

Article **Studies of Deformed Halo Structures of ³⁹Na and ⁴²Mg**

Qingzhen Chai ¹ , Hongxing Chen ¹ , Minghao Zha ¹ , Junchen Pei 2,* and Furong Xu [2](https://orcid.org/0000-0001-6699-0965)

- ¹ Department of Physics, Henan University of Technology, Zhengzhou 450001, China; chaiqz@haut.edu.cn (Q.C.); 201821020111@stu.haut.edu.cn (H.C.); icydonutlcp@gmail.com (M.Z.)
- ² State Key Laboratory of Nuclear Physics and Technology, School of Physics, Peking University, Beijing 100871, China; frxu@pku.edu.cn
- ***** Correspondence: peij@pku.edu.cn

Abstract: Background: The recent experimental discovery of drip-line nucleus³⁹Na has attracted great interest in theoretical studies of exotic nuclear structures in this mass region. **Methods:** We solve the Skyrme–Hartree–Fock–Bogoliubov (Skyrme-HFB) equation within deformed coordinatespaces. The present approach is suitable for descriptions of weakly bound deformed nuclei with continuum effects and deformed halo structures. **Results:** The systematical two-neutron separation energies are obtained with the $\text{SkM}_{\text{ext1}}^*$ and $\text{UNEDF0}_{\text{ext1}}$ forces for Na and Mg isotopes close to the neutron drip line. The density distributions show that ³⁹Na and ⁴²Mg have deformed halo structures. Furthermore, there are significant influences of various pairing interactions on halo shapes at large distances. **Conclusions:** Both ³⁹Na and ⁴²Mg are very weakly bound with well prolate deformed cores. However, their surface halo structures are dependent on the choices of pairing interactions. The volume-type pairing interaction tends to predict a prolate deformed halo, while the halo deformations at large distances are reduced by adopting the surface pairing. We demonstrate that 39 Na and $42Mg$ are promising candidates for two-neutron deformed halo nuclei.

Keywords: deformed halos; Hartree–Fock–Bogoliubov; pairing interaction

Citation: Chai, Q.; Chen, H.; Zha, M.; Pei, J.; Xu, F. Studies of Deformed Halo Structures of ³⁹Na and ⁴²Mg. *Symmetry* **2022**, *14*, 215. [https://](https://doi.org/10.3390/sym14020215) doi.org/10.3390/sym14020215

Academic Editors: Floyd W. Stecker and Ignatios Antoniadis

Received: 17 December 2021 Accepted: 20 January 2022 Published: 23 January 2022

Publisher's Note: MDPI stays neutral with regard to jurisdictional claims in published maps and institutional affiliations.

Copyright: © 2022 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license [\(https://](https://creativecommons.org/licenses/by/4.0/) [creativecommons.org/licenses/by/](https://creativecommons.org/licenses/by/4.0/) $4.0/$).

1. Introduction

Exploring exotic nuclear structures far away from the *β*-decay stable line is one of the major scientific goals in rare-isotope beam facilities around the world. Of particular interest is the halo structure, which is characterized by significant extensions of neutron or proton density distributions at nuclear surfaces [\[1–](#page-6-0)[6\]](#page-6-1). There were extensive experimental studies to look for halo structures [\[7\]](#page-6-2). Some one-neutron and two-neutron halos have been discovered by experiments, which have attracted strong theoretical interests. However, deformed halo structures are rare, and two-neutron deformed halos are still absent in nature.

Recently, the drip-line nuclei ${}^{31}F$ and ${}^{34}Ne$ have been reconfirmed by experiments in RIKEN [\[8\]](#page-6-3). In this experiment, one event of 39 Na was also observed. 39 Na is mostly likely to be the drip line of sodium isotopes and is a promising two-neutron halo nucleus. Besides, the neighboring 40 Mg has been observed [\[9\]](#page-6-4) and the drip line of magnesium is still not known. Both ³⁹Na and ⁴⁰Mg have a magic neutron number $N = 28$, but this neuron shell is broken near the neutron drip line $[10]$. Previous theoretical studies have consistently predicted that ⁴⁰Mg is a prolate deformed nuclei [\[10](#page-6-5)[–12\]](#page-6-6). In deformed weakly bound nuclei, the interplay between continuum, pairing correlations, halo structures and deformations is particularly intriguing.

The suitable microscopic tool for descriptions of weakly bound deformed nuclei in the framework of nuclear density functional theory is the coordinate-space formulation of the Hartree–Fock–Bogoliubov (HFB) method [\[13,](#page-6-7)[14\]](#page-6-8). The ab initio methods can also describe weakly bound spherical nuclei, but this is difficult for deformed shapes. The coordinatespace HFB can discretize the quasiparticle continuum and resonances rather precisely and is able to describe deformed halo asymptotics within large boxes. In HFB calculations, Skyrme forces [\[15\]](#page-6-9) are commonly adopted as non-relativistic effective nucleon–nucleon interactions. There are many Skyrme parameterizations developed with different physics purposes. For example, SkM^{*} forces [\[16\]](#page-6-10) are widely used in fission studies for reasonable descriptions of fission barriers. SLy4 force [\[17\]](#page-6-11) has good isotopic properties towards drip lines and is widely used in descriptions of neutron stars. UNEDF0 force [\[18\]](#page-6-12) has a very small rms of 1.455 MeV regarding global binding energies. Furthermore, extended Skyrme forces with additional higher-order density dependent terms are expected to improve the global theoretical descriptions [\[19,](#page-6-13)[20\]](#page-6-14). Generally, different effective interactions can lead to very different drip-line locations although they all are good at descriptions of stable nuclei.

In the present work, we aim to study deformed halo structures of 39 Na and 42 Mg. The calculations are performed based on the self-consistent Skyrme-HFB approach in a large, axially symmetric coordinate-space. We expect the two-neutron deformed halos could be confirmed experimentally in the near future.

2. The Theoretical Framework

The Skyrme-HFB calculations are performed with the HFB-AX solver based on the B-spline method in a two-dimensional coordinate space [\[13\]](#page-6-7). Such an approach has been successfully used to describe the properties of weakly bound deformed nuclei around the neutron drip line. Generally, with a sufficiently large box and small mesh sizes, the description of the continuum spectrum would be very precise [\[14\]](#page-6-8). In present calculations, we adopt a coordinate-space of 21 fm and a maximum lattice mesh of 0.6 fm, respectively.

The HFB equation in the coordinate-space can be written as [\[13,](#page-6-7)[21\]](#page-6-15):

$$
\begin{bmatrix} h(r) - \lambda & \Delta(r) \\ \Delta^*(r) & h(r) - \lambda \end{bmatrix} \begin{bmatrix} U_k(r) \\ V_k(r) \end{bmatrix} = E_k \begin{bmatrix} U_k(r) \\ V_k(r) \end{bmatrix},
$$
\n(1)

where *h* is the single-particle Hamiltonian; λ is the Fermi surface energy or chemical potential; Δ is the pairing potential; U_k and V_k are the upper and lower components of quasi-particle wave functions; and *E^k* denotes the quasi-particle energy. For the particle-hole interaction channel, the SkM^{*}_{ext1} and UNEDF0_{ext1} [\[20\]](#page-6-14) forces are adopted in $h(r)$, respectively. Note that SkM^*_{ext1} and $\overline{UNEDF0_{ext1}}$ forces include an additional higher-order density-dependent term as in Ref. [\[19\]](#page-6-13). In the particle-particle channel, a density-dependent pairing interaction is adopted as follows [\[22](#page-6-16)[,23\]](#page-6-17),

$$
\Delta(r) = V_0 \left\{ 1 - \eta \left[\frac{\rho(r)}{\rho_0(r)} \right]^{\gamma} \right\},\tag{2}
$$

where $\rho_0(r)$ is the nuclear saturation density of 0.16 fm³ and *V*₀, *η*, γ are three adjustable parameters. For different *η*, *γ*, there are four types of pairing interactions:

- 1. Volume pairing: $n = 0$;
- 2. Surface pairing: $\eta = 1, \gamma = 1$;
- 3. Mix pairing: *η* = 0.5, *γ* = 1;
- 4. New pairing: $\eta = 0.8$, $\gamma = 0.7$.

With a pairing window of 60 MeV, we adjusted pairing strength V_0 with the SkM $_{\text{ext1}}^*$ force to regularize the neutron pairing gap of ¹²⁰Sn to be $\Delta_n = 1.245$ MeV. The adjusted pairing strengths V_0 are 146.5 MeV, 467.5 MeV, 233.5 MeV, and 389.8 MeV corresponding to volume, surface, mix, and new pairing types, respectively. Note that results with different pairing interactions are more or less similar for stable nuclei. However, the pairing gaps could be very different towards drip lines by using different pairing interaction forms [\[23\]](#page-6-17). The surface pairing would lead to very large pairing gaps towards the neutron drip line, while volume pairing is on the contrary. The mixing pairing and new pairing are between the surface and volume pairings [\[23\]](#page-6-17).

The particle density $\rho(r)$ and the pairing density $\tilde{\rho}(r)$ of even–even nuclei are expressed as,

$$
\rho(r) = 2 \sum_{k} V_k^*(r) V_k(r),
$$

\n
$$
\tilde{\rho}(r) = -2 \sum_{k} V_k(r) U_k^*(r),
$$
\n(3)

where in the sum, the quasiparticle energy cutoff is taken as $(60 - \lambda)$ MeV. To describe the odd-mass nuclei, the quasiparticle blocking is invoked. By blocking quasiparticles at the state μ , the density ρ_{B}^{μ} \int_B^{μ} (r) and the pairing density $\tilde{\rho}_{B}^{\mu}$ $B_{B}^{\mu}(r)$ of odd-mass nuclei become [\[24\]](#page-6-18),

$$
\rho_B^{\mu}(r) = U_{\mu}(r)U_{\mu}^*(r) + V_{\mu}^*(r)V_{\mu}(r) + 2\sum_{k \neq \mu} V_k^*(r)V_k(r),
$$

\n
$$
\tilde{\rho}_B^{\mu}(r) = -2\sum_{k \neq \mu} V_k(r)U_k^*(r).
$$
\n(4)

Besides, the particle number equation for the odd-mass nuclei has to be modified. The quasiparticle blocking is necessary for descriptions of ³⁹Na.

3. Results and Discussion

Figure [1](#page-3-0) shows the two-neutron separation energies S_{2n} of sodium isotopes and magnesium isotopes based on the $\text{SkM}_{\text{ext1}}^*$ force and $\text{UNEDF0}_{\text{ext1}}$ force with the new pairing interaction. As expected, calculated S_{2n} decreases gradually towards the drip line. In both calculations, 39 Na is the drip line nucleus. In experiments, 39 Na is mostly likely to be the drip line since only one event was observed [\[8\]](#page-6-3). For magnesium isotopes, SkM $_{\text{ext1}}^*$ predicts ⁴²Mg as the drip-line nucleus, while UNEDF0_{ext1} predicts ⁴⁴Mg as the drip line. UNEDF0_{ext1} also predicts a slightly larger S_{2n} for ³⁹Na than SkM $_{\text{ext1}}^*$. In fact, the drip-line locations of Na and Mg isotopes are very different by using various mass models. In the macroscopic–microscopic model, the drip line locations are at 39 Na and 40 Mg [\[25\]](#page-6-19). The WS4 model predicts that 37 Na and 40 Mg are the drip line nuclei [\[26\]](#page-6-20), which slightly underestimates the neutron drip lines. The Gogny-HFB calculations with the D1S force [\[27\]](#page-6-21) predict that the drip lines are at 35 Na and 40 Mg. The HFB-21 calculations [\[28\]](#page-6-22) predict 37 Na and 42 Mg. The Relativistic-Hartree–Bogoliubov [\[29\]](#page-7-0) predict 45 Na and 46 Mg as the drip lines, which significantly overestimate the neutron drip lines. Thus, ³⁹Na as the drip line provides a stringent constraint on various theoretical models. With this constraint, we can more confidently infer that ^{42}Mg is also weakly bound. However, it is difficult to predict whether 44 Mg barely exists or not.

The halo properties of 39 Na and 42 Mg are dependent on the choices of Skyrme forces. We display the rms radii of neutron distributions in Na and Mg isotopes in Figure [2.](#page-3-1) As shown in Figure [2,](#page-3-1) the neutron radii by $\text{UNEDF0}_{\text{ext1}}$ force increase almost linearly with increasing neutron numbers. It can be seen that the neutron radii by $\text{SkM}_{\text{ext1}}^*$ are particularly enhanced for ³⁹Na. Note that UNEDF0_{ext1} predicts larger S_{2n} than that by SkM_{ext1}. Thus the larger neutron radii by $\text{UNEDF0}_{\text{ext1}}$ are not due to weakly bound effects, but due to larger deformations.

Figure 1. The two-neutron separation energies S_{2n} of Na and Mg isotopes obtained by the SkM $_{\rm ext1}^*$ force and UNEDF0_{ext1} force with the new pairing interaction. Experimental data labeled by solid symbols are taken from Ref. [\[30\]](#page-7-1).

Figure 2. The calculated r.m.s. neutron radius in Na (**a**) and Mg (**b**) isotopes. Calculations are performed with $\text{SkM}_{\text{ext1}}^*$ and $\text{UNEDF0}_{\text{ext1}}$ forces, respectively.

It is expected that the pairing is crucial for descriptions of weakly bound nuclei. Presently, four types of pairing interactions are used: volume, surface, mix, and new pairing interactions. For ³⁹Na, the neutron Fermi energy λ_n and two-neutron separation energies S_{2n} are −0.20 MeV (0.82 MeV), −0.48 MeV (0.67 MeV), −0.16 MeV (0.63 MeV), and −0.26 MeV (0.27 MeV), corresponding to volume, surface, mix, and new pairing interactions, respectively. Figure [3](#page-4-0) displays the deformed halo structures of density distributions by different pairing interactions and the SkM $_{\rm ext1}^*$ force for 39 Na. The mixed pairing results

in the highest neutron Fermi surface energy λ_n , and it predicts the most significant halo structures. On the contrary, the surface pairing has a lowest λ_n , and it is less weakly bound with a less prominent halo structure. It is understandable that the surface pairing can bring more binding effects for weakly bound nuclei. The density distributions are shown in the cylindrical coordinates. The differences along *z*-axis and *r*-axis reflect the deformation effects in density distributions. We see that all the core density distributions have good prolate deformations. However, the surface halo shapes are very different. The surface pairing leads to the smallest surface deformations, while the volume pairing leads to the largest surface deformations. Hence, the surface halo deformations are very much dependent on the density dependencies of pairing interactions.

Figure 3. The density distributions of ³⁹Na obtained with various pairing interactions in the cylindrical coordinate spaces *ρ*(*r*, *z*). Panel lists as (**a**) volume, (**b**) surface, (**c**) mix, and (**d**) new pairing interactions. The solid lines and dotted lines denote the density profiles along *z*-axis and *r*-axis.

Figure [4](#page-5-0) shows the density distributions of ^{42}Mg by calculations with the Sk M_{ext1}^{*} force and various pairing interactions. Note that ⁴²Mg is slightly unbound with volume and mixing pairing interactions. For ^{42}Mg , the neutron Fermi energy λ_n (and two-neutron separation energies S_{2n}) is −0.43 MeV (−0.20 MeV), −0.50 MeV (0.66 MeV), −0.37 MeV (−0.12 MeV), and −0.32 MeV (0.29 MeV), corresponding to volume, surface, mix, and new pairing interactions, respectively. It is consistent with ³⁹Na that the surface pairing results in more binding energies and could slightly extend the neutron drip line. The core densities all have good prolate deformations. However, the surface halos by surface pairing and new pairing are almost spherical. This is similar to the situation of 39 Na. The different deformation between the core and the halo in weakly bound nuclei is called shape decoupling effect. Exotic shape decoupling effects in deformed halo nuclei are of strong interest [\[31\]](#page-7-2). The pygmy dipole resonance is a possible probe of shape decoupling effects [\[23\]](#page-6-17).

Figure 4. Similar to Figure [3,](#page-4-0) but for ^{42}Mg . Panel lists as (a) volume, (b) surface, (c) mix, and (d) new pairing interactions.

4. Conclusions

We have studied the deformed halo structures of drip-line nuclei 39 Na and 42 Mg, based on coordinate-space Skyrme-HFB calculations. The experimental discovery of ³⁹Na provides a stringent constraint on various theoretical models. With this constraint, we infer that ⁴²Mg is weakly bound. The problem is quite interesting because two-neutron deformed halos have still not been discovered yet. Both 39 Na and 42 Mg have a prolate deformed core, but their halo deformations could be different by adopting different pairing interactions. The surface pairing can bring more binding energies to weakly bound nuclei and tends to obtain spherical halos. The volume pairing results in significant deformed halos. In conclusion, the surface halo shapes are very much dependent on the density dependencies of pairing interactions.

Author Contributions: Conceptualization, J.P.; methodology, J.P.; software, J.P.; validation, J.P. and F.X.; formal analysis, Q.C.; investigation, Q.C.; resources, Q.C.; data curation, Q.C.; writing—original draft preparation, Q.C.; writing—review and editing, J.P.; visualization, H.C. and M.Z.; supervision, Q.C.; project administration, J.P.; funding acquisition, Q.C., J.P. and F.X. All authors have read and agreed to the published version of the manuscript.

Funding: This research was funded by the National Natural Science Foundation of China under Grants No. 12047504, 11975032, 11835001, and 11961141003, the China Postdoctoral Science Foundation under Grant No. 2020M670012, the National Key R&D Program of China (Contract No. 2018YFA0404403), and the Launching Fund of Henan University of Technology (31401409).

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: Data is contained within the article. If you could not reproduce any figures, please contact chaiqz@haut.edu.cn.

Acknowledgments: The authors acknowledge the computations in the present calculations by using Tianhe-1A located in Tianjin.

Conflicts of Interest: The authors declare no conflict of interest.

References

- 1. Tanihata, I. Structure of neutron-rich nuclei studied by radioactive beams: Neutron halo and soft E1 excitation. *Nucl. Phys. A* **1991**, *522*, 275c–292c. [\[CrossRef\]](http://doi.org/10.1016/0375-9474(91)90062-B)
- 2. Fukuda, M.; Ichihara, T.; Inabe, N.; Kubo, T.; Kumagai, H.; Nakagawa, T.; Yano, Y.; Tanihata, I.; Adachi, M.; Asahi, K.; et al. Neutron halo in ¹¹Be studied via reaction cross sections. *Phys. Lett. B* **1991**, *268*, 339–344. [\[CrossRef\]](http://dx.doi.org/10.1016/0370-2693(91)91587-L)
- 3. Meng, J.; Ring, P. Relativistic Hartree-Bogoliubov description of the neutron halo in ¹¹Li. *Phys. Rev. Lett.* **1996**, *77*, 3963–3966. [\[CrossRef\]](http://dx.doi.org/10.1103/PhysRevLett.77.3963) [\[PubMed\]](http://www.ncbi.nlm.nih.gov/pubmed/10062353)
- 4. Nakamura, T.; Kobayashi, N.; Kondo, Y.; Satou, Y.; Aoi, N.; Baba, H.; Deguchi, S.; Fukuda, N.; Gibelin, J.; Inabe, N.; et al. Halo structure of the island of inversion nucleus ³¹Ne. *Phys. Rev. Lett.* **2009**, *103*, 262501. [\[CrossRef\]](http://dx.doi.org/10.1103/PhysRevLett.103.262501)
- 5. Kobayashi, N.; Nakamura, T.; Kondo, Y.; Tostevin, J.A.; Utsuno, Y.; Aoi, N.; Baba, H.; Barthelemy, R.; Famiano, M.A.; Fukuda, N.; et al. Observation of a *p*-wave one-neutron halo configuration in ³⁷Mg. *Phys. Rev. Lett.* **2014**, *112*, 242501. [\[CrossRef\]](http://dx.doi.org/10.1103/PhysRevLett.112.242501)
- 6. Jian, H.; Gao, Y.; Dai, F.; Liu, J.; Xu, X.; Yuan, C.; Kaneko, K.; Sun, Y.; Liang, P.; Shi, G.; et al. *β*-delayed *γ* emissions of ²⁶P and its mirror asymmetry. *Symmetry* **2021**, *13*, 2278. [\[CrossRef\]](http://dx.doi.org/10.3390/sym13122278)
- 7. Tanihata, I.; Savajols, H.; Kanungo, R. Recent experimental progress in nuclear halo structure studies. *Prog. Part. Nucl. Phys.* **2013**, *68*, 215–313. [\[CrossRef\]](http://dx.doi.org/10.1016/j.ppnp.2012.07.001)
- 8. Ahn, D.S.; Fukuda, N.; Geissel, H.; Inabe, N.; Iwasa, N.; Kubo, T.; Kusaka, K.; Morrissey, D.J.; Murai, D.; Nakamura, T.; et al. Location of the neutron dripline at fluorine and neon. *Phys. Rev. Lett.* **2019**, *123*, 212501. [\[CrossRef\]](http://dx.doi.org/10.1103/PhysRevLett.123.212501) [\[PubMed\]](http://www.ncbi.nlm.nih.gov/pubmed/31809143)
- 9. Baumann, T.; Amthor, A.M.; Bazin, D.; Brown, B.A.; Folden, C.M., III; Gade, A.; Ginter, T.N.; Hausmann, M.; Matoš, M.; Morrissey, D.J.; et al. Discovery of ⁴⁰Mg and ⁴²Al suggests neutron drip-line slant towards heavier isotopes. *Nature* **2007**, *449*, 1022–1024. [\[CrossRef\]](http://dx.doi.org/10.1038/nature06213)
- 10. Chai, Q.Z.; Pei, J.C.; Fei, N.; Guan, D.W. Constraints on the neutron drip line with the newly observed ³⁹Na. *Phys. Rev. C* **2020**, *102*, 014302. [\[CrossRef\]](http://dx.doi.org/10.1103/PhysRevC.102.014312)
- 11. Terasaki, J.; Flocard, H.; Heenen, P.H.; Bonche, P. Deformation of nuclei close to the two-neutron drip line in the Mg region. *Nucl. Phys. A* **1997**, *621*, 706–718. [\[CrossRef\]](http://dx.doi.org/10.1016/S0375-9474(97)00183-8)
- 12. Crawford, H.L.; Fallon, P.; Macchiavelli, A.O.; Doornenbal, P.; Aoi, N.; Browne, F.; Campbell, C.M.; Chen, S.; Clark, R.M.; Cortés, M.L.; et al. First spectroscopy of the near drip-line nucleus ⁴⁰Mg. *Phys. Rev. Lett.* **2019**, *122*, 052501. [\[CrossRef\]](http://dx.doi.org/10.1103/PhysRevLett.122.052501) [\[PubMed\]](http://www.ncbi.nlm.nih.gov/pubmed/30822018)
- 13. Pei, J.C.; Stoitsov,M.V.; Fann, G.I.; Nazarewicz, W.; Schunck, N.; Xu, F.R. Deformed coordinate-space Hartree-Fock-Bogoliubov approach to weakly bound nuclei and large deformations. *Phys. Rev. C* **2008**, *78*, 064306. [\[CrossRef\]](http://dx.doi.org/10.1103/PhysRevC.78.064306)
- 14. Zhang, Y.N.; Pei, J.C.; Xu, F.R. Hartree-Fock-Bogoliubov descriptions of deformed weakly bound nuclei in large coordinate spaces. *Phys. Rev. C* **2013**, *88*, 054305. [\[CrossRef\]](http://dx.doi.org/10.1103/PhysRevC.88.054305)
- 15. Skyrme, T. CVII. The nuclear surface. *Phil. Mag.* **1956**, *1*,1043–1054. [\[CrossRef\]](http://dx.doi.org/10.1080/14786435608238186)
- 16. Bartel, J.; Quentin, P.; Brack, M.; Guet, C.; Håkansson, H.B. Towards a better parametrisation of Skyrme-like effective forces: A critical study of the SkM force. *Nucl. Phys. A* **1982**, *386*, 79–100. [\[CrossRef\]](http://dx.doi.org/10.1016/0375-9474(82)90403-1)
- 17. Chabanat, E.; Bonche, P.; Haensel, P.; Meyer, J.; Schaeffer, R. A Skyrme parametrization from subnuclear to neutron star densities Part II. Nuclei far from stabilities. *Nucl. Phys. A* **1998**, *635*, 231–256. [\[CrossRef\]](http://dx.doi.org/10.1016/S0375-9474(98)00180-8)
- 18. Kortelainen, M.; Lesinski, T.; Moré, J.; Nazarewicz, W.; Sarich, J.; Schunck, N.; Stoitsov, M.V.; Wild, S. Nuclear energy density optimization. *Phys. Rev. C* **2010**, *82*, 024313. [\[CrossRef\]](http://dx.doi.org/10.1103/PhysRevC.82.024313)
- 19. Xiong, X.Y.; Pei, J.C.; Chen, W.J. Extension and parametrization of high-order density dependence in Skyrme forces. *Phys. Rev. C* **2016**, *93*, 024311. [\[CrossRef\]](http://dx.doi.org/10.1103/PhysRevC.93.024311)
- 20. Zuo, Z.W.; Pei, J.C., Xiong, X.Y.; Zhu, Y. Global analysis of Skyrme forces with high-order density dependencies. *Chin. Phys. C* **2018**, *42*, 064106. [\[CrossRef\]](http://dx.doi.org/10.1088/1674-1137/42/6/064106)
- 21. Dobaczewski, J.; Flocard, H.; Treiner, J. Hartree-Fock-Bogoliubov description of nuclei near the neutron-drip line. *Nucl. Phys. A* **1984**, *422*, 103–139. [\[CrossRef\]](http://dx.doi.org/10.1016/0375-9474(84)90433-0)
- 22. Pastore, A.; Barranco, F.; Broglia, R.A.; Vigezzi, E. Microscopic calculation and local approximation of the spatial dependence of the pairing field with bare and induced interactions. *Phys. Rev. C* **2008**, *78*, 024315. [\[CrossRef\]](http://dx.doi.org/10.1103/PhysRevC.78.024315)
- 23. Wang, K.; Kortelainen, M.; Pei, J.C. Probing surface quantum flows in deformed pygmy dipole modes. *Phys. Rev. C* **2017**, *96*, 031301(R). [\[CrossRef\]](http://dx.doi.org/10.1103/PhysRevC.96.031301)
- 24. Bertsch, G.; Dobaczewski, J.; Nazarewicz, W.; Pei, J.C. Hartree-Fock-Bogoliubov theory of polarized Fermi systems. *Phys. Rev. A* **2009**, *79*, 043602. [\[CrossRef\]](http://dx.doi.org/10.1103/PhysRevA.79.043602)
- 25. Mo¨ller, P.; Sierk, A.J.; Ichikawa, T.; Sagawa, H. Nuclear ground-state masses and deformations: FRDM(2012). *At. Data Nucl. Data Tables* **2016**, *109*, 1–204. [\[CrossRef\]](http://dx.doi.org/10.1016/j.adt.2015.10.002)
- 26. Wang, N.; Liu, M.; Wu, X.Z.; Meng, J. Surface diffuseness correction in global mass formula. *Phys. Lett. B* **2014**, *734*, 215–219. [\[CrossRef\]](http://dx.doi.org/10.1016/j.physletb.2014.05.049)
- 27. Hilaire, S.; Girod, M. Large-scale mean-field calculations from proton to neutron drip lines using the D1S Gogny force. *Eur. Phys. J. A* **2007**, *33*, 237–241. [\[CrossRef\]](http://dx.doi.org/10.1140/epja/i2007-10450-2)
- 28. Goriely, S.; Chamel, N.; Pearson, J.M. Further explorations of Skyrme-Hartree-Fock-Bogoliubov mass formulas. XII. Stiffness and stability of neutron-star matter. *Phys. Rev. C* **2010**, *82*, 035804. [\[CrossRef\]](http://dx.doi.org/10.1103/PhysRevC.82.035804)
- 29. Xia, X.W.; Lim, Y.; Zhao, P.W.; Liang, H.Z.; Qu, X.Y.; Chen, Y.; Liu, H.; Zhang, L.F.; Zhang, S.Q.; Kim, Y.; et al. The limits of the nuclear landscape explored by the relativistic continuum Hartree CBogoliubov theory. *At. Data Nucl. Data Tables* **2018**, *121*, 1–215. [\[CrossRef\]](http://dx.doi.org/10.1016/j.adt.2017.09.001)
- 30. Wang, M.; Audi, G.; Kondev, G.G.; Huang, W.J.; Naimi, S.; Xu, X. The AME2016 atomic mass evaluation (II). Tables, graphs and references. *Chin. Phys. C* **2017**, *41*, 030003. [\[CrossRef\]](http://dx.doi.org/10.1088/1674-1137/41/3/030003)
- 31. Zhou, S.G.; Meng, J.; Ring, P.; Zhao, E.G. Neutron halo in deformed nuclei. *Phys. Rev. C* **2010**, *82*, 011301. [\[CrossRef\]](http://dx.doi.org/10.1103/PhysRevC.82.011301)