline{\overline{\mathcal{M}}}_{\overline{\overline{I}}_j}\right)}, \overline{\overline{\mathcal{N}}}_{\overline{\overline{R}}_j} e^{i2\pi\left(\overline{\overline{\mathcal{N}}}_{\overline{\overline{I}}_j}\right)}\right)$, $j = 1, 2, \ldots, n$, *If* $\overline{\overline{\mathfrak{C}_{C_j}}} = \overline{\overline{\mathfrak{C}}}$*, then*

$$CIFAAHG\left(\overline{\overline{\mathfrak{C}_{C_1}}}, \overline{\overline{\mathfrak{C}_{C_2}}}, \ldots, \overline{\overline{\mathfrak{C}_{C_n}}}\right) = \overline{\overline{\mathfrak{C}}} \tag{40}$$

Proof. Omitted. □

Proposition 14. *By considering any group of CIFNs* $\overline{\overline{\mathfrak{C}_{C_j}}} = \left(\overline{\overline{\mathcal{M}}}_{\overline{\overline{R}}_j} e^{i2\pi\left(\overline{\overline{\mathcal{M}}}_{\overline{\overline{I}}_j}\right)}, \overline{\overline{\mathcal{N}}}_{\overline{\overline{R}}_j} e^{i2\pi\left(\overline{\overline{\mathcal{N}}}_{\overline{\overline{I}}_j}\right)}\right)$, $j = 1, 2, \ldots, n$, *If* $\overline{\overline{\mathfrak{C}}}^- = \left(\min_j \overline{\overline{\mathcal{M}}}_{\overline{\overline{R}}_j} e^{i2\pi\left(\min_j \overline{\overline{\mathcal{M}}}_{\overline{\overline{I}}_j}\right)}, \max_j \overline{\overline{\mathcal{N}}}_{\overline{\overline{R}}_j} e^{i2\pi\left(\max_j \overline{\overline{\mathcal{N}}}_{\overline{\overline{I}}_j}\right)}\right)$ *and* $\overline{\overline{\mathfrak{C}}}^+ = \left(\max_j \overline{\overline{\mathcal{M}}}_{\overline{\overline{R}}_j} e^{i2\pi\left(\max_j \overline{\overline{\mathcal{M}}}_{\overline{\overline{I}}_j}\right)}, \min_j \overline{\overline{\mathcal{N}}}_{\overline{\overline{R}}_j} e^{i2\pi\left(\min_j \overline{\overline{\mathcal{N}}}_{\overline{\overline{I}}_j}\right)}\right)$*, then*

$$\overline{\overline{\mathfrak{C}}}^- \leq CIFAAHG\left(\overline{\overline{\mathfrak{C}_{C_1}}}, \overline{\overline{\mathfrak{C}_{C_2}}}, \ldots, \overline{\overline{\mathfrak{C}_{C_n}}}\right) \leq \overline{\overline{\mathfrak{C}}}^+ \tag{41}$$

Proof. Omitted. □

Proposition 15. *By considering any group of CIFNs* $\overline{\overline{\mathfrak{C}_{C_j}}} = \left(\overline{\overline{\mathcal{M}_{\overline{\overline{R_j}}}}} e^{i2\pi(\overline{\overline{\mathcal{M}_{\overline{\overline{I_j}}}}})}, \overline{\overline{\mathcal{N}_{\overline{\overline{R_j}}}}} e^{i2\pi(\overline{\overline{\mathcal{N}_{\overline{\overline{I_j}}}}})}\right)$, $j = 1, 2, \ldots, n$*, If* $\overline{\overline{\mathfrak{C}_{C_j}}} \leq \overline{\overline{\mathfrak{C}_{C_j}}}' = \left(\overline{\overline{\mathcal{M}_{\overline{\overline{R_j}}}}}' e^{i2\pi(\overline{\overline{\mathcal{M}_{\overline{\overline{I_j}}}}}')}, \overline{\overline{\mathcal{N}_{\overline{\overline{R_j}}}}}' e^{i2\pi(\overline{\overline{\mathcal{N}_{\overline{\overline{I_j}}}}}')}\right)$*, then*

$$CIFAAHG\left(\overline{\overline{\mathfrak{C}_{C_1}}}, \overline{\overline{\mathfrak{C}_{C_2}}}, \ldots, \overline{\overline{\mathfrak{C}_{C_n}}}\right) \leq CIFAAHG\left(\overline{\overline{\mathfrak{C}_{C_1}}}', \overline{\overline{\mathfrak{C}_{C_2}}}', \ldots, \overline{\overline{\mathfrak{C}_{C_n}}}'\right) \quad (42)$$

Proof. Omitted. □

5. MADM Procedures under Investigated Operators

The major contribution of this analysis is to choose the MADM technique under CIFS for determining the beneficial optimal from the family of complex intuitionistic fuzzy information. Several intellectuals have utilized the assumption of CIFS in the region of individual situations. To investigate the beneficial optimal, we elaborated a decision-making process; for this, we considered the group alternatives and their attributes in the shape of $\overline{\overline{\mathfrak{C}_C}} = \left\{\overline{\overline{\mathfrak{C}_{C_1}}}, \overline{\overline{\mathfrak{C}_{C_2}}}, \ldots, \overline{\overline{\mathfrak{C}_{C_m}}}\right\}$, $\overline{\overline{\mathfrak{C}'_C}} = \left\{\overline{\overline{\mathfrak{C}'_{C_1}}}, \overline{\overline{\mathfrak{C}'_{C_2}}}, \ldots, \overline{\overline{\mathfrak{C}'_{C_n}}}\right\}$ with weight vector $\overline{\overline{\mathfrak{W}}} = \left(\overline{\overline{\mathfrak{W}}}_1, \overline{\overline{\mathfrak{W}}}_2, \ldots, \overline{\overline{\mathfrak{W}}}_n\right)^T$, with a rule $\sum_{j=1}^{n} \overline{\overline{\mathfrak{W}}}_j = 1$. For this, we suppose that $\overline{\overline{\mathfrak{C}_{C_{jk}}}} = \left(\overline{\overline{\mathcal{M}_{\overline{\overline{R_{jk}}}}}} e^{i2\pi(\overline{\overline{\mathcal{M}_{\overline{\overline{I_{jk}}}}}})}, \overline{\overline{\mathcal{N}_{\overline{\overline{R_{jk}}}}}} e^{i2\pi(\overline{\overline{\mathcal{N}_{\overline{\overline{I_{jk}}}}}})}\right), j, k = 1, 2, \ldots, n, m.$ Where $\overline{\overline{\mathcal{M}_{\overline{\overline{\mathfrak{C}_C}}}}}(\widetilde{\chi_E}) = \overline{\overline{\mathcal{M}_{\overline{\overline{R}}}}}(\widetilde{\chi_E}) e^{i2\pi(\overline{\overline{\mathcal{M}_{\overline{\overline{I}}}}}(\widetilde{\chi_E}))}$ and $\overline{\overline{\mathcal{N}_{\overline{\overline{\mathfrak{C}_C}}}}}(\widetilde{\chi_E}) = \overline{\overline{\mathcal{N}_{\overline{\overline{R}}}}}(\widetilde{\chi_E}) e^{i2\pi(\overline{\overline{\mathcal{N}_{\overline{\overline{I}}}}}(\widetilde{\chi_E}))}$, then the TD and FD with $0 \leq \overline{\overline{\mathcal{M}_{\overline{\overline{R}}}}}(\widetilde{\chi_E}) + \overline{\overline{\mathcal{N}_{\overline{\overline{R}}}}}(\widetilde{\chi_E}) \leq 1$ and $0 \leq \overline{\overline{\mathcal{M}_{\overline{\overline{I}}}}}(\widetilde{\chi_E}) + \overline{\overline{\mathcal{N}_{\overline{\overline{I}}}}}(\widetilde{\chi_E}) \leq 1$. From the above theory, we are able to determine the beneficial optimal, and we can construct the decision-making technique.

A. *Decision-Making Technique*

For investigating the beneficial optimal, we constructed the decision-making algorithm in the shape of the following stages:

Stage 1: Under the different sort of CIFNs which cover cost sort and benefit sorts in the shape of a matrix.

Stage 2: If the matrix covers the cost sort of data, then by using the below theory, we scandalized the matrix such that

$$D = \begin{cases} \left(\overline{\overline{\mathcal{M}_{\overline{\overline{R_{jk}}}}}} e^{i2\pi(\overline{\overline{\mathcal{M}_{\overline{\overline{I_{jk}}}}}})}, \overline{\overline{\mathcal{N}_{\overline{\overline{R_{jk}}}}}} e^{i2\pi(\overline{\overline{\mathcal{N}_{\overline{\overline{I_{jk}}}}}})}\right) & for\ benefit\ sort\ of\ data \\ \left(\overline{\overline{\mathcal{N}_{\overline{\overline{R_{jk}}}}}} e^{i2\pi(\overline{\overline{\mathcal{N}_{\overline{\overline{I_{jk}}}}}})}, \overline{\overline{\mathcal{M}_{\overline{\overline{R_{jk}}}}}} e^{i2\pi(\overline{\overline{\mathcal{M}_{\overline{\overline{I_{jk}}}}}})}\right) & for\ cost\ sort\ of\ data \end{cases}$$

If the matrix covers the benefits sort of data, then leave them.

Stage 3: Under Equation (17) (CIFAAWA) and Equation (32) (CIFAAWG), we accomplished the matrix to analyze the CIFN.

Stage 4: Under Equation (6) (if Equation (6) failed, then we use Equation (7)), we investigated the SV of the CIFNs.

Stage 5: From the SV, we explored the ranking technique of the SVs to elaborate on the beneficial optimal.

B. *Illustrated Example*

The major finding of this analysis was to analyze the elaborated operators in the circumstances of the MADM procedure. For this, we discuss some practical data to determine the feasibility and possibility of the presented works.

C. *Explanation of the Problem*

Considering the extending competition of globalization and fast mechanical overhauls, worldwide markets are compelling organizations to supply the best-quality goods and organizations. This must be cultivated through the cooperation of suitable representatives. Worker's choice is a cycle choice of individuals who have the essential capabilities to play out specific work in a best case scenario. It picks the data idea of workers and plays out a critical job in staffing the board. Developing competition in overall business sectors empowers associations with more prominent accentuation on the enrollment cycle. Huge issues such as changes in affiliations, society, work, governance, and commercializing impact affect the determination of faculty. A few organizations settle on the essential choice to choose the best job hunter, utilizing thorough and costly ID techniques. Since various individual characteristics considered for workers' decisions show irregularity and imprecision, thusly, the CIFS speculation is a helpful medium to give a design that incorporates uncertain choices in the worker's decision technique.

Allow us to contemplate a creation organization that intends on enlisting an advertising administrator for an empty post [43]. In the wake of pre-screening, five candidates $\overline{\overline{\mathfrak{C}_{C_j}}}, j = 1,2,3,4,5$ have been assigned for additional assessment. You need to choose depending on the accompanying four credits: $\overline{\overline{\mathfrak{C}'_{C_1}}}$: Oral introduction ability; $\overline{\overline{\mathfrak{C}'_{C_2}}}$: History; $\overline{\overline{\mathfrak{C}'_{C_3}}}$: Overall inclination; $\overline{\overline{\mathfrak{C}'_{C_4}}}$: self-assurance. The weight vectors for attribute values are illustrated as followed: $0.4, 0.3, 0.2, 0.1$. The five experts $\overline{\overline{\mathfrak{C}_{C_j}}}, j = 1,2,3,4,5$ are to manage ambiguity under complex intuitionistic fuzzy data by using decision-making procedures. The data provided by experts are illustrated in the shape of Table 1.

Table 1. The complex intuitionistic fuzzy information.

Alternatives/Attributes	$\overline{\overline{\mathfrak{C}}}'_{C_1}$	$\overline{\overline{\mathfrak{C}}}'_{C_2}$
$\overline{\overline{\mathfrak{C}}}_{C_1}$	$\left(0.5e^{i2\pi(0.4)}, 0.3e^{i2\pi(0.4)}\right)$	$\left(0.51e^{i2\pi(0.41)}, 0.31e^{i2\pi(0.41)}\right)$
$\overline{\overline{\mathfrak{C}}}_{C_2}$	$\left(0.4e^{i2\pi(0.1)}, 0.1e^{i2\pi(0.1)}\right)$	$\left(0.41e^{i2\pi(0.11)}, 0.11e^{i2\pi(0.11)}\right)$
$\overline{\overline{\mathfrak{C}}}_{C_3}$	$\left(0.5e^{i2\pi(0.2)}, 0.4e^{i2\pi(0.3)}\right)$	$\left(0.51e^{i2\pi(0.21)}, 0.41e^{i2\pi(0.31)}\right)$
$\overline{\overline{\mathfrak{C}}}_{C_4}$	$\left(0.3e^{i2\pi(0.5)}, 0.1e^{i2\pi(0.2)}\right)$	$\left(0.31e^{i2\pi(0.51)}, 0.11e^{i2\pi(0.21)}\right)$
$\overline{\overline{\mathfrak{C}}}_{C_5}$	$\left(0.5e^{i2\pi(0.5)}, 0.1e^{i2\pi(0.3)}\right)$	$\left(0.51e^{i2\pi(0.51)}, 0.11e^{i2\pi(0.31)}\right)$
Alternatives/Attributes	$\overline{\overline{\mathfrak{C}}}'_{C_3}$	$\overline{\overline{\mathfrak{C}}}'_{C_4}$
$\overline{\overline{\mathfrak{C}}}_{C_1}$	$\left(0.52e^{i2\pi(0.42)}, 0.32e^{i2\pi(0.42)}\right)$	$\left(0.53e^{i2\pi(0.43)}, 0.33e^{i2\pi(0.43)}\right)$
$\overline{\overline{\mathfrak{C}}}_{C_2}$	$\left(0.42e^{i2\pi(0.12)}, 0.12e^{i2\pi(0.12)}\right)$	$\left(0.43e^{i2\pi(0.13)}, 0.13e^{i2\pi(0.13)}\right)$
$\overline{\overline{\mathfrak{C}}}_{C_3}$	$\left(0.52e^{i2\pi(0.22)}, 0.42e^{i2\pi(0.32)}\right)$	$\left(0.53e^{i2\pi(0.23)}, 0.43e^{i2\pi(0.33)}\right)$
$\overline{\overline{\mathfrak{C}}}_{C_4}$	$\left(0.32e^{i2\pi(0.52)}, 0.12e^{i2\pi(0.22)}\right)$	$\left(0.33e^{i2\pi(0.53)}, 0.13e^{i2\pi(0.23)}\right)$
$\overline{\overline{\mathfrak{C}}}_{C_5}$	$\left(0.52e^{i2\pi(0.52)}, 0.12e^{i2\pi(0.32)}\right)$	$\left(0.53e^{i2\pi(0.53)}, 0.13e^{i2\pi(0.33)}\right)$

D. *Method under CIFAAWA and CIFAAWG Operators*

On the path to determine the most beneficial person from the group of persons (five candidates) by using the MADM technique under CIFAAWA and CIFAAWG operators., for investigating the beneficial optimal, we constructed the decision-making algorithm in the shape of the following stages:

Stage 1: Under the different sort of CIFNs which cover cost sort and benefit sorts in the shape of a matrix (Table 1).

Stage 2: If the matrix covers the cost sort of data, then by using the below theory, we scandalized the matrix, such that

$$D = \begin{cases} \left(\overline{\overline{\mathcal{M}_{\overline{\overline{R_{jk}}}}}} e^{i2\pi\left(\overline{\overline{\mathcal{M}_{\overline{\overline{I_{jk}}}}}}\right)}, \overline{\overline{\mathcal{N}_{\overline{\overline{R_{jk}}}}}} e^{i2\pi\left(\overline{\overline{\mathcal{N}_{\overline{\overline{I_{jk}}}}}}\right)} \right) & for\ benefit\ sort\ of\ data \\ \left(\overline{\overline{\mathcal{N}_{\overline{\overline{R_{jk}}}}}} e^{i2\pi\left(\overline{\overline{\mathcal{N}_{\overline{\overline{I_{jk}}}}}}\right)}, \overline{\overline{\mathcal{M}_{\overline{\overline{R_{jk}}}}}} e^{i2\pi\left(\overline{\overline{\mathcal{M}_{\overline{\overline{I_{jk}}}}}}\right)} \right) & for\ cost\ sort\ of\ data \end{cases}$$

If the matrix covers the benefits sort of data, then leave them. The data in Table 1 cover all the beneficial sorts of data, so they do not need to be standardized.

Stage 3: Under Equation (17) (CIFAAWA) and Equation (32) (CIFAAWG), we accomplished the matrix to analyze the CIFN, which is illustrated in the shape of Table 2.

Table 2. The aggregated operators for $\psi = 1$.

Alternatives	CIFAAWA Operator	CIFAAWG Operator
$\overline{\overline{\mathfrak{C}}}_{C_1}$	$\left(0.2665e^{i2\pi(0.2048)}, 0.6012e^{i2\pi(0.6789)}\right)$	$\left(0.7464e^{i2\pi(0.6789)}, 0.1489e^{i2\pi(0.2048)}\right)$
$\overline{\overline{\mathfrak{C}}}_{C_2}$	$\left(0.2048e^{i2\pi(0.0494)}, 0.3828e^{i2\pi(0.3828)}\right)$	$\left(0.6789e^{i2\pi(0.3828)}, 0.0494e^{i2\pi(0.0494)}\right)$
$\overline{\overline{\mathfrak{C}}}_{C_3}$	$\left(0.2665e^{i2\pi(0.0973)}, 0.6789e^{i2\pi(0.6012)}\right)$	$\left(0.7464e^{i2\pi(0.5075)}, 0.2048e^{i2\pi(0.1489)}\right)$
$\overline{\overline{\mathfrak{C}}}_{C_4}$	$\left(0.1489e^{i2\pi(0.2665)}, 0.3828e^{i2\pi(0.5075)}\right)$	$\left(0.6012e^{i2\pi(0.7464)}, 0.0494e^{i2\pi(0.0973)}\right)$
$\overline{\overline{\mathfrak{C}}}_{C_5}$	$\left(0.2665e^{i2\pi(0.2665)}, 0.3828e^{i2\pi(0.3828)}\right)$	$\left(0.7464e^{i2\pi(0.7464)}, 0.0494e^{i2\pi(0.0494)}\right)$

Stage 4: Under Equation (6) (if Equation (6) failed, then we use Equation (7)), we investigated the SV of the CIFNs illustrated in the shape of Table 3.

Table 3. The Score values.

Alternatives	CIFAAWA Operator	CIFAAWG Operator
$\overline{\overline{\mathfrak{C}}}_{C_1}$	−0.4044	0.5358
$\overline{\overline{\mathfrak{C}}}_{C_2}$	−0.2557	0.4814
$\overline{\overline{\mathfrak{C}}}_{C_3}$	−0.4581	0.4501
$\overline{\overline{\mathfrak{C}}}_{C_4}$	−0.2375	0.6004
$\overline{\overline{\mathfrak{C}}}_{C_5}$	−0.1163	0.697

Stage 5: From the SV, we explored the ranking technique of the SVs to elaborate on the beneficial optimal illustrated in the shape of Table 4.

Table 4. The ranking values.

CIFAAWA Operator	$\overline{\overline{\mathfrak{C}}}_{C_5} > \overline{\overline{\mathfrak{C}}}_{C_4} > \overline{\overline{\mathfrak{C}}}_{C_1} > \overline{\overline{\mathfrak{C}}}_{C_2} > \overline{\overline{\mathfrak{C}}}_{C_3}$
CIFAAWG Operator	$\overline{\overline{\mathfrak{C}}}_{C_5} > \overline{\overline{\mathfrak{C}}}_{C_4} > \overline{\overline{\mathfrak{C}}}_{C_2} > \overline{\overline{\mathfrak{C}}}_{C_1} > \overline{\overline{\mathfrak{C}}}_{C_3}$

Under the different sorts of operators, we have gotten the same beneficial optimal $\overline{\overline{\mathfrak{C}}}_{C_5}$. Additionally, we enhanced the quality of the research works under distinct sorts with the help of some massive examples. The data provided by experts are illustrated in the shape of Table 5.

Table 5. The intuitionistic fuzzy information.

Alternatives/Attributes	$\overline{\overline{\mathfrak{C}}}'_{C_1}$	$\overline{\overline{\mathfrak{C}}}'_{C_2}$	$\overline{\overline{\mathfrak{C}}}'_{C_3}$	$\overline{\overline{\mathfrak{C}}}'_{C_4}$
$\overline{\overline{\mathfrak{C}}}_{C_1}$	$(0.5, 0.3)$	$(0.51, 0.31)$	$(0.52, 0.32)$	$(0.53, 0.33)$
$\overline{\overline{\mathfrak{C}}}_{C_2}$	$(0.4, 0.1)$	$(0.41, 0.11)$	$(0.42, 0.12)$	$(0.43, 0.13)$
$\overline{\overline{\mathfrak{C}}}_{C_3}$	$(0.5, 0.4)$	$(0.51, 0.41)$	$(0.52, 0.42)$	$(0.53, 0.43)$
$\overline{\overline{\mathfrak{C}}}_{C_4}$	$(0.3, 0.1)$	$(0.31, 0.11)$	$(0.32, 0.12)$	$(0.33, 0.13)$
$\overline{\overline{\mathfrak{C}}}_{C_5}$	$(0.5, 0.1)$	$(0.51, 0.11)$	$(0.52, 0.12)$	$(0.53, 0.13)$

E. *Method under CIFAAWA and CIFAAWG Operators*

On the path to determine the beneficial person from the group of persons (five candidates) by using the MADM technique under CIFAAWA and CIFAAWG operators, for investigating the beneficial optimal, we constructed the decision-making algorithm in the shape of the following stages:

Stage 1: Under the different sort of CIFNs, which cover cost sort and benefit sorts in the shape of a matrix (Table 5).

Stage 2: If the matrix covers the cost sort of data, then by using the below theory, we scandalized the matrix, such that

$$D = \begin{cases} \left(\overline{\overline{\mathcal{M}}}_{\overline{\overline{R}}_{jk}} e^{i2\pi\left(\overline{\overline{\mathcal{M}}}_{\overline{\overline{I}}_{jk}}\right)}, \overline{\overline{\mathcal{N}}}_{\overline{\overline{R}}_{jk}} e^{i2\pi\left(\overline{\overline{\mathcal{N}}}_{\overline{\overline{I}}_{jk}}\right)} \right) & for\ benefit\ sort\ of\ data \\ \left(\overline{\overline{\mathcal{N}}}_{\overline{\overline{R}}_{jk}} e^{i2\pi\left(\overline{\overline{\mathcal{N}}}_{\overline{\overline{I}}_{jk}}\right)}, \overline{\overline{\mathcal{M}}}_{\overline{\overline{R}}_{jk}} e^{i2\pi\left(\overline{\overline{\mathcal{M}}}_{\overline{\overline{I}}_{jk}}\right)} \right) & for\ cost\ sort\ of\ data \end{cases}$$

If the matrix covers the benefits sort of data, then leave them. The data in Table 5 cover all the beneficial sorts of data, so they do not need to be standardized.

Stage 3: Under Equation (17) (CIFAAWA) and Equation (32) (CIFAAWG), we accomplished the matrix to analyze the CIFN, which is illustrated in the shape of Table 6.

Under the different sorts of operators, we have gotten the same beneficial optimal $\overline{\overline{\mathfrak{C}}}_{C_5}$.

F. *Influence of Parameter*

The major aim of this study was to check the fluency of the parameter ψ, to investigate the consistency of the elaborated works. For this, the data in Table 1 were considered for determining the fluency of the parameter, stated in Table 9 under both CIFAAWA and CIFAAWG operators.

Table 6. The aggregated operators for $\psi = \mathbf{1}$.

Alternatives	CIFAAWA Operator	CIFAAWG Operator
$\overline{\overline{\mathfrak{C}}}_{C_1}$	$(0.2665, 0.6012)$	$(0.7464, 0.1489)$
$\overline{\overline{\mathfrak{C}}}_{C_2}$	$(0.2048, 0.3828)$	$(0.6789, 0.0494)$
$\overline{\overline{\mathfrak{C}}}_{C_3}$	$(0.2665, 0.6789)$	$(0.7464, 0.2048)$
$\overline{\overline{\mathfrak{C}}}_{C_4}$	$(0.1489, 0.3828)$	$(0.6012, 0.0494)$
$\overline{\overline{\mathfrak{C}}}_{C_5}$	$(0.2665, 0.3828)$	$(0.7464, 0.0494)$

Stage 4: Under Equation (6) (if Equation (6) failed, then we use Equation (7)), we investigated the SV of the CIFNs illustrated in the shape of Table 7.

Table 7. The Score values.

Alternatives	CIFAAWA Operator	CIFAAWG Operator
$\overline{\overline{\mathfrak{C}}}_{C_1}$	-0.1674	0.2988
$\overline{\overline{\mathfrak{C}}}_{C_2}$	-0.089	0.3147
$\overline{\overline{\mathfrak{C}}}_{C_3}$	-0.2062	0.2708
$\overline{\overline{\mathfrak{C}}}_{C_4}$	-0.1169	0.2759
$\overline{\overline{\mathfrak{C}}}_{C_5}$	-0.0581	0.3485

Stage 5: From the SV, we explored the ranking technique of the SVs to elaborate the beneficial optimal illustrated in the shape of Table 8.

Table 8. The ranking values.

CIFAAWA Operator	$\overline{\overline{\mathfrak{C}}}_{C_5} > \overline{\overline{\mathfrak{C}}}_{C_2} > \overline{\overline{\mathfrak{C}}}_{C_1} > \overline{\overline{\mathfrak{C}}}_{C_4} > \overline{\overline{\mathfrak{C}}}_{C_3}$
CIFAAWG Operator	$\overline{\overline{\mathfrak{C}}}_{C_5} > \overline{\overline{\mathfrak{C}}}_{C_2} > \overline{\overline{\mathfrak{C}}}_{C_1} > \overline{\overline{\mathfrak{C}}}_{C_4} > \overline{\overline{\mathfrak{C}}}_{C_3}$

Table 9. The fluency for the different values of parameter ψ by using the data in Table 1.

Parameter	Operator	Score Values	Ranking Values
$\psi = \mathbf{1}$	CIFAAWA Operator	$-0.4044, -0.2557, -0.4581, -0.2375, -0.1163$	$\overline{\overline{\mathfrak{C}}}_{C_5} > \overline{\overline{\mathfrak{C}}}_{C_4} > \overline{\overline{\mathfrak{C}}}_{C_1} > \overline{\overline{\mathfrak{C}}}_{C_2} > \overline{\overline{\mathfrak{C}}}_{C_3}$
	CIFAAWG Operator	$0.5358, 0.4814, 0.4501, 0.6004, 0.697$	$\overline{\overline{\mathfrak{C}}}_{C_5} > \overline{\overline{\mathfrak{C}}}_{C_4} > \overline{\overline{\mathfrak{C}}}_{C_2} > \overline{\overline{\mathfrak{C}}}_{C_1} > \overline{\overline{\mathfrak{C}}}_{C_3}$
$\psi = \mathbf{5}$	CIFAAWA Operator	$-0.4036, -0.2538, -0.4572, -0.2362, -0.1147$	$\overline{\overline{\mathfrak{C}}}_{C_5} > \overline{\overline{\mathfrak{C}}}_{C_4} > \overline{\overline{\mathfrak{C}}}_{C_1} > \overline{\overline{\mathfrak{C}}}_{C_2} > \overline{\overline{\mathfrak{C}}}_{C_3}$
	CIFAAWG Operator	$0.535, 0.4797, 0.4492, 0.5993, 0.6957$	$\overline{\overline{\mathfrak{C}}}_{C_5} > \overline{\overline{\mathfrak{C}}}_{C_4} > \overline{\overline{\mathfrak{C}}}_{C_2} > \overline{\overline{\mathfrak{C}}}_{C_1} > \overline{\overline{\mathfrak{C}}}_{C_3}$
$\psi = 11$	CIFAAWA Operator	$-0.4024, -0.2512, -0.4559, -0.2343, -0.1126$	$\overline{\overline{\mathfrak{C}}}_{C_5} > \overline{\overline{\mathfrak{C}}}_{C_4} > \overline{\overline{\mathfrak{C}}}_{C_1} > \overline{\overline{\mathfrak{C}}}_{C_2} > \overline{\overline{\mathfrak{C}}}_{C_3}$
	CIFAAWG Operator	$0.5338, 0.4773, 0.4479, 0.5976, 0.6938$	$\overline{\overline{\mathfrak{C}}}_{C_5} > \overline{\overline{\mathfrak{C}}}_{C_4} > \overline{\overline{\mathfrak{C}}}_{C_2} > \overline{\overline{\mathfrak{C}}}_{C_1} > \overline{\overline{\mathfrak{C}}}_{C_3}$
$\psi = 51$	CIFAAWA Operator	$-0.3957, -0.2409, -0.449, -0.2253, -0.1024$	$\overline{\overline{\mathfrak{C}}}_{C_5} > \overline{\overline{\mathfrak{C}}}_{C_4} > \overline{\overline{\mathfrak{C}}}_{C_1} > \overline{\overline{\mathfrak{C}}}_{C_2} > \overline{\overline{\mathfrak{C}}}_{C_3}$
	CIFAAWG Operator	$0.5275, 0.4684, 0.4409, 0.5906, 0.687$	$\overline{\overline{\mathfrak{C}}}_{C_5} > \overline{\overline{\mathfrak{C}}}_{C_4} > \overline{\overline{\mathfrak{C}}}_{C_2} > \overline{\overline{\mathfrak{C}}}_{C_1} > \overline{\overline{\mathfrak{C}}}_{C_3}$
$\psi = 101$	CIFAAWA Operator	$-0.3913, -0.2363, -0.445, -0.2202, -0.0967$	$\overline{\overline{\mathfrak{C}}}_{C_5} > \overline{\overline{\mathfrak{C}}}_{C_4} > \overline{\overline{\mathfrak{C}}}_{C_1} > \overline{\overline{\mathfrak{C}}}_{C_2} > \overline{\overline{\mathfrak{C}}}_{C_3}$
	CIFAAWG Operator	$0.5236, 0.4652, 0.4366, 0.5875, 0.6846$	$\overline{\overline{\mathfrak{C}}}_{C_5} > \overline{\overline{\mathfrak{C}}}_{C_4} > \overline{\overline{\mathfrak{C}}}_{C_2} > \overline{\overline{\mathfrak{C}}}_{C_1} > \overline{\overline{\mathfrak{C}}}_{C_3}$

Under the different sorts of operators, we have gotten the same beneficial optimal $\overline{\overline{\mathfrak{C}}}_{C_5}$ by using the distinct values of the parameter. For this, the data in Table 5 were considered for determining the fluency of the parameter, stated in Table 10 under both CIFAAWA and CIFAAWG operators.

Table 10. The fluency for the different values of parameter $\boldsymbol{\psi}$ by using the data in Table 5.

Parameter	Operator	Score Values	Ranking Values
$\boldsymbol{\psi} = \mathbf{1}$	CIFAAWA Operator	$-0.1674, -0.089, -0.2062, -0.1169, -0.0581$	$\overline{\overline{\mathfrak{C}}}_{C_5} > \overline{\overline{\mathfrak{C}}}_{C_2} > \overline{\overline{\mathfrak{C}}}_{C_1} > \overline{\overline{\mathfrak{C}}}_{C_4} > \overline{\overline{\mathfrak{C}}}_{C_3}$
	CIFAAWG Operator	$0.2988, 0.3147, 0.2708, 0.2759, 0.3485$	$\overline{\overline{\mathfrak{C}}}_{C_5} > \overline{\overline{\mathfrak{C}}}_{C_2} > \overline{\overline{\mathfrak{C}}}_{C_1} > \overline{\overline{\mathfrak{C}}}_{C_4} > \overline{\overline{\mathfrak{C}}}_{C_3}$
$\boldsymbol{\psi} = \mathbf{5}$	CIFAAWA Operator	$-0.1669, -0.0882, -0.2058, -0.1161, -0.0574$	$\overline{\overline{\mathfrak{C}}}_{C_5} > \overline{\overline{\mathfrak{C}}}_{C_2} > \overline{\overline{\mathfrak{C}}}_{C_1} > \overline{\overline{\mathfrak{C}}}_{C_4} > \overline{\overline{\mathfrak{C}}}_{C_3}$
	CIFAAWG Operator	$0.2984, 0.3141, 0.2704, 0.2752, 0.3479$	$\overline{\overline{\mathfrak{C}}}_{C_5} > \overline{\overline{\mathfrak{C}}}_{C_2} > \overline{\overline{\mathfrak{C}}}_{C_1} > \overline{\overline{\mathfrak{C}}}_{C_4} > \overline{\overline{\mathfrak{C}}}_{C_3}$
$\boldsymbol{\psi} = 11$	CIFAAWA Operator	$-0.1663, -0.0871, -0.2052, -0.115, -0.0563$	$\overline{\overline{\mathfrak{C}}}_{C_5} > \overline{\overline{\mathfrak{C}}}_{C_2} > \overline{\overline{\mathfrak{C}}}_{C_1} > \overline{\overline{\mathfrak{C}}}_{C_4} > \overline{\overline{\mathfrak{C}}}_{C_3}$
	CIFAAWG Operator	$0.2978, 0.3131, 0.2698, 0.2742, 0.3469$	$\overline{\overline{\mathfrak{C}}}_{C_5} > \overline{\overline{\mathfrak{C}}}_{C_2} > \overline{\overline{\mathfrak{C}}}_{C_1} > \overline{\overline{\mathfrak{C}}}_{C_4} > \overline{\overline{\mathfrak{C}}}_{C_3}$
$\boldsymbol{\psi} = 51$	CIFAAWA Operator	$-0.1629, -0.0821, -0.202, -0.1099, -0.0512$	$\overline{\overline{\mathfrak{C}}}_{C_5} > \overline{\overline{\mathfrak{C}}}_{C_2} > \overline{\overline{\mathfrak{C}}}_{C_1} > \overline{\overline{\mathfrak{C}}}_{C_4} > \overline{\overline{\mathfrak{C}}}_{C_3}$
	CIFAAWG Operator	$0.2946, 0.3096, 0.2668, 0.2705, 0.3435$	$\overline{\overline{\mathfrak{C}}}_{C_5} > \overline{\overline{\mathfrak{C}}}_{C_2} > \overline{\overline{\mathfrak{C}}}_{C_1} > \overline{\overline{\mathfrak{C}}}_{C_4} > \overline{\overline{\mathfrak{C}}}_{C_3}$
$\boldsymbol{\psi} = 101$	CIFAAWA Operator	$-0.1605, -0.0794, -0.1997, -0.1074, -0.0483$	$\overline{\overline{\mathfrak{C}}}_{C_5} > \overline{\overline{\mathfrak{C}}}_{C_2} > \overline{\overline{\mathfrak{C}}}_{C_1} > \overline{\overline{\mathfrak{C}}}_{C_4} > \overline{\overline{\mathfrak{C}}}_{C_3}$
	CIFAAWG Operator	$0.2928, 0.3083, 0.2648, 0.269, 0.3423$	$\overline{\overline{\mathfrak{C}}}_{C_5} > \overline{\overline{\mathfrak{C}}}_{C_2} > \overline{\overline{\mathfrak{C}}}_{C_1} > \overline{\overline{\mathfrak{C}}}_{C_4} > \overline{\overline{\mathfrak{C}}}_{C_3}$

Under the different sorts of operators, we have gotten the same sort of beneficial optimal $\overline{\overline{\mathfrak{C}}}_{C_5}$ by using the distinct values of the parameter.

G. *Sensitive Analysis*

The main contribution of this analysis is to choose some prevailing operators under different types of ideas, for illustration, aggregation operators [43], geometric aggregation operators [44], information aggregation operators [45], Einstein geometric aggregation operators [46], Hamacher aggregation operators [47], Dombi aggregation operators [48] under the IFSs, aggregation operators [31], and geometric aggregation operators based on CIFSs. Under the above analysis, we employed the resultant values for the data in Table 1, illustrated in Table 11.

Table 11. The sensitivity analysis by using the data in Table 1.

Methods	Score Values	Ranking Values
Xu [43]	× × × × × × × × × × × × × × × × ×	× × × × × × × × × × × × × × × × ×
Xu and Yager [44]	× × × × × × × × × × × × × × × × ×	× × × × × × × × × × × × × × × × ×
Wang and Liu [45]	× × × × × × × × × × × × × × × × ×	× × × × × × × × × × × × × × × × ×
Wang and Liu [46]	× × × × × × × × × × × × × × × × ×	× × × × × × × × × × × × × × × × ×
Huang [47]	× × × × × × × × × × × × × × × × ×	× × × × × × × × × × × × × × × × ×
Seikh and Mandal [48]	× × × × × × × × × × × × × × × × ×	× × × × × × × × × × × × × × × × ×
Garg and Rani [31]	$0.5349, 0.4805, 0.4502, 0.6003, 0.688$	$\overline{\overline{\mathfrak{C}}}_{C_5} > \overline{\overline{\mathfrak{C}}}_{C_4} > \overline{\overline{\mathfrak{C}}}_{C_2} > \overline{\overline{\mathfrak{C}}}_{C_1} > \overline{\overline{\mathfrak{C}}}_{C_3}$
Garg and Rani [32]	$0.4257, 0.3713, 0.3400, 0.5003, 0.586$	$\overline{\overline{\mathfrak{C}}}_{C_5} > \overline{\overline{\mathfrak{C}}}_{C_4} > \overline{\overline{\mathfrak{C}}}_{C_2} > \overline{\overline{\mathfrak{C}}}_{C_1} > \overline{\overline{\mathfrak{C}}}_{C_3}$
CIFAAWA operator	$-0.4044, -0.2557, -0.4581, -0.2375, -0.1163$	$\overline{\overline{\mathfrak{C}}}_{C_5} > \overline{\overline{\mathfrak{C}}}_{C_4} > \overline{\overline{\mathfrak{C}}}_{C_2} > \overline{\overline{\mathfrak{C}}}_{C_1} > \overline{\overline{\mathfrak{C}}}_{C_3}$
CIFAAWG operator	$0.5358, 0.4814, 0.4501, 0.6004, 0.697$	$\overline{\overline{\mathfrak{C}}}_{C_5} > \overline{\overline{\mathfrak{C}}}_{C_4} > \overline{\overline{\mathfrak{C}}}_{C_2} > \overline{\overline{\mathfrak{C}}}_{C_1} > \overline{\overline{\mathfrak{C}}}_{C_3}$

Under the different sorts of operators, we have gotten the same sort of beneficial optimal $\overline{\overline{\mathfrak{C}_{C_5}}}$. Under the above analysis, we employed the resultant values for the data in Table 5, illustrated in Table 12.

Table 12. The sensitivity analysis by using the data in Table 5.

Methods	Score Values	Ranking Values
Xu [43]	$0.1877, 0.2036, 0.1607, 0.1648, 0.2374$	$\overline{\overline{\mathfrak{C}_{C_5}}} > \overline{\overline{\mathfrak{C}_{C_2}}} > \overline{\overline{\mathfrak{C}_{C_1}}} > \overline{\overline{\mathfrak{C}_{C_4}}} > \overline{\overline{\mathfrak{C}_{C_3}}}$
Xu and Yager [44]	$0.2946, 0.3096, 0.2668, 0.2705, 0.3435$	$\overline{\overline{\mathfrak{C}_{C_5}}} > \overline{\overline{\mathfrak{C}_{C_2}}} > \overline{\overline{\mathfrak{C}_{C_1}}} > \overline{\overline{\mathfrak{C}_{C_4}}} > \overline{\overline{\mathfrak{C}_{C_3}}}$
Wang and Liu [45]	$-0.1563, -0.078, -0.2051, -0.1158, -0.0570$	$\overline{\overline{\mathfrak{C}_{C_5}}} > \overline{\overline{\mathfrak{C}_{C_2}}} > \overline{\overline{\mathfrak{C}_{C_1}}} > \overline{\overline{\mathfrak{C}_{C_4}}} > \overline{\overline{\mathfrak{C}_{C_3}}}$
Wang and Liu [46]	$0.5944, 0.6102, 0.5703, 0.5704, 0.6441$	$\overline{\overline{\mathfrak{C}_{C_5}}} > \overline{\overline{\mathfrak{C}_{C_2}}} > \overline{\overline{\mathfrak{C}_{C_1}}} > \overline{\overline{\mathfrak{C}_{C_4}}} > \overline{\overline{\mathfrak{C}_{C_3}}}$
Huang [47]	$0.3957, 0.4097, 0.3679, 0.3716, 0.4446$	$\overline{\overline{\mathfrak{C}_{C_5}}} > \overline{\overline{\mathfrak{C}_{C_2}}} > \overline{\overline{\mathfrak{C}_{C_1}}} > \overline{\overline{\mathfrak{C}_{C_4}}} > \overline{\overline{\mathfrak{C}_{C_3}}}$
Seikh and Mandal [48]	$0.6955, 0.7113, 0.6714, 0.6715, 0.7452$	$\overline{\overline{\mathfrak{C}_{C_5}}} > \overline{\overline{\mathfrak{C}_{C_2}}} > \overline{\overline{\mathfrak{C}_{C_1}}} > \overline{\overline{\mathfrak{C}_{C_4}}} > \overline{\overline{\mathfrak{C}_{C_3}}}$
Garg and Rani [31]	$0.2977, 0.3136, 0.2707, 0.2748, 0.3474$	$\overline{\overline{\mathfrak{C}_{C_5}}} > \overline{\overline{\mathfrak{C}_{C_2}}} > \overline{\overline{\mathfrak{C}_{C_1}}} > \overline{\overline{\mathfrak{C}_{C_4}}} > \overline{\overline{\mathfrak{C}_{C_3}}}$
Garg and Rani [32]	$0.1899, 0.2058, 0.1619, 0.1669, 0.2396$	$\overline{\overline{\mathfrak{C}_{C_5}}} > \overline{\overline{\mathfrak{C}_{C_2}}} > \overline{\overline{\mathfrak{C}_{C_1}}} > \overline{\overline{\mathfrak{C}_{C_4}}} > \overline{\overline{\mathfrak{C}_{C_3}}}$
CIFAAWA operator	$-0.1674, -0.089, -0.2062, -0.1169, -0.0581$	$\overline{\overline{\mathfrak{C}_{C_5}}} > \overline{\overline{\mathfrak{C}_{C_2}}} > \overline{\overline{\mathfrak{C}_{C_1}}} > \overline{\overline{\mathfrak{C}_{C_4}}} > \overline{\overline{\mathfrak{C}_{C_3}}}$
CIFAAWG operator	$0.2988, 0.3147, 0.2708, 0.2759, 0.3485$	$\overline{\overline{\mathfrak{C}_{C_5}}} > \overline{\overline{\mathfrak{C}_{C_2}}} > \overline{\overline{\mathfrak{C}_{C_1}}} > \overline{\overline{\mathfrak{C}_{C_4}}} > \overline{\overline{\mathfrak{C}_{C_3}}}$

Under the different sorts of operators, we have gotten the same sort of beneficial optimal $\overline{\overline{\mathfrak{C}_{C_5}}}$. Therefore, we have discussed all the possibilities under the different types of circumstances. The beneficial optimal is the same, which is $\overline{\overline{\mathfrak{C}_{C_5}}}$. From the above discussion, we conclude that our presented works are massive dominant, and consistent as compared to prevailing works.

6. Conclusions

The mathematical and theoretical representation of the CIF set is very powerful and reliable to manage awkward and vague information because the grades contained in the CIF set are computed in the shape of a complex number, which can easily manage two-dimensional information in the shape of a single set. The impacts of the derived analysis are listed below:

1. Firstly, we derived valuable AA operational laws under the presence of CIF information.
2. Secondly, we discovered the idea of aggregation operators based on AA operations for the CIF set then developed the ideas of CIFAAWA, CIFAAOWA, CIFAAHA, CIFAAWG, CIFAAOWG, CIFAAHG and their beneficial results.
3. Thirdly, we illustrated a MADM technique for derived operators and justified it with the help of some suitable examples.
4. Finally, we compared the derived work with some existing results to enhance the worth and capability of the pioneered operators.

The theory of AA aggregation operators based on the CIF set is a very effective and dominant idea, which can be used for aggregating the collection of information into a single set. The proposed theory can easily resolve many real-life problems, for instance, artificial intelligence in education, in healthcare, in business, in manufacturing, roads, machine learning, game theory, computer science, and many more.

In the upcoming times, we will modify our principles in this analysis based on CIFSs for complex q-rung orthopair fuzzy sets [49,50], complex spherical fuzzy sets [51,52], T-spherical fuzzy sets [53], correlation measures [54], Einstein aggregation operators [55], algebraic aggregation operators [56], logarithmic aggregation operators [57], entropy measures [58], linear Diophantine fuzzy sets [59,60], spherical Diophantine fuzzy sets [61,62], and m-polar fuzzy sets to further fill the gap of the research works.

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Conflicts of Interest: The authors declare no conflict of interest.

Appendix A

Proof. Under Equations (12)–(15), we prove that the above points, such that

1. Suppose $\overline{\overline{\mathfrak{C}_{C_1}}} \oplus \overline{\overline{\mathfrak{C}_{C_2}}}$, then

$$\overline{\overline{\mathfrak{C}_{C_1}}} \oplus \overline{\overline{\mathfrak{C}_{C_2}}} = \begin{pmatrix} \left(1-e^{-\left(\left(-\log\left(1-\overline{\overline{\mathcal{M}}}_{\overline{\overline{R_1}}}\right)\right)^{\psi}+\left(-\log\left(1-\overline{\overline{\mathcal{M}}}_{\overline{\overline{R_2}}}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right)e^{i2\pi\left(1-e^{-\left(\left(-\log\left(1-\overline{\overline{\mathcal{M}}}_{\overline{\overline{R_1}}}\right)\right)^{\psi}+\left(-\log\left(1-\overline{\overline{\mathcal{M}}}_{\overline{\overline{R_2}}}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right)}, \\ \left(e^{-\left(\left(-\log\left(\overline{\overline{\mathcal{N}}}_{\overline{\overline{R_1}}}\right)\right)^{\psi}+\left(-\log\left(\overline{\overline{\mathcal{N}}}_{\overline{\overline{R_2}}}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right)e^{i2\pi\left(e^{-\left(\left(-\log\left(\overline{\overline{\mathcal{N}}}_{\overline{\overline{I_1}}}\right)\right)^{\psi}+\left(-\log\left(\overline{\overline{\mathcal{N}}}_{\overline{\overline{I_2}}}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right)} \end{pmatrix}$$

$$= \begin{pmatrix} \left(1-e^{-\left(\left(-\log\left(1-\overline{\overline{\mathcal{M}}}_{\overline{\overline{R_2}}}\right)\right)^{\psi}+\left(-\log\left(1-\overline{\overline{\mathcal{M}}}_{\overline{\overline{R_1}}}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right)e^{i2\pi\left(1-e^{-\left(\left(-\log\left(1-\overline{\overline{\mathcal{M}}}_{\overline{\overline{R_2}}}\right)\right)^{\psi}+\left(-\log\left(1-\overline{\overline{\mathcal{M}}}_{\overline{\overline{R_1}}}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right)}, \\ \left(e^{-\left(\left(-\log\left(\overline{\overline{\mathcal{N}}}_{\overline{\overline{R_2}}}\right)\right)^{\psi}+\left(-\log\left(\overline{\overline{\mathcal{N}}}_{\overline{\overline{R_1}}}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right)e^{i2\pi\left(e^{-\left(\left(-\log\left(\overline{\overline{\mathcal{N}}}_{\overline{\overline{I_2}}}\right)\right)^{\psi}+\left(-\log\left(\overline{\overline{\mathcal{N}}}_{\overline{\overline{I_1}}}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right)} \end{pmatrix}$$

$$= \overline{\overline{\mathfrak{C}_{C_2}}} \oplus \overline{\overline{\mathfrak{C}_{C_1}}}.$$

   Hence, we investigated $\overline{\overline{\mathfrak{C}_{C_1}}} \oplus \overline{\overline{\mathfrak{C}_{C_2}}} = \overline{\overline{\mathfrak{C}_{C_2}}} \oplus \overline{\overline{\mathfrak{C}_{C_1}}}$.

2. Omitted.
3. Suppose $\overline{\overline{\theta_S}}\left(\overline{\overline{\mathfrak{C}_{C_1}}} \oplus \overline{\overline{\mathfrak{C}_{C_2}}}\right)$, then

$$\overline{\overline{\theta_S}}\left(\overline{\overline{\mathfrak{C}_{C_1}}} \oplus \overline{\overline{\mathfrak{C}_{C_2}}}\right) = \overline{\overline{\theta_S}}\left(\begin{array}{c} \left(1 - e^{-\left(\left(-\log\left(1-\overline{\overline{\mathcal{M}_{\overline{\overline{R}}_1}}}\right)\right)^{\psi} + \left(-\log\left(1-\overline{\overline{\mathcal{M}_{\overline{\overline{R}}_2}}}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right) e^{i2\pi\left(1-e^{-\left(\left(-\log\left(1-\overline{\overline{\mathcal{M}_{\overline{\overline{R}}_1}}}\right)\right)^{\psi} + \left(-\log\left(1-\overline{\overline{\mathcal{M}_{\overline{\overline{R}}_2}}}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right)}, \\ \left(e^{-\left(\left(-\log\left(\overline{\overline{\mathcal{N}_{\overline{\overline{R}}_1}}}\right)\right)^{\psi} + \left(-\log\left(\overline{\overline{\mathcal{N}_{\overline{\overline{R}}_2}}}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right) e^{i2\pi\left(e^{-\left(\left(-\log\left(\overline{\overline{\mathcal{N}_{\overline{\overline{I}}_1}}}\right)\right)^{\psi} + \left(-\log\left(\overline{\overline{\mathcal{N}_{\overline{\overline{I}}_2}}}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right)} \end{array}\right)$$

$$= \left(\begin{array}{c} \left(1 - e^{-\left(\overline{\overline{\theta_S}}\left(-\log\left(1-\overline{\overline{\mathcal{M}_{\overline{\overline{R}}_1}}}\right)\right)^{\psi} + \left(-\log\left(1-\overline{\overline{\mathcal{M}_{\overline{\overline{R}}_2}}}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right) e^{i2\pi\left(1-e^{-\left(\overline{\overline{\theta_S}}\left(-\log\left(1-\overline{\overline{\mathcal{M}_{\overline{\overline{R}}_1}}}\right)\right)^{\psi} + \left(-\log\left(1-\overline{\overline{\mathcal{M}_{\overline{\overline{R}}_2}}}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right)}, \\ \left(e^{-\left(\overline{\overline{\theta_S}}\left(-\log\left(\overline{\overline{\mathcal{N}_{\overline{\overline{R}}_1}}}\right)\right)^{\psi} + \left(-\log\left(\overline{\overline{\mathcal{N}_{\overline{\overline{R}}_2}}}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right) e^{i2\pi\left(e^{-\left(\overline{\overline{\theta_S}}\left(-\log\left(\overline{\overline{\mathcal{N}_{\overline{\overline{I}}_1}}}\right)\right)^{\psi} + \left(-\log\left(\overline{\overline{\mathcal{N}_{\overline{\overline{I}}_2}}}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right)} \end{array}\right)$$

$$= \left(\begin{array}{c} \left(1 - e^{-\left(\overline{\overline{\theta_S}}\left(-\log\left(1-\overline{\overline{\mathcal{M}_{\overline{\overline{R}}_1}}}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right) e^{i2\pi\left(1-e^{-\left(\overline{\overline{\theta_S}}\left(-\log\left(1-\overline{\overline{\mathcal{M}_{\overline{\overline{I}}_1}}}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right)}, \\ \left(e^{-\left(\overline{\overline{\theta_S}}\left(-\log\left(\overline{\overline{\mathcal{N}_{\overline{\overline{R}}_1}}}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right) e^{i2\pi\left(e^{-\left(\overline{\overline{\theta_S}}\left(-\log\left(\overline{\overline{\mathcal{N}_{\overline{\overline{I}}_1}}}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right)} \end{array}\right) \oplus \left(\begin{array}{c} \left(1 - e^{-\left(\overline{\overline{\theta_S}}\left(-\log\left(1-\overline{\overline{\mathcal{M}_{\overline{\overline{R}}_2}}}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right) e^{i2\pi\left(1-e^{-\left(\overline{\overline{\theta_S}}\left(-\log\left(1-\overline{\overline{\mathcal{M}_{\overline{\overline{I}}_2}}}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right)}, \\ \left(e^{-\left(\overline{\overline{\theta_S}}\left(-\log\left(\overline{\overline{\mathcal{N}_{\overline{\overline{R}}_2}}}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right) e^{i2\pi\left(e^{-\left(\overline{\overline{\theta_S}}\left(-\log\left(\overline{\overline{\mathcal{N}_{\overline{\overline{I}}_2}}}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right)} \end{array}\right)$$

$$= \overline{\overline{\theta_S \mathfrak{C}_{C_1}}} \oplus \overline{\overline{\theta_S \mathfrak{C}_{C_2}}}.$$

Hence, $\overline{\overline{\theta_S}}\left(\overline{\overline{\mathfrak{C}_{C_1}}} \oplus \overline{\overline{\mathfrak{C}_{C_2}}}\right) = \overline{\overline{\theta_S \mathfrak{C}_{C_1}}} \oplus \overline{\overline{\theta_S \mathfrak{C}_{C_2}}}$.

4. Omitted.
5. Suppose $\left(\overline{\overline{\mathfrak{C}_{C_1}}} \otimes \overline{\overline{\mathfrak{C}_{C_2}}}\right)^{\overline{\overline{\theta_S}}}$, then

$$\left(\overline{\overline{\mathfrak{C}_{C_1}}} \otimes \overline{\overline{\mathfrak{C}_{C_2}}}\right)^{\overline{\overline{\theta_S}}} = \left(\begin{array}{c} \left(e^{-\left(\left(-\log\left(\overline{\overline{\mathcal{M}_{\overline{\overline{R_1}}}}}\right)\right)^{\psi}+\left(-\log\left(\overline{\overline{\mathcal{M}_{\overline{\overline{R_2}}}}}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right) e^{i2\pi\left(e^{-\left(\left(-\log\left(\overline{\overline{\mathcal{M}_{\overline{\overline{I_1}}}}}\right)\right)^{\psi}+\left(-\log\left(\overline{\overline{\mathcal{M}_{\overline{\overline{I_2}}}}}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right)}, \\ \left(1-e^{-\left(\left(-\log\left(1-\overline{\overline{\mathcal{N}_{\overline{\overline{R_1}}}}}\right)\right)^{\psi}+\left(-\log\left(1-\overline{\overline{\mathcal{N}_{\overline{\overline{R_2}}}}}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right) e^{i2\pi\left(1-e^{-\left(\left(-\log\left(1-\overline{\overline{\mathcal{N}_{\overline{\overline{R_1}}}}}\right)\right)^{\psi}+\left(-\log\left(1-\overline{\overline{\mathcal{N}_{\overline{\overline{R_2}}}}}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right)} \end{array}\right)^{\overline{\overline{\theta_S}}}$$

$$= \left(\begin{array}{c} \left(e^{-\left(\overline{\overline{\theta_S}}\left(-\log\left(\overline{\overline{\mathcal{M}_{\overline{\overline{R_1}}}}}\right)\right)^{\psi}+\left(-\log\left(\overline{\overline{\mathcal{M}_{\overline{\overline{R_2}}}}}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right) e^{i2\pi\left(e^{-\left(\overline{\overline{\theta_S}}\left(-\log\left(\overline{\overline{\mathcal{M}_{\overline{\overline{I_1}}}}}\right)\right)^{\psi}+\left(-\log\left(\overline{\overline{\mathcal{M}_{\overline{\overline{I_2}}}}}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right)}, \\ \left(1-e^{-\left(\overline{\overline{\theta_S}}\left(-\log\left(1-\overline{\overline{\mathcal{N}_{\overline{\overline{R_1}}}}}\right)\right)^{\psi}+\left(-\log\left(1-\overline{\overline{\mathcal{N}_{\overline{\overline{R_2}}}}}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right) e^{i2\pi\left(1-e^{-\left(\overline{\overline{\theta_S}}\left(-\log\left(1-\overline{\overline{\mathcal{N}_{\overline{\overline{R_1}}}}}\right)\right)^{\psi}+\left(-\log\left(1-\overline{\overline{\mathcal{N}_{\overline{\overline{R_2}}}}}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right)} \end{array}\right)$$

$$= \left(\begin{array}{c} \left(e^{-\left(\overline{\overline{\theta_S}}\left(-\log\left(\overline{\overline{\mathcal{M}_{\overline{\overline{R_1}}}}}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right) e^{i2\pi\left(e^{-\left(\overline{\overline{\theta_S}}\left(-\log\left(\overline{\overline{\mathcal{M}_{\overline{\overline{I_1}}}}}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right)}, \\ \left(1-e^{-\left(\overline{\overline{\theta_S}}\left(-\log\left(1-\overline{\overline{\mathcal{N}_{\overline{\overline{R_1}}}}}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right) e^{i2\pi\left(1-e^{-\left(\overline{\overline{\theta_S}}\left(-\log\left(1-\overline{\overline{\mathcal{N}_{\overline{\overline{I_1}}}}}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right)} \end{array}\right) \otimes \left(\begin{array}{c} \left(e^{-\left(\overline{\overline{\theta_S}}\left(-\log\left(\overline{\overline{\mathcal{M}_{\overline{\overline{R_2}}}}}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right) e^{i2\pi\left(e^{-\left(\overline{\overline{\theta_S}}\left(-\log\left(\overline{\overline{\mathcal{M}_{\overline{\overline{I_2}}}}}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right)}, \\ \left(1-e^{-\left(\overline{\overline{\theta_S}}\left(-\log\left(1-\overline{\overline{\mathcal{N}_{\overline{\overline{R_2}}}}}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right) e^{i2\pi\left(1-e^{-\left(\overline{\overline{\theta_S}}\left(-\log\left(1-\overline{\overline{\mathcal{N}_{\overline{\overline{I_2}}}}}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right)} \end{array}\right)$$

$$= \overline{\overline{\mathfrak{C}_{C_1}}}^{\overline{\overline{\theta_S}}} \otimes \overline{\overline{\mathfrak{C}_{C_2}}}^{\overline{\overline{\theta_S}}}$$

Hence, $\left(\overline{\overline{\mathfrak{C}_{C_1}}} \otimes \overline{\overline{\mathfrak{C}_{C_2}}}\right)^{\overline{\overline{\theta_S}}} = \overline{\overline{\mathfrak{C}_{C_1}}}^{\overline{\overline{\theta_S}}} \otimes \overline{\overline{\mathfrak{C}_{C_2}}}^{\overline{\overline{\theta_S}}}$.

6. Omitted. □

Appendix B

Proof. Based on the initiated laws and mathematical induction, we prove Equation (17); first we have a check for $n = 2$, then

$$\overline{\overline{\mathfrak{W}_1 \mathfrak{C}_{C_1}}} = \left(\begin{array}{c} \left(1-e^{-\left(\overline{\overline{\mathfrak{W}_1}}\left(-\log\left(1-\overline{\overline{\mathcal{M}_{\overline{\overline{R_1}}}}}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right) e^{i2\pi\left(1-e^{-\left(\overline{\overline{\mathfrak{W}_1}}\left(-\log\left(1-\overline{\overline{\mathcal{M}_{\overline{\overline{I_1}}}}}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right)}, \\ \left(e^{-\left(\overline{\overline{\mathfrak{W}_1}}\left(-\log\left(\overline{\overline{\mathcal{N}_{\overline{\overline{R_1}}}}}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right) e^{i2\pi\left(e^{-\left(\overline{\overline{\mathfrak{W}_1}}\left(-\log\left(\overline{\overline{\mathcal{N}_{\overline{\overline{I_1}}}}}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right)} \end{array}\right)$$

and

$$\overline{\overline{\mathfrak{W}_2 \mathfrak{C}_{C_2}}} = \left(\begin{array}{c} \left(1-e^{-\left(\overline{\overline{\mathfrak{W}_2}}\left(-\log\left(1-\overline{\overline{\mathcal{M}_{\overline{\overline{R_2}}}}}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right) e^{i2\pi\left(1-e^{-\left(\overline{\overline{\mathfrak{W}_2}}\left(-\log\left(1-\overline{\overline{\mathcal{M}_{\overline{\overline{I_2}}}}}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right)}, \\ \left(e^{-\left(\overline{\overline{\mathfrak{W}_2}}\left(-\log\left(\overline{\overline{\mathcal{N}_{\overline{\overline{R_2}}}}}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right) e^{i2\pi\left(e^{-\left(\overline{\overline{\mathfrak{W}_2}}\left(-\log\left(\overline{\overline{\mathcal{N}_{\overline{\overline{I_2}}}}}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right)} \end{array}\right)$$

then

$$CIFAAWA\left(\overline{\overline{\mathfrak{C}_{C_1}}}, \overline{\overline{\mathfrak{C}_{C_2}}}\right) = \overline{\overline{\mathfrak{W}_1\mathfrak{C}_{C_1}}} \oplus \overline{\overline{\mathfrak{W}_2\mathfrak{C}_{C_2}}}$$

$$= \begin{pmatrix} \left(1 - e^{-\left(\overline{\overline{\mathfrak{W}_1}}\left(-\log\left(1-\overline{\mathcal{M}_{\overline{\overline{R_1}}}}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right) e^{i2\pi\left(1-e^{-\left(\overline{\overline{\mathfrak{W}_1}}\left(-\log\left(1-\overline{\mathcal{M}_{\overline{\overline{I_1}}}}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right)}, \\ \left(e^{-\left(\overline{\overline{\mathfrak{W}_1}}\left(-\log\left(\overline{\mathcal{N}_{\overline{\overline{R_1}}}}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right) e^{i2\pi\left(e^{-\left(\overline{\overline{\mathfrak{W}_1}}\left(-\log\left(\overline{\mathcal{N}_{\overline{\overline{I_1}}}}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right)} \end{pmatrix}$$

$$\oplus \begin{pmatrix} \left(1 - e^{-\left(\overline{\overline{\mathfrak{W}_2}}\left(-\log\left(1-\overline{\mathcal{M}_{\overline{\overline{R_2}}}}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right) e^{i2\pi\left(1-e^{-\left(\overline{\overline{\mathfrak{W}_2}}\left(-\log\left(1-\overline{\mathcal{M}_{\overline{\overline{I_2}}}}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right)}, \\ \left(e^{-\left(\overline{\overline{\mathfrak{W}_2}}\left(-\log\left(\overline{\mathcal{N}_{\overline{\overline{R_2}}}}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right) e^{i2\pi\left(e^{-\left(\overline{\overline{\mathfrak{W}_2}}\left(-\log\left(\overline{\mathcal{N}_{\overline{\overline{I_2}}}}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right)} \end{pmatrix}$$

$$\begin{pmatrix} \left(1 - e^{-\left(\sum_{j=1}^{2}\overline{\overline{\mathfrak{W}_j}}\left(-\log\left(1-\overline{\mathcal{M}_{\overline{\overline{R_j}}}}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right) e^{i2\pi\left(1-e^{-\left(\sum_{j=1}^{2}\overline{\overline{\mathfrak{W}_j}}\left(-\log\left(1-\overline{\mathcal{M}_{\overline{\overline{I_j}}}}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right)}, \\ \left(e^{-\left(\sum_{j=1}^{2}\overline{\overline{\mathfrak{W}_j}}\left(-\log\left(\overline{\mathcal{N}_{\overline{\overline{R_j}}}}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right) e^{i2\pi\left(e^{-\left(\sum_{j=1}^{2}\overline{\overline{\mathfrak{W}_j}}\left(-\log\left(\overline{\mathcal{N}_{\overline{\overline{I_j}}}}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right)} \end{pmatrix}$$

Further, we suggest that if Equation (17) is okay for $n = k$, then

$$CIFAAWA\left(\overline{\overline{\mathfrak{C}_{C_1}}}, \overline{\overline{\mathfrak{C}_{C_2}}}, \ldots, \overline{\overline{\mathfrak{C}_{C_k}}}\right) = \begin{pmatrix} \left(1 - e^{-\left(\sum_{j=1}^{k}\overline{\overline{\mathfrak{W}_j}}\left(-\log\left(1-\overline{\mathcal{M}_{\overline{\overline{R_j}}}}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right) e^{i2\pi\left(1-e^{-\left(\sum_{j=1}^{k}\overline{\overline{\mathfrak{W}_j}}\left(-\log\left(1-\overline{\mathcal{M}_{\overline{\overline{I_j}}}}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right)}, \\ \left(e^{-\left(\sum_{j=1}^{k}\overline{\overline{\mathfrak{W}_j}}\left(-\log\left(\overline{\mathcal{N}_{\overline{\overline{R_j}}}}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right) e^{i2\pi\left(e^{-\left(\sum_{j=1}^{k}\overline{\overline{\mathfrak{W}_j}}\left(-\log\left(\overline{\mathcal{N}_{\overline{\overline{I_j}}}}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right)} \end{pmatrix}$$

Then, we will prove that Equation (17) is also held for $n = k + 1$, we have

$$CIFAAWA\left(\overline{\overline{\mathfrak{C}_{C_1}}},\overline{\overline{\mathfrak{C}_{C_2}}},\ldots,\overline{\overline{\mathfrak{C}_{C_{k+1}}}}\right)=\oplus_{j=1}^{k+1}\left(\overline{\overline{\mathfrak{W}_j\mathfrak{C}_{C_j}}}\right)=\oplus_{j=1}^{k}\left(\overline{\overline{\mathfrak{W}_j\mathfrak{C}_{C_j}}}\right)\oplus\overset{=}{\mathfrak{W}}_{k+1}\overline{\overline{\mathfrak{C}_{C_{k+1}}}}$$

$$=\left(\begin{array}{c}\left(1-e^{-\left(\sum_{j=1}^{k}\overline{\overline{\mathfrak{W}_j}}\left(-\log\left(1-\overline{\overline{\mathcal{M}_{\overline{\overline{R}}_j}}}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right)e^{i2\pi\left(1-e^{-\left(\sum_{j=1}^{k}\overline{\overline{\mathfrak{W}_j}}\left(-\log\left(1-\overline{\overline{\mathcal{M}_{\overline{\overline{I}}_j}}}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right)},\\ \left(e^{-\left(\sum_{j=1}^{k}\overline{\overline{\mathfrak{W}_j}}\left(-\log\left(\overline{\overline{\mathcal{N}_{\overline{\overline{R}}_j}}}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right)e^{i2\pi\left(e^{-\left(\sum_{j=1}^{k}\overline{\overline{\mathfrak{W}_j}}\left(-\log\left(\overline{\overline{\mathcal{N}_{\overline{\overline{I}}_j}}}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right)}\end{array}\right)$$

$$\oplus\left(\begin{array}{c}\left(1-e^{-\left(\overset{=}{\mathfrak{W}}_{k+1}\left(-\log\left(1-\overset{=}{\mathcal{M}}_{\overset{=}{R}_{k+1}}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right)e^{i2\pi\left(1-e^{-\left(\overset{=}{\mathfrak{W}}_{k+1}\left(-\log\left(1-\overset{=}{\mathcal{M}}_{\overset{=}{I}_{k+1}}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right)},\\ \left(e^{-\left(\overset{=}{\mathfrak{W}}_{k+1}\left(-\log\left(\overset{=}{\mathcal{N}}_{\overset{=}{R}_{k+1}}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right)e^{i2\pi\left(e^{-\left(\overset{=}{\mathfrak{W}}_{k+1}\left(-\log\left(\overset{=}{\mathcal{N}}_{\overset{=}{I}_{k+1}}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right)}\end{array}\right)$$

$$=\left(\begin{array}{c}\left(1-e^{-\left(\sum_{j=1}^{k+1}\overline{\overline{\mathfrak{W}_j}}\left(-\log\left(1-\overline{\overline{\mathcal{M}_{\overline{\overline{R}}_j}}}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right)e^{i2\pi\left(1-e^{-\left(\sum_{j=1}^{k+1}\overline{\overline{\mathfrak{W}_j}}\left(-\log\left(1-\overline{\overline{\mathcal{M}_{\overline{\overline{I}}_j}}}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right)},\\ \left(e^{-\left(\sum_{j=1}^{k+1}\overline{\overline{\mathfrak{W}_j}}\left(-\log\left(\overline{\overline{\mathcal{N}_{\overline{\overline{R}}_j}}}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right)e^{i2\pi\left(e^{-\left(\sum_{j=1}^{k+1}\overline{\overline{\mathfrak{W}_j}}\left(-\log\left(\overline{\overline{\mathcal{N}_{\overline{\overline{I}}_j}}}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right)}\end{array}\right)$$

Hence, the result is investigated correctly. □

Appendix C

Proof. Suppose $\overline{\overline{\mathfrak{C}_{C_j}}}=\overline{\overline{\mathfrak{C}}}=\left(\overline{\overline{\mathcal{M}_{\overline{\overline{R}}}}}e^{i2\pi\left(\overline{\overline{\mathcal{M}_{\overline{\overline{I}}}}}\right)},\overline{\overline{\mathcal{N}_{\overline{\overline{R}}}}}e^{i2\pi\left(\overline{\overline{\mathcal{N}_{\overline{\overline{I}}}}}\right)}\right)$, then

$$CIFAAWA\left(\overline{\overline{\mathfrak{C}_{C_1}}},\overline{\overline{\mathfrak{C}_{C_2}}},\ldots,\overline{\overline{\mathfrak{C}_{C_n}}}\right)=\left(\begin{array}{c}\left(1-e^{-\left(\sum_{j=1}^{n}\overline{\overline{\mathfrak{W}_j}}\left(-\log\left(1-\overline{\overline{\mathcal{M}_{\overline{\overline{R}}_j}}}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right)e^{i2\pi\left(1-e^{-\left(\sum_{j=1}^{n}\overline{\overline{\mathfrak{W}_j}}\left(-\log\left(1-\overline{\overline{\mathcal{M}_{\overline{\overline{I}}_j}}}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right)},\\ \left(e^{-\left(\sum_{j=1}^{n}\overline{\overline{\mathfrak{W}_j}}\left(-\log\left(\overline{\overline{\mathcal{N}_{\overline{\overline{R}}_j}}}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right)e^{i2\pi\left(e^{-\left(\sum_{j=1}^{n}\overline{\overline{\mathfrak{W}_j}}\left(-\log\left(\overline{\overline{\mathcal{N}_{\overline{\overline{I}}_j}}}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right)}\end{array}\right)$$

$$=\left(\begin{array}{c}\left(1-e^{-\left(\left(-\log\left(1-\overline{\overline{\mathcal{M}_{\overline{\overline{R}}}}}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right)e^{i2\pi\left(1-e^{-\left(\left(-\log\left(1-\overline{\overline{\mathcal{M}_{\overline{\overline{I}}}}}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right)},\\ \left(e^{-\left(\left(-\log\left(\overline{\overline{\mathcal{N}_{\overline{\overline{R}}}}}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right)e^{i2\pi\left(e^{-\left(\left(-\log\left(\overline{\overline{\mathcal{N}_{\overline{\overline{I}}}}}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right)}\end{array}\right)$$

$$=\left(\begin{array}{c}\left(1-e^{\log\left(1-\overline{\overline{\mathcal{M}_{\overline{\overline{R}}}}}\right)}\right)e^{i2\pi\left(1-e^{\log\left(1-\overline{\overline{\mathcal{M}_{\overline{\overline{I}}}}}\right)}\right)},\\ \left(e^{\log\left(\overline{\overline{\mathcal{N}_{\overline{\overline{R}}}}}\right)}\right)e^{i2\pi\left(e^{\log\left(\overline{\overline{\mathcal{N}_{\overline{\overline{I}}}}}\right)}\right)}\end{array}\right)=\left(\overline{\overline{\mathcal{M}_{\overline{\overline{R}}}}}e^{i2\pi\left(\overline{\overline{\mathcal{M}_{\overline{\overline{I}}}}}\right)},\overline{\overline{\mathcal{N}_{\overline{\overline{R}}}}}e^{i2\pi\left(\overline{\overline{\mathcal{N}_{\overline{\overline{I}}}}}\right)}\right).$$

□

Appendix D

Proof. Suppose $\overline{\overline{\mathfrak{C}_{C_j}}} = \left(\overline{\overline{\mathcal{M}}}_{\overline{\overline{R}}_j} e^{i2\pi(\overline{\overline{\mathcal{M}}}_{\overline{\overline{I}}_j})}, \overline{\overline{\mathcal{N}}}_{\overline{\overline{R}}_j} e^{i2\pi(\overline{\overline{\mathcal{N}}}_{\overline{\overline{I}}_j})}\right), j = 1, 2, \ldots, n,$ If $\overline{\overline{\mathfrak{C}}}^{-} = \left(\min_j \overline{\overline{\mathcal{M}}}_{\overline{\overline{R}}_j} e^{i2\pi(\min_j \overline{\overline{\mathcal{M}}}_{\overline{\overline{I}}_j})}, \max_j \overline{\overline{\mathcal{N}}}_{\overline{\overline{R}}_j} e^{i2\pi(\max_j \overline{\overline{\mathcal{N}}}_{\overline{\overline{I}}_j})}\right)$ and $\overline{\overline{\mathfrak{C}}}^{+} = \left(\max_j \overline{\overline{\mathcal{M}}}_{\overline{\overline{R}}_j} e^{i2\pi(\max_j \overline{\overline{\mathcal{M}}}_{\overline{\overline{I}}_j})}, \min_j \overline{\overline{\mathcal{N}}}_{\overline{\overline{R}}_j} e^{i2\pi(\min_j \overline{\overline{\mathcal{N}}}_{\overline{\overline{I}}_j})}\right)$, by using inequality, we have

$$1 - e^{-\left(\sum_{j=1}^{n} \overline{\overline{\mathfrak{W}}}_j \left(-\log\left(1 - \overline{\overline{\mathcal{M}}}_{\overline{\overline{R}}_j}^{-}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}} \leq 1 - e^{-\left(\sum_{j=1}^{n} \overline{\overline{\mathfrak{W}}}_j \left(-\log\left(1 - \overline{\overline{\mathcal{M}}}_{\overline{\overline{R}}_j}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}} \leq 1 - e^{-\left(\sum_{j=1}^{n} \overline{\overline{\mathfrak{W}}}_j \left(-\log\left(1 - \overline{\overline{\mathcal{M}}}_{\overline{\overline{R}}_j}^{+}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}$$

$$1 - e^{-\left(\sum_{j=1}^{n} \overline{\overline{\mathfrak{W}}}_j \left(-\log\left(1 - \overline{\overline{\mathcal{M}}}_{\overline{\overline{I}}_j}^{-}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}} \leq 1 - e^{-\left(\sum_{j=1}^{n} \overline{\overline{\mathfrak{W}}}_j \left(-\log\left(1 - \overline{\overline{\mathcal{M}}}_{\overline{\overline{I}}_j}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}} \leq 1 - e^{-\left(\sum_{j=1}^{n} \overline{\overline{\mathfrak{W}}}_j \left(-\log\left(1 - \overline{\overline{\mathcal{M}}}_{\overline{\overline{I}}_j}^{+}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}$$

$$e^{-\left(\sum_{j=1}^{n} \overline{\overline{\mathfrak{W}}}_j \left(-\log\left(\overline{\overline{\mathcal{M}}}_{\overline{\overline{R}}_j}^{-}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}} \leq e^{-\left(\sum_{j=1}^{n} \overline{\overline{\mathfrak{W}}}_j \left(-\log\left(\overline{\overline{\mathcal{M}}}_{\overline{\overline{R}}_j}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}} \leq e^{-\left(\sum_{j=1}^{n} \overline{\overline{\mathfrak{W}}}_j \left(-\log\left(\overline{\overline{\mathcal{M}}}_{\overline{\overline{R}}_j}^{+}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}$$

$$e^{-\left(\sum_{j=1}^{n} \overline{\overline{\mathfrak{W}}}_j \left(-\log\left(\overline{\overline{\mathcal{M}}}_{\overline{\overline{I}}_j}^{-}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}} \leq e^{-\left(\sum_{j=1}^{n} \overline{\overline{\mathfrak{W}}}_j \left(-\log\left(\overline{\overline{\mathcal{M}}}_{\overline{\overline{I}}_j}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}} \leq e^{-\left(\sum_{j=1}^{n} \overline{\overline{\mathfrak{W}}}_j \left(-\log\left(\overline{\overline{\mathcal{M}}}_{\overline{\overline{I}}_j}^{+}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}$$

Therefore, we obtained

$$\overline{\overline{\mathfrak{C}}}^{-} \leq CIFAAWA\left(\overline{\overline{\mathfrak{C}_{C_1}}}, \overline{\overline{\mathfrak{C}_{C_2}}}, \ldots, \overline{\overline{\mathfrak{C}_{C_n}}}\right) \leq \overline{\overline{\mathfrak{C}}}^{+}.$$

□

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