Article

The Cosmological Constant as Event Horizon

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Abstract: General Relativity allows for a cosmological constant (Λ) which has inspired models of cosmic Inflation and Dark Energy. We show instead that $r_\Lambda = \sqrt{3/\Lambda}$ corresponds to an event horizon: a causal boundary term in the action. Our Universe is expanding inside its Schwarzschild radius $r_S = r_\Lambda = 2GM$, which could have originated from a uniform free falling cloud of mass M that collapsed as a Black Hole (BH) 25 Gyrs ago. Such a BH Universe allows for large-scale structure formation without the need of Inflation or Dark Energy.

Keywords: cosmology; Dark Energy; general relativity; black holes

1. Introduction

The cosmological constant Λ has a very interesting history. For a review, see [1–3]. In 1917, right after the first publications of General Relativity (GR) [4], Albert Einstein applied GR to the Universe as a whole [5]. This was before the discovery of cosmic expansion [6]. Einstein realized that to account for the whole Universe, we need to include a cosmological constant, Λ, as a new universal constant of nature. At that time, most scientists believed that the Universe has to be in static equilibrium on large scales. This is what is usually assumed for large physical systems that are allowed to evolve in isolation for a long time. However, gravity is always attractive and it is therefore hard to imagine how you can reach such an equilibrium on the largest scales where other forces are negligible. This is why Einstein had to invoke Λ.

We can see this by considering the GR version of the Poisson equation for a perfect fluid with density ρ and pressure p (see Equation (12) in [7]):

$$\nabla_\mu g^\mu = \frac{d\Theta}{d\tau} + \frac{1}{3}\Theta^2 = R_{\mu\nu}u^\mu u^\nu = \Lambda - 4\pi G(\rho + 3p) \tag{1}$$

where g^μ is the geodesic acceleration [8]. This is also the Raychaudhuri equation for a shear-free, non-rotating fluid where $\Theta = \nabla_\nu u^\nu$ and u^ν is the 4-velocity. The above equation is purely geometric: it describes the evolution in proper time τ of the dilatation coefficient Θ of a bundle of nearby geodesics. Note that without Λ, the acceleration is always negative and there is no way to reach an equilibrium, unless $p < -1/3\rho$ which is what we call Dark Energy today.

The Λ term in the above equation is equivalent to a vacuum or a ground state of constant density (and negative pressure). It also corresponds to a centrifugal (or Hooke's) force [9,10] in the Newtonian potential (or metric element): $2\Phi = -\Lambda r^2/3$ that opposes Newtonian gravity, $2\Phi = -2GM/r$. Equilibrium happens when both forces are equal, similar to the circular Kepler orbits in Newtonian dynamics.

The Λ term was mostly abandoned after the Hubble–Lemaitre expansion law was accepted, which took some time [6]. In 1931, an unpublished manuscript of Einstein [11] considered the possibility of a Universe that expands with Λ but remains static due to a con-tinuous formation of matter from empty space, as in the Steady State Cosmology ([12,13]). This has the advantage of a Perfect Cosmological Principle, where the universe is homoge-neous, isotropic and static. This is in better agreement with the relativity principle than

the regular Cosmological Principle, which only applies to space. The key to this idea is the static deSitter (dS) metric:

$$2\Phi = 2\Psi = -r^2/r_*^2 \tag{2}$$

where r_* is a constant that we will identify later as $r_* = r_\Lambda \equiv 1/H_\Lambda \equiv \sqrt{\Lambda/3}$ in our Universe. Φ and Ψ are the gravitational potentials of a generic metric with spherical symmetry in physical or Schwarzschild (SW) coordinates (t, r, θ, φ) [8,14]:

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = -(1 + 2\Psi)dt^2 + \frac{dr^2}{1 + 2\Phi} + r^2 d\Omega^2 \tag{3}$$

where we have introduced the solid angle: $d\Omega^2 = d\theta^2 + \sin\theta^2 d\varphi^2$.

The idea of the Steady State Cosmology solution was rejected because of the lack of observational evidence for continuous matter creation (to compensate the dilution by the expansion). However, this was a misunderstanding. There is no need for ad hoc matter creation to reconcile the observed cosmic expansion with Λ. The frame duality [15]:

$$-d\tau^2 + a^2(\tau)d\chi^2 + r^2 d\Omega^2 = -(1 + 2\Psi)dt^2 + \frac{dr^2}{1 - r^2 H^2} + r^2 d\Omega^2 \tag{4}$$

shows how the Friedmann–Lemaitre–Robertson–Walker (FLRW) in comoving coordinates $(\tau, \chi, \theta, \varphi)$ (for an observer comoving with the Hubble–Lemaitre expansion law) is the same time a quasi-static Universe in the SW frame. This is similar to the dS metric but with $H = H(\tau) = H(t, r)$. Under this duality, the Steady State Cosmology is not so different from the Big Bang FLRW expansion with Λ. Indeed, our FLRW metric (to the left of Equation (4) above) asymptotically approaches the dS metric in Equation (2). As H becomes dominated by Λ, the expansion becomes static (see also [16]) in the SW frame.

Regardless of these considerations, it turns out that we need a Λ term again today to explain the latest measurements of cosmic accelerated expansion [17,18]. An effective Λ term, similar to Dark Energy, might also be neeeded to understand cosmic Inflation ([19–22]). The Big Bang, Dark Energy, Λ, Inflation and BHs are puzzles we do not yet understand at any fundamental level. Are they somehow connected? What is the meaning of the measured Λ?

2. The Action of GR and the Λ Term

Consider the Einstein–Hilbert action ([8,23]):

$$S = \int_{V_4} dV_4 \left[\frac{R - 2\Lambda}{16\pi G} + \mathcal{L} \right], \tag{5}$$

where $dV_4 = \sqrt{-g}d^4x$ is the invariant volume element, V_4 is the volume of the 4D spacetime manifold, $R = R_\mu^\mu = g^{\mu\nu}R_{\mu\nu}$ is the Ricci scalar curvature and $\mathcal{L}$ the Lagrangian of the energy–matter content. We can obtain Einstein's field equations (EFE) for the metric field $g_{\mu\nu}$ from this action by requiring S to be stationary $\delta S = 0$ under arbitrary variations of the metric $\delta g^{\mu\nu}$. The solution is ([4,8]):

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G\, T_{\mu\nu} \equiv -\frac{16\pi G}{\sqrt{-g}} \frac{\delta(\sqrt{-g}\mathcal{L})}{\delta g^{\mu\nu}}, \tag{6}$$

where $G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R$. For a perfect fluid in spherical coordinates:

$$T_{\mu\nu} = (\rho + p)u_\mu u_\nu + p g_{\mu\nu} \tag{7}$$

where ρ, and p are the energy–matter density and pressure. This fluid can be made of several components, each with a different equation of state $p = \omega\rho$. For such a perfect fluid, the solution that minimizes the action (called action on-shell S^{on-sh}) results in a boundary term [7]:

$$S^{on-sh} = \int_{V_4} dV_4 \frac{\nabla_\mu g^\mu}{8\pi G} = \oint_{\partial V_4} \frac{dV_\mu g^\mu}{8\pi G} = < \frac{\Lambda}{4\pi G} - (\rho + 3p) >_{V_4} \tag{8}$$

where, in the last step, we have used the Raychaudhuri equation (Equation (1)). This is the situation when the evolution happens inside a BH. We want this boundary term to vanish (see below), so that its contribution to the action on-shell is zero: $S^{on-sh} = 0$ [7]. The observational fact that $\Lambda \neq 0$ implies that we live inside a boundary and to cancel the boundary term we need:

$$\Lambda = 4\pi G < (\rho + 3p) >_{V_4} \tag{9}$$

which basically tells us that Λ is just given by the matter content inside such boundary. This is similar to what happens with the event horizon of a BH. This gives an answer to the so-called coincidence problem [3]: the coincidence between the value of $\rho_\Lambda \equiv \Lambda/(8\pi G)$ and ρ matter today [7,10].

Another way to see this is to note that Equation (6) requires that boundary terms vanish (e.g., see [8,24,25]). Otherwise, we need to add a Gibbons–Hawking–York (GHY) boundary term [26–28] to the action:

$$S = \int_{V_4} dV_4 \left[\frac{R - 2\Lambda}{16\pi G} + \mathcal{L} \right] + \frac{1}{8\pi G} \oint_{\partial V_4} d^3y \sqrt{-h}\, K. \tag{10}$$

where K is the trace of the extrinsic curvature at the boundary ∂V_4 and h is the induced metric. We will show explicitly in Section 2.2 that the GHY boundary results in a Λ term when the evolution happens following an FLRW metric inside an expanding BH event horizon. To cancel the GHY term, we need $r_\Lambda = r_S$. That Λ is a GHY term was first proposed in [7] and has also been later interpreted as a boundary entropy term by [29–31].

2.1. The Black Hole Universe (BHU) Solution

Observations show that the expansion rate today is dominated by Λ. This indicates that the FLRW metric is inside a trapped surface $r_\Lambda \equiv 1/H_\Lambda = (8\pi G\rho_\Lambda/3)^{-1/2}$, which behaves like the interior of a BH. To see this, consider the maximum distance r_* traveled by an outgoing radial null geodesic (the Event Horizon at τ, [32]) in the FLRW metric:

$$r_* \equiv a\chi_* = a \int_\tau^\infty \frac{d\tau}{a(\tau)} = a \int_a^\infty \frac{d\ln a}{aH(a)} < \frac{1}{H_\Lambda} \equiv r_\Lambda \tag{11}$$

where $\chi_*(a)$ is the corresponding comoving scale. For small a, the value of χ_* is fixed to a constant $\chi_* \simeq 3r_\Lambda$. Thus, the physical trapped surface radius r_* increases with time. As we approach $a \simeq 1$, the Hubble rate becomes constant and r_* freezes to a constant value $r_* = r_\Lambda$. No signal from inside r_* can reach the outside, just like in the interior of a BH. In fact, according to the Birkhoff theorem (see [33]), the metric outside should be exactly that of the SW BH in the limit of an empty outside space. So, the FLRW metric is a SW BH from the outside with $r_S = r_\Lambda$. This breaks homogeneity (on scales larger than r_Λ), but this is needed if we want causality. Homogeneity is inconsistent with a causal origin.

We next look for a solution to EFE with spherical symmetry in the SW frame where we have matter $\rho_m = \rho_m(t, r)$ and radiation $\rho_R = \rho_R(t, r)$ inside some radius R and empty space outside:

$$\rho(t, r) = \begin{cases} 0 & \text{for } r > R \\ \rho_m + \rho_R & \text{for } r < R \end{cases}. \tag{12}$$

The solution can be given in terms of the metric in Equation (3) with:

$$2\Phi(t, r) = \begin{cases} -r_S/r & \text{for } r > R \\ -r^2 H^2(t, r) & \text{for } r < R \end{cases} \tag{13}$$

where for $r > R$, we have the SW metric. At the junction $r = R$, we find that:

$$R = [r_H^2 r_S]^{1/3}. \tag{14}$$

Using Equation (4), we can immediately see that the solution is $H(t, r) = H(\tau)$ and $R(\tau) = [r_S/H^2(\tau)]^{1/3}$, where τ is the proper time of an observer comoving with the fluid. Given $\rho(\tau)$ and $p(\tau)$ in the interior, we can find $H(\tau)$ and $R(\tau)$ as:

$$H^2(\tau) = \frac{8\pi G}{3}\rho(\tau) = \frac{r_S}{R^3(\tau)} \tag{15}$$

This corresponds to a homogeneous FLRW cloud of fixed mass $M = r_S/2G$ in free-fall inside $R(\tau)$. For constant $H(\tau) = H_\Lambda \equiv r_\Lambda^{-1}$, the FLRW metric becomes the dS metric; so, the interior solution becomes Equation (2) with $R = r_\Lambda = r_S = r_H$. Such solutions are illustrated in Figure 1.
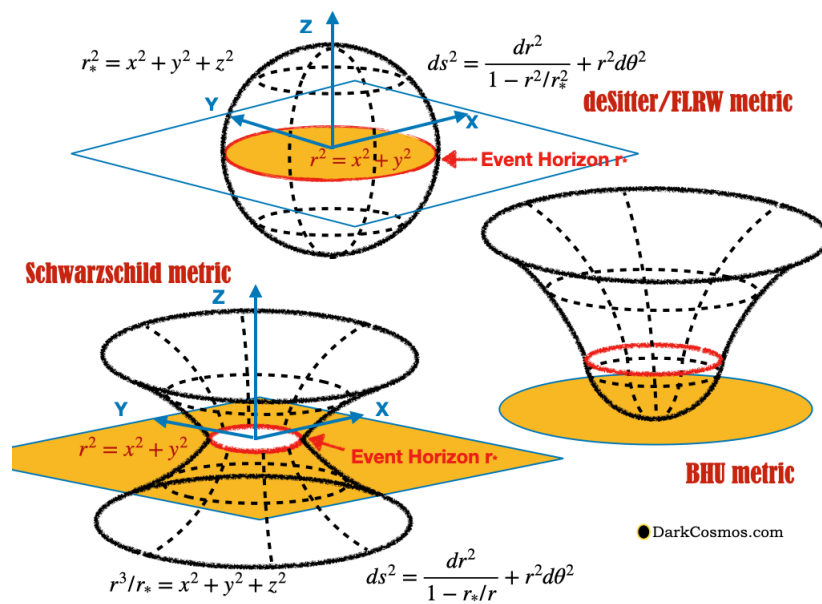
Figure 1. Spatial 2D polar representation of $ds^2 = (1 + 2\Phi)^{-1}dr^2 + r^2d\theta^2$ metric embedded in a 3D flat space. deSitter space (top, $2\Phi = -r^2/r_*^2$) corresponds to a 2D sphere (S2) in 3D flat space. Yellow region shows the projected coverage in the true (x, y)-plane. Coordinate z is an auxiliary extra dimension for illustration ($z^2 = r_*^2 - r^2$ for deSitter). The true 3D space has an additional angle in $d\Omega$, which is not shown. Schwarzschild metric (bottom left) corresponds to $2\Phi = -r_*/r$ and the combined BHU metric is shown in the middle right.

The corresponding comoving radius to $R(\tau)$ is $\chi_*(\tau) \equiv R(\tau)/a(\tau)$. We can see how R can be related with a free-fall geodesic radial shell:

$$\frac{dR}{d\tau} = a\frac{d\chi_*}{d\tau} + \chi_*\frac{da}{d\tau} = V_0 + HR = V_0 + (r_S/R)^{1/2} \tag{16}$$

where $V_0 \equiv a\dot{\chi}_*$. For a time-like geodesic of constant χ_* ($d\chi = 0$), we have $V_0 = 0$. For a null-like geodesic ($ad\chi = d\tau$), $V_0 = 1$. This shows that a solution for $R(\tau)$ exists for any content inside R. Israel junction conditions [34] for continuity of the metric and their derivative (the extrinsic curvature) are fulfilled for Equation (14) with no surface terms (see [15]).

Note that for $R < r_S$ (i.e., inside the BH), Equation (14) indicates that there is a region with no matter, $r_S > r > R$, and a region with matter outside the Hubble horizon, $R > r > r_H$. This region is dynamically frozen because the time a perturbation takes to travel that distance is larger than the expansion time [14]. The frozen region increases during collapse inside r_S and re-enters r_H during expansion. This is a potential source of frozen perturbations, which acts very much like Cosmic Inflation. This is illustrated by a yellow shading in Figure 2.

2.2. The GHY Boundary Term

The expansion that happens inside an isolated BH is bounded by its event horizon $r < r_S$ and we need to add the GHY boundary term S_{GHY} to the action in Equation (10), where:

$$S_{GHY} = \frac{1}{8\pi G} \oint_{\partial V_4} d^3y \sqrt{-h}\, K \tag{17}$$

The integral is over the induced metric at ∂V_4, which for a time-like junction $d\chi = 0$ corresponds to $R = r_S$:

$$ds^2_{\partial V_4} = h_{\alpha\beta} dy^\alpha dy^\beta = -d\tau^2 + r_S^2 d\Omega^2 \tag{18}$$

So, the only remaining degrees of freedom in the action are time τ and the angular coordinates. We can use this metric and the trace of the extrinsic curvature at $R = r_S$ to estimate $K = -2/r_S$ [15]. This result is also valid for a null geodesic. We then have:

$$S_{GHY} = \frac{1}{8\pi G} \int d\tau\, 4\pi r_S^2\, K = -\frac{r_S}{G} \tau \tag{19}$$

The Λ contribution to the action in Equation (10) is: $S_\Lambda = -\Lambda V_4/(8\pi G) = -r_S^3 \Lambda \tau/3G$ where we have estimated the total 4D volume V_4 as that bounded by ∂V_4 inside $r < r_S$, i.e., $V_4 = 2V_3\tau$, where the factor 2 accounts for the fact that $V_3 = 4\pi r_S^3/3$ is covered twice, first during collapse and again during expansion. Comparing the two terms, we can see that we need $\Lambda = 3r_S^{-2}$ or, equivalently, $r_\Lambda = r_S$ to cancel the boundary term. In other words, the evolution inside a BH event horizon induces a Λ term in the EFE even when there is no Λ term to start with. Such an event horizon becomes a boundary for outgoing geodesics, i.e., expanding solutions. This provides a fundamental interpretation to the observed Λ as a causal boundary [7,10].
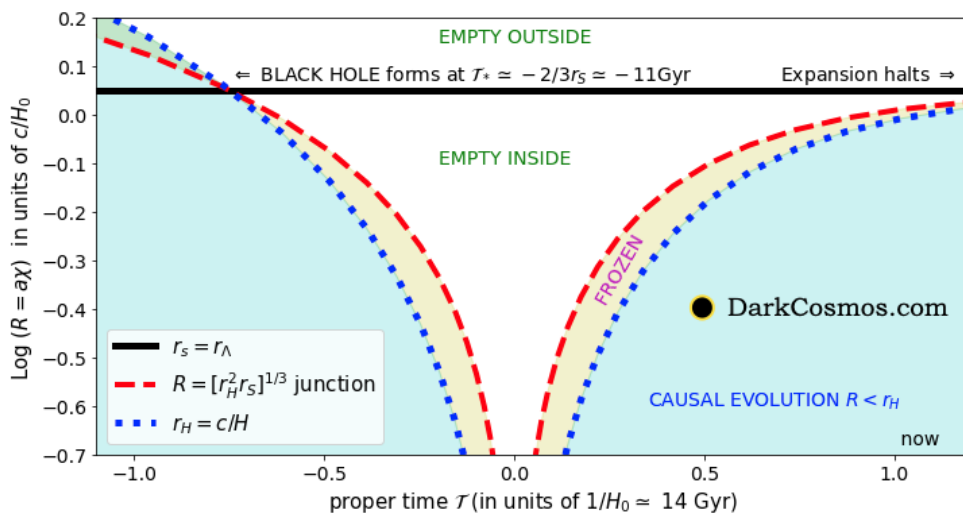
Figure 2. Physical radius R of a homogeneous spherical cloud as a function of proper (or comoving) time τ. A cloud of radius R and mass M collapses in free-fall. When it reaches $R = r_S = 2GM$, at $\tau_* \simeq 2r_S/3 \simeq -11$Gyr, it becomes a BH. The collapse proceeds inside the BH until it bounces producing an expansion (the hot Big Bang). The event horizon r_S behaves like a cosmological constant with $\Lambda = 3/r_S^2$ so that the expansion freezes before it reaches back to $r_S = r_\Lambda$. The calculation corresponds to $\Omega_\Lambda = 0.7$ with $R = [r_H^2 r_S]^{1/3}$. Structure in between R and r_H is frozen (yellow shading) and seeds structure formation in our Universe, which could include smaller BHUs, CMB, stars and galaxies. Blue shading indicates causal evolution ($R < r_H$).

3. How Λ Emerges

We will start assuming that $\Lambda = 0$ and show how Λ emerges because of the dynamics inside the BH event horizon. Consider a large cloud dominated by dust (ρ_m with $p = 0$) with radius R and mass M, surrounded by a region of empty space. This is a good approximation for our Universe with $M \simeq 5 \times 10^{22} M_\odot$, which corresponds to the mass inside our future FLRW event horizon r_Λ in Equation (11). The resulting BH density is very low, $\rho_{BH} = 3r_S^{-2}/8\pi G = \rho_\Lambda$, which is 25% lower than the critical density today $\rho = 3H_0^2/8\pi G$ (a few protons per cubic meter).

Gravity will make such a dust cloud collapse following a free-fall time-like geodesic of Equation (14): $R = (r_H^2 r_S)^{1/3}$. As there is no pressure support, it will eventually collapse into a BH when $R = r_S = r_H = 2GM$. This happened 25 Gyrs ago according to the numerical result in Figure 2. The collapse continues free-fall inside the BH as illustrated in the left half of Figure 2.

As the density inside gets larger, matter ionizes and radiation resists collapse, as it happens in stars. After 11Gyrs, the density reaches the density of a neutron star. This happens well before Planck times (Quantum Gravity). The Hubble radius at that time corresponds to a solar mass, so the situation is similar to the interior of a collapsing star. This should lead to a bounce, either because of supernova explosion or because of the elasticity of the neutron density [35]. This Big Bounce generates a hot Big Bang, similar to the bouncing of a ball on the floor.

The resulting radiation and plasma fluids cool down as they expand into the standard FLRW evolution: Nucleosynthesis and Cosmic Microwave Background (CMB) decoupling. Frozen perturbations from outside r_H re-enter the horizon as the expansion proceeds. These are the seeds for new structure (Baryon Acoustic Oscillations or BAO and galaxies) that grow under gravitational instability. Finally, the expansion freezes (exponential inflation) as it reaches back to the original BH event horizon $R = r_S$. As detailed in Section 2.2, the expansion inside a BH event horizon induces a Λ term in EFE, even when there is none to start with. The expansion comes to a halt in the physical frame as nothing can come out of a BH. This provides a fundamental interpretation to the observed Λ as causal boundary [7,10] and also for the origin of the Big Bang.

4. Discussion and Conclusions

The observed cosmic acceleration directly implies that the FLRW metric has an Event Horizon $r_S = r_\Lambda$ in Equation (11), which agrees well with the BHU idea [15]. Moreover, the measured cosmic density ρ and expansion rate also match well those of a BH with $r_S = r_\Lambda = 2GM$ and BH density $\rho = 3r_S^{-2}/(8\pi G)$. Thus, the BHU provides a fundamental explanation to the meaning and origin of the Λ term and the hot Big Bang, as shown in Figure 2.

Inflation ([19–22]) is an important ingredient in the standard cosmological model. For a review, see [14,36]. It solves several problems, the most relevant here are: (1) the horizon problem; (2) the source of structure in the universe (such BAO); (3) the flatness problem. The horizon and structure problems arise because much of the structure that we observe today, e.g., BAO in CMB maps, was outside the Hubble horizon r_H at the time of light emission and therefore could not have had a causal origin if there was nothing before the Big Bang. Inflation solves all these problem at the prize of introducing several new ad hoc ingredients to the early universe that are hard to test.

As shown by the yellow shaded region in Figure 2, a large fraction of the mass M that collapsed into our BHU is outside r_H. This solves the horizon problem. The same mass outside r_H can be the source of structure as it re-enters r_H. Harrison [37] and Zel'dovich [38] independently proposed in 1970's (see also [39]) that gravitational instability of regular matter alone can generate a scale-invariant spectrum of fluctuations, very similar to that produced by models of Inflation.

We have assumed here a global flat topology $k = \Lambda = 0$ because this reproduces the Minkowski metric for empty space. However, the BHU solutions are also exact GR

solutions for $k \neq \Lambda \neq 0$ by just replacing $r_H^{-2} \equiv H^2 + k/a^2 + \Lambda/3$ in Equation (14). Given some energy content, EFE cannot be used to decide the topology k of the metric or the value of the effective Λ. This is a global property that is either assumed or directly measured. So, any choice other than $\Lambda = k = 0$ would require some justification that is outside GR. In that respect, we believe there is no flatness or Λ problem within GR.

At the time of CMB last scattering, R corresponds to an angle $\theta \simeq 60$ deg. So, we can actually observe perturbations larger than R. Scales that are not causally connected! There is good observational evidence for homogeneity and lack of correlations in the CMB at $r > R$ (see [40] and references therein). This could be related to the so-called CMB anomalies (i.e., apparent deviations with respect to simple predictions from Λ Cold Dark Matter, see [7,41,42] and references therein), or the apparent tensions in measurements from vastly different cosmic scales or times (e.g., [17,18,43,44]). This requires further investigation that will be presented in a separate publication.

Conflicts of Interest: The author declares no conflict of interest.

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