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Assessing Parameters of the Coplanar Components of Perturbing Accelerations Using the Minimal Number of Optical Observations

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Abstract: The method presented in this paper is developed to assess the parameters (the application moment and the magnitude of a velocity impulse) of a maneuver-like perturbation of motion of the center of mass of a spacecraft in a near-circular orbit. The assessment is based on the information on the spacecraft's trajectory before the maneuver and the optical observations of the spacecraft's angular position (right ascension and declination angles) after the maneuver. This study considers the cases of solely transversal (in-track) or transversal and radial components of the velocity increment vector. A single pair of the values of angles is used for the assessment of the single transversal maneuver parameters and two pairs are used in the other cases. The method also makes it possible to estimate the parameters of a continuous maneuver performed with low-thrust engines. For this case the property of its symmetry is used. The approach described in this article makes it possible to determine the spacecraft's orbit after the maneuver much faster and more accurately in comparison to traditional methods.

Keywords: maneuver; spacecraft; maneuver assessment; geostationary orbit; optic observations; perturbing accelerations; nonmodeling perturbations assessment; continuous maneuver symmetry

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Citation: Baranov, A.; Agapov, V.; Golosova, N.; Karatunov, M. Assessing Parameters of the Coplanar Components of Perturbing Accelerations Using the Minimal Number of Optical Observations. *Symmetry* **2022**, *14*, 2564. https://doi.org/10.3390/sym14122564

Academic Editor: Juan Luis García Guirao

Received: 7 October 2022
Accepted: 30 November 2022
Published: 5 December 2022

1. Introduction

Presently, the orbital information on approximately 26,000 space objects in near-Earth orbits is available from public sources accessible via the Internet. It is relatively easy to propagate motion for most of them since all considerable natural perturbing factors which define this motion are described with acceptable accuracy by the known motion models. However, there are space objects whose motion is heavily influenced by additional perturbations. These perturbations are hard to account for directly in the process of the determination and propagation of the parameters of motion of the space object's center of mass. First of all, it concerns the maneuvering spacecraft. At present, there are more than five thousand such objects in orbit and their number is increasing fast due to very large spacecraft constellation deployments. These objects perform maneuvers to maintain their orbits or change them for the fulfillment of new tasks. As a rule, only the operators of these maneuvering spacecraft have a priori and actual data about the orbit parameter alterations. Furthermore, maneuvering spacecraft of some design have the ability to calculate their maneuvers on board. In the course of the space object catalogue maintenance process, one has to wait for the accumulation of the sufficient amount of observation data for the accurate determination of new orbits of these objects after the maneuver is performed.

The use of the observation information for the assessment of parameters of motion of the center of mass of maneuvering spacecraft, simultaneously with the assessment of parameters of the performed maneuver, allows speeding up the process of orbits updating in the space object catalogue. This catalogue is necessary for solving different tasks. For example, it allows to predict and assess the potentially hazardous close approaches between space objects with the maneuvering spacecraft being one of them.

Numerous modern approaches to the determination and assessment of the maneuver parameters of spacecraft are based on the improvement and modification of Kalman type filters, which were actively developed at the end of the last century for atmospheric aircraft [1,2]. Some of the first works to take into account the peculiarities of near-Earth space flight were [3,4]. The work [3] shows the feasibility of the application of a sequential filter for the procession of radiolocation measurements for the determination of long-duration and impulsive maneuver parameters. Woodburn et al. in [4] considers the maneuvers of small duration and uses the radiolocation measurements as preliminary ones. The solution is based on the application of the sequential filter for the assessment of the parameters of motion from the initial point after the moment of the impulse application to the moment of the last measurement. Then, the state vectors before and after the maneuver allow assessing the parameters of the impulsive maneuvers. As the approaches to the solution of the cataloging problem of the maneuvering objects developed, the methods allowing distinguishing the fact of maneuvering were divided into separate groups [5–9]. The methods of the maneuver parameter assessment can be divided into the following criteria: the processing of data on state vectors and covariance matrices exclusively [10,11] or the inclusion of trajectory measurements [3,4,12–17]—the usage of radiolocation measurements [12–14] or optical measurements [15–17]. The methods based on radar measurements are aimed to raise the resistance of filters to the occurrence of unaccounted accelerations. This can be achieved through the combination and modification of Kalman filters: extended Kalman filter (EKF), state smoothing, limited batch least-squares data reduction (BLSQ) in [13] and predictive corrective iterations for the EKF and unscented Kalman filter (UKF) in [14]. The work [14] describes the concept of the variable structure estimator (VSE), which also uses the EKF and belongs to the multiple model adaptive estimation (MMAE) family. The works [15–17] should be noted as they present the problem statement most close to the statement of this research. G. Escribano et al. [15] suggests a new method based on the presentation of the space of states as a stochastic hybrid system, which allows implementing the methods of successive Monte Carlo filtration, with which the maneuver parameters are determined during the process of Bayesian inference. The lack of dynamical constrictions and effectiveness of the problem solution from the fuel expenditures point of view can be treated as a disadvantage of this method. In the meantime, the work [16] contains an approach for narrowing the space of permissible solutions on the basis of the limitation of the finite energy expenditures. This approach is complemented with the algorithm of analyses of the previous spacecraft maneuvers which allows propagating the probability density of the alteration of the orbital parameters for the given moments of time after the last realized maneuver. K. Hill [17] suggests the two angle pairs initial orbit with conjunction analysis method (TAPIOCA), which consists of the preliminary assessment of the orbit on the basis of two angular measurements after the maneuver realization and successive search for the point of the closest approach between the maneuvering spacecraft before and after the maneuver. The obtained maneuver parameters are used in the least squares method for the refinement of the orbit after the maneuver. The inability of the assessment of the long-duration maneuver parameters during measurement gathering in the process of the dynamic operation fulfillment can be treated as a disadvantage of this method. The simultaneous maneuver assessment along with the orbit determination after the maneuver are also used in [18]. Although many methods have been proposed, they all come down to filtering in one form or another, which has significant computational costs.

The maneuvers of space objects in near-circular orbits are assessed in this paper and the preceding works of the authors. This simplifies the problem and allows suggesting semianalytical methods of its solution, which decreases the problem solution time substantially. The crucial importance of the performance arises from the necessity of regular operative assessments of maneuvers of a large number of real maneuvering spacecraft.

Especially important is the problem of the maneuver parameter assessment [19] when the orbit shaped by the maneuver is determined with the use of observation data with considerable uncertainties. These uncertainties arise due to the small number of used observations with low accuracy. An example of solving a similar problem using the method developed in [19] as applied to a real maneuvering spacecraft is provided in Table 1.

Table 1. Assessment of one single-impulse coplanar maneuver with the presence of errors in orbit determination.

Maneuver Parameters	Traditional Method	With the Uncertainty Accountancy	Reference (Actual Values)
ΔV_t (m/s)	−0.392	0.419	0.419
Maneuver application time	18 h 7 min 33 s	18 h 8 min 17 s	18 h 9 min 54 s

Table 1 contains the results of the assessment of a single-impulse coplanar maneuver of a spacecraft in a geostationary orbit with errors in the orbit determination [19]. The second column indicates the result of the traditional method of maneuver assessment (the velocity difference in the point of maximum proximity) obtained without the accountancy of orbit determination errors. The third one shows the assessment result obtained with the accountancy of the determination error. The fourth one provides the magnitude of the real implemented velocity impulse. In the case when the orbit after the maneuver was determined with the use of the observation information of the short observation interval with substantial uncertainties, the traditional method provides an assessment of the maneuver parameters which is far from the actual values. The method described in [19] allows increasing considerably the accuracy of the maneuver parameters assessment (the magnitudes and the application moments). Furthermore, one can obtain the terminal orbit considerably closer to the real orbit than the orbit obtained with the help of observations on the short observation interval if the estimated velocity impulse was applied to the initial orbit.

The use of the refined orbital parameters after performing the maneuver makes it possible to significantly increase the speed of the reliable recognition and evaluation of potentially dangerous encounters of space objects (one of them being a maneuvering spacecraft).

The next step is taken in this work. The maneuver parameters are assessed directly from the minimal number of observations without waiting for the sufficient number of observations needed for the initial approximate orbit determination to accumulate. The developed method of the maneuver assessment allows substantially speeding up the process of the acquisition of reliable and accurate enough assessments of the motion parameters of these objects. This result is possible thanks to the use of the minimal number of pairs α and δ (the angles of the right ascension and declination), which define the direction from the observer on the Earth's surface to the points on the celestial sphere. At these points, the maneuvering spacecraft is detected at the corresponding moments of time during the optical monitoring session.

The suggested method of the maneuver assessment allows determining the parameters of the single-impulse maneuvers with different attitudes of the thrust vector in the case of the use of the high- and low-thrust engines.

2. Materials and Methods

2.1. Variants of the Problems Being Solved

2.1.1. Basic Problem Statement

Let the state vector of the maneuvering space object $X = X(t_0)$ be accurately known at the moment t_0. The observers' positions on the Earth's surface $Y = Y(t_i)$ at the moment t_i ($t_i > t_0$) and the angles α_i and δ_i, which set the direction from the observer to the maneuvering space object at this moment, are known. It is necessary to determine the moment of application, the magnitude and the attitude of the velocity impulse for maneuvers close to impulses, and for maneuvers performed with a low-thrust engine, to evaluate the moments of switching the propulsion system on and off and the attitude of the thrust vector, as well as the acceleration caused by the operation of the propulsion system.

2.1.2. Equations for the Velocity Impulse Influence

Orbit parameters change instantaneously if the interval of the engines' work is small with respect to the maneuvering space object's orbit period.

The spacecraft moves along an orbit that is close to a circular orbit (geostationary orbit), which we call the reference orbit. This allows one to write down the equations of motion in deviations of the real orbit from a circular one in a cylindrical coordinate frame [20]. The solution of the linearized equations allows one to write down the influence of velocity impulses.

Each of these velocity impulses, applied at the point with the angles φ_i defined as an argument of latitude on the reference orbit at time t_i ($i = 1, \ldots, N$), causes the alteration of orbit elements at the point defined by the angle φ_f. The sum of these deviations caused by N velocity impulses can be written as [21]:

$$\sum_{i=1}^{N} r_0\left(\frac{\Delta V_{ri}}{V_0} sin(\varphi_f - \varphi_i) + 2\frac{\Delta V_{ti}}{V_0}(1 - cos(\varphi_f - \varphi_i))\right) = \Delta r, \quad (1)$$

$$\textstyle\sum_{i=1}^{N} (\Delta V_{ri} cos(\varphi_f - \varphi_i) + 2\Delta V_{ti} sin(\varphi_f - \varphi_i)) = \Delta V_r, \quad (2)$$

$$\sum_{i=1}^{N} (-\Delta V_{ri} sin(\varphi_f - \varphi_i) - \Delta V_{ti}(1 - 2cos(\varphi_f - \varphi_i))) = \Delta V_t, \quad (3)$$

$$\sum_{i=1}^{N} r_0\left(-2\frac{\Delta V_{ri}}{V_0}(1 - cos(\varphi_f - \varphi_i)) - \frac{\Delta V_{ti}}{V_0}(3(\varphi_f - \varphi_i) - 4sin(\varphi_f - \varphi_i))\right) = \Delta n, \quad (4)$$

$$\sum_{i=1}^{N} r_0 \frac{\Delta V_{zi}}{V_0} sin(\varphi_f - \varphi_i) = z, \quad (5)$$

$$\sum_{i=1}^{N} \Delta V_{zi} cos(\varphi_f - \varphi_i) = V_z. \quad (6)$$

Here, r_0 and V_0 are the radius and velocity of the reference circular orbit in the vicinity of which the motion occurs; ΔV_{ri}, ΔV_{ti} and ΔV_{zi} are the radial, transversal and lateral components of the i-th velocity impulse, correspondingly; Δr, ΔV_r, ΔV_t and Δn are the deviation along the radius vector, deviations of the radial and transversal components, and deviation along the orbit caused by the velocity impulses; and the angles φ_i and φ_f are measured from the maneuvering space object's position at the moment t_0 to the direction of the maneuvering space object's motion.

2.1.3. Determination of Deviations Along the Radius and Along the Orbit Using Observations

In order to solve the stated problem, one should first calculate the deviations caused by the maneuver at the moment t_i along the radius Δr_i and along the orbit Δn_i using the information about the known angles α_i and δ_i at the moment t_i.

For this purpose, at first, it is necessary to find the intersection point of the beam originating at the observer's position on the Earth's surface with the right ascension and declination angles α_i and δ_i and the plane of the initial maneuvering space object's orbit. This plane's orientation can be determined with the help of numerical integration with the use of the initial conditions vector $X(t_0)$ at the moment t_i. Note that the numerical propagation of the orbit from time t_0 to t_i should be performed with a fairly accurate model that incorporates major disturbances. Then, $\Delta r_i = r_p(t_i) - r_{orbi}$ is calculated, where $r_p(t_i)$ is the magnitude of the vector $r_p(t_i)$, which is pointed out of the Earth's center to the point of intersection (this vector lies in the plane of the initial maneuvering space object's orbit) and r_{orbi} is the magnitude of the radius vector of the initial orbit aimed along the vector $r_p(t_i)$. Therefore, Δr_i indicates the difference in the radius vector magnitude of the real observed orbit, which exhibits the yet undefined impulse and propagates the initial orbit without the impulse. In addition, the distance along the orbit Δn_i between the position of the maneuvering space object in the initial orbit propagated at the moment t_i (it is determined by the vector $r(t_i)$) and its position in the orbit shaped by the maneuver at this moment (it is set by the vector $\boldsymbol{r_p}(t_i)$) is calculated. To calculate Δn_i, the angle between the vectors $r(t_i)$ and $r_p(t_i)$ is multiplied by r_0 in a framework of linearized equations of relative motion.

Further on, we work with Δr and Δn directly. The technical part of the work described in this section requires the main efforts in writing programs.

2.1.4. Possible Variants of the Problems Being Solved

As the different variants of maneuver realizations are possible, there are several problems that need to be solved. The methods of solving these problems and the number of Δr and Δn (α and δ) pairs used also differ. Only the problem statements relating to coplanar maneuvers are provided.

These problems are:

1. A transversal velocity impulse is performed. Its application angle φ and the magnitude of the transversal component ΔV_t should be determined. There are two unknown variables in the problem, hence, one pair of Δr and Δn (α and δ) is enough for its solution. It is the most frequently met problem;
2. The same as above but the impulse is radial. The problem in this statement is solved in some rare special cases;
3. A velocity impulse with the transversal and radial components is performed. Its application angle φ and the magnitudes of the transversal and radial components ΔV_t and ΔV_r should be determined. There are three unknown variables in the problem, hence, two pairs of deviations Δr and Δn are needed for its solution;
4. A long-duration transversal maneuver is performed. The angular duration of the maneuver $\Delta\varphi$, the angle which corresponds to its medium point φ_m and the constant acceleration of the maneuvering space object during the maneuver w_t should be found. There are three unknown variables in the problem, hence, two pairs of deviations Δr and Δn are needed for its solution;
5. A long-duration maneuver with the transversal and radial components is performed. The angular duration of the maneuver $\Delta\varphi$, the angle which corresponds to its medium point φ_m and the transversal w_t and radial w_r constant accelerations are to be derived. There are four unknown variables in the problem, hence, two pairs of deviations Δr and Δn (α and δ) are needed for its solution.

There are only three unknown variables in the third and the fourth problems. Two pairs of Δr and Δn are used which provide four equations using Equations (1) and (4). Three equations out of four are directly used to find problem parameters, while the fourth equation can be used for a selection of a more accurate solution. This is discussed in detail when solving specific problems.

2.2. Small-Duration Maneuver Parameters Assessment

2.2.1. Transversal and Radial Velocity Impulse Assessment

The equations for the determination of the velocity transversal impulse application angle φ and the magnitude ΔV_t can be described as:

$$2r_0\frac{\Delta V_t}{V_0}(1-cos(\varphi_f-\varphi)) = \Delta r,$$
$$r_0\frac{\Delta V_t}{V_0}(3(\varphi_f-\varphi)-4sin(\varphi_f-\varphi)) = \Delta n.$$

Here, the angle φ_f corresponds to the point in which the deviations Δr and Δn are calculated.

Figure 1 shows a change in the shape of the orbit, accompanied by a deviation along the radius caused by the transversal component of the velocity impulse. Figure 2 shows the deviation along the orbit depending on the angular distance from the moment of application of the velocity impulse.

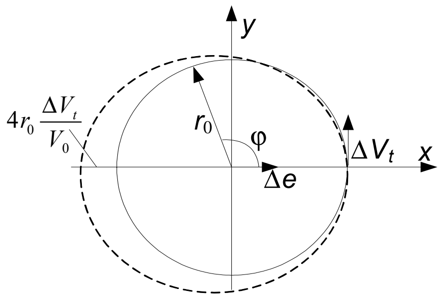

Figure 1. Orbit shape change after the application of the transversal component of the velocity impulse.

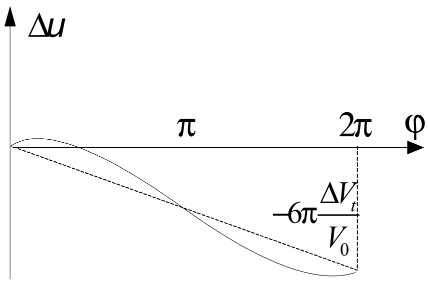

Figure 2. Along-the-orbit shift after the application of the transversal component of the velocity impulse.

The deviations along the radius Δr and along the orbit $\Delta n = \Delta u \cdot \mathrm{r}_0$ shown in Figures 1 and 2 can be calculated in the program using the angles α_i and δ_i. Then, they themselves are used to determine the maneuver parameters.

The parameter ΔV_t in Equation (7) is determined from the first equation and is used in the second one. This leads to Equation (8) for the determination of φ, which is in turn used in Equation (7) so ΔV_t can be determined.

$$\Delta V_t = V_0 \frac{\Delta r}{2r_0(1 - cos(\varphi_f - \varphi))}, \tag{7}$$

$$\Delta r \frac{3(\varphi_f - \varphi) - 4sin(\varphi_f - \varphi)}{2 - 2cos(\varphi_f - \varphi)} = \Delta n. \tag{8}$$

This is the simplest problem in which the unique solution can be found. The accuracy of the deviation calculations Δr and Δn effect the solution quality.

Similarly, one can obtain formulas for determining the parameters of the radial velocity impulse.

$$\Delta V_r = \frac{V_0 \Delta r}{r_0 sin(\varphi_f - \varphi)}, \tag{9}$$

$$\frac{\Delta r(1 - cos(\varphi_f - \varphi))}{sin(\varphi_f - \varphi)} = \Delta n. \tag{10}$$

This is also a relatively easy problem, in which the unique solution can also be found fast. The radial velocity impulses can rarely be met, but the possibility of obtaining the parameters of such a maneuver should be provided.

2.2.2. Assessment of the Velocity Impulse with Transversal and Radial Components

There are three unknown variables in the problem: the application angle φ and the magnitudes of the transversal and radial components ΔV_t and ΔV_r, hence, two pairs of deviations Δr and Δn calculated at the given moments φ_1 and φ_2 should be used for its solution.

The equations for the determination of the velocity impulse application angle φ and the magnitudes of the transversal and radial components of the velocity impulse ΔV_t and ΔV_r are:

$$r_0(\frac{\Delta V_r}{V_0}sin(\varphi_1 - \varphi) + 2\frac{\Delta V_t}{V_0}(1 - cos(\varphi_1 - \varphi))) = \Delta r_1, \quad (11)$$

$$r_0(\frac{\Delta V_r}{V_0}sin(\varphi_2 - \varphi) + 2\frac{\Delta V_t}{V_0}(1 - cos(\varphi_2 - \varphi))) = \Delta r_2, \quad (12)$$

$$r_0(-2\frac{\Delta V_r}{V_0}(1 - cos(\varphi_1 - \varphi)) - \frac{\Delta V_t}{V_0}(3(\varphi_1 - \varphi) - 4sin(\varphi_1 - \varphi))) = \Delta n_1, \quad (13)$$

$$r_0(-2\frac{\Delta V_r}{V_0}(1 - cos(\varphi_2 - \varphi)) - \frac{\Delta V_t}{V_0}(3(\varphi_2 - \varphi) - 4sin(\varphi_2 - \varphi))) = \Delta n_2. \quad (14)$$

We determine ΔV_t in Equation (15) and ΔV_r in Equation (16) from Equations (11) and (12) and use them in Equation (14). Equation (14) is preferred over Equation (13) because it has a greater deviation of Δn than in Equation (13), which reduces the effect of the error in the determination of the value of Δn. We obtain Equation (17) for determining φ. Having determined φ, we use it in Equations (15) and (16) and determine ΔV_t and ΔV_r.

$$\Delta V_t = \frac{V_0(\Delta r_1 sin(\varphi_2 - \varphi) - \Delta r_2 sin(\varphi_1 - \varphi))}{2r_0((1 - cos(\varphi_1 - \varphi))sin(\varphi_2 - \varphi) - (1 - cos(\varphi_2 - \varphi))sin(\varphi_1 - \varphi))}, \quad (15)$$

$$\Delta V_r = \frac{V_0(\Delta r_1(1 - cos(\varphi_2 - \varphi)) - \Delta r_2(1 - cos(\varphi_1 - \varphi)))}{r_0(sin(\varphi_1 - \varphi)(1 - cos(\varphi_2 - \varphi)) - sin(\varphi_2 - \varphi)(1 - cos(\varphi_1 - \varphi)))}, \quad (16)$$

$$\Delta n_2 = -\frac{2(\Delta r_1(1-cos(\varphi_2-\varphi))-\Delta r_2(1-cos(\varphi_1-\varphi)))}{r_0(sin(\varphi_1-\varphi)(1-cos(\varphi_2-\varphi)-sin(\varphi_2-\varphi)(1-cos(\varphi_1-\varphi))}(1 - cos(\varphi_2 - \varphi)) - \frac{(\Delta r_1 sin(\varphi_2-\varphi)-\Delta r_2 sin(\varphi_1-\varphi))(3(\varphi_2-\varphi)-4sin(\varphi_2-\varphi))}{2((1-cos(\varphi_1-\varphi))sin(\varphi_2-\varphi)-(1-cos(\varphi_2-\varphi))sin(\varphi_1-\varphi))}). \quad (17)$$

Equation (17) sometimes has several solutions for this problem. In this case, the velocity impulse parameters for each solution are used in Equation (13). The solution for which the right and the left parts of Equation (13) match with higher accuracy is taken as the final solution of the problem.

By applying the calculated velocity impulse to the initial orbit, we can obtain the orbit after the maneuver. This orbit can already be used to calculate the rendezvous with this object and for other purposes.

2.3. Long-Duration Maneuver Parameters Assessment

For determination of the influence of transversal and radial thrust vector components on deviations along the radius vector and along the orbit, it is assumed that the thrust and the acceleration created by it are constant over the entire interval of the engine operation time.

First, let us find the influence of the maneuver performed with the low-thrust engine, directed along the transversal component in the orbital coordinate frame, on the deviation along the radius vector at the moment set by the angle φ_f.

It is assumed that the maneuver has the angular duration $\Delta\varphi$, the middle of the maneuver is set by the angle φ_m and the constant acceleration of the maneuvering space object within the maneuver is w_t. The key is the assumption that the maneuver is symmetric about its center. By using Equation (1), along with the assumption of maneuver symmetry, one can find the deviation along the radius produced after the ΔV_t application ($\Delta V_t = w_t \Delta t$

and Δt is the operating time of the propulsion system), which is equally distributed on the interval of the latitude argument $\Delta\varphi$:

$$\Delta r_t = 2\frac{r_0}{V_0}\frac{\Delta V_t}{\Delta\phi}\int_{-\Delta\varphi/2}^{\Delta\varphi/2}(1 - cos(\varphi_f - \varphi_m - \varphi))d\varphi = 2\frac{r_0}{V_0}\frac{\Delta V_t}{\Delta\varphi}(\Delta\varphi - 2sin\frac{\Delta\varphi}{2}cos(\varphi_f - \varphi_m)).$$

By using the equation $\Delta\varphi = \lambda_0\Delta t = k\frac{\Delta V}{V_0} = \frac{w_c}{w}\frac{\Delta V}{V_0}$, where $\lambda_0 = \frac{V_0}{r_0}$, $k = \frac{mV_0^2}{P\,r_0} = \frac{w_c}{w}$, w_c is the centripetal acceleration of the reference circular orbit ($w_c = \frac{V_0^2}{r_0}$) and w_t is the acceleration produced by the spacecraft's engine ($w = \frac{P}{m}$), we obtain the final formula for the deviation:

$$\Delta r_t = 2r_0\frac{w_t}{w_c}(\Delta\varphi - 2sin\frac{\Delta\varphi}{2}cos(\varphi_f - \varphi_m)). \quad (18)$$

Similarly, the influence of the radial component distributed on the interval of the latitude argument $\Delta\varphi$ on the deviation along the radius can be found:

$$\Delta r_r = \frac{r_0}{V_0}\frac{\Delta V_r}{\Delta\phi}\int_{-\Delta\varphi/2}^{\Delta\varphi/2}sin(\varphi_f - \varphi_m - \varphi)d\varphi = 2\frac{r_0}{V_0}\frac{\Delta V_r}{\Delta\varphi}sin\frac{\Delta\varphi}{2}sin(\varphi_f - \varphi_m),$$

$$\Delta r_r = 2r_0\frac{w_r}{w_c}sin\frac{\Delta\varphi}{2}sin(\varphi_f - \varphi_m). \quad (19)$$

By using Equation (4), we can find the influence of the maneuver performed with the low-thrust engine, the orientation of which is fixed by the transversal component in the orbital coordinate frame, on the deviation along the orbit. Just as before it is assumed that the maneuver has the angular duration $\Delta\varphi$, the middle of the maneuver is determined by the angle φ_m:

$$\begin{aligned}\Delta n_t &= -\tfrac{r_0}{V_0}\tfrac{\Delta V_t}{\Delta\phi}\int_{-\Delta\varphi/2}^{\Delta\varphi/2}(3(\varphi_f - \varphi_m - \varphi) - 4sin(\varphi_f - \varphi_m - \varphi))d\varphi \\ &= \tfrac{r_0}{V_0}\tfrac{\Delta V_t}{\Delta\phi}(3(\varphi_f - \varphi_m)\Delta\varphi - 8sin\tfrac{\Delta\varphi}{2}sin(\varphi_f - \varphi_m)),\end{aligned}$$

$$\Delta n_t = r_0\frac{w_t}{w_c}(3(\varphi_f - \varphi_m)\Delta\varphi - 8sin\frac{\Delta\varphi}{2}sin(\varphi_f - \varphi_m)). \quad (20)$$

Similarly, the influence of the radial component distributed on the interval of the latitude argument $\Delta\varphi$ on the deviation along the orbit can be found:

$$\Delta n_r = -2\frac{r_0}{V_0}\frac{\Delta V_r}{\Delta\phi}\int_{-\frac{\Delta\varphi}{2}}^{\frac{\Delta\varphi}{2}}(1 - cos(\varphi_f - \varphi_m - \varphi))d\varphi = -2\frac{r_0}{V_0}\frac{\Delta V_r}{\Delta\varphi}(\Delta\varphi + 2sin\frac{\Delta\varphi}{2}cos(\varphi_f - \varphi_m)),$$

$$\Delta n_r = -2r_0\frac{w_r}{w_c}(\Delta\varphi + 2sin\frac{\Delta\varphi}{2}cos(\varphi_f - \varphi_m)), \quad (21)$$

where φ_f is the point in which the deviations are calculated.

In the problem of determining the parameters of the maneuver performed with a low-thrust engine, there are four unknown variables φ_m, $\Delta\varphi$, w_t and w_r, hence, it is necessary to use two pairs of deviations Δr and Δn (α and δ) for its solution. By solving the equation system of four equations, in which the influence of the engines' work is determined by Equations (18)–(21), we obtain the values of all unknowns. If it is known that the object uses only transversal orientation while performing the maneuver, there are only three unknown variables φ_m, $\Delta\varphi$ and w_t. It is enough to use three deviations: Δn for the first point and Δr and Δn for the second one. The deviations along the orbit are always more preferable to use, as they can be determined in a more accurate fashion, also it is more preferable to use the deviations which correspond to the more distant point from the maneuver.

By changing the elements of the orbit at the second point by the magnitude of the calculated deviations [21], we obtain an orbit taking into account the influence of the long-term operation of the propulsion system.

3. Results

The examples with the known realized velocity impulses from Table 2 were taken in order to obtain the results which allowed us to assess the accuracy of the found solution.

Table 2. Example 1: Real maneuver parameters.

Parameter	Value
Time of the initial conditions setting	t_0 = 2022.04.11 16:37:08.950 (GMT+3)
Time of the maneuver	2022.04.12 08:18:25.000
Maneuver magnitude	$\Delta V_t = -0.112$ m/s
The angle between the maneuver and the observation	$\Delta\varphi = 178.381°$

Optical measurements were performed with errors, which, for a considered mission, did not exceed five arcseconds. It could be seen that due to these errors, the deviations measured at different time points differed, although the measurement times were very close to each other.

All observations were conducted within an interval of less than two minutes, therefore, ten observations were replaced by one average measurement.

We obtained the mean values $\Delta r = -5927$ (km) and Δn = 13,761 (km).

For comparison purpose, we calculated using Equations (1)–(4) the deviations caused by the real velocity impulse $\Delta V_t = -0.112$ m/s applied at $\Delta\varphi = 178.381°$ to be $\Delta r = -6144$ (km) and Δn = 14,176 (km).

It can be seen that the averaged deviations calculated from Table 3 prove to be quite close to the theoretical ones.

Table 3. Example 1: Series of very close measurements of the position of an object.

Date	Time	Δr	Δn
12 April 2022	20:08:18.543	−6.05975	13.73975
12 April 2022	20:08:29.543	−5.59847	13.83167
12 April 2022	20:08:40.543	−5.67197	13.78066
12 April 2022	20:08:51.543	−5.81142	13.64288
12 April 2022	20:09:02.543	−5.93997	13.86445
12 April 2022	20:09:13.543	−6.30605	13.82325
12 April 2022	20:09:24.543	−5.61857	13.73580
12 April 2022	20:09:35.543	−6.14954	13.73975
12 April 2022	20:09:46.543	−6.10882	13.66814
12 April 2022	20:09:57.543	−6.01462	13.79292

The solution using the averages of Δr and Δn was $\Delta\varphi = -178.858$ and $\Delta V_t = -0.108$ m/s. The derived maneuver magnitude and application angle are very close to the real maneuver parameters in Table 2.

Table 4 provides parameters of the second example. In this case the maneuver was not exactly an impulsive one. Instead, it took approximately 8 min. However, the maneuver could be treated as an impulsive one due to the fact that the maneuver performance duration was much shorter in comparison with the orbit period.

Table 4. Example 2: Real maneuver parameters.

Parameter	Value
Performance date	22 June 2021
Time of the maneuver start	23:17:30
Duration of the burn	465.5 s
Velocity impulse magnitude	0.246 m/s
Time of the impulsive maneuver (the middle of the burn duration)	23:21:22.75
Maneuver application angle (from the moment of the initial conditions setting)	170.75°
Acceleration	0.000529 m/s^2

The run contained ten observations, the duration of the run was 40 s, the average time of the run was 23 June 2021 20:57:52.89, the average angle was 496° (136°) and the mean values of the deviations were Δr = 0.521 km and Δn = −65.623 km.

The theoretical values of the deviations for the angle between the moment of application of the real velocity impulse and the moment of the optic observations φ = 325.031° and ΔV_t = 0.246 m/s were Δr = 1218 km and Δn = −65,130 km.

The found solution was the velocity impulse application angle of 160.49° (φ = −345.84° from the observed point) and the velocity impulse magnitude ΔV_t = 0.249 m/s.

The maneuver magnitude was determined with high accuracy in both examples, even for the second one with the continuous, although relatively short, maneuver. This meant that the semimajor axis of the orbit after the maneuver was now known practically with the same accuracy as the accuracy with which the semimajor axis of the initial orbit was determined. The error in the measurement of the semimajor axis was approximately Δa = 100 m and the error in the measurement of the eccentricity was Δe = 0.000002.

It can be noted that the deviations determined by the observations and the deviations determined analytically differ from each other especially in the second example. It is mainly related to the deviations along the radius. This is the main problem in the use of this method connected with the present errors of the observations. The given approach cannot be used when the orbit observations are situated closely to the moment of the maneuver performance. In this case the deviations caused by the velocity impulse were less than the observation errors themselves.

In the second example, the angle between the moment of the velocity impulse application and the average moment of the observations was substantial enough and comprised 325°. The considerable deviation along the orbit (the theoretical deviation was Δn = −65,130 km and the value measured with the observations was Δn = −65,623 km) was accumulated for this period of time. The 0.5 km error was not crucial compared to the deviation itself of 66.5 km. However, the error along the radius vector of magnitude 0.7 km was substantial (the theoretical deviation was Δr = 1218 km and the deviation calculated with the measurements was Δr = 0.521 km), which was commensurable with the determined value. This error caused the error in the determination of the velocity impulse application moment. Furthermore, if we just took the average value of all 10 observations from the orbit observation run, we obtained the value Δr = 0.214 km. However, the considerable negative deviation along the orbit indicated that the positive velocity impulse was performed. Hence, all deviations along the radius were positive. This allowed the removal of four observations with negative deviations. Figure 1 illustrates this discussion, as a positive transversal impulse led to the radius vector being larger than the initial radius vector at any point on the orbit. As a result, the mean deviation increased from Δr = 0.214 km to Δr = 0.521 km. While using the deviation along the orbit for the correction of the deviation along the radius vector, one should bear in mind that the arousing (after the positive transversal velocity impulse) shift along the orbit was positive at the start and then reached its maximum at $\Delta n \approx 0.4776 \frac{r_0}{V_0} \Delta V_t$ with φ = $41°24'35''$ (Figure 2) [22], then the shift started to decrease and after $\varphi = 73°05'32''$ the retardation from the unperturbed motion started. The retardation per one revolution was $\Delta n = -6\pi \frac{r_0}{V_0} \Delta V_t$. This possibility of the alteration of the sign of the deviation along the orbit needed to be accounted for. The number of deviations along the radius with different signs, as well as the magnitude and the sign of the deviation along the orbit itself were considered for the removal of the erroneous deviations along the radius.

Avoiding the use of the radial deviations was the simplest way to remove errors in the radial direction. In this case, two deviations along the orbit Δn were used. The solution for this option for Example 1 is provided below.

The real velocity impulse parameters derived from Table 2 were $\Delta V_t = -0.112$ m/s and $\varphi = 235.92°$.

The found solution was $\Delta V_t = -0.112$ m/s and $\varphi = 235.69°$.

The magnitudes of the velocity impulse coincided and the angles of application of the velocity impulse almost coincided as well. The angles φ of the velocity application differ from those provided in Table 1 ($\varphi = 178.38°$) because they are measured from the second compact group of observations additionally used in this example.

The new solution confirms the one observation series solution. The accuracy of the solution increased due to the fact that the used deviation along the orbit increased and the error in determining this value remained within the same limits. However, the solution was obtained approximately six hours later. However, even in this case, we obtained the new orbit significantly earlier in comparison to the use of the usual technique of determining the orbit with observations after the maneuver and with a higher accuracy.

4. Conclusions

In the geostationary orbit, a quite frequent situation occurs when several spacecraft which maintain their orbits via maneuvers operate at almost the same point. It is essential to quickly and accurately predict the motion of each spacecraft and its neighbors in order to firmly protect each spacecraft from collisions. A similar problem arises when the spacecraft is transferred to the operating point.

This paper describes the method that allows the assessment of the maneuvers performed in the plane of a geostationary orbit using no more than two compact measurements of the right ascension and declination angles. The provided examples demonstrate high accuracy of this method. The traditional technique usually uses four spaced revolutions in order to determine the orbit after the maneuver with tolerable accuracy. Thus, the method from this work helps to reduce the time of the spacecraft's orbit determination after the maneuver by at least two times, while the new orbit is determined with almost the same high accuracy with which it is determined before the maneuver.

Unlike traditional methods in which the orbit after the maneuver is first determined and then the maneuver parameters are estimated, in this work, the maneuver parameters are estimated directly from the measurements, and then using this information, the parameters of the formed orbit are determined. This increases the speed of determining the parameters of the formed orbit and the accuracy of determining its parameters.

Maneuver evaluation provides a double effect. One can identify the possibility of a collision with this object along with the determination of the purpose of the maneuver (maintenance of the orbit, transfer to a new operating point or transfer to a disposal orbit). The evaluation of the maneuvers of your own spacecraft makes it possible to determine the health of the propulsion system and whether the implemented maneuver matches to the planned one.

It is also possible to determine the velocity impulse caused by an imbalance in the operation of the orientation motors. As a result of this imbalance, along with the given rotation of the spacecraft, there is an undesirable movement of its center of mass. The evaluation of the emerging velocity impulse allows predicting the undesirable movement of the center of mass.

Author Contributions: Conceptualization, A.B. and V.A.; methodology, A.B.; software, N.G. and M.K.; validation, A.B. and V.A.; investigation, N.G. and M.K.; writing—original draft preparation, A.B.; writing—review and editing, V.A. All authors have read and agreed to the published version of the manuscript.

Funding: This work was supported by the Moscow Center of Fundamental and Applied Mathematics, in agreement with the Ministry of Science and Higher Education of the Russian Federation, No. 075-15-2022-283.

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: Not applicable.

Conflicts of Interest: The authors declare no conflict of interest.

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