symmetry

MDPI

Article

Fermatean Neutrosophic Topological Spaces and an Application of Neutrosophic Kano Method

Nazmiye Gonul Bilgin [1,*], Dragan Pamučar [2] and Muhammad Riaz [3]

[1] Department of Mathematics, Zonguldak Bulent Ecevit University, Zonguldak 67100, Turkey
[2] Department of Operations Research and Statistics, Faculty of Organizational Sciences, University of Belgrade, 11000 Belgrade, Serbia
[3] Department of Mathematics, University of the Punjab, Lahore 54590, Pakistan
* Correspondence: nazmiyegonul@beun.edu.tr

Abstract: The main objective of this paper is to redefine the concept of Fermatean neutrosophic sets as well as to introduce topological structure on Fermatean neutrosophic sets. The idea of Fermatean neutrosophic sets is the hybrid model of Fermatean fuzzy sets and neutrosophic sets to utilize key features of these structures. Topological data analysis for indeterminate and uncertain information is a rapidly developing field. Motivated by this recent trend, the idea of Fermatean neutrosophic topology is proposed, which is an extension of neutrosophic topology and Fermatean fuzzy topology. Some fundamental properties of Fermatean neutrosophic topology are explored and related results are investigated. Certain properties provided in the classical topological space that may not be valid in this space is one of the factors that makes the study important. Moreover, an application is made for the problem of seeking reasonable solutions to customer expectations by using the neutrosophic Kano method, which is an interesting illustration of neutrosophic decision making.

Keywords: Fermatean set; neutrosophic topological spaces; Fermatean neutrosophic set

MSC: 54F65; 03E75; 54D99

Citation: Gonul Bilgin, N.; Pamučar, D.; Riaz, M. Fermatean Neutrosophic Topological Spaces and an Application of Neutrosophic Kano Method. *Symmetry* **2022**, *14*, 2442. https://doi.org/10.3390/sym14112442

Academic Editor: José Carlos R. Alcantud

Received: 15 October 2022
Accepted: 9 November 2022
Published: 17 November 2022

1. Introduction

The subject of symmetry, which is important for many areas of mathematics, has recently begun to be transferred to the neutrosophic structure, fuzzy sets and intuitionistic fuzzy sets. Decision-making, which combines mathematics with other areas, is an interesting field of study in which concepts in mathematics are applied. In multi-criteria decision-making (MCDM) problems, incomplete and unclear information in the available data have revealed the variable of approximate reasoning and pushed the decision-making process to new developments and innovation. Various MCDM techniques have been developed to deal with indeterminate and vague information, and new, theoretical studies have been carried out [1–4]. Recent developments in MCDM methods have been studied by various researchers for different extensions of fuzzy sets [5–8].

Smarandache [9] introduced the concept of neutrosophic structure, which enables new beginnings in decision making and in other fields. This structure is named neutrosophic sets (NSs) and argues that every concept with a certain degree of accuracy contains inaccuracy and indeterminacy. For this purpose, Smarandache extended concepts of intuitionistic fuzzy sets to a deeper understanding and interpretation in terms of truthfulness, indeterminacy and falsity. Membership and non-membership are dependent components in intuitionistic fuzzy sets. Smarandache brought flexibility to this situation in neutrosophic sets by considering that the three components of NSs are independent. There are a lot of applications of this concept in many areas such as engineering and philosophy.

In this regard, it is inevitable to evaluate the decision-making process together with neutrosophic logic, in which the components can be characterized as dependent or independent. Especially, studies dealing with dependent and independent variables in an unusual way make a great contribution to the related fields. NS is a strong model that brings a new perspective to strategic development, decision-making and analysis in many areas of daily life, and substantial studies have emerged in many fields [10–17].

The most important field in which NS are examined is topology and topological spaces. Neutrosophic topological spaces have been initiated due to the inadequacies in fuzzy and intuitionistic fuzzy concepts given in [18–20]. In 2020, Senapati and Yager established a new extension of fuzzy sets named Fermatean fuzzy sets [21]. Some important studies on Fermatean fuzzy sets for MCDM problems have been conducted by various researchers [22–25]. Recently, Ibrahim defined Fermatean fuzzy topological spaces in [26]. Fermatean fuzzy topological space and some important identities for the closure and interior of sets and the neighborhoods were investigated in [26]. Certain novel concepts of Pythagorean fuzzy topological space were introduced by Olgun et al. [27]. Antony et al. [28] proposed Fermatean neutrosophic sets [28]. Arokiarani et al. [29] studied functions on NS topological spaces. Ajay [30] studied alpha-open sets, and continuity was defined on Pythagorean fuzzy topological space. Iswarya et al. [31] studied neutrosophic frontier, neutrosophic semi-frontier and neutrosophic topological space with the idea of pre-alpha and pre-beta irresolute open mapping defined in [32]. Fuzzy strongly alpha-irresolute maps were studied on fuzzy topological spaces in [33]. Extensions of fuzzy sets to uncertain real-life circumstances have been examined by many researchers [33–39]. The concept of neutrosophic sets and their components have interesting applications in decision-making problems [40–44].

Neutrosophic set structures have a large number of applications in various fields such as medical diagnosis, information analysis, artificial intelligence and image processing. In all these fields, the main objectives are to seek drawbacks of existing decision-making processes and develop new MCDM methods. The NS concept is also a strong model for economics and business management. Overcoming the economic crisis in many countries can be achieved by the correct management of decision-making processes. The correct use of money is crucial for individual economies in the periods of pandemic crisis in the world. This requires the development of the right strategies from the establishment for businesses working in every sector. Choosing the place where the business will be established and its size are among the strategic decisions that are effective in increasing competitiveness. This situation becomes even more important in companies and businesses working in one-on-one relationships with customers. It is predicted that customers prefer businesses that can meet a certain standard and meet many criteria apart from their price expectations. Considering the asset prospects of a business, it may not always be possible to meet all of the customers' expected criteria. However, it is necessary for the enterprise to meet these expectations to the highest extent in terms of competition and survival. In this respect, it is important to present the situation correctly as a decision problem.

For this purpose, one can start by mentioning the reason for choosing the method to be applied for the practical part of our study. The decision-making process can be grouped according to the characteristics of the decision makers, the number of determinants and the information topology. The importance of the method used for the correct functioning of the process is obvious. The number of techniques related to the multivariate decision-making process increases day by day.

The Kano model, which is a clustering method, has been used by different researchers in decision-making processes in many different fields, especially in the service sector. Wang and Wang used a fuzzy Kano model while taking into account customer perceptions when deciding on the key features that should be in new product design in [44]. However, there are hardly any studies in which the Kano model is used in neutrosophic decision-making processes in the literature. In [45], Egilmez sought a solution to the supplier selection problem of a raw material supplier company by combining the neutrosophic approach and the Kano method. She defined the neutrosophic Kano model in her paper, which was a study that synthesized the Kano model and multi-criteria decision making. Garg et al. [46] introduced spherical fuzzy soft topology and Kausar et al. [47] proposed LAM and SIR methods for topological data analysis of m-polar spherical fuzzy information. This manuscript has multiple objectives that are described in the following paragraphs.

The idea of Fermatean neutrosophic sets is to form a robust combination of Fermatean fuzzy sets and neutrosophic sets to utilize their key features. The Fermatean fuzzy set is redefined to analyze the importance of product features offered to its customers by a restaurant serving local food. The Fermatean set structure will be examined in such a way that the indecision and uncertainty of the human thinking mechanism, the product features being important for the customer, are not ignored while the restaurant is presenting the product. We assume that arising customer requirements in this direction are the quality, naturalness, freshness, taste and presentation style of the products. This area was chosen because the indecision encountered from people when making judgmental decisions can be modeled with neutrosophic set theory.

In this work, the suitability of the neutrosophic Kano method to the Fermatean sets is examined in a decision-making process based on customer satisfaction. This method was preferred in order to better understand customer expectations. During the determination of the criteria that represent customer expectations, it is of great importance to examine the customer interviews and literature on the situation. However, it is not possible to include all these criteria in the decision-making process. Nevertheless, this process should be managed by using the largest number of variables that will allow correct decision-making.

The main objective of this manuscript is to extend the concept of Fermatean neutrosophic sets to introduce the idea of Fermatean neutrosophic topology. Topological data analysis for indeterminate and uncertain information is a rapidly developing field. Motivated by this recent trend, the idea of Fermatean neutrosophic topology is proposed, which is an extension of neutrosophic topology and Fermatean fuzzy topology. The basic properties of topology such as Fermatean neutrosophic interior and closure are determined and related concepts such as Fermatean neutrosophic pre-open and semi-open sets are given.

This manuscript is arranged as follows. First, we review some existing sets and models including Fermatean fuzzy sets [21], neutrosophic sets [9], neutrosophic topological spaces [19,20] and Fermatean neutrosophic sets [28]. These studies are essential to develop novel concepts in this manuscript. In Section 2, the concepts of Fermatean neutrosophic sets and topological structure on Fermatean neutrosophic sets are proposed. In Section 3, the idea of Fermatean neutrosophic continuity is explored. In Section 4, an application of the neutrosophic Kano method is presented. In Section 5, a brief discussion about Fermatean neutrosophic sets and their topological structure is presented. The conclusion of this manuscript is given in Section 6.

Definition 1. *(Fermatean fuzzy set)* [21]. *Let $\mathfrak{U} \neq \varnothing$ and $I = [0,1]$. A Fermatean fuzzy set A_f has the form $A_f = \{(s, \varphi_A(s), \psi_A(s)) : s \in \mathfrak{U}\}$. Here, φ_A, $\psi_A : \mathfrak{U} \to [0,1]$ demonstrate the grade of membership and non-membership of all $s \in \mathfrak{U}$ to A_f; additionally, for every $s \in \mathfrak{U}$, $0 \leq {\varphi_A}^3(s) + {\psi_A}^3(s) \leq 1$.*

Definition 2. *(Neutrosophic set)* [9]. *Let $I = [0,1]$ and $\mathfrak{U} \neq \varnothing$. A Neutrosophic set A_f has the notation $A_f = \{(s, \varphi_A(s), \psi_A(s), \zeta_A(s)) : s \in \mathfrak{U}\}$. Here, φ_A, $\zeta_A, \psi_A : \mathfrak{U} \to [0,1]$ demonstrate the grade of membership, indeterminacy and non-membership of all $s \in \mathfrak{U}$ to A_f; further, for each $s \in \mathfrak{U}$, $0 \leq \varphi_A(s) + \psi_A(s) + \zeta_A(s) \leq 1$.*

Definition 3. (*Neutrosophic Topological Spaces)* [19,20]. *Let $\mathfrak{U} \neq \varnothing$, τ_N be a collection of neutrosophic subsets of $\mathfrak{U}$ and τ_N satisfy the next properties; then, it is a neutrosophic topology.*

$$(\tau_N 1)\ \ddot{0},\ \ddot{1} \in \tau_N,$$

$$(\tau_N 2)\quad D_1, D_2 \in \tau_N, (D_1 \cap D_2) \in \tau_N,$$

$$(\tau_N 3) \quad i \in \mathbb{N},\ \{D_i\} \in \tau_N, \text{then}(\cup D_i) \in \tau_N$$

Definition 4. *(Fermatean Neutrosophic set)* [28]. *Let $\mathcal{Y} \neq \varnothing$. A Fermatean neutrosophic set $A_f = \{(s, \varphi_A(s), \psi_A(s), \zeta_A(s)) : s \in \mathcal{Y}\}$, where φ_A, $\psi_A, \zeta_A : \mathcal{Y} \to [0,1]$ and, for each $s \in \mathcal{Y}$, $0 \leq {\varphi_A}^3(s) + {\psi_A}^3(s) \leq 1$ and $0 \leq {\zeta_A}^3(s) \leq 1$, then for each $s \in \mathcal{Y}$, $0 \leq {\varphi_A}^3(s) + {\psi_A}^3(s) + {\zeta_A}^3(s) \leq 2$. $\zeta_A(s)$ is an independent component; $\varphi_A(s)$ and $\psi_A(s)$ are dependent components.*

2. Materials and Methods

Before the current neutrosophic structure was established, researchers attempted to generalize the concept of intuitionistic fuzzy sets and establish their structure. However, this approach was not widely accepted due to the fact that the states of belonging, not belonging and uncertainty in daily life could change independently of each other. Besides, problems caused by imprecise and incomplete information can be eliminated more effectively with Fermatean neutrosophic sets than with intuitionistic fuzzy and Pythagorean neutrosophic sets. Moreover, as a result of taking partially dependent components in the definition given in [26], the intuitionistic fuzzy structure could not be moved far away, and the flexibility in the neutrosophic structure decreased. Moreover, this inadequacy may continue in the application of addition and multiplication operations and in the results to be obtained.

So, first of all, the Fermatean neutrosophic definition that will be used for the construction of the topological spaces definition will be modified and re-expressed.

A Fermatean Neutrosophic Set has three components: membership, non-membership and indeterminacy. Membership and non-membership are dependent components but indeterminacy is an independent component. The sum of the cubes of membership and non-membership is less than 1 to satisfy the Fermatean set property. Then, the sum of the cubes of membership, non-membership and indeterminacy is less than or equal to 2.

Then, the definition of topology using the Fermatean neutrosophic set structure and important theorems using this definition will be given.

Definition 5. *Let $\mathfrak{U} \neq \varnothing$ and $I = [0,1]$. A Fermatean neutrosophic set $\dddot{A}_{fn}$ has the form*

$$\dddot{A}_{fn} = \left\{ \left(\mathfrak{r}, \varphi_{\dddot{A}_{fn}}(\mathfrak{r}), \psi_{\dddot{A}_{fn}}(\mathfrak{r}), \zeta_{\dddot{A}_{fn}}(\mathfrak{r}) \right) : \mathfrak{r} \in \mathfrak{U} \right\},$$

where $\zeta_{\dddot{A}_{fn}}$, $\varphi_{\dddot{A}_{fn}}, \psi_{\dddot{A}_{fn}} : \mathfrak{U} \to I$ demonstrate the degree of indeterminacy, membership and non-membership of all $\mathfrak{r} \in \mathfrak{U}$ to $\dddot{A}_{fn}$, such that membership and non-membership are dependent components and indeterminacy is an independent component for all $\mathfrak{r} \in \mathfrak{U}$, $0 \leq {\varphi_{\dddot{A}_{fn}}}^3(\mathfrak{r}) + {\psi_{\dddot{A}_{fn}}}^3(\mathfrak{r}) \leq 1$, and for all $\mathfrak{r} \in \mathfrak{U}$, such that

$$0 \leq {\varphi_{\dddot{A}_{fn}}}^3(\mathfrak{r}) + {\psi_{\dddot{A}_{fn}}}^3(\mathfrak{r}) + {\zeta_{\dddot{A}_{fn}}}^3(\mathfrak{r}) \leq 2.$$

Example 1. *Let $\mathfrak{U} = \{\mho_1, \mho_2, \mho_3\}$ and $I = [0,1]$. $\dddot{A}_{fn}$ is a Fermatean neutrosophic set that can be written as*

$$\begin{aligned} \dddot{A}_{fn} &= \left\{ \begin{array}{c} \left(\mho_1, \varphi_{\dddot{A}_{fn}}(\mho_1), \psi_{\dddot{A}_{fn}}(\mho_1), \zeta_{\dddot{A}_{fn}}(\mho_1)\right), \left(\mho_2, \varphi_{\dddot{A}_{fn}}(\mho_2), \psi_{\dddot{A}_{fn}}(\mho_2), \zeta_{\dddot{A}_{fn}}(\mho_2)\right), \\ \left(\mho_2, \varphi_{\dddot{A}_{fn}}(\mho_2), \psi_{\dddot{A}_{fn}}(\mho_2), \zeta_{\dddot{A}_{fn}}(\mho_2)\right) \end{array} \right\} \\ &= \{(\mho_1, 0.8,\ 0.4,\ 0.1), (\mho_2, 0.99,\ 0.99,\ 0.99), (\mho_3, 0,\ 1,\ 0.4)\} \end{aligned}$$

Here, for all $\mho_\mho \in \{\mho_1, \mho_2, \mho_3\}$, $0 \leq {\varphi_{\dddot{A}_{fn}}}^3(\mho_\mho) + {\psi_{\dddot{A}_{fn}}}^3(\mho_\mho) \leq 1$, $0 \leq {\varphi_{\dddot{A}_{fn}}}^3(\mho_\mho) + {\psi_{\dddot{A}_{fn}}}^3(\mho_\mho) + {\zeta_{\dddot{A}_{fn}}}^3(\mho_\mho) \leq 2$.

The definitions of $\ddot{1}_{fn}$ and $\ddot{0}_{fn}$ that will be needed before proceeding to set operations will be given. In [6], possible definitions of $\ddot{1}_{fn}$ and $\ddot{0}_{fn}$ neutrosophic sets are given. In this paper, the theory will be constructed by defining $\ddot{0}_{fn}$ and $\ddot{1}_{fn}$ Fermatean neutrosophic sets in a single way. $\ddot{0}_{fn}$ and $\ddot{1}_{fn}$ are defined as

$$\ddot{0}_{fn} = \{(s, 0, 0, 1) : s \in \mathfrak{U}\} \text{ and } \ddot{1}_{fn} = \{(s, 1, 1, 0) : s \in \mathfrak{U}\}. \quad (1)$$

Now, the union, intersection and complement definitions necessary for the definition of the topological space will be given. These definitions are given in several different ways in classical neutrosophic spaces in [18]; to avoid confusion here, only one method will be given for sets with Fermatean structure, and this method is different from the method chosen in [19].

Definition 6. *For Fermatean neutrosophic sets* $\ddot{A}_{fn}$ *and* $\ddot{B}_{fn}$, $\ddot{K}_{fn} = \ddot{A}_{fn} \ddot{\cup} \ddot{B}_{fn}$, $\ddot{L}_{fn} = \ddot{A}_{fn} \ddot{\cap} \ddot{B}_{fn}$ *and* $\ddot{M}_{fn} = \complement \ddot{A}_{fn}$ *is defined as*

$$\ddot{K}_{fn} = \left\{ \left(s,\ max\left\{ \varphi_{\ddot{A}_{fn}}(s), \varphi_{\ddot{B}_{fn}}(s), \right\},\ max\left\{ \psi_{\ddot{A}_{fn}}(s), \psi_{\ddot{B}_{fn}}(s), \right\}\ min\left\{ \zeta_{\ddot{A}_{fn}}(s), \zeta_{\ddot{B}_{fn}}(s), \right\} \right) \right\},$$

$$\ddot{L}_{fn} = \left\{ \left(s,\ min\left\{ \varphi_{\ddot{A}_{fn}}(s), \varphi_{\ddot{B}_{fn}}(s), \right\},\ min\left\{ \psi_{\ddot{A}_{fn}}(s), \psi_{\ddot{B}_{fn}}(s), \right\}\ max\left\{ \zeta_{\ddot{A}_{fn}}(s), \zeta_{\ddot{B}_{fn}}(s), \right\} \right) \right\}, \quad (2)$$

$$\ddot{M}_{fn} = \left\{ \left(\mathfrak{z}, 1 - \varphi_{\ddot{A}_{fn}}(\mathfrak{z}), 1 - \psi_{\ddot{A}_{fn}}(\mathfrak{z}), 1 - \zeta_{\ddot{A}_{fn}}(\mathfrak{z}) \right) : \mathfrak{z} \in \mathfrak{U} \right\}. \quad (3)$$

Definition 7. *Let* $\mathfrak{U} \neq \varnothing$ *and* $\ddot{A}_{fn}$, $\ddot{B}_{fn}$ *be Fermatean neutrosophic subsets in* $\mathfrak{U}$ *with the notation* $\ddot{A}_{fn} = \left\{ \left(\mathfrak{z}, \varphi_{\ddot{A}_{fn}}(\mathfrak{z}), \psi_{\ddot{A}_{fn}}(\mathfrak{z}), \zeta_{\ddot{A}_{fn}}(\mathfrak{z}) \right) : \mathfrak{z} \in \mathfrak{U} \right\}$ *and* $\ddot{B}_{fn} = \left\{ \left(\mathfrak{z}, \varphi_{\ddot{B}_{fn}}(\mathfrak{z}), \psi_{\ddot{B}_{fn}}(\mathfrak{z}), \zeta_{\ddot{B}_{fn}}(\mathfrak{z}) \right) : \mathfrak{z} \in \mathfrak{U} \right\}$. *If* $\varphi_{\ddot{A}_{fn}}(\mathfrak{z}) \leq \varphi_{\ddot{B}_{fn}}(\mathfrak{z})$, $\psi_{\ddot{B}_{fn}}(\mathfrak{z}) \geq \psi_{\ddot{A}_{fn}}(\mathfrak{z})$ *and* $\zeta_{\ddot{B}_{fn}}(\mathfrak{z}) \leq \zeta_{\ddot{A}_{fn}}(\mathfrak{z})$, *then it is denoted with* $\ddot{A}_{fn} \subseteq \ddot{B}_{fn}$.

Definition 8. *Let* $\mathfrak{U}_1, \mathfrak{U}_2 \neq \varnothing$, $\mathfrak{f}_{fn} : \mathfrak{U}_1 \to \mathfrak{U}_2$ *be a function and let* $\mathfrak{V}, \mathfrak{Y}$ *be Fermatean neutrosophic sets* $\mathfrak{V} \subseteq \mathfrak{U}_1$ *and* $\mathfrak{Y} \subseteq \mathfrak{U}_2$. *The grade of membership, non-membership and indeterminacy of image of* $\mathfrak{V}$ in accordance with $\mathfrak{f}_{fn}$, *demonstrated with* $\mathfrak{f}_{fn}(\mathfrak{V})$, *is defined by*

$$\varphi_{\ddot{A}_{fn},\ \mathfrak{f}_{fn}(\mathfrak{V})}(\mathfrak{u}_2) = \begin{cases} \sup\limits_{\mathfrak{u} \in \mathfrak{f}^{-1}{}_{fn}(\mathfrak{u}_2)} \varphi_{\ddot{A}_{fn}}(\mathfrak{u}), & if\ \mathfrak{f}^{-1}{}_{fn}(\mathfrak{u}_2) \neq \varnothing \\ 0, & if\ \mathfrak{f}^{-1}{}_{fn}(\mathfrak{u}_2) = \varnothing \end{cases} \quad (4)$$

$$\psi_{\ddot{A}_{fn},\ \mathfrak{f}_{fn}(\mathfrak{V})}(\mathfrak{u}_2) = \begin{cases} \inf\limits_{\mathfrak{u} \in \mathfrak{f}^{-1}{}_{fn}(\mathfrak{u}_2)} \psi_{\ddot{A}_{fn}}(\mathfrak{u}), & if\ \mathfrak{f}^{-1}{}_{fn}(\mathfrak{u}_2) \neq \varnothing \\ 1, & if\ \mathfrak{f}^{-1}{}_{fn}(\mathfrak{u}_2) = \varnothing \end{cases} \quad (5)$$

and

$$\zeta_{\ddot{A}_{fn},\ \mathfrak{f}_{fn}(\mathfrak{V})}(\mathfrak{u}_2) = \begin{cases} \inf\limits_{\mathfrak{u} \in \mathfrak{f}^{-1}{}_{fn}(\mathfrak{u}_2)} \zeta_{\ddot{A}_{fn}}(\mathfrak{u}), & if\ \mathfrak{f}^{-1}{}_{fn}(\mathfrak{u}_2) \neq \varnothing \\ 1, & if\ \mathfrak{f}^{-1}{}_{fn}(\mathfrak{u}_2) = \varnothing \end{cases} \quad (6)$$

Here, $\mathfrak{f}_{fn}(\mathfrak{V})$ is a *Fermatean neutrosophic subset. The degree of membership, non-membership and indeterminacy of the pre-image of* $\mathfrak{Y}$ *according to* $\mathfrak{f}_{fn}$ *demonstrated with* $\mathfrak{f}^{-1}{}_{fn}(\mathfrak{Y})$ *is defined by*

$$\varphi_{\ddot{A}_{fn},\ \mathfrak{f}^{-1}{}_{fn}(\mathfrak{Y})}(\mathfrak{u}_1) = \varphi_{\ddot{A}_{fn},\ \mathfrak{Y}}\left(\mathfrak{f}_{fn}(\mathfrak{u}_1) \right),\ \psi_{\ddot{A}_{fn},\ \mathfrak{f}^{-1}{}_{fn}(\mathfrak{Y})}(\mathfrak{u}_1) = \psi_{\ddot{A}_{fn},\ \mathfrak{Y}}\left(\mathfrak{f}_{fn}(\mathfrak{u}_1) \right) \quad (7)$$

and

$$\zeta_{\ddot{A}_{fn},\ \mathfrak{f}^{-1}{}_{fn}(\mathfrak{Y})}(\mathfrak{u}_1) = \zeta_{\ddot{A}_{fn},\ \mathfrak{Y}}\left(\mathfrak{f}_{fn}(\mathfrak{u}_1) \right) \quad (8)$$

At the same time, the set $\mathfrak{f}^{-1}{}_{fn}(\mathfrak{Y})$ is also a Fermatean neutrosophic subset.

Lemma 1. *Let $\mathfrak{U}_1$, $\mathfrak{U}_2 \neq \varnothing$, $\mathfrak{f}_{fn} : \mathfrak{U}_1 \to \mathfrak{U}_2$ be a function and let $\mathfrak{Y}_1, \mathfrak{Y}_2 \subseteq \mathfrak{U}_2$ be Fermatean neutrosophic sets, $\mathfrak{Y}_1 \subseteq \mathfrak{Y}_2$. Then, $\mathfrak{f}^{-1}{}_{fn}(\mathfrak{Y}_1) \subseteq \mathfrak{f}^{-1}{}_{fn}(\mathfrak{Y}_2)$.*

Proof. Let $\mathfrak{Y}_1 \subseteq \mathfrak{Y}_2$ and, for every $\mathfrak{u}_1 \in \mathfrak{U}_1$, it is written as

$$\varphi_{\dddot{A}_{fn}\ ,\ \mathfrak{f}^{-1}{}_{fn}(\mathfrak{Y}_1)}(\mathfrak{u}_1) = \varphi_{\dddot{A}_{fn}\ ,\ \mathfrak{Y}_1}\Big(\mathfrak{f}_{fn}(\mathfrak{u}_1)\Big) \leq \varphi_{\dddot{A}_{fn}\ ,\ \mathfrak{Y}_2}\Big(\mathfrak{f}_{fn}(\mathfrak{u}_1)\Big) = \varphi_{\dddot{A}_{fn}\ ,\ \mathfrak{f}^{-1}{}_{fn}(\mathfrak{Y}_2)}(\mathfrak{u}_1),$$
$$\psi_{\dddot{A}_{fn}\ ,\ \mathfrak{f}^{-1}{}_{fn}(\mathfrak{Y}_1)}(\mathfrak{u}_1) = \psi_{\dddot{A}_{fn}\ ,\ \mathfrak{Y}_1}\Big(\mathfrak{f}_{fn}(\mathfrak{u}_1)\Big) \geq \psi_{\dddot{A}_{fn}\ ,\ \mathfrak{Y}_2}\Big(\mathfrak{f}_{fn}(\mathfrak{u}_1)\Big) = \psi_{\dddot{A}_{fn}\ ,\ \mathfrak{f}^{-1}{}_{fn}(\mathfrak{Y}_2)}(\mathfrak{u}_1)$$

and

$$\zeta_{\dddot{A}_{fn}\ ,\ \mathfrak{f}^{-1}{}_{fn}(\mathfrak{Y}_1)}(\mathfrak{u}_1) = \zeta_{\dddot{A}_{fn}\ ,\ \mathfrak{Y}_1}\Big(\mathfrak{f}_{fn}(\mathfrak{u}_1)\Big) \leq \zeta_{\dddot{A}_{fn}\ ,\ \mathfrak{Y}_2}\Big(\mathfrak{f}_{fn}(\mathfrak{u}_1)\Big) = \zeta_{\dddot{A}_{fn}\ ,\ \mathfrak{f}^{-1}{}_{fn}(\mathfrak{Y}_2)}(\mathfrak{u}_1).$$

So, $\mathfrak{f}^{-1}{}_{fn}(\mathfrak{Y}_1) \subseteq \mathfrak{f}^{-1}{}_{fn}(\mathfrak{Y}_2)$. □

Definition 9. ***(Fermatean Neutrosophic Topological Spaces).*** *Let $\mathfrak{U} \neq \varnothing$ and τ_{fn} be a collection of Fermatean neutrosophic subsets of $\mathfrak{U}$. If τ_{fn} satisfies the next properties, it is called a Fermatean neutrosophic topology.*

$$(\tau_N 1)\ \ddot{0}_{fn},\ \ddot{1}_{fn} \in \tau_{fn},$$
$$(\tau_N 2)\ \text{For all}\ \dddot{A}_{fn}\ ,\dddot{B}_{fn}\ \in \tau_{fn},\ \Big(\dddot{A}_{fn} \cap \dddot{B}_{fn}\Big) \in \tau_{fn},$$
$$(\tau_N 3)\ \text{For all}\ i \in \mathbb{N},\ \Big\{\dddot{A}_{fn_i}\Big\} \in \tau_{fn}\ \text{then}\ \Big(\cup \dddot{A}_{fn_i}\Big) \in \tau_{fn}.$$

Then, $\Big(\mathfrak{U}, \tau_{fn}\Big)$ is called a Fermatean neutrosophic topological space.

The component of τ_{fn} is named open Fermatean neutrosophic sets and a Fermatean neutrosophic set $\dddot{A}_{fn}$ is closed if the complement of $\dddot{A}_{fn}$ is Fermatean neutrosophic open. Now, a Fermatean neutrosophic topological space will be given:

Example 2. *Let $\mathfrak{U} = \{\mho_1, \mho_2\}$ for all $k \in \{1,2,3,4\}$ $\dddot{A}_{fn_k}$ be Fermatean neutrosophic sets:*

$$\dddot{A}_{fn_1} = \{(\mho_1, 0.9, 0.4, 0.1), (\mho_2, 0.99, 0.99, 0.99)\}, \dddot{A}_{fn_2} = \{(\mho_1, 0, 0, 1), (\mho_2, 0, 0, 1)\},$$
$$\dddot{A}_{fn_3} = \{(\mho_1, 0.2, 0.4\ ,\ 0.6), (\mho_2, 0.1, 0.9, 0.99)\}, \dddot{A}_{fn_4} = \{(\mho_1, 1, 1, 0), (\mho_2, 1, 1, 0)\}$$

where for all $j \in \{1,2\}$, $0 \leq \varphi_{\dddot{A}_{fn_k}}{}^3(\mho_j) + \psi_{\dddot{A}_{fn_k}}{}^3(\mho_j) \leq 1$ and $0 \leq \varphi_{\dddot{A}_{fn_k}}{}^3(\mho_j)$ $+ \psi_{\dddot{A}_{fn_k}}{}^3(\mho_j) + \zeta_{\dddot{A}_{fn_k}}{}^3(\mho_j) \leq 2$. In this case, $\tau_{fn} = \Big\{\ddot{0}_{fn},\ \ddot{1}_{fn}, \dddot{A}_{fn_1}, \dddot{A}_{fn_2},\ \dddot{A}_{fn_3}, \dddot{A}_{fn_2}\Big\}$ is a Fermatean neutrosophic topology. Here, $\dddot{A}_{fn_1} \cup \dddot{A}_{fn_2} = \dddot{A}_{fn_1} \cup \dddot{A}_{fn_3} = \dddot{A}_{fn_1}$, $\dddot{A}_{fn_1} \cup \dddot{A}_{fn_4} = \dddot{A}_{fn_2} \cup \dddot{A}_{fn_4} = \dddot{A}_{fn_3} \cup \dddot{A}_{fn_4} = \dddot{A}_{fn_4}$, $\dddot{A}_{fn_2} \cup \dddot{A}_{fn_3} = \dddot{A}_{fn_3}$ and $\dddot{A}_{fn_1} \cup \dddot{A}_{fn_2} \cup \dddot{A}_{fn_3} = \dddot{A}_{fn_1}$, $\dddot{A}_{fn_1} \cup \dddot{A}_{fn_2} \cup \dddot{A}_{fn_4} = \dddot{A}_{fn_2} \cup \dddot{A}_{fn_3} \cup \dddot{A}_{fn_4} = \dddot{A}_{fn_1} \cup \dddot{A}_{fn_2} \cup \dddot{A}_{fn_3} \cup \dddot{A}_{fn_4} = \dddot{A}_{fn_4}$.

Further, $\dddot{A}_{fn_1} \cap \dddot{A}_{fn_2} = \dddot{A}_{fn_2} \cap \dddot{A}_{fn_3} = \dddot{A}_{fn_2} \cap \dddot{A}_{fn_4} = \dddot{A}_{fn_2}, \dddot{A}_{fn_1} \cap \dddot{A}_{fn_3} = \dddot{A}_{fn_3} \cap \dddot{A}_{fn_4} = \dddot{A}_{fn_3}, \dddot{A}_{fn_1} \cap \dddot{A}_{fn_4} = \dddot{A}_{fn_1}, \dddot{A}_{fn_1} \cap \dddot{A}_{fn_2} \cap \dddot{A}_{fn_3} = \dddot{A}_{fn_1} \cap \dddot{A}_{fn_2} \cap \dddot{A}_{fn_4} = \dddot{A}_{fn_1} \cap \dddot{A}_{fn_2} \cap \dddot{A}_{fn_3}$
$\cap \dddot{A}_{fn_4} = \dddot{A}_{fn_2}$ and $\ddot{0}_{fn} = \dddot{A}_{fn_2}$, $\ddot{1}_{fn} = \dddot{A}_{fn_4}$.

Definition 10. *Let $\Big(\mathfrak{U}, \tau_{fn_1}\Big)$, $\Big(\mathfrak{U}, \tau_{fn_2}\Big)$ be two Fermatean neutrosophic topological spaces and $\tau_{fn_1} \subset \tau_{fn_2}$. Then, the τ_{fn_2} topology is called to be a finer Fermatean neutrosophic topology than τ_{fn_1}.*

Remark 1. *Let $\mathfrak{U} \neq \varnothing$ and J be an index set. For all $j \in J$, if τ_{fn_j} is a Fermatean neutrosophic topology on $\mathfrak{U}$, then $\tau = \cap_{j\in J}\tau_{fn_j}$ is a Fermatean neutrosophic topology on $\mathfrak{U}$.*

Definition 11. *On $\left(\mathfrak{U}, \tau_{fn}\right)$, let $\ddot{A}_{fn} = \left\{\left(s, \varphi_{\ddot{A}_{fn}}(s), \psi_{\ddot{A}_{fn}}(s), \zeta_{\ddot{A}_{fn}}(s)\right) : s \in \mathfrak{U}\right\}$ be a Fermatean neutrosophic set. In this case, the Fermatean neutrosophic interior and closure for $\ddot{A}_{fn}$ are defined with*

1. $Int_{fn}\left(\ddot{A}_{fn}\right) = \cup\left\{\ddot{O}_{fn_i} : \ddot{O}_{fn_i} \subset \ddot{A}_{fn_i}, \ddot{O}_{fn_i}\right.$ *is open Fermatean neutrosophic sets*$\left.\right\}$,
2. $Cl_{fn}\left(\ddot{A}_{fn}\right) = \cap\left\{\ddot{C}_{fn_i} : \ddot{A}_{fn_i} \subset \ddot{C}_{fn_i}, \ddot{C}_{fn_i}\right.$ *is closed Fermatean neutrosophic sets*$\left.\right\}$

Theorem 1. *Let $\ddot{A}_{fn}$ be a Fermatean neutrosophic set on $\left(\mathcal{U}, \tau_{fn}\right)$. In this case, the following four properties hold:*

1. $Cl_{fn}\left(\ddot{A}_{fn}\right)$ *is a closed Fermatean neutrosophic set.*
2. $Cl_{fn}\left(\ddot{1}_{fn}\right) = \ddot{1}_{fn}$, $Cl_{fn}\left(\ddot{0}_{fn}\right) = \ddot{0}_{fn}$.
3. $Int_{fn}\left(\ddot{A}_{fn}\right)$ *is an open Fermatean neutrosophic set.*
4. $Int_{fn}\left(\ddot{1}_{fn}\right) = \ddot{1}_{fn}$, $Int_{fn}\left(\ddot{0}_{fn}\right) = \ddot{0}_{fn}$.

Proof. Properties are easily obtained from Definition 11. □

Lemma 2. *Let $\ddot{A}_{fn}$ be a Fermatean neutrosophic set on $\mathcal{U}$. On Fermatean neutrosophic topological space $\left(\mathcal{U}, \tau_{fn}\right)$, the following properties hold:*

1. $\complement\left(Int_{fn}\left(\ddot{A}_{fn}\right)\right) = Cl_{fn}\left(\complement\ddot{A}_{fn}\right)$,
2. $Int_{fn}\left(\mathcal{C}\ddot{A}_{fn}\right) = \complement\left(Cl_{fn}\left(\ddot{A}_{fn}\right)\right)$.

Proof. Let $\left(\mathcal{U}, \tau_{fn}\right)$ be a Fermatean neutrosophic topological space and $\ddot{A}_{fn}$ be a Fermatean neutrosophic set where $\ddot{A}_{fn} = \left\{\left(\mathfrak{z}, \varphi_{\ddot{A}_{fn}}(\mathfrak{z}), \psi_{\ddot{A}_{fn}}(\mathfrak{z}), \zeta_{\ddot{A}_{fn}}(\mathfrak{z})\right) : \mathfrak{z} \in \mathfrak{U}\right\}$. Using the

$$Cl_{fn}\left(\ddot{A}_{fn}\right) = \bigcap_{\substack{\ddot{A}_{fn_i} \subset \ddot{C}_{fn_i} \\ \complement(\ddot{C}_{fn_i}) \in \tau_{fn}}} \ddot{C}_{fn_i},$$

let us assume that the collection of open Fermatean neutrosophic sets included in $\ddot{A}_{fn}$ is indexed by $\left\{\left(\mathfrak{z}, \varphi_{\ddot{G}_{fn_j}}(\mathfrak{z}), \psi_{\ddot{G}_{fn_j}}(\mathfrak{z}), \zeta_{\ddot{G}_{fn_j}}(\mathfrak{z})\right) : \mathfrak{z} \in \mathfrak{U}\right\}$.

In this case, $Int\ddot{A}_{fn} = \left\{\left(\mathfrak{z}, \vee\varphi_{\ddot{G}_{fn_j}}(\mathfrak{z}), \vee\psi_{\ddot{G}_{fn_j}}(\mathfrak{z}), \wedge\zeta_{\ddot{G}_{fn_j}}(\mathfrak{z})\right) : j \in \Lambda\right\}$, $\complement\left(Int\ddot{A}_{fn}\right)$ $= \left\{\left(\mathfrak{z}, \wedge\varphi_{\ddot{G}_{fn_j}}(\mathfrak{z}), \vee\psi_{\ddot{G}_{fn_j}}(\mathfrak{z}), \vee\zeta_{\ddot{G}_{fn_j}}(\mathfrak{z})\right) : j \in \Lambda\right\}$.

Using $\complement\ddot{A}_{fn} = \left\{\left(\mathfrak{z}, 1-\varphi_{\ddot{A}_{fn}}(\mathfrak{z}), 1-\psi_{\ddot{A}_{fn}}(\mathfrak{z}), 1-\zeta_{\ddot{A}_{fn}}(\mathfrak{z})\right) : \mathfrak{z} \in \mathfrak{U}\right\}$ and for all $\varphi_{\ddot{G}_{fn_j}}(\mathfrak{z}) \leq \varphi_{\ddot{A}_{fn}}(\mathfrak{z})$, $\psi_{\ddot{A}_{fn}}(\mathfrak{z}) \leq \psi_{\ddot{G}_{fn_j}}(\mathfrak{z})$ and $\zeta_{\ddot{A}_{fn}}(\mathfrak{z}) \leq \zeta_{\ddot{G}_{fn_j}}(\mathfrak{z})$. It is written as $Cl_{fn}\left(\complement\ddot{A}_{fn}\right) = \left\{\left(\mathfrak{z}, \wedge\varphi_{\ddot{G}_{fn_j}}(\mathfrak{z}), \vee\psi_{\ddot{G}_{fn_j}}(\mathfrak{z}), \vee\zeta_{\ddot{G}_{fn_j}}(\mathfrak{z})\right) : j \in \Lambda\right\}$. □

Remark 2. *Let. and $\ddot{B}_{fn}$ be Fermatean neutrosophic sets on $\left(\mathfrak{U}, \tau_{fn}\right)$. In this case,*

1. $Int_{fn}\left(\ddot{A}_{fn}\right) \cup Int_{fn}\left(\ddot{B}_{fn}\right) \neq Int_{fn}\left(\ddot{A}_{fn} \cup \ddot{B}_{fn}\right)$.

2. $Cl_{fn}\left(\ddot{A}_{fn}\right) \cap Cl_{fn}\left(\ddot{B}_{fn}\right) \neq Cl_{fn}\left(\ddot{A}_{fn} \cup \ddot{B}_{fn}\right)$.

Definition 12. *Let $\left(\mathcal{Y}, \tau_{fn}\right)$ be a Fermatean neutrosophic topological space and $\mathfrak{V}$, $\mathcal{Y}$ be Fermatean neutrosophic sets in $\mathcal{Y}$. If there is an open Fermatean neutrosophic subset $\mathfrak{S}$ such that $\mathfrak{V} \subset \mathfrak{S} \subset \mathcal{Y}$, it is said that $\mathcal{Y}$ is a neighborhood of $\mathfrak{V}$.*

Lemma 3. *Let $\left(\mathcal{Y}, \tau_{fn}\right)$ be Fermatean neutrosophic topological space and $\ddot{A}_{fn}$ be a Fermatean neutrosophic subset of $\mathcal{Y}$. A is open on a Fermatean neutrosophic topological space if it contains a neighborhood of its every subset.*

Proof. It is easily obtained using Definition 12. □

Definition 13. *Let $\ddot{A}_{fn}$ be a Fermatean neutrosophic subset on $\left(\mathcal{U}, \tau_{fn}\right)$.*

3. *If $\ddot{A}_{fn} \subseteq Cl_{fn}\left(Int_{fn}\left(\ddot{A}_{fn}\right)\right)$, it is said to be a Fermatean neutrosophic semi-open set. Besides, the complement of the Fermatean neutrosophic semi-open set is said to be the Fermatean neutrosophic semi-closed set.*
4. *If $\ddot{A}_{fn} \subseteq Cl_{fn}\left(Int_{fn}\left(Cl_{fn}\left(\ddot{A}_{fn}\right)\right)\right)$, it is said to be a Fermatean neutrosophic β-open set. Additionally, the complement of a Fermatean neutrosophic β-open set is said to be a Fermatean neutrosophic β-closed set.*

Lemma 4. *Let $\left(\mathfrak{U}, \tau_{fn}\right)$ be a Fermatean neutrosophic topological space,* and *$\ddot{A}_{fn}$ and $\ddot{B}_{fn}$ be Fermatean neutrosophic β-open subsets of $\mathfrak{U}$. However, $\ddot{A}_{fn} \cap \ddot{B}_{fn}$ is not a Fermatean neutrosophic β-open set.*

Proof. It is clearly obtained if $\ddot{A}_{fn}$ and $\ddot{B}_{fn}$ are chosen such that $\ddot{A}_{fn} \subseteq Cl_{fn}\left(Int_{fn}\left(Cl_{fn}\left(\ddot{A}_{fn}\right)\right)\right) = \ddot{1}_{fn}$, $\ddot{B}_{fn} \subseteq Cl_{fn}\left(Int_{fn}\left(Cl_{fn}\left(\ddot{B}_{fn}\right)\right)\right) = \ddot{1}_{fn}$ and $\ddot{A}_{fn} \cap \ddot{B}_{fn}$ is not a Fermatean neutrosophic β-open set in a Fermatean neutrosophic topological space. □

Definition 14. *Let $\ddot{A}_{fn}$ be a Fermatean neutrosophic subset on $\left(\mathfrak{U}, \tau_{fn}\right)$. If $\ddot{A}_{fn} \subseteq Int_{fn}\left(Cl_{fn}\left(Int_{fn}\left(\ddot{A}_{fn}\right)\right)\right)$, it is named a Fermatean neutrosophic α-open set. In addition, the complement of a Fermatean neutrosophic α-open set is said a Fermatean neutrosophic α-closed set.*

Lemma 5. *Let $\ddot{A}_{fn}$ be a Fermatean neutrosophic subset on $\left(\mathfrak{U}, \tau_{fn}\right)$.*

1. *For all $i \in \mathbb{N}$,* if *$\left\{\ddot{A}_{fn_i}\right\}$ is a Fermatean neutrosophic α-open set, then $\left(\cup \ddot{A}_{fn_i}\right)$ is a Fermatean neutrosophic α-open set.*
2. *For all $i \in \mathbb{N}$,* if *$\left\{\ddot{A}_{fn_i}\right\}$ is a Fermatean neutrosophic α-closed set, then $\left(\cap \ddot{A}_{fn_i}\right)$ is a Fermatean neutrosophic α-closed set.*

Proof. Let $\ddot{A}_{fn_i}$ be a family of Fermatean neutrosophic α-open sets. $\ddot{A}_{fn_i} \subseteq Int_{fn}\left(Cl_{fn}\left(Int_{fn}\left(\ddot{A}_{fn_i}\right)\right)\right) \Rightarrow \cup \ddot{A}_{fn_i} \subseteq Int_{fn}\left(Cl_{fn}\left(Int_{fn}\left(\cup \ddot{A}_{fn_i}\right)\right)\right)$, where all $i \in \mathbb{N}$. So, $\cup \ddot{A}_{fn_i}$ is a Fermatean neutrosophic α-open set. Other properties can be easily proven by a similar method. □

Proposition 1. *Let $\ddot{A}_{fn}$ be a Fermatean neutrosophic α-open set on $\left(\mathfrak{U}, \tau_{fn}\right)$. In this case, this set is a Fermatean neutrosophic semi-open set.*

Proof. It is clear from Definition 14. □

3. Results

In this section, a transition to the concept of pre-continuous mapping has been made by constructing the definition of continuous transformation with the help of the Fermatean neutrosophic open set definition. In addition, many important coverage situations are clarified based on the Fermatean neutrosophic point concept.

Definition 15. *Let $\left(\mathfrak{U}_1, \tau_{fn}\right)$, $\left(\mathfrak{U}_2, \tau_{fn}\right)$ be Fermatean neutrosophic topological spaces. If the inverse image of all Fermatean neutrosophic open sets on $\mathfrak{U}_2$ is a Fermatean neutrosophic α-open set on $\mathfrak{U}_1$, then $\eth_{fn} : \mathfrak{U}_1 \to \mathfrak{U}_2$ is called a Fermatean neutrosophic α-continuous mapping.*

Lemma 6. *Let $\left(\mathfrak{U}_1, \tau_{fn}\right)$ and $\left(\mathfrak{U}_2, \tau_{fn}\right)$ be Fermatean neutrosophic topological spaces and $\eth_{fn} : \mathfrak{U}_1 \to \mathfrak{U}_2$. If $\eth_{fn}$ is a Fermatean neutrosophic α-open set, then $\eth_{fn}$ is a Fermatean neutrosophic semi-open set.*

Proof. For a Fermatean neutrosophic α-open set $\dddot{A}_{fn}$, using definition, $\dddot{A}_{fn} \subseteq Int_{fn}\left(Cl_{fn}\left(Int_{fn}\left(\dddot{A}_{fn}\right)\right)\right)$ is written. So, $\dddot{A}_{fn} \subseteq Cl_{fn}\left(Int_{fn}\left(\dddot{A}_{fn}\right)\right)$—that is, $\eth_{fn}$ is neutrosophic semi-open. □

Lemma 7. *Let $\left(\mathfrak{U}_1, \tau_{fn}\right)$ and $\left(\mathfrak{U}_2, \tau_{fn}\right)$ be Fermatean neutrosophic topological spaces and $\eth_{fn} : \mathfrak{U}_1 \to \mathfrak{U}_2$. If $\eth_{fn}$ is a Fermatean neutrosophic α-continuous mapping, then for every $\dddot{A}_{fn} \subset \mathfrak{U}_1$, $\eth_{fn}\left(Cl_{fn}\left(Int_{fn}\left(Cl_{fn}\left(\dddot{A}_{fn}\right)\right)\right)\right) \subseteq Cl_{fn}\left(\eth_{fn}\left(\dddot{A}_{fn}\right)\right)$.*

Proof. For a Fermatean neutrosophic subset of $\mathfrak{U}_1$, $\dddot{A}_{fn}$ and $\eth_{fn}$ are a Fermatean neutrosophic α-continuous mapping. Given that $Cl_{fn}\left(\eth_{fn}\left(\dddot{A}_{fn}\right)\right)$ is a Fermatean neutrosophic closed set on $\mathfrak{U}_2$, it can be said that ${\eth_{fn}}^{-1}\left(Cl_{fn}\left(\eth_{fn}\left(\dddot{A}_{fn}\right)\right)\right)$ is a Fermatean neutrosophic α-closed set on $\mathfrak{U}_1$. In this case,

$$\begin{aligned} Cl_{fn}\left(Int_{fn}\left(Cl_{fn}\left(\dddot{A}_{fn}\right)\right)\right) &= Cl_{fn}\left(Int_{fn}\left(Cl_{fn}\left(Cl_{fn}\left(\dddot{A}_{fn}\right)\right)\right)\right) \\ &\subseteq Cl_{fn}\left(Int_{fn}\left(Cl_{fn}\left({\eth_{fn}}^{-1}\left(Cl_{fn}\right.\right.\right.\right. \\ &\left.\left.\left.\left.\left(\eth_{fn}\left(\dddot{A}_{fn}\right)\right)\right)\right)\right)\right) \subseteq {\eth_{fn}}^{-1}\left(Cl_{fn}\left(\eth_{fn}\left(\dddot{A}_{fn}\right)\right)\right). \end{aligned}$$

Thus, $\eth_{fn}\left(Cl_{fn}\left(Int_{fn}\left(Cl_{fn}\left(\dddot{A}_{fn}\right)\right)\right)\right) \subseteq Cl_{fn}\left(\eth_{fn}\left(\dddot{A}_{fn}\right)\right)$. □

Definition 16. *Let $\left(\mathfrak{U}_1, \tau_{fn}\right)$, $\left(\mathfrak{U}_2, \tau_{fn}\right)$ be Fermatean neutrosophic topological spaces. If the inverse image of all Fermatean neutrosophic α-open sets on $\mathfrak{U}_2$ is a Fermatean neutrosophic α-open in $\mathfrak{U}_1$, then $\eth_{fn} : \mathfrak{U}_1 \to \mathfrak{U}_2$ is called a Fermatean neutrosophic α-irresolute mapping.*

Definition 17. *Let $\left(\mathfrak{U}_1, \tau_{fn}\right)$ and $\left(\mathfrak{U}_2, \tau_{fn}\right)$ be Fermatean neutrosophic topological spaces. If the inverse image of all Fermatean neutrosophic α-open sets in $\mathfrak{U}_2$ is a Fermatean neutrosophic open set on $\mathfrak{U}_1$, then $\eth_{fn} : \mathfrak{U}_1 \to \mathfrak{U}_2$ is called a Fermatean neutrosophic strongly α-irresolute mapping.*

Proposition 2. *Let $\left(\mathfrak{U}_1, \tau_{fn}\right)$ and $\left(\mathfrak{U}_2, \tau_{fn}\right)$ be Fermatean neutrosophic topological spaces and $\eth_{fn} : \mathfrak{U}_1 \to \mathfrak{U}_2$. If $\eth_{fn}$ is a Fermatean neutrosophic strongly α-irresolute mapping, then, for every Fermatean neutrosophic α-closed set on $\mathfrak{U}_2$, ${\eth_{fn}}^{-1}$ is a Fermatean neutrosophic closed set on $\mathfrak{U}_1$.*

Proof. Let $\ddot{B}_{fn}$ be Fermatean neutrosophic α-closed sets in $\mathfrak{U}_2$. So, $\complement\ddot{B}_{fn} = \mathfrak{U}_2 \setminus \ddot{B}_{fn}$ Fermatean neutrosophic α-open sets in this space. Using the mapping $\eth_{fn}$ as Fermatean neutrosophic strongly α-irresolute, ${\eth_{fn}}^{-1}\left(\complement\ddot{B}_{fn}\right) = {\eth_{fn}}^{-1}\left(\mathfrak{U}_2 \setminus \ddot{B}_{fn}\right)$ is Fermatean neutrosophic open in $\mathfrak{U}_1$. □

Definition 18. *Let $\ddot{A}_{fn}$ be a Fermatean neutrosophic set on $\left(\mathfrak{U}, \tau_{fn}\right)$. In this case, the Fermatean neutrosophic α-interior and α-closure of $\ddot{A}_{fn}$ are defined by*

$$Int^{\alpha}_{fn}\left(\ddot{A}_{fn}\right) = \cup\left\{\ddot{O}_{fn_i} : \ddot{O}_{fn_i} \subset \ddot{A}_{fn_i}, \ \ddot{O}_{fn_i} \text{ is } \alpha \text{ open Fermatean neutrosophic sets}\right\}$$

$$Cl^{\alpha}_{fn}\left(\ddot{A}_{fn}\right) = \cap\left\{\ddot{C}_{fn_i} : \ddot{A}_{fn_i} \subset \ddot{C}_{fn_i}, \ \ddot{C}_{fn_i} \text{ is } \alpha \text{ closed Fermatean neutrosophic sets}\right\}$$

Lemma 8. *Let $\left(\mathfrak{U}_1, \tau_{fn}\right)$, $\left(\mathfrak{U}_2, \tau_{fn}\right)$ be Fermatean neutrosophic topological spaces and $\eth_{fn} : \mathfrak{U}_1 \to \mathfrak{U}_2$. If $\eth_{fn}$ is a Fermatean neutrosophic α-continuous mapping, then, for every $\ddot{B}_{fn} \subset \mathfrak{U}_2$, $Cl^{\alpha}_{fn}\left({\eth_{fn}}^{-1}\left(\ddot{B}_{fn}\right)\right) \subseteq {\eth_{fn}}^{-1}\left(Cl_{fn}\left(\ddot{B}_{fn}\right)\right)$.*

Proof. It is clear from Proposition 2 and Definition 18. □

Definition 19. *Let $\left(\mathfrak{U}_1, \tau_{fn}\right)$ and $\left(\mathfrak{U}_2, \tau_{fn}\right)$ be Fermatean neutrosophic topological spaces. If the inverse image of all Fermatean neutrosophic pre-open sets on $\mathfrak{U}_2$ is a Fermatean neutrosophic pre-open set on $\mathfrak{U}_1$, then $\eth_{fn} : \mathfrak{U}_1 \to \mathfrak{U}_2$ is called a Fermatean neutrosophic pre-continuous mapping.*

Definition 20. *Let $\left(\mathfrak{U}_1, \tau_{fn}\right)$ and $\left(\mathfrak{U}_2, \tau_{fn}\right)$ be Fermatean neutrosophic topological spaces. If the image of all Fermatean neutrosophic open sets in $\mathfrak{U}_1$ is a Fermatean neutrosophic pre-open set on $\mathfrak{U}_2$, then $\eth_{fn} : \mathfrak{U}_1 \to \mathfrak{U}_2$ is called a Fermatean neutrosophic pre-open.*

Remark 3. *Let $\left(\mathfrak{U}_1, \tau_{fn}\right)$ and $\left(\mathfrak{U}_2, \tau_{fn}\right)$ be Fermatean neutrosophic topological spaces and $\eth_{fn} : \mathfrak{U}_1 \to \mathfrak{U}_2$ be a Fermatean neutrosophic α-open set, then $\eth_{fn}$ is a Fermatean neutrosophic pre-open set.*

Lemma 9. *Let $\left(\mathfrak{U}_1, \tau_{fn}\right)$ and $\left(\mathfrak{U}_2, \tau_{fn}\right)$ be Fermatean neutrosophic topological spaces and $\eth_{fn} : \mathfrak{U}_1 \to \mathfrak{U}_2$ be a Fermatean neutrosophic α-continuous mapping, then $\eth_{fn}$ is a Fermatean neutrosophic pre-continuous mapping.*

Proof. It can be easily obtained with the help of Definition 19. □

Before moving on to the concept of frontier in Fermatean neutrosophic topological spaces, the previously given point concept for classical neutrosophic topological spaces should be adapted to the space studied.

Definition 21. *Let $\ddot{A}_{fn}$ be a Fermatean neutrosophic set on $\left(\mathcal{Y}, \tau_{fn}\right)$. For each $\varphi, \psi, \zeta : \mathcal{Y} \to [0,1]$, $0 \leq {\varphi_{\ddot{A}_{fn}}}^3(\mathfrak{k}) + {\psi_{\ddot{A}_{fn}}}^3(\mathfrak{k}) \leq 1$ and $0 \leq \varphi^3(\mathfrak{k}) + \psi^3(\mathfrak{k}) + \zeta^3(\mathfrak{k}) \leq 2$, a Fermatean neutrosophic point is the form that is given as*

$$t_{fn}(\mathfrak{k}) = \begin{cases} (\mathfrak{k}, \varphi(\mathfrak{k}), \psi(\mathfrak{k}), \zeta(\mathfrak{k})), & t = \mathfrak{k} \\ (\mathfrak{k}, 0, 0, 1), & t \neq \mathfrak{k}. \end{cases}$$

If $\varphi(\mathfrak{k}) \leq \varphi_{\ddot{A}_{fn}}(\mathfrak{k})$, $\psi(\mathfrak{k}) \leq \psi_{\ddot{A}_{fn}}(\mathfrak{k})$ and *$\zeta_{\ddot{A}_{fn}}(\mathfrak{k}) \leq \zeta(\mathfrak{k})$, then $t_{fn}(\mathfrak{k})$ is said to belong to the Fermatean neutrosophic set $\ddot{A}_{fn}$.*

Example 3. *Let* $\mathfrak{U} = \{\mho_1, \mho_2\}$, $\left(\mathfrak{U}, \tau_{fn}\right)$ *be Fermatean neutrosophic topological spaces.* $\ddot{A}_{fn}$ *is a Fermatean neutrosophic set: for* $j \in \{1,\ 2\}$, $\ddot{A}_{fn} = \{\ (\mho_1,\ 0.9,\ 0.4,\ 0.1),\ (\mho_2,\ 0.99,\ 0.99,\ 0.99)\ \}$ *where* $0 \le \varphi_{\ddot{A}_{fn}}{}^3(\mho_j) + \psi_{\ddot{A}_{fn}}{}^3(\mho_j) \le 1, 0 \le \varphi_{\ddot{A}_{fn}}{}^3(\mho_j) + \psi_{\ddot{A}_{fn}}{}^3(\mho_j) + \zeta_{\ddot{A}_{fn}}{}^3(\mho_j) \le 2$. *In this case,* $a_{1fn}(\mho_1) = (\mho_1,\ 0.9,\ 0.4,\ 0.1)$ *and* $a_{2fn}(\mho_2) = (\mho_2, 0, 0, 1)$.

Definition 22. *Let* $\ddot{A}_{fn}$ *be a Fermatean neutrosophic set on* $\left(\mathfrak{U}, \tau_{fn}\right)$. *If a Fermatean neutrosophic point* $t_{fn}(s) \in \left(Cl_{fn}\left(\ddot{A}_{fn}\right) \cap Cl_{fn}\left(\complement\ddot{A}_{fn}\right)\right)$, *then* $t_{fn}(s)$ *is called a Fermatean neutrosophic frontier point of* $\ddot{A}_{fn}$. *The set of all Fermatean neutrosophic frontier points of* $\ddot{A}_{fn}$ *is denoted with* $Fr_{fn}\left(\ddot{A}_{fn}\right)$. *So,*

$$Fr_{fn}\left(\ddot{A}_{fn}\right) = \left(Cl_{fn}\left(\ddot{A}_{fn}\right) \cap Cl_{fn}\left(\complement\ddot{A}_{fn}\right)\right)$$

Theorem 2. *Let* $\ddot{A}_{fn}$ *be a Fermatean neutrosophic set on* $\left(\mathfrak{U}, \tau_{fn}\right)$. *At that time,*

$$Fr_{fn}\left(\ddot{A}_{fn}\right) = Fr_{fn}\left(\complement\ddot{A}_{fn}\right)$$

Proof. Let $\ddot{A}_{fn}$ be a Fermatean neutrosophic subset of $\mathfrak{U}$. Using the definition of frontier point, it is written as follows:

$$\begin{aligned} Fr_{fn}\left(\ddot{A}_{fn}\right) &= \left(Cl_{fn}\left(\ddot{A}_{fn}\right) \cap Cl_{fn}\left(\complement\ddot{A}_{fn}\right)\right) \\ &= \left(Cl_{fn}\left(\complement\ddot{A}_{fn}\right) \cap Cl_{fn}\left(\complement\left(\complement\ddot{A}_{fn}\right)\right)\right) \\ &= Fr_{fn}\left(\complement\ddot{A}_{fn}\right). \end{aligned}$$

□

Lemma 10. Let $\ddot{A}_{fn}$ *be a Fermatean neutrosophic subset on* $\left(\mathfrak{U}, \tau_{fn}\right)$. *Then, the next equalities do not have to be valid in* $\left(\mathfrak{U}, \tau_{fn}\right)$.

1. $Fr_{fn}\left(\ddot{A}_{fn}\right) \cap Int_{fn}\left(\ddot{A}_{fn}\right) = \ddot{0}_{fn}$.
2. $Fr_{fn}\left(\ddot{A}_{fn}\right) \cup Int_{fn}\left(\ddot{A}_{fn}\right) = Cl_{fn}\left(\ddot{A}_{fn}\right)$.

Proof. It can be easily done with the method used in Remark 2.19 given in [31]. □

Lemma 11. *Let* $\left(\mathfrak{U}, \tau_{fn}\right)$ *be Fermatean neutrosophic topological spaces and* $\ddot{A}_{fn}, \ddot{B}_{fn}$ *be Fermatean neutrosophic subsets of* $\mathfrak{U}$. *Then, the next inclusions do not have to be valid in* $\left(\mathfrak{U}, \tau_{fn}\right)$.

1. $Fr_{fn}\left(\ddot{A}_{fn}\right) \cap Fr_{fn}\left(\ddot{B}_{fn}\right) \subseteq Fr_{fn}\left(\ddot{A}_{fn} \cap \ddot{B}_{fn}\right)$.
2. $Fr_{fn}\left(\ddot{A}_{fn} \cap \ddot{B}_{fn}\right) \subseteq Fr_{fn}\left(\ddot{A}_{fn}\right) \cap Fr_{fn}\left(\ddot{B}_{fn}\right)$.

Proof. It can be shown that inclusion is not achieved by selecting sets in accordance with Definition 22. □

Lemma 12. *Let* $\ddot{E}_{fn}$ *and* $\ddot{F}_{fn}$ *be Fermatean neutrosophic subsets on* $\left(\mathfrak{U}, \tau_{fn}\right)$. *In this case,* $Fr_{fn}\left(\ddot{E}_{fn}\right) \cup Fr_{fn}\left(\ddot{F}_{fn}\right) \supseteq Fr_{fn}\left(\ddot{E}_{fn} \cap \ddot{F}_{fn}\right)$.

Proof. Let $\ddot{A}_{fn}, \ddot{B}_{fn}$ be Fermatean neutrosophic subsets of $\mathfrak{U}$, then

$$\begin{aligned}\left(Fr_{fn}\left(\ddot{E}_{fn}\right)\cup Fr_{fn}\left(\ddot{F}_{fn}\right)\right) &\supseteq \left(Fr_{fn}\left(\ddot{E}_{fn}\right)\cap Cl_{fn}\left(\ddot{F}_{fn}\right)\right)\cup\left(Cl_{fn}\left(\ddot{E}_{fn}\right)\cap Fr_{fn}\left(\ddot{F}_{fn}\right)\right)\\ &\supseteq Cl_{fn}\left(\ddot{E}_{fn}\cap\ddot{F}_{fn}\right)\cap Cl_{fn}\left(\complement\left(\ddot{E}_{fn}\right)\cup\complement\left(\ddot{F}_{fn}\right)\right)\\ &= Cl_{fn}\left(\ddot{E}_{fn}\cap\ddot{F}_{fn}\right)\cap Cl_{fn}\left(\complement\left(\ddot{E}_{fn}\cap\ddot{F}_{fn}\right)\right)\\ &= Fr_{fn}\left(\ddot{E}_{fn}\cap\ddot{F}_{fn}\right).\end{aligned}$$

□

After the definition of the neutrosophic topological space, great progress was made in the development of the theory with the studies carried out by different researchers. In the study, important concepts and basic properties of the Fermatean neutrosophic topological space—which are defined in this direction and based on the fact that the three components of the neutrosophic structure, namely, membership and non-membership, can be independent from each other—are given.

4. An Application of Neutrosophic Kano Method

In this part, the neutrosophic Kano method—mentioned in the introduction, which is thought to be useful for decision-making processes—and the evaluation of Fermatean structures within this method will be made. We will evaluate the neutrosophic Kano model steps given in [45]. According to this method, the answer given by a decision maker to the positive question in the Kano rubric is determined by the linguistic term P, and the answer given to the negative question by the linguistic term P. With this in mind, let us recall some concepts and equations in [45]. Let the single-valued neutrosophic number equivalents of these linguistic terms be $\text{P}^j = \left\langle \text{Ŧ}_{\text{P}^j}, \text{Ƒ}_{\text{P}^j}, \text{ɨ}_{\text{P}^j} \right\rangle$ and $\text{N}^j = \left\langle \text{Ŧ}_{\text{N}^j}, \text{Ƒ}_{\text{N}^j}, \text{ɨ}_{\text{N}^j} \right\rangle$, respectively. Here, P^j and N^j are the neutrosophic number equivalents of the answer given by the *j*th decision maker to the positive and negative questions, respectively, in terms of the relevant criteria. In order to obtain the Kano rating code, the answers given to the positive and negative questions are multiplied by the neutrosophic number equivalent P^j and N^j, respectively, and the single-valued neutrosophic number equivalents for the corresponding category in the Kano rating are found with the formulas

$$\text{P}^j = \left\langle \text{Ŧ}_{\text{P}^j}, \text{ɨ}_{\text{P}^j}, \text{Ƒ}_{\text{P}^j} \right\rangle$$

and

$$\text{N}^j = \left\langle \text{Ŧ}_{\text{N}^j}, \text{Ƒ}_{\text{N}^j}, \text{ɨ}_{\text{N}^j} \right\rangle,$$

$$\text{P}^j \bigotimes \text{N}^j = \left\langle \text{Ŧ}_{\text{P}^j}\text{Ŧ}_{\text{N}^j}, \text{ɨ}_{\text{P}^j} + \text{ɨ}_{\text{N}^j} - \text{ɨ}_{\text{P}^j}\text{ɨ}_{\text{N}^j}, \text{Ƒ}_{\text{P}^j} + \text{Ƒ}_{\text{N}^j} - \text{Ƒ}_{\text{P}^j}\text{Ƒ}_{\text{N}^j} \right\rangle$$

$$\text{P}^j \bigotimes \text{N}^j = \left\langle \text{Ŧ}_{\text{PN}^j}, \text{ɨ}_{\text{PN}^j}, \text{Ƒ}_{\text{PN}^j} \right\rangle.$$

The score value of the obtained neutrosophic number $\left\langle \text{Ŧ}_{\text{PN}^j}, \text{ɨ}_{\text{PN}^j}, \text{Ƒ}_{\text{PN}^j} \right\rangle$ is obtained by the following equation. This value is the score value of the category to which the positive and negative questions correspond. Here, C_i^j is the consensus value of the *j*th decision maker in terms of the *i*th criterion.

$$\left(\text{C}_i^j\right) = \frac{2 + \text{Ŧ}_{\text{PN}^j} - \text{ɨ}_{\text{PN}^j} - \text{Ƒ}_{\text{PN}^j}}{3},\ 0 \le \left(\text{C}_i^j\right) \le 1.$$

A consensus score value is obtained for each category. Kano category values, the geometric mean of the decision makers who prefer the relevant Kano category, are found with the formula below:

$$(\mathrm{C}_i) = \prod_{j=1}^{\surd}\left(\left(\mathrm{C}_i^j\right)\right)^{\frac{1}{\surd}}$$

Finally, customer satisfaction and dissatisfaction coefficients are obtained with the help of the following two equations:

$$CS = \frac{E+L}{E+L+B+C}(\text{customer satisfaction}),$$

$$CDS = \frac{B+L}{(-1)(E+L+B+C)}(\text{customer dissatisfaction})$$

Here, E, S, L, C, N and B represents the exciting, suspicious, linear, contrary, neutral and basic requirement, respectively. These two values indicate the level of need for the relevant requirement by the customer. Generally, requirements with coefficients above 0.5 are included in the exciting or linear category. Thus, these categories are taken into account in determining the requirements suitable for the customers' idea. In the given example, suppose a 5-point Likert type scale is used to reveal customer needs. This survey was named the Kano model survey in [45]. In [34], linguistic terms and neutrosophic number equivalents for the 5-point Likert Scale given by Biswas are as follows:

I would be pleased → $\langle 0.9,\ 0.1,\ 0.1 \rangle$
I expect it to be → $\langle 0.8,\ 0.2,\ 0.15 \rangle$
It doesn't matter → $\langle 0.5,\ 0.4,\ 0.45 \rangle$
I'm not satisfied but I can stand → $\langle 0.35,\ 0.6,\ 0.7 \rangle$
I'm not satisfied → $\langle 0.35,\ 0.6,\ 0.7 \rangle$.

All these Neutrosophic sets are Fermatean Neutrosophic sets and satisfy the next inequalities given in Definition 5.

$$0 \le {\varphi_{\ddot{A}_{fn}}}^3 + {\psi_{\ddot{A}_{fn}}}^3 \le 1$$

and

$$0 \le {\varphi_{\ddot{A}_{fn}}}^3 + {\psi_{\ddot{A}_{fn}}}^3 + {\zeta_{\ddot{A}_{fn}}}^3 \le 2.$$

In the exemplary decision-making problem, it can be aimed to develop a mechanism that allows choosing among different businesses that offer local products and to determine the criteria that are decisive for this selection. A two-way question pair should be prepared for each requirement, and the pair of positive–negative questions should determine how the customer will feel, respectively, according to whether the feature that represents the expectation is in the product or not. In the given decision case, if the decision maker selected the option "I am satisfied" for the positive question and "I am not satisfied but I can stand" for the negative question, they can be considered to be in the category of "exciting requirements" denoted by E.

If the decision maker ticks "I am satisfied" for the positive question and "I am not satisfied" for the negative question on the two-way question, it can be considered that the category is linear-one-dimensional, indicated by L.

For example, if the decision maker chooses "I would be satisfied" for the positive question and "I would not be satisfied" for the negative question, the Kano categories of quality, naturalness of products, freshness and flavor requirements would be linear.

For each of these four requirements, the Fermatean neutrosophic equivalents of "I am satisfied" and "I am not satisfied" would be $u_1 = \langle 0.9,\ 0.1,\ 0.1 \rangle$ and $u_2 = \langle 0.1,\ 0.8,\ 0.9 \rangle$, respectively. Here, $u_1 \otimes u_2 = \langle 0.09,\ 0.82,\ 0.91 \rangle$ is the Kano value in the linear category of the four requirements mentioned. Further, $u_1 \otimes u_2$ are Fermatean neutrosophic sets. The single value of the neutrosophic number obtained for the requirements of quality, naturalness of products, freshness and taste is 0.12. Since the answers to these four requirements are the same for all decision makers, the geometric mean result of the linear category's Kano value is again 0.12.

In addition, the customer satisfaction coefficient (CSC) and customer dissatisfaction coefficient (CSDC) for quality, naturalness of products, freshness and taste are 1 and -1, respectively. This situation can be interpreted as if the quality, naturalness, freshness and taste needs are met, the customer satisfaction will increase by 100%; if not, the dissatisfaction will increase 100%.

5. Discussion

The idea of Neutrosophic topological space, which was first given in [19], was studied again in [20]; then, classical set theory and fuzzy–intuitionistic fuzzy set theory concepts were quickly transferred to this space. In this study, we aimed to define a topological space in which the concepts in the aforementioned theories can be applied by taking the Neutrosophic topological space structure one step further.

In the studies given in [24–26], which are the starting point of the study idea, fuzzy topological space, which deals only with the degree of membership of an member, and the notion of Fermatean are combined. In the solution of daily life problems, fuzzy and even intuitionistic fuzzy concepts, which are prepared by using neutrosophic structures that have emerged to eliminate inadequacy and deficiencies, are a more preferable study for problem solving with this aspect.

In [27], Pythagorean fuzzy topological spaces are studied. Appropriate modifications of the concepts of pre-image and continuity examined in this study have been defined and it has been obtained that these concepts can also be studied for the new space defined. Open set types are defined in neutrosophic topological space [29] and Pythagorean fuzzy topological space [27]. Such open set types are defined for the studied space and their important properties are examined.

Neutrosophic point and frontier are given in [31]. In this study, the appropriate equivalents of these concepts, which are important for topological space research, are also included.

In [32] on neutrosophic topological space and in [33] on fuzzy topological space, the types of irresolute open mapping are defined. In this sense, explicit transformation types are defined in the space constructed on the basis of the Fermatean structure.

The results obtained can be transferred to different topological spaces, as in [35,36], by new researchers.

In order to explain the problems in daily life, many important researches have been made and continue to be done with different perspectives on decision-making processes and their modifications, which are popular topics of recent days. Studies such as [4–8], in which solutions to fuzzy-based decision making problems are sought and very important findings related to daily life are obtained, can be designed to include uncertainty and indecision situations by using neutrosophic set theory. The studies given in [11–16], where contradictory and incomplete data on neutrosophic decision-making processes based on uncertainty and instability modeling can be explained in a sense and where important studies have been made from different perspectives, can be evaluated by including a topological context and Fermatean structure in these studies. When [21–24] are examined, very important ideas may emerge for studies in which studies that synthesize the Kano model and multi-criteria decision-making can be applied.

When the studies examining the intersections of medical, linguistic and theoretical mathematics subjects with the decision-making processes in [37–39] are evaluated together with the neutrosophic Kano method mentioned above, it can give important ideas about the determination of different decision-making processes for studies investigating customer satisfaction.

6. Conclusions

This manuscript is designed to redefine the notion of Fermatean neutrosophic sets to utilize characteristics of Fermatean fuzzy sets and neutrosophic sets. This manuscript provides useful concepts to the researchers, which contain fundamental properties of neutrosophic topological spaces. A decision-making problem is tackled with the neutrosophic Kano method. While evaluating a problem in neutrosophic sets, accuracy, inaccuracy and uncertainty are handled at the same time, making it easier to explain the uncertainties we encounter. In this study, prepared in this direction by defining Fermatean neutrosophic topological space, important concepts are given in this space and important theoretical information is formed with the help of these definitions. In addition, the idea of Fermatean neutrosophic sets can be carried to different directions, including new MCDM methods such as TOPSIS, VIKOR, ELECTRE, AHP, BWM, etc. Some symmetry-based structures can be developed together with the Fermatean neutrosophic information aggregation.

Author Contributions: Conceptualization, N.G.B., D.P. and M.R.; investigation, N.G.B., D.P. and M.R.; writing—original draft preparation, N.G.B.; methodology, N.G.B., D.P. and M.R. All authors have read and agreed to the published version of the manuscript.

Funding: This study has not received any outside funding.

Data Availability Statement: Not applicable.

Acknowledgments: The authors would like to thank the editors and referees who reviewed the study.

Conflicts of Interest: The authors declare no conflict of interest.

References

1. Akram, M.; Naz, S.; Smarandache, F. Generalization of maximizing deviation and TOPSIS method for MADM in simplified neutrosophic hesitant fuzzy environment. *Symmetry* **2019**, *11*, 1058. [CrossRef]
2. Akram, M.; Smarandache, F. Decision-making with bipolar neutrosophic TOPSIS and bipolar neutrosophic ELECTRE-I. *Axioms* **2018**, *7*, 33. [CrossRef]
3. Zhang, X.; Xu, Z. Extension of TOPSIS to multiple criteria decision making with Pythagorean fuzzy sets. *Int. J. Intell. Syst.* **2014**, *29*, 1061–1078. [CrossRef]
4. Peng, X.; Yang, Y. Some results for Pythagorean fuzzy sets. *Int. J. Intell. Syst.* **2015**, *30*, 1133–1160. [CrossRef]
5. Riaz, M.; Hashmi, M.R. Linear Diophantine fuzzy set and its applications towards multi-attribute decision-making problems. *J. Intell. Fuzzy Syst.* **2019**, *37*, 5417–5439. [CrossRef]
6. Riaz, M.; Hashmi, M.R.; Pamucar, D.; Chu, Y.M. Spherical linear Diophantine fuzzy sets with modeling uncertainties in MCDM. *Comput. Modeling Eng. Sci.* **2021**, *126*, 1125–1164. [CrossRef]
7. Eraslan, S.; Karaaslan, F. A group decision making method based on TOPSIS under fuzzy soft environment. *J. New Theory* **2015**, *3*, 30–40.
8. Riaz, M.; Farid, H.M.A. Multicriteria decision-making with proportional distribution based spherical fuzzy fairly aggregation operators. *Int. J. Intell. Syst.* **2022**, 7079–7109. [CrossRef]
9. Smarandache, F. A unifying field in logics. In *Neutrosophy: Neutrosophic Probability, Set and Logic*; American Research Press: Rehoboth, DE, USA, 1999; pp. 1–144.
10. Krishankumar, R.; Pamucar, D.; Cavallaro, F.; Ravichandran, K.S. Clean energy selection for sustainable development by using entropy-based decision model with hesitant fuzzy information. *Environ. Sci. Pollut. Res.* **2022**, *29*, 42973–42990. [CrossRef]
11. Almagrabi, A.O.; Abdullah, S.; Shams, M.; Al-Otaibi, Y.D.; Ashraf, S. A new approach to q-linear Diophantine fuzzy emergency decision support system for COVID19. *J. Ambient. Intell. Humaniz. Comput.* **2022**, *13*, 1687–1713. [CrossRef]
12. Riaz, M.; Almalki, Y.; Batool, S.; Tanveer, S. Topological Structure of Single-Valued Neutrosophic Hesitant Fuzzy Sets and Data Analysis for Uncertain Supply Chains. *Symmetry* **2022**, *14*, 1382. [CrossRef]
13. Kamacı, H.; Garg, H.; Petchimuthu, S. Bipolar trapezoidal neutrosophic sets and their Dombi operators with applications in multicriteria decision making. *Soft Comput.* **2021**, *25*, 8417–8440. [CrossRef]
14. Riaz, M.; Hashmi, M.R. *m*-polar neutrosophic soft mapping with application to multiple personality disorder and its associated mental disorders. *Artif. Intell. Rev.* **2021**, *54*, 2717–2763. [CrossRef]
15. Aslam, M. Design of a new Z-test for the uncertainty of Covid-19 events under Neutrosophic statistics. *BMC Med. Res. Methodol.* **2022**, *22*, 99. [CrossRef] [PubMed]
16. Kisi, O.; Gurdal, V. Triple Lacunary Delta-Statistical Convergence in Neutrosophic Normed Spaces. *Konuralp J. Math.* **2022**, *10*, 127–133.

17. Mallik, S.; Mohanty, S.; Mishra, B.S. Neutrosophic Logic and Its Scientific Applications. In *Biologically Inspired Techniques in Many Criteria Decision Making*; Springer: Singapore, 2022; pp. 415–432.
18. Bilgin, N.G. Hibrid Δ-statistical convergence for Neutrosophic normed space. *J. Math.* **2022**, *2022*, 3890308.
19. Salma, A.A.; Alblowi, S.A. Neutrosophic set and neutrosophic topological spaces. *IOSR J. Appl. Math.* **2012**, *3*, 31–35. [CrossRef]
20. Karatas, S.; Kuru, C. Neutrosophic topology. *Neutrosophic Sets Syst.* **2016**, *13*, 90–95.
21. Senapati, T.; Yager, R.R. Fermatean fuzzy sets. *J. Ambient. Intell. Humaniz. Comput.* **2020**, *11*, 663–674. [CrossRef]
22. Saha, A.; Pamucar, D.; Gorcun, O.F.; Mishra, A.R. Warehouse site selection for the automotive industry using a fermatean fuzzy-based decision-making approach. *Expert Syst. Appl.* **2022**, *211*, 118497. [CrossRef]
23. Wei, D.; Meng, D.; Rong, Y.; Liu, Y.; Garg, H.; Pamucar, D. Fermatean Fuzzy Schweizer–Sklar Operators and BWM-Entropy-Based Combined Compromise Solution Approach: An Application to Green Supplier Selection. *Entropy* **2022**, *24*, 776. [CrossRef] [PubMed]
24. Mishra, A.R.; Rani, P.; Saha, A.; Hezam, I.M.; Pamucar, D.; Marinović, M.; Pandey, K. Assessing the Adaptation of Internet of Things (IoT) Barriers for Smart Cities' Waste Management Using Fermatean Fuzzy Combined Compromise Solution Approach. *IEEE Access* **2022**, *10*, 37109–37130. [CrossRef]
25. Rani, P.; Mishra, A.R.; Saha, A.; Hezam, I.M.; Pamucar, D. Fermatean fuzzy Heronian mean operators and MEREC-based additive ratio assessment method: An application to food waste treatment technology selection. *Int. J. Intell. Syst.* **2022**, *37*, 2612–2647. [CrossRef]
26. Ibrahim, H.Z.J. Fermatean fuzzy topological space. *J. Appl. Math. Inform.* **2022**, *40*, 85–98.
27. Olgun, M.; Unver, M.; Yardımcı, S. Pythagorean fuzzy topological spaces. *Complex Intell. Syst.* **2019**, *5*, 177–183. [CrossRef]
28. Sweety, C.A.C.; Jansi, R. Fermatean Neutrosophic Sets. *Int. J. Adv. Res. Comput. Commun. Eng.* **2021**, *10*, 24–27.
29. Arokiarani, I.; Dhavaseelan, R.; Jafari, S.; Parimala, M. On some new notions and functions in neutrosophic topological spaces. *Neutrosophic Sets Syst.* **2017**, *16*, 16–19.
30. Ajay, D. Pythagorean fuzzy α-continuity. *J. Comput. Math.* **2020**, *4*, 10–15.
31. Iswarya, P.; Bageerathi, K. A study on neutrosophic frontier and neutrosophic semi-frontier in neutrosophic topological spaces. *Neutrosophic Sets Syst.* **2017**, *16*, 6–15.
32. Kannan, T.R.; Chandrasekar, S. Neutrosophic pre-α, semi-α and pre-β irresolute open and closed mappings in Neutrosophic topological spaces. *Malaya J. Mat.* **2020**, *8*, 1795–1806.
33. Shukla, M. On fuzzy strongly α-irresolute mappings. *Int. J. Fuzzy Math. Syst.* **2012**, *2*, 141–147.
34. Biswas, P.; Pramanik, S.; Giri, B.C. TOPSIS method for multi-attribute group decision-making under single-valued neutrosophic environment. *Neural Comput. Applic* **2016**, *27*, 727–737. [CrossRef]
35. Shabir, M.; Naz, M. On soft topological spaces. *Comput. Math. Appl.* **2011**, *61*, 1786–1799. [CrossRef]
36. Cagman, N.; Karatas, S.; Enginoglu, S. Soft topology. *Comput. Math. Appl.* **2011**, *62*, 351–358. [CrossRef]
37. Biswas, S.; Pamucar, D.; Mukhopadhyaya, J.N. A multi-criteria-based analytical study of the impact of COVID-19 on ELSS fund performance. *Int. J. Manag. Decis. Mak.* **2022**, *21*, 339–378. [CrossRef]
38. Mahmood, T.; Ali, Z.; Aslam, M.; Chinram, R. Generalized Hamacher aggregation operators based on linear Diophantine uncertain linguistic setting and their applications in decision-making problems. *IEEE Access* **2021**, *9*, 126748–126764.
39. Kamaci, H. Linear Diophantine fuzzy algebraic structures. *J. Ambient. Intell. Humaniz. Comput.* **2021**, *12*, 10353–10373. [CrossRef]
40. Wei, G.; Garg, H.; Gao, H.; Wei, C. Interval-valued Pythagorean fuzzy Maclaurin symmetric mean operators in multiple attribute decision making. *IEEE Access* **2018**, *6*, 67866–67884. [CrossRef]
41. Saqlain, M.; Jafar, N.; Moin, S.; Saeed, M.; Broumi, S. Single and multi-valued neutrosophic hypersoft set and tangent similarity measure of single valued neutrosophic hypersoft sets. *Neutrosophic Sets Syst.* **2020**, *32*, 317–329.
42. Saqlain, M.; Saeed, M.; Ahmad, M.R.; Smarandache, F. Generalization of TOPSIS for Neutrosophic Hypersoft set using Accuracy Function and its Application. *Neutrosophic Sets Syst.* **2019**, *27*, 131–138.
43. Zulqarnain, R.M.; Xin, X.L.; Saqlain, M.; Smarandache, F.; Ahamad, M.I. An integrated model of neutrosophic TOPSIS with application in multi-criteria decision-making problem. *Neutrosophic Sets Syst.* **2021**, *40*, 253–269.
44. Wang, C.H.; Wang, J. Combining fuzzy AHP and fuzzy kano to optimize product varieties for smart cameras: A zero-one integer programming perspective. *Appl. Soft Comput.* **2014**, *22*, 410–416. [CrossRef]
45. Egilmez, B. *Supplier Selection for a Copper Production Company with an Integrated Quality Function Deployment Approach Using Neutrosophic Numbers Based Kano Model and MULTIMOORA Methods*; Eskisehir Osmangazi University, Institute of Science and Technology, Department of Statistics: Eskişehir, Turkey, 2022.
46. Garg, H.; Perveen PA, F.; John, S.J.; Perez-Dominguez, L. Spherical Fuzzy Soft Topology and Its Application in Group Decision-Making Problems. *Math. Probl. Eng.* **2022**, *2022*, 1007133. [CrossRef]
47. Kausar, R.; Tanveer, S.; Riaz, M.; Pamucar, D.; Goran, C. Topological Data Analysis of m-Polar Spherical Fuzzy Information with LAM and SIR Models. *Symmetry* **2022**, *14*, 2216. [CrossRef]