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# Choquet-Integral-Based Data Envelopment Analysis with Stochastic Multicriteria Acceptability Analysis

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**Abstract:** Data envelopment analysis (DEA) is a non-parametric method for measuring the efficiencies of decision-making units (DMUs) by using a set of inputs and a set of outputs. However, traditional DEA models always assume that the inputs or outputs are independent of each other, which is unrealistic in practical problems. To reflect the interactions between inputs or outputs, the Choquet integral is employed in DEA models. The traditional DEA models are usually used to find some specific input and output weights of DMUs to optimize the efficiency score of DMUs, but the corresponding input and output weights for the optimal efficiency score of a DMU may not be distributed symmetrically, that is to say, the space of weights may be different for different DMUs. Instead of finding the self-efficiency score and the cross-efficiency score of a DMU in traditional DEA models based on some specific input and output weights, stochastic multicriteria acceptability analysis is used to explore the input or output evaluation space and weight space to calculate the Choquet-integral-based acceptability indices of DMUs. The proposed method considers the interactions between inputs or outputs, which can make more DMUs efficient and can also measure the acceptability of a DMU to become an efficient one by exploring the supporting information space. Examples are given to illustrate the proposed method.

**Keywords:** data envelopment analysis; stochastic multicriteria acceptability analysis; Choquet integral; interactions between variables

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## 1. Introduction

Data envelopment analysis (DEA) [1] is a non-parametric method for measuring the efficiencies of decision-making units (DMUs) by finding the most favorable inputs and outputs for them. Such a self-evaluation can classify all the DMUs into efficient ones and inefficient ones, but efficient DMUs are not further discriminated [2,3]. To enhance the discrimination power of the original DEA, the cross-efficiency evaluation was then developed to calculate the cross-efficiency scores of DMUs linked to all DMUs [4]. However, the cross-efficiency evaluations obtained from the original DEA are generally not unique due to the optimal solution to the DEA linear program not being unique. Sexton et al. [4] and Doyle and Green [5] proposed the secondary goals (the aggressive and benevolent formulations) to deal with this issue. Several authors [6,7] extended Doyle and Green's model by introducing a number of different secondary objective functions.

The above DEA models are focused on the calculation of the self-efficiency scores or cross-efficiency scores of DMUs, which are based on some specific weight information. However, the weight information space or the evaluation information space that makes a DMU efficient is not distributed symmetrically. For two efficient DMUs, the corresponding weight information space or evaluation information space is different, and the efficient DMU with a bigger information space should be better than the one with a smaller information space [8–10]. Therefore, the corresponding information space distributed symmetrically can be used to discriminate efficient DMUs, which is not considered in the classical DEA models. Stochastic multicriteria acceptability analysis (SMAA) [9] is used to find the information space that supports each alternative for the best ranking. Lahdelma and Salminen [8] introduced the SMAA-2 method, which extends SMAA by considering all the rankings in the analysis.

However, these methods are not suitable to deal with DEA problems. Lahdelma and Salminen [10] presented the SMAA-DEA method, which is a combination of DEA and SMAA-2 and is intended for evaluating the efficiencies of DMUs by exploring the corresponding weight spaces, according to which, clearly efficient and barely efficient DMUs can be identified. Yang et al. [11] considered all possible weights in the weight space when computing the cross-efficiency, and each DMU was given an interval cross-efficiency. By using the SMAA-2 method, all DMUs in the interval cross-efficiency matrix could be fully ranked according to acceptability indices. However, in Lahdelma and Salminen's method [10] and Yang et al.'s method [11], the inputs and outputs of DMUs were assumed to be independent of each other. By considering the interactions between criteria, Angilella et al. [12,13] applied the Choquet integral [14] to the SMAA-2 method.

In most of the existing DEA methods, the inputs and outputs are assumed independent. Actually, there exist interactions between inputs or outputs in many practical problems [15–17]. Recently, Ji et al. [18] and Xia and Chen [19] gave the efficiency evaluation model with interactive inputs and outputs, in which the Choquet integral was used to aggregate the multiple inputs and outputs into a single efficiency index. Pereira et al. [20] used a Choquet-integral-based approach for incorporating decision-maker's preference judgments in DEA. Ji et al.'s [18] method and Xia and Chen's [19] method were based on the CCR model [1], while Pereira et al.'s [20] method was based on the value-based ADD model [21]. However, their method also has multiple solutions, and the calculated self-efficiency and cross-efficiency scores of DMUs were also based on some specific input and output weight vectors. Their method can identify the efficient and inefficient DMUs, but cannot further identify which is better in the efficient DMUs. In addition, their method can only deal with the DMUs with determined input and output evaluations, which is not common in practical problems, because there always exists uncertainty, fuzziness, or randomness in the process of estimating input and output evaluations due to its inherent stochastic nature or specification errors [22,23].

Based on the above analysis, we can find that the DEA models taking into account the interactive variables do not consider the information space when calculating the efficiency scores of DMUs, while the DEA models taking into account the information space do not consider the interactions between inputs and outputs. This paper fills this gap by using the SMAA-2 method to deal with DEA models with interactive variables to explore the information space that is favorable for a DMU at any ranking. In the process, the Choquet integral is employed to reflect the interactions between inputs or outputs. The contributions of this paper are given as follows:

(1) The proposed method gives a combined method, which not only considers the interactions between inputs and outputs, but also can discriminate the efficient DMUs by exploring the corresponding supported information space;
(2) Compared to the DEA models with interactive variables, the proposed method can explore the information space that supports each DMU, which not only can discriminate the efficient DMUs, but also can give a ranking of efficient DMUs according to the supported information space;
(3) Compared to the DEA models with interactive variables, the proposed method not only can deal with the DEA problem with determined input and output evaluations, but also can deal with the DEA problem, in which the weight vector and evaluations of the input and output are stochastic;
(4) Compared to the DEA models with dependent input and output evaluations, the proposed method can deal with the DEA problems in which the inputs and outputs are interactive.

The remainder of this paper is constructed as follows: Section 2 introduces the Choquet-based DEA models; Section 3 gives the CH-SMAA-DEA method to measure DMUs in terms of rank acceptability indices, central weights, and confidence factors. Examples are given in Section 4 to compare the proposed method with the existing ones. Section 5 gives the conclusions.

## 2. DEA Models Based on the Choquet Integral

Suppose there are $m$ decision-making units (DMUs) with $h$ inputs and $s$ outputs. Let $x_{gi}(g=1,2,\cdots,h)$ and $x_{ri}(r=h+1,h+2,\cdots,h+s)$ be the input and output values of DMU$_i$, respectively. Let $\varpi_g>0(g=1,2,\cdots,h)$ and $\varpi_r>0(r=h+1,h+2,\cdots,h+s)$ be the input and output weights, respectively. Then, the efficiency $\bar{E}_i$ of DMU$_i$ can be calculated by the ratio of its weighted score for the output criteria to its weighted score for the input criteria:

$$\bar{E}_i(x_i,\varpi)=\frac{\sum\limits_{r=h+1}^{h+s}\varpi_r x_{ri}}{\sum\limits_{g=1}^{h}\varpi_g x_{gi}},i=1,2,\cdots,m \tag{1}$$

For convenience, let $\bar{W}=\{\varpi=(\varpi_1,\varpi_2,\cdots,\varpi_{h+s})|\varpi_j>0,j=1,2,\cdots,h+s\}$ be the set of input and output weight information and $X=\{(x_1,x_2,\cdots,x_m)^{\mathrm{T}}|x_{ji}>0,i=1,\cdots,2,m,j=1,2,\cdots,h+s\}$ be the set of all input and output evaluations with $x_i=(x_{1i},x_{2i},\cdots,x_{(h+s)i})$ being the evaluation vector corresponding to DMU$_i$.

The DEA model aims to maximize each DMU's self-efficiency score by finding the most favorable weights, then the maximum efficiency score of DMU$_d$ can be calculated by the following CCR model [1]:

$$\begin{aligned}(\text{MOD 1})\quad \max\quad & \bar{E}_d=\bar{E}_d(x,\varpi)\\ s.t.\quad & \bar{E}_i(x_i,\varpi)\leqslant 1,i=1,2,\cdots,m\\ & \varpi\in\bar{W},x_i\in X\end{aligned}$$

Based on Equation (1), (MOD 1) can be written as the following linear programming model:

$$\begin{aligned}(\text{MOD 2})\quad \max\quad & \sum_{r=h+1}^{h+s}\varpi_r x_{rd}\\ s.t.\quad & \sum_{g=1}^{h}\varpi_g x_{gd}=1\\ & \sum_{r=h+1}^{h+s}\varpi_r x_{ri}\leqslant\sum_{g=1}^{h}\varpi_g x_{gi},i=1,2,\cdots,m\\ & \varpi\in\bar{W},x_i\in X\end{aligned}$$

Suppose the optimal value of (MOD 2) is denoted by $\bar{E}_d'$. If $\bar{E}_d'=1$, then DMU$_i$ is efficient in the CCR model [1]. (MOD 2) is a self-efficiency model, which can identify the efficient DMUs, but cannot further discriminate between efficient DMUs. The cross-efficiency is defined by considering all the DMUs. For DMU$_d(d=1,2,\cdots,n)$, a group of optimal weights $\varpi_{gd}^*,g=1,2,\cdots,h$ and $\varpi_{rd}^*,r=h+1,h+2,\cdots,h+s$ can be obtained by solving (MOD 1), and its cross-efficiency of DMU$_i$ to DMU$_d$, namely $\bar{E}_{di}$, can be calculated by using the weights of DMU$_d$.

$$\bar{E}_{di}=\frac{\sum\limits_{r=h+1}^{h+s}\varpi_{rd}^* x_{ri}}{\sum\limits_{g=1}^{h}\varpi_{gd}^* x_{gi}},d,i=1,2,\cdots,m \tag{2}$$

Then, the average of all $\bar{E}_{di}(d = 1, 2, \cdots, n)$ can be calculated as:

$$\overline{CE}_i = \frac{1}{m}\sum_{d=1}^{m} \bar{E}_{di}, (i = 1, 2, \cdots, m) \tag{3}$$

which is called the cross-efficiency score of DMU$_i$.

It is noticed that (MOD 1) may have multiple optimal solutions, and the cross-efficiency calculated by Equation (3) is referred to as an arbitrary strategy. To resolve this problem, one remedy suggested by Sexton et al. [4] is to introduce a secondary goal to choose the one from multiple optimal solutions while keeping the self-efficiency obtained by (MOD 2) unchanged. Many other strategies [5,6,24,25] have also been developed about the secondary goal. However, when different strategies are used, different results may be obtained.

It is noted that the classical DEA models assume that the inputs and outputs are independent. However, many authors [13,18] showed that there are interactions between inputs or outputs. The Choquet integral [14] is the generally used technique to reflect the interactions between criteria. Before introducing the concept of the Choquet integral, several definitions are given first:

**Definition 1** ([26]). *A fuzzy measure $\mu$ on $Y = \{y_1, y_2, \cdots, y_n\}$ is a function $\mu$: $P(Y) \to [0, 1]$, satisfying the axioms: (i) $\mu(\varphi) = 0$; (ii) $A \subset B \subset Y$ implies $\mu(A) \leqslant \mu(B)$.*

**Definition 2** ([14]). *Let $f$ be a positive real-valued function on $Y = \{y_1, y_2, \cdots, y_n\}$ and $\mu$ be a fuzzy measure on $Y$. The discrete Choquet integral of a function $f : Y \to R^+$ with respect to $\mu$ is defined by:*

$$c_\mu = \sum_{i=1}^{n} (\mu(A_{(i)}) - \mu(A_{(i+1)}))f(y_{(i)})$$

*where $(i)$ indicates that the indices have been permuted so that $0 \leqslant f(y_{(1)}) \leqslant \cdots \leqslant f(y_{(n)})$, and $A_{(i)} = \{y_{(i)}, \cdots, y_{(n)}\}$ is the set of $y_{(k)}, k = i, \cdots, n$, and let $A_{(n+1)} = \varphi$ here.*

As the Choquet integral takes into account the interactions between criteria, Ji et al. [18] utilized it to aggregate the input and output evaluations of DMU$_i$:

$$E_i(x_i, \mu) = \frac{\sum_{r=h+1}^{h+s} (\mu(B_{\rho(r)}) - \mu(B_{\rho(r+1)}))x_{\rho(r)i}}{\sum_{g=1}^{h} (\mu(A_{\sigma(g)}) - \mu(A_{\sigma(g+1)}))x_{\sigma(g)i}} \tag{4}$$

where $\sigma(r)$ indicates that the indices have been permuted so that $0 \leqslant x_{\sigma(1)i} \leqslant \cdots \leqslant x_{\sigma(h)i}$, $A_{\sigma(i)} = \{y_{\sigma(i)}, \cdots, y_{\sigma(h)}\}$ is the set of $y_{\sigma(k)}, k = i, \cdots, h$ and $A_{(h+1)} = \varphi$ and $\rho(r)$ indicates that the indices have been permuted so that $0 \leqslant x_{\rho(h+1)i} \leqslant \cdots \leqslant x_{\rho(h+s)i}$ and $B_{\rho(i)} = \{y_{\rho(i)}, \cdots, y_{\rho(h+s)}\}$ is the set of $y_{\rho(k)}, k = i, \cdots, h+s$ and $B_{(h+s+1)} = \varphi$.

For DMU 1 and DMU 20 in Example 2 (see Table A5), we have:

$$E_1(x_1, \mu) = \frac{181(\mu(y_4,y_5) - \mu(y_5)) + 231\mu(y_5)}{236(\mu(y_1,y_2,y_3) - \mu(y_2,y_3)) + 266(\mu(y_2,y_3) - \mu(y_3)) + 302\mu(y_3)}$$

and:

$$E_{20}(x_{20}, \mu) = \frac{191(\mu(y_5,y_4) - \mu(y_4)) + 232\mu(y_4)}{269(\mu(y_2,y_1,y_3) - \mu(y_1,y_3)) + 298(\mu(y_1,y_3) - \mu(y_3)) + 338\mu(y_3)}$$

However, it is noted that the input and output evaluations of DMUs should be ordered before being aggregated. Therefore, it is not convenient when there are many input and output evaluations to be ordered. Especially when the input and output evaluations are not expressed exactly, it is hard to give an exact order. In addition, if there are two DMUs and the orders of their input and output evaluations are not the same, then the corresponding weight vectors associated with these two DMUs are not the same. This means that we have to determine the corresponding weight vector for each DMU, which makes it difficult in modeling and calculation, especially when the fuzzy measures are unknown and should be determined from the known information.

To deal with such issues, another form of the Choquet integral can be defined in the following.

**Definition 3** ([27–29]). *The Möbius transform of* $\mu$ *is a function on* $Y = \{y_1, y_2, \cdots, y_n\}$ *defined as* $\eta(A) = \sum_{B \subset A} (-1)^{|A \backslash B|} \mu(B), \forall A \subset Y$; $A \backslash B$ *is the set of elements in* $A$ *excluding the elements in* $B$; $|A \backslash B|$ *is the cardinality of* $A \backslash B$.

In terms of Möbius representation, (i) and (ii) can be represented by [27,28]: (iii) $\eta(\varphi) = 0$; (iv) $\forall i \in Y$ and $\forall S \subseteq Y \backslash \{i\}$, $\sum_{T \subseteq S} \eta(T \cup \{i\}) \geqslant 0$, where $S$ is the subset of $Y$ excluding $i$ and $T$ is the subset of $S$.

The Choquet integral can be redefined in terms of the Möbius representation, without reordering the aggregated values [27,28,30]:

**Definition 4** ([27,28,30]). *With respect to the Möbius representation, the Choquet integral defined in Definition 2 can be rewritten as:*

$$c_m = \sum_{T \subseteq N} \eta(T) \min_{i \in T} \{f(y_i)\} \tag{5}$$

*where* $\eta(\varphi) = 0$, $\sum_{T \subseteq N} \eta(T) = 1$, $\forall i \in N$, $\forall S \subseteq N \backslash \{i\}$, $\sum_{T \subseteq S} \eta(T \cup \{i\}) \geqslant 0$, *and* $N = \{1, 2, \cdots, n\}$.

Different from Definition 2, Definition 4 does not have to reorder the aggregated arguments, and the weight vector is associated with the aggregated arguments, but not the position, and therefore would be easy to use. In Definition 4, for any two alternatives, the corresponding weight vectors are the same, which provides much convenience in deriving the unknown weight vector.

Let $H = \{1, 2, \cdots, h\}$, $S = \{h+1, h+2, \cdots, h+s\}$, $w_T = \eta(T)$, $T \subseteq NorS$ and $W'$ be the set of input and output weight information with interactions, then:

$$W' = \left\{ w = \{w_G, G \subseteq H, w_R, R \subseteq S\} | w_\varphi = 0, \sum_{G \subseteq H'} w_{G \cup \{i\}} \geqslant 0, \forall i \in H, \forall H' \subseteq H \backslash \{i\}; \right.$$

$$\left. \sum_{R \subseteq S'} w_{R \cup \{i\}} \geqslant 0, \forall i \in S, \forall S' \subseteq S \backslash \{i\} \right\}$$

where $W'$ includes the input weights and output weights with interactions between each other, respectively.

By considering the interactions between inputs or between outputs, the efficiency of $\text{DMU}_i$ can be written as [19]

$$E_i(x_i, w) = \frac{\sum_{R \subseteq S} w_R \min_{r \in R} \{x_{ri}\}}{\sum_{G \subseteq H} w_G \min_{g \in G} \{x_{gi}\}} \tag{6}$$

In Example 2, take DMU 1 and DMU 20 as an example (see Table A5); we have:

$$E_1(x_1, w) = \frac{181w_4 + 232w_5 + 181w_{\{4,5\}}}{236w_1 + 266w_2 + 302w_3 + 236w_{\{1,2\}} + 236w_{\{1,3\}} + 266w_{\{2,3\}} + 236w_{\{1,2,3\}}}$$

$$E_{20}(x_{20}, w) = \frac{232w_4 + 191w_5 + 191w_{\{4,5\}}}{298w_1 + 269w_2 + 338w_3 + 269w_{\{1,2\}} + 298w_{\{1,3\}} + 269w_{\{2,3\}} + 269w_{\{1,2,3\}}}$$

It is noted that, when calculating the efficiency scores of DMUs, we do not have to reorder the aggregated input and output evaluations, and the corresponding input and output weight vectors will not change as the DMU changes, which provides much convenience in modeling and calculation.

The difference between Equation (6) and Equation (1) is that the former considers the interactions between inputs or outputs, but the latter does not. Especially, if the interactions between inputs or outputs are not considered, that is $w_T = 0, t \geqslant 2$, where $t$ is the cardinality of the coalition $T$ and $t = |T|$, then Equation (6) reduces to Equation (1). Comparing Equation (6) with Equation (4) given by [18], both of them are based on the Choquet integral and can be converted between each other; this is because Definition 2 and Definition 4 can be converted between each other. However, they are based on different forms of the Choquet integral: Ji et al.'s model [18] is based on Definition 2, while Equation (6) is based on Definition 4. The most important is that Equation (6) does not have to reorder the input or output evaluations and the corresponding input and output weight vectors will not change as the DMUs change, which provides much convenience in the process of deriving the efficiency scores of DMUs.

Based on Equation (3) and (MOD 2), the DEA model considering interactions between inputs and outputs can be established as [19]:

$$\begin{aligned} \text{(MOD 3)} \quad \max \quad & \sum_{R \subseteq S} w_R \min_{r \in R}\{x_{rd}\} \\ s.t. \quad & \sum_{G \subseteq H} w_G \min_{g \in G}\{x_{gd}\} = 1 \\ & \sum_{R \subseteq S} w_R \min_{r \in R}\{x_{ri}\} \leqslant \sum_{G \subseteq H} w_G \min_{g \in G}\{x_{gi}\}, i = 1, 2, \cdots, m \\ & w \in W', x_i \in X \end{aligned}$$

where $w = \{w_G, G \subseteq H, w_R, R \subseteq S\} \in W'$ is the Choquet integral input and output weights associated with $\text{DMU}_d$. Since $x$ is determined, (MOD 3) is linear. Similar to (MOD 2), the solution of (MOD 3) may not be unique. By solving (MOD 3) using Lingo 14, we obtain a set of optimal input and output interactive weights $w_d^* \in W'$ for each $\text{DMU}_d$. In (MOD 3), each DMU is self-evaluated and termed efficient if and only if the optimal objective function is equal to 1. The cross-efficiency of $\text{DMU}_i$ using the weights of $\text{DMU}_d$, namely $E_{di}$, can be calculated as:

$$E_{di} = \frac{\sum_{R \subseteq S} w_{Rd}^* \min_{r \in R}\{x_{ri}\}}{\sum_{G \subseteq H} w_{Gd}^* \min_{g \in G}\{x_{gi}\}}, d, i = 1, 2, \cdots, m \tag{7}$$

For $\text{DMU}_i$, the average of all $E_{di}$, namely $CE_i$, can be considered as the cross-efficiency score of $\text{DMU}_i$:

$$CE_i = \frac{1}{m} \sum_{d=1}^{m} E_{di}, i = 1, 2, \cdots, m \tag{8}$$

Suppose the optimal value of (MOD 3) is denoted by $E_i^{'}$. By comparing (MOD 2) and (MOD 3), it is noted that all the feasible solutions of (MOD 2) are also those of (MOD 3), which indicates that the feasible region of (MOD 3) is not smaller than that of (MOD 2). Therefore, the optimal solution of (MOD 3), $E_i^{'}$, is not smaller than that of (MOD 2), $\bar{E}_i^{'}$, that is $E_i^{'} \geqslant \bar{E}_i^{'}$. That is because (MOD 3) takes into account the interactions between inputs and outputs, while (MOD 2) does not. Especially, if $w_T = 0, t \geqslant 2$, then (MOD 3) reduces to (MOD 2), and Equation (8) reduces to Equation (3).

It has been proven that Definitions 2 and 4 can be transformed between each other; therefore, Xia and Chen's model [19] based on Definition 4 is equivalent to the one given by Ji et al. [18], which is based on Definition 2. The only difference is that Xia and Chen's model [19] does not have to reorder the input and output evaluations when aggregating them, and the corresponding input and output weight vectors will remain unchanged with different DMUs. We denote the Choquet-integral-based DEA model as CH-DEA hereafter.

However, the disadvantage of (MOD 2) and (MOD 3) is that they can only discriminate efficient DMUs from inefficient ones, but cannot further identify between efficient ones. Although the cross-efficiency scores calculated by Equations (3) and (8) can give a ranking of DMUs, different results may be obtained when the input and output weights are calculated by using different strategies. Actually, self-efficiency scores and cross-efficiency scores are all based on some specific weight vectors and do not consider the whole set of the information space. In addition, neither of them can provide the acceptability of an efficient DMU. For two efficient DMUs, one may correspond to a large space of weight information, and the other may correspond to a smaller one, which indicates the former should be better than the latter. Therefore, these two efficient DMUs should be discriminated, which will be discussed in the following section.

## 3. SMAA-DEA Based on the Choquet Integral

By exploring the corresponding information spaces of DMUs, Lahdelma and Salminen [10] presented the SMAA-DEA method, in which the DMUs are evaluated by using several indices including the acceptability index, the central weight vector, the confidence factor, the maximum efficiency, the central efficiency, and the average efficiency. However, their method does not consider the interactions between inputs or outputs. In this section, the Choquet-integral-based SMAA-DEA method (CH-SMAA-DEA) is proposed to explore the information space of each DMU by taking into account the interactions between inputs or outputs.

In DEA models, suppose the input and output evaluations of $\text{DMU}_i$ are represented by the stochastic variables $\xi_{ji}(j = 1, 2, \cdots, h, h+1, \cdots, h+s)$ with the probability distribution $f(\xi)$ over the space $X \subseteq R_{m\times(h+s)}$. Similarly, the decision-makers' unknown or partially known preference about the input and output weights is represented by a stochastic weight vector $w$ with joint density function $f(w)$ in the feasible input and output weight space $W$. $f(w)$ can be given by the decision-makers; we assumed that it is an independent uniform distribution in this paper. In the SMAA-DEA method, the input and output weights are normalized to give a finite information space for simple computation. Similarly, the Choquet-based input and output weights in space $W$ are also normalized, respectively, as:

$$W = \left\{ w = \{w_G, G \subseteq H, w_R, R \subseteq S\} | w_\varphi = 0, \sum_{R\subseteq S} w_R = 1, \sum_{R\subseteq S'} w_{R\cup\{i\}} \geqslant 0, \forall i \in S, \forall S' \subseteq S\backslash\{i\}; \right.$$

$$\left. \sum_{G\subseteq H} w_G = 1, \sum_{G\subseteq H'} w_{G\cup\{i\}} \geqslant 0, \forall i \in H, \forall H' \subseteq H\backslash\{i\} \right\}$$

For each $\xi$ in $X$ and each $w$ in $W$, the efficient score $u'(i, \xi, w)$ of $\text{DMU}_i$ can be denoted by the following formula:

$$u'(i, \xi, w) = \frac{\sum\limits_{R\subseteq S} w_R \min\limits_{r\in R} \xi_{ri}}{\sum\limits_{G\subseteq H} w_G \min\limits_{g\in G} \xi_{gi}} \tag{9}$$

One difference between Equations (6) and (9) is that the input and output evaluations are determined in Equation (6), while they are stochastic in Equation (9). When the input and output evaluations are determined in Equation (9), the difference between Equations (6) and (9) is that the input and output weights are respectively normalized in Equation (9), but are not in Equation (6). For a DMU, the biggest efficiency score derived from Equation (6) is 1, but may not be 1 in Equation (9). To deal with such a situation, we first normalize the values of $u'(i, \xi, w)$ into the interval $[0, 1]$, that is:

$$u(i, \xi, w) = u'(i, \xi, w) / \max_{k=1,2,\cdots,m} \{u'(k, \xi, w)\}$$

For each $\xi$ in $X$ and each $w$ in $W$, $u(i,\xi,w)$ provides a complete ranking of alternatives, then the rank of DMU$_i$ is denoted by:

$$rank(i,\xi,w) = 1 + \sum_{k \neq i} \rho(u(k,\xi,w) \geqslant u(i,\xi,w)) \tag{10}$$

where $\rho(true) = 1$ and $\rho(false) = 0$.

For each $\xi \in X$, suppose DMU$_i$ ranks $r$th; we can compute the set of the possible input and output weight space based on SMAA-2 [8]:

$$W_i^r(\xi) = \{w \in W, rank(i,\xi,w) = r\} \tag{11}$$

which is called the favorable weights of DMU$_i$ ranking $r$th. $W_i^r(\xi)$ contains all the input and output weights that make DMU$_i$ rank $r$th. It is noted that $W_i^r(\xi)$ is distributed asymmetrically. If $W_i^r(\xi) \neq 0$, then it is possible that DMU$_i$ ranks $r$th, and the bigger the space $W_i^r(\xi)$, the bigger the likelihood that DMU$_i$ ranks $r$th for $\xi \in X$. Considering all the $\xi$ in $X$, an index can be given to measure the acceptability of DMU$_i$ ranking $r$th.

On the basis of the favorable weight information space $W_i^r(\xi)$ and all input and output evaluations $\xi \in X$, the Choquet-integral-based acceptability index for DMU$_i$ ranking $r$th is given as:

$$b_i^r = \int_{\xi \in X} f_X(\xi) \int_{w \in W_i^r(\xi)} f_W(w) dw d\xi \tag{12}$$

which is described by the shared information space that supports DMU$_i$ ranking $r$th over all the information space; in particular, $b_i^1$ measures the shared information space making DMU$_i$ the most preferred one. If $b_i^1 \neq 0$, then DMU$_i$ is efficient according to the CCR model; otherwise, DMU$_i$ is not. The bigger the $b_i^1$, the more efficient DMU$_i$ is. Therefore, $b_i^1$ can not only discriminate the efficient DMUs, but also can measure the acceptability of efficiency.

For efficient DMUs, to describe which weight vector supports DMU$_i$ ranking first, the Choquet-integral-based central weight vector can be defined as:

$$w_i^c = \frac{1}{b_i^1} \int_{\xi \in X} f_X(\xi) \int_{w \in W_i^1(\xi)} f_W(w) w dw d\xi \tag{13}$$

The Choquet-integral-based central weight vector describes the preference of a typical weight vector that makes DMU$_i$ the most preferred one, which can help decision-makers understand which weights support which alternative. For inefficient DMUs, we have $b_i^1$ = 0; suppose their Choquet-integral-based central weight vector is that which makes them attain their maximum efficiency or attain their best rank $r_i^*$, that is:

$$w_i^c = \frac{1}{b_i^{r_i^*}} \int_{\xi \in X} f_X(\xi) \int_{w \in W_i^{r_i^*}(\xi)} f_W(w) w dw d\xi \tag{14}$$

Based on the Choquet-integral-based central weight vector, the Choquet-integral-based confidence factor is defined as:

$$p_i^c = \int_{\xi \in X : w \in W_i^1(\xi)} f(\xi) d\xi$$

which measures the likelihood of DMU$_i$ becoming the best one when the Choquet-integral-based central weight vector is used.

It is noted that the above measures are all based on the calculation of the information space for supporting DMUs. Except for the above indices, the following stochastic efficiency measures can be defined to reflect the efficiencies of DMUs from different views.

The Choquet-integral-based maximum efficiency $E_i^{\max}$ is the best efficiency score for DMU$_i$ and can be calculated by maximizing the efficiency score over all the stochastic evaluation values and weight values.

$$E_i^{\max} = \max_{\xi \in X} \max_{w \in W} u(\xi_i, \xi, w)$$

The process to calculate the Choquet-integral-based maximum efficiency is to find the most favorable evaluations and weights for a DMU.

The Choquet-integral-based central efficiency $E_i^c$ is the expected efficiency score of $\text{DMU}_i$ when the Choquet-integral-based central weight vector is used:

$$E_i^c = \int_{\xi \in X} f(\xi) u(\xi_i, \xi, w_i^c) d\xi$$

which can estimate the average performance of a DMU when the most favorable weight vector is used.

The Choquet-integral-based average efficiency $E_i^{ave}$ is the Choquet-integral-based expected efficiency score of $\text{DMU}_i$ over all the stochastic evaluation values and weights:

$$E_i^{ave} = \int_{\xi \in X} f(\xi) \int_{w \in W} f(w) u(\xi_i, \xi, w) dw d\xi$$

According to $E_i^{ave}$, the average performance of $\text{DMU}_i$ can be estimated when all possible evaluations and weights are considered. The flowchart of the proposed method is illustrated in Figure A1 in Appendix A.

Ji et al. [18] calculated the self-efficiencies and cross-efficiencies of DMUs considering the interactions between inputs and outputs. However, their method has multiple solutions, and the calculated self-efficiency and cross-efficiency scores of DMUs are based on some specific input and output weight vectors. Their method can identify the efficient and inefficient DMUs, but cannot provide the acceptability of efficient DMUs. Although the cross-efficiencies of DMUs can distinguish efficient DMUs, different optimal input and output weights will produce different cross-efficiencies of DMUs, which will produce different results in distinguishing efficient DMUs. In addition, their method can only deal with the DMUs with determined input and output evaluations and will be invalid when the input and output evaluations are uncertain.

Lahdelma and Salminen [10] developed the SMAA-DEA method to derive the acceptability indices of DMUs by exploring the information space that supports the ranking of DMUs, but they did not consider the interactions between inputs or outputs. Especially, if the interactions between inputs or outputs are not considered, that is $w_T = 0$, $t \geqslant 2$, then the proposed method reduces to Lahdelma and Salminen's method [10] and the proposed indices reduce to the ones defined by Lahdelma and Salminen [10].

## 4. Examples

In this section, two examples are given to compare the proposed methods with the ones given by Lahdelma and Salminen [10] and Ji et al. [18].

**Example 1** ([10])**.** *Consider eight DMUs A, B, C, D, E, F, G, and H with one input and two outputs as listed in Table A1.*

Lahdelma and Salminen [10] assumed that there was no interaction between inputs and outputs. As discussed in the Introduction, it is reasonable to assume that the there exist interactions between inputs and outputs, which is also the assumption in this paper.

First, we treat the problem as deterministic. SMAA-DEA [10] and CH-SMAA-DEA were implemented by Monte Carlo simulation in the MATLAB environment. The results obtained by the classical DEA model [1], the SMAA-DEA [10], the CH-DEA [18], and the CH-SMAA-DEA are given in Table A2. Based on the optimal weights ($\bar{w}$ and $w$) and the efficiency scores ($\bar{E}_i'$ and $E_i'$) obtained by the DEA and CH-DEA methods, we can find that A, B, C, and D are all efficient DMUs, but the DEA and CH-DEA method cannot further describe which is better. Based on the acceptability indices ($a_i$) and the confidence factors ($p_i^c$) obtained by the SMAA-DEA and CH-SMAA-DEA methods, it is shown that SMAA-DEA identified A, B, and C as efficient DMUs with 100% confidence and acceptability indices 33%, 38%, and 29%, correspondingly, while CH-SMAA-DEA identified A, B, and C as efficient DMUs with 100% confidence and acceptability indices 13%, 77%, and 10%, correspondingly. It is obvious that the deviation of the acceptability indices between the efficient DMUs (A, B, and C) obtained by CH-SMAA-DEA was bigger than that obtained by SMAA-DEA, which indicates that the CH-SMAA-DEA method has better discriminability than the SMAA-DEA method. It was found that B has the highest acceptability both in the SMAA-DEA and the CH-SMAA-DEA method. Further, the values of the maximum, central, and average efficiencies ($E_i^{\max}$, $E_i^c$, and $E_i^{ave}$) obtained by CH-SMAA-DEA were not smaller than those obtained by SMAA-DEA, that is because CH-SMAA-DEA considers the interactions between inputs or outputs and can provide better results.

Next, we introduce uncertainty to the problem in Example 1. The inputs are accurate, but the outputs follow an independent uniform distribution $f(\xi_{ij})$ in the range $[x_{ij} - \Delta x_{ij}, x_{ij} + \Delta x_{ij}]$ with $\Delta x_{ij} = 0.5$. Ji et al.'s method [18] will be invalid in this situation, because their method is only used for the DEA problems with determined input and output values. The rank acceptability indices obtained by SMAA-DEA and CH-SMAA-DEA are illustrated in Table A3; here, $a_i = b_i^1$. We can find that A, B, C, D, and E are classified as efficient DMUs by both SMAA-DEA and CH-SMAA-DEA, and B has the highest acceptability, while F has the lowest one. However, the ranking acceptability indices are different by using SMAA-DEA and CH-SMAA-DEA; for example, the acceptability of B is 36% by SMAA-DEA, which is slightly bigger that of A with 33%, while the acceptability of B is 74% by CH-SMAA-DEA, which is much bigger than other ones. The acceptability of F is 0.05% by SMAA-DEA and 0.00015% by CH-SMAA-DEA, which shows that CH-SMAA-DEA can discriminate the efficient DMUs better than SMAA-DEA.

The central weight vector ($w_i^c$), confidence factors ($p_i^c$), and maximum, central, and average efficiencies ($E_i^{\max}$, $E_i^c$, $E_i^{ave}$) obtained by SMAA-DEA and CH-SMAA-DEA are listed in Table A4, from which it was found that most of the values of the maximum, central, and average efficiencies obtained by CH-SMAA-DEA are not smaller than those obtained by SMAA-DEA. This is because CH-SMAA-DEA considers the interactions between inputs and outputs, which can enlarge the information space.

**Example 2** ([18])**.** *The Community Health Center (DMU) of Hebei Province in China was evaluated. The evaluated input indices were the public expenditure (CNY 10,000 Yuan), the number of medical staff, and the fixed assets (CNY 10,000); the output indices were the number of medical services (thousands) (including inpatient service and childhood immunization) and the number of managed of chronic diseases (thousands). The data are shown in Table A5.*

Ji et al. [18] assumed that there exist low the interactions (correlations) between the input (output) variables, but they did not give the evidence to show that the interactions between input (output) variables are low. It is usually known that there exist interactions between inputs and outputs, but it is not easy to give exactly the interactions between them. Therefore, we assumed that there exist interactions between inputs and outputs, but we do not know whether the interactions are low or high.

The results obtained by the CCR, CH-CCR, SMAA-DEA, and CH-SMAA-DEA methods are listed in Table A6, from which we can find that DMUs 2, 3, 6, 9, 10, 12, 16, 18, 19, and 20 were classified as efficient DMUs by the CCR, CH-CCR, SMAA-DEA, and CH-SMAA-DEA methods. DMU 18 was efficient by CCR and CH-CCR methods and was almost efficient by the SMAA-DEA and CH-SMAA-DEA methods with maximum efficiency scores of 0.9997 and 0.9963, respectively. This may be because CCR and CH-CCR calculate the efficiency scores of DMUs based on the optimization programming with the whole feasible region, while SMAA-DEA and CH-SMAA-DEA derive the maximum efficiency scores of DMUs based on Monte Carlo simulation, which is a sampling analysis. SMAA-DEA and CH-SMAA-DEA can measure DMUs from different views, such as confidence factors, maximum efficiencies, confidence efficiencies, and average efficiencies, while CCR and CH-CCR measure DMUs based on self-efficiency scores and cross-efficiency scores. Most of the results obtained by CH-CCR were not smaller than those obtained by CCR, and most of the values of $p_i^c$, $E_i^{\max}$, $E_i^c$, and $E_i^{ave}$ obtained by CH-SMAA-DEA were not smaller than those obtained by SMAA-DEA.

It was found that the measures in Table A6, i.e., $E_i$, $E_i^{\max}$, and $E_i^c$, can classify DMUs into efficient and inefficient ones, but cannot further discriminate between efficient ones, and the measures $CE_i$ and $E_i^{ave}$ can give a ranking of DMUs, but cannot identify which DMU is efficient. All of the measures in Table A6 cannot give the acceptability of an efficient DMU. Then, the rank acceptability indices of DMUs were calculated by CH-SMAA-DEA and SMAA-DEA, and are listed in Tables A7 and A8 and Figures A2 and A3, and the central weight vectors are listed in Table A9. From the data, we can find that DMUs 2, 3, 6, 9, 10, 12, 16, 18, 19, and 20 are efficient DMUs with different rank acceptability indices; DMU 10 had the biggest acceptability 62% by CH-SMAA-DEA and 48% by SMAA-DEA; DMU 3 had the smallest acceptability 0.001% by CH-SMAA-DEA; DMU 9 had the smallest acceptability 0.003% by SMAA-DEA. The results showed that CH-SMAA-DEA can discriminate the efficient DMUs better than SMAA-DEA. In other rankings, the acceptability indices of DMUs obtained by CH-SMAA-DEA and SMAA-DEA were different. For example, DMU 14 had the highest acceptability with 18% for ranking sixth by CH-SMAA-DEA, but DMU 1 had the highest acceptability with 35% for ranking sixth by SMAA-DEA.

## 5. Conclusions

This paper investigated DEA with interactive inputs and outputs by SMAA. The CH-DEA model was introduced to reflect the interactions between inputs or outputs. To discriminate efficient DMUs, the SMAA method was used to explore the information space that supports the ranking of DMUs. To give a further analysis, several indices were defined to compare different DMUs, such as the Choquet-integral-based acceptability index, the Choquet-integral-based confidence factor, and the Choquet-integral-based central weight vector, which describe the DMUs based on the statistic analysis, the Choquet-integral-based maximum efficiency score, the Choquet-integral-based confidence efficiency score, and the Choquet-integral-based average efficiency score, which describe the DMUs based on the optimal analysis. Examples were given to compare the proposed method with the existing ones. Compared to the SMAA-DEA method, the CH-SMAA-DEA method can better discriminate DMUs by considering the interactions between inputs or outputs. Compared to the CH-DEA method, the proposed method can deal with the stochastic situation and propose the acceptability indices of efficient DMUs by exploring the information space supporting each of them. The disadvantage of the proposed method is that it may need more computation, but it can provide more information for the decision-makers. In the future, we will investigate the algorithms for CH-SMAA-DEA to reduce the computation and improve the accuracy.

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## Appendix A

**Table A1.** DMUs and their inputs and outputs.

| DMU | Input 1 | Output 1 | Output 2 |
|---|---|---|---|
| A | 1 | 3 | 9 |
| B | 1 | 7 | 7 |
| C | 1 | 9 | 2 |
| D | 1 | 9 | 1 |
| E | 1 | 7 | 6 |
| F | 1 | 3 | 8 |
| G | 1 | 8 | 1 |
| H | 1 | 5 | 6 |

**Table A2.** Different measure indices obtained by DEA and CH-SMAA-DEA.

| | DEA | | | | SMAA-DEA | | | | | CH-DEA | | | | | CH-SMAA-DEA | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $\bar{w}_1$ | $\bar{w}_2$ | $\bar{w}_3$ | $\bar{E}_i'$ | $a_i$ | $p_i^c$ | $E_i^{\max}$ | $E_i^c$ | $E_i^{ave}$ | $w_1$ | $w_2$ | $w_3$ | $w_{23}$ | $E_i'$ | $a_i$ | $p_i^c$ | $E_i^{\max}$ | $E_i^c$ | $E_i^{ave}$ |
| A | 1 | 0.05 | 0.10 | 1 | 33 | 100 | 1 | 1 | 0.79 | 1 | 0.0001 | 0.11 | 0 | 1 | 13 | 100 | 1 | 1 | 0.80 |
| B | 1 | 0.05 | 0.10 | 1 | 38 | 100 | 1 | 1 | 0.93 | 1 | 0.0001 | 0.0001 | 0.14 | 1 | 77 | 100 | 1 | 1 | 0.93 |
| C | 1 | 0.10 | 0.04 | 1 | 29 | 100 | 1 | 1 | 0.73 | 1 | 0.11 | 0.0001 | 0 | 1 | 10 | 100 | 1 | 1 | 0.76 |
| D | 1 | 0.11 | 0.0001 | 1.00 | 0 | 0 | 1.00 | 1.00 | 0.66 | 1 | 0.11 | 0.04 | 0 | 1 | 0 | 0 | 1 | 1 | 0.69 |
| E | 1 | 0.10 | 0.04 | 0.96 | 0 | 0 | 0.96 | 0.96 | 0.86 | 1 | 0.1 | 0.0001 | 0.04 | 0.96 | 0 | 0 | 0.96 | 0.96 | 0.86 |
| F | 1 | 0.05 | 0.10 | 0.90 | 0 | 0 | 0.90 | 0.90 | 0.73 | 1 | 0.0001 | 0.1 | 0.05 | 0.90 | 0 | 0 | 0.90 | 0.90 | 0.74 |
| G | 1 | 0.11 | 0.0001 | 0.89 | 0 | 0 | 0.89 | 0.89 | 0.60 | 1 | 0.11 | 0.0001 | 0 | 0.89 | 0 | 0 | 0.89 | 0.89 | 0.62 |
| H | 1 | 0.05 | 0.10 | 0.81 | 0 | 0 | 0.81 | 0.81 | 0.73 | 1 | 0.0001 | 0.1 | 0.05 | 0.81 | 0 | 0 | 0.81 | 0.81 | 0.73 |

**Table A3.** Rank acceptability obtained by SMAA-DEA and CH-SMAA-DEA.

| | SMAA-DEA | | | | | | | | CH-SMAA-DEA | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $b^1$ | $b^2$ | $b^3$ | $b^4$ | $b^5$ | $b^6$ | $b^7$ | $b^8$ | $b^1$ | $b^2$ | $b^3$ | $b^4$ | $b^5$ | $b^6$ | $b^7$ | $b^8$ |
| A | 33 | 9 | 10 | 3 | 3 | 3 | 26 | 12 | 14 | 10 | 18 | **20** | 11 | 8 | 13 | 6 |
| B | **36** | 19 | **28** | 10 | 7 | 0 | 0 | 0 | **74** | 10 | 8 | 4 | 2 | 1 | 0 | 0 |
| C | 19 | 14 | 12 | 5 | 2 | **48** | 1 | 0 | 6 | 6 | 16 | 15 | 9 | **49** | 1 | 0 |
| D | 10 | 16 | 5 | 12 | 2 | 3 | **40** | 12 | 3 | 5 | 3 | 10 | 11 | 7 | **47** | 14 |
| E | 2 | **23** | 18 | **40** | 18 | 0 | 0 | 0 | 3 | 64 | 12 | 14 | 5 | 2 | 1 | 0 |
| F | 0.05 | 20 | 14 | 14 | 4 | 3 | 15 | 30 | 0.00015 | 5 | 7 | 17 | **29** | 11 | 13 | 17 |
| G | 0 | 0 | 13 | 5 | 19 | 3 | 15 | **46** | 0 | 0 | 3 | 2 | 8 | 10 | 20 | **56** |
| H | 0 | 0 | 0 | 11 | **44** | 40 | 3 | 0 | 0 | 0 | **32** | 19 | 25 | 12 | 5 | 7 |

Note: The numbers in bold indicate the biggest acceptability indices.

**Table A4.** The central weights, confidence factors, maximum efficiency, central efficiency, and average efficiency of DMUs.

| | SMAA-DEA | | | | | | | CH-SMAA-DEA | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $\bar{w}_1$ | $\bar{w}_2$ | $\bar{w}_3$ | $p_i^c$ | $E_i^{\max}$ | $E_i^c$ | $E_i^{ave}$ | $w_1$ | $w_2$ | $w_3$ | $w_{23}$ | $p_i^c$ | $E_i^{\max}$ | $E_i^c$ | $E_i^{ave}$ |
| A | 1 | 0.17 | 0.83 | 99.8 | 1 | 0.99999 | 0.79 | 1 | 0.4235 | 0.8391 | −0.2626 | 99.4907 | 1 | 1.0000 | 0.7881 |
| B | 1 | 0.52 | 0.48 | 95 | 1 | 0.999 | 0.92 | 1 | 0.3531 | 0.3405 | 0.3064 | 98.4421 | 1 | 0.9998 | 0.9326 |
| C | 1 | 0.84 | 0.16 | 67 | 1 | 0.99 | 0.72 | 1 | 0.8507 | 0.4194 | −0.2702 | 68.6795 | 1 | 0.9912 | 0.7205 |
| D | 1 | 0.88 | 0.12 | 38 | 1 | 0.97 | 0.66 | 1 | 0.8909 | 0.4403 | −0.3312 | 36.2799 | 1 | 0.9733 | 0.6521 |
| E | 1 | 0.61 | 0.39 | 10 | 1 | 0.94 | 0.86 | 1 | 0.5719 | 0.2800 | 0.1481 | 6.8233 | 1 | 0.9366 | 0.8651 |
| F | 1 | 0.25 | 0.75 | 0.3 | 1 | 0.90 | 0.72 | 1 | 0.3464 | 0.7756 | −0.1219 | 0.3187 | 1 | 0.8998 | 0.7236 |
| G | 1 | 0.9991 | 0.0009 | 0 | 0.99 | 0.87 | 0.59 | 1 | 0.9812 | 0.2856 | −0.2668 | 0 | 0.9936 | 0.8703 | 0.5875 |
| H | 1 | 0.33 | 0.67 | 0 | 0.92 | 0.80 | 0.72 | 1 | 0.1975 | 0.6271 | 0.1754 | 0 | 0.9293 | 0.8002 | 0.7316 |

**Table A5.** DMUs and their inputs and outputs.

| DMU | Input 1 | Input 2 | Input 3 | Output 1 | Output 2 |
|---|---|---|---|---|---|
| 1 | 236 | 266 | 302 | 181 | 231 |
| 2 | 254 | 229 | 269 | 164 | 239 |
| 3 | 379 | 213 | 268 | 179 | 176 |
| 4 | 308 | 306 | 366 | 221 | 222 |
| 5 | 312 | 260 | 332 | 188 | 221 |
| 6 | 298 | 398 | 279 | 211 | 311 |
| 7 | 286 | 329 | 368 | 231 | 267 |
| 8 | 279 | 306 | 399 | 198 | 243 |
| 9 | 305 | 332 | 297 | 238 | 275 |
| 10 | 288 | 309 | 308 | 243 | 292 |
| 11 | 246 | 336 | 332 | 190 | 242 |

**Table A5.** *Cont.*

| DMU | Input 1 | Input 2 | Input 3 | Output 1 | Output 2 |
|---|---|---|---|---|---|
| 12 | 214 | 320 | 309 | 188 | 283 |
| 13 | 269 | 303 | 298 | 209 | 204 |
| 14 | 288 | 296 | 336 | 194 | 268 |
| 15 | 332 | 380 | 312 | 203 | 235 |
| 16 | 268 | 288 | 359 | 239 | 206 |
| 17 | 256 | 269 | 378 | 216 | 173 |
| 18 | 299 | 271 | 319 | 228 | 188 |
| 19 | 245 | 332 | 277 | 231 | 219 |
| 20 | 298 | 269 | 338 | 232 | 191 |

**Table A6.** Results obtained by different models.

| DMU | CCR | | CH-CCR | | SMAA-DEA | | | | CH-SMAA-DEA | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0.92449 | 0.79411 (8) | 0.92491 | 0.84765 (8) | 0 | 0.9231 | 0.9231 | 0.8448 (7) | 0 | 0.9261 | 0.9261 | 0.8547 (7) |
| 2 | 1 | 0.81906 (5) | 1 | 0.87617 (5) | 100 | 1.0000 | 1.0000 | 0.8818 (5) | 100 | 1.0000 | 1.0000 | 0.8894 (5) |
| 3 | 1 | 0.82673 (4) | 1 | 0.88293 (4) | 100 | 1.0000 | 1.0000 | 0.6964 (20) | 100 | 1.0000 | 1.0000 | 0.7022 (20) |
| 4 | 0.87827 | 0.72672 (16) | 0.91799 | 0.80108 (16) | 0 | 0.8777 | 0.8777 | 0.7489 (16) | 0 | 0.9111 | 0.9111 | 0.7616 (15) |
| 5 | 0.87934 | 0.72156 (17) | 0.91372 | 0.79985 (17) | 0 | 0.9098 | 0.9098 | 0.7490 (15) | 0 | 0.9065 | 0.9065 | 0.7529 (17) |
| 6 | 1 | 0.76411 (13) | 1 | 0.82717 (13) | 100 | 1.0000 | 1.0000 | 0.8835 (4) | 100 | 1.0000 | 1.0000 | 0.9004 (4) |
| 7 | 0.93242 | 0.79292 (9) | 0.942698 | 0.84666 (9) | 0 | 0.9302 | 0.9302 | 0.8367 (8) | 0 | 0.9344 | 0.9344 | 0.8455 (8) |
| 8 | 0.84351 | 0.71915 (18) | 0.842796 | 0.78986 (18) | 0 | 0.8426 | 0.8426 | 0.7417 (17) | 0 | 0.8433 | 0.8433 | 0.7546 (16) |
| 9 | 1 | 0.82856 (3) | 1 | 0.88653 (3) | 100 | 1.0000 | 1.0000 | 0.9066 (3) | 100 | 1.0000 | 1.0000 | 0.9186 (3) |
| 10 | 1 | 0.90077 (1) | 1 | 0.96356 (1) | 100 | 1.0000 | 1.0000 | 0.9744 (1) | 100 | 1.0000 | 1.0000 | 0.9833 (1) |
| 11 | 0.86797 | 0.71865 (19) | 0.87946 | 0.75164 (19) | 0 | 0.8659 | 0.8659 | 0.7799 (12) | 0 | 0.8745 | 0.8745 | 0.7812 (12) |
| 12 | 1 | 0.83276 (2) | 1 | 0.91485 (2) | 100 | 1.0000 | 1.0000 | 0.9207 (2) | 100 | 1.0000 | 1.0000 | 0.9209 (2) |
| 13 | 0.89589 | 0.74091 (14) | 0.90078 | 0.81457 (14) | 0 | 0.8937 | 0.8937 | 0.7856 (11) | 0 | 0.8981 | 0.8981 | 0.7919 (11) |
| 14 | 0.91996 | 0.76867 (11) | 0.83342 | 0.83461 (11) | 0 | 0.9186 | 0.9186 | 0.8262 (9) | 0 | 0.9527 | 0.9527 | 0.8384 (9) |
| 15 | 0.81219 | 0.63864 (20) | 0.81226 | 0.71519 (20) | 0 | 0.8105 | 0.8105 | 0.7066 (19) | 0 | 0.8099 | 0.8099 | 0.7163 (19) |
| 16 | 1 | 0.80526 (7) | 1 | 0.86374 (7) | 100 | 1.0000 | 1.0000 | 0.8091 (10) | 100 | 1.0000 | 1.0000 | 0.8200 (10) |
| 17 | 0.96309 | 0.73989 (15) | 0.9676 | 0.80638 (15) | 0 | 0.9612 | 0.9612 | 0.7204 (18) | 0 | 0.9648 | 0.9648 | 0.7324 (18) |
| 18 | 1 | 0.76761 (12) | 1 | 0.8305 (12) | 0 | 0.9997 | 0.9997 | 0.7784 (13) | 0 | 0.9963 | 0.9963 | 0.7824 (14) |
| 19 | 1 | 0.8085 (6) | 1 | 0.86891 (6) | 100 | 1.0000 | 1.0000 | 0.8743 (6) | 100 | 1.0000 | 1.0000 | 0.8856 (6) |
| 20 | 1 | 0.77637 (10) | 1 | 0.8385 (10) | 100 | 1.0000 | 1.0000 | 0.7782 (14) | 100 | 1.0000 | 1.0000 | 0.7837 (13) |

Note: The numbers in parentheses indicate the rankings of DMUs.

**Table A7.** Rank acceptability indices obtained by CH-SMAA-DEA.

| DMU | $b^1$ | $b^2$ | $b^3$ | $b^4$ | $b^5$ | $b^6$ | $b^7$ | $b^8$ | $b^9$ | $b^{10}$ | $b^{11}$ | $b^{12}$ | $b^{13}$ | $b^{14}$ | $b^{15}$ | $b^{16}$ | $b^{17}$ | $b^{18}$ | $b^{19}$ | $b^{20}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0 | 0 | 0.158 | 1.957 | 8.238 | 15.069 | **35.229** | 16.185 | 6.230 | 5.812 | 5.827 | 3.751 | 1.075 | 0.263 | 0.028 | 0 | 0 | 0 | 0 | 0 |
| 2 | 1.850 | 9.248 | 16.215 | 19.305 | 12.069 | 10.175 | 4.201 | 4.922 | 4.885 | 3.930 | 3.657 | 3.393 | 2.821 | 1.908 | 0.962 | 0.419 | 0.298 | 0.024 | 0 | 0 |
| 3 | 0.001 | 0.074 | 0.103 | 0.407 | 0.442 | 0.513 | 0.726 | 0.865 | 1.036 | 1.585 | 1.867 | 2.319 | 3.244 | 3.851 | 4.277 | 5.049 | 6.216 | 7.850 | 15.735 | **43.600** |
| 4 | 0 | 0 | 0.002 | 0.030 | 0.158 | 0.527 | 0.802 | 1.202 | 2.232 | 3.263 | 4.998 | 7.127 | 8.995 | **13.195** | **14.633** | 12.458 | 13.452 | 11.463 | 4.956 | 0.604 |
| 5 | 0 | 0 | 0 | 0 | 0.006 | 0.007 | 0.027 | 0.139 | 1.022 | 2.512 | 4.599 | 6.504 | 8.703 | 9.753 | **14.436** | **14.675** | **14.416** | 15.768 | 6.420 | 1.171 |
| 6 | 11.130 | 11.688 | 13.617 | 16.532 | 10.294 | 4.821 | 4.886 | 4.539 | 3.483 | 2.829 | 2.069 | 2.596 | 2.370 | 1.953 | 1.873 | 1.837 | 1.511 | 1.174 | 0.638 | 0 |
| 7 | 0 | 0 | 0.192 | 2.090 | 7.737 | 10.019 | 11.161 | **30.579** | **27.326** | 6.843 | 2.355 | 1.494 | 0.222 | 0.019 | 0.003 | 0 | 0 | 0 | 0 | 0 |
| 8 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.019 | 1.507 | 3.223 | 9.043 | 8.797 | 7.644 | 8.227 | 8.615 | 12.058 | 12.855 | 12.781 | 10.629 | 4.816 |
| 9 | 0.011 | 20.420 | **26.072** | **20.666** | **22.989** | 5.972 | 3.407 | 0.486 | 0.255 | 0.059 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 10 | **61.576** | **28.282** | 9.356 | 0.773 | 0.010 | 0.002 | 0.002 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 11 | 0 | 0 | 0 | 0.004 | 0.073 | 0.408 | 0.478 | 1.911 | 7.363 | **18.437** | 10.127 | 6.638 | 6.547 | 8.410 | 7.338 | 6.914 | 8.141 | 7.995 | 6.428 | 2.559 |
| 12 | 19.098 | 15.656 | 13.748 | 12.843 | 11.109 | 5.740 | 2.805 | 2.372 | 2.703 | 2.126 | 1.940 | 2.032 | 1.840 | 1.762 | 1.639 | 1.162 | 0.790 | 0.290 | 0.024 | 0 |
| 13 | 0 | 0 | 0.002 | 1.788 | 2.386 | 2.762 | 4.574 | 4.545 | 5.395 | 8.446 | 12.962 | 10.938 | **13.611** | 10.825 | 7.457 | 6.969 | 4.224 | 2.676 | 0.730 | 0.066 |
| 14 | 0 | 0 | 0.235 | 2.246 | 4.540 | **18.420** | 12.837 | 8.664 | 9.531 | 8.573 | 5.313 | 5.739 | 6.785 | 6.310 | 5.164 | 3.207 | 1.329 | 0.396 | 0.086 | 0 |
| 15 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.003 | 0.009 | 0.132 | 0.926 | 1.979 | 2.449 | 3.201 | 5.212 | 8.508 | 9.181 | 15.255 | **29.832** | 23.309 |
| 16 | 0.351 | 2.648 | 3.642 | 7.818 | 5.397 | 5.517 | 4.789 | 5.371 | 7.753 | 13.785 | **12.989** | **11.999** | 6.905 | 4.744 | 3.345 | 2.083 | 0.719 | 0.101 | 0 | 0 |
| 17 | 0 | 0.001 | 0.114 | 0.307 | 0.674 | 0.853 | 1.769 | 1.243 | 1.484 | 1.667 | 2.556 | 3.293 | 5.637 | 5.323 | 7.244 | 7.028 | 7.855 | 9.767 | 19.539 | 23.674 |
| 18 | 0 | 0.050 | 0.240 | 1.870 | 2.842 | 4.110 | 3.961 | 3.029 | 3.293 | 4.834 | 7.705 | 10.456 | 10.321 | 8.873 | 8.828 | 8.306 | 10.323 | 7.747 | 2.910 | 0.201 |
| 19 | 5.940 | 11.758 | 15.571 | 10.146 | 7.135 | 11.105 | 5.010 | 10.635 | 10.923 | 7.133 | 2.260 | 1.462 | 0.691 | 0.498 | 0.264 | 0.122 | 0.056 | 0.006 | 0 | 0 |
| 20 | 0.043 | 0.175 | 0.733 | 1.218 | 3.901 | 3.980 | 3.336 | 3.291 | 3.570 | 4.811 | 8.807 | 9.483 | 10.140 | 10.885 | 8.682 | 9.205 | 8.634 | 6.707 | 2.073 | 0 |

Note: The numbers in bold indicate the biggest acceptability indices.

**Table A8.** Rank acceptability indices obtained by SMAA-DEA.

| DMU | $b^1$ | $b^2$ | $b^3$ | $b^4$ | $b^5$ | $b^6$ | $b^7$ | $b^8$ | $b^9$ | $b^{10}$ | $b^{11}$ | $b^{12}$ | $b^{13}$ | $b^{14}$ | $b^{15}$ | $b^{16}$ | $b^{17}$ | $b^{18}$ | $b^{19}$ | $b^{20}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0 | 0 | 0.782 | 6.410 | 11.356 | **12.578** | **25.890** | 12.238 | 4.721 | 6.241 | 8.948 | 5.662 | 3.465 | 2.002 | 0.074 | 0 | 0 | 0 | 0 | 0 |
| 2 | 6.356 | 11.412 | 11.670 | 11.615 | 12.608 | 11.415 | 3.846 | 4.563 | 4.806 | 3.725 | 4.265 | 4.514 | 3.288 | 2.389 | 2.035 | 0.782 | 1.298 | 0 | 0 | 0 |
| 3 | 0.013 | 0.198 | 0.857 | 1.215 | 1.148 | 1.366 | 2.308 | 2.203 | 2.350 | 3.946 | 2.525 | 2.935 | 3.924 | 5.258 | 3.016 | 3.650 | 3.776 | 5.888 | 14.254 | **38.827** |
| 4 | 0 | 0 | 0 | 0 | 0 | 0 | 0.002 | 0.035 | 0.271 | 1.003 | 1.666 | 4.443 | 7.154 | **16.766** | **25.558** | **27.040** | 11.686 | 4.209 | 0.006 | 0 |
| 5 | 0 | 0 | 0 | 0.003 | 0.028 | 0.206 | 1.051 | 1.919 | 5.524 | 3.522 | 6.390 | 6.115 | 10.425 | 10.431 | 9.899 | 7.745 | 10.469 | **18.835** | 7.922 | 0 |
| 6 | 10.656 | 9.319 | 12.502 | 14.548 | 8.015 | 4.208 | 4.625 | 5.325 | 3.257 | 3.139 | 1.973 | 2.653 | 2.694 | 2.483 | 1.886 | 1.329 | 3.665 | 4.279 | 3.083 | 0.003 |
| 7 | 0 | 0 | 0 | 0.419 | 2.549 | 8.829 | 14.413 | **33.352** | **25.440** | 10.391 | 3.750 | 0.865 | 0.108 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 8 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.010 | 1.448 | 4.369 | 11.617 | 7.921 | 7.663 | 7.048 | 6.893 | 6.954 | **13.423** | 10.397 | 11.590 | 10.625 |
| 9 | 0.004 | 10.354 | **24.047** | **21.556** | **20.182** | 11.106 | 8.368 | 2.721 | 1.297 | 0.079 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 10 | **47.991** | **40.076** | 11.236 | 0.805 | 0.166 | 0.019 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 11 | 0 | 0 | 0 | 0.066 | 0.795 | 0.978 | 1.131 | 3.151 | 10.910 | **19.220** | 9.047 | 7.065 | 5.232 | 6.060 | 4.398 | 6.571 | 5.827 | 9.723 | 9.871 | 0.053 |
| 12 | 22.938 | 11.774 | 15.939 | 12.025 | 11.350 | 3.104 | 1.735 | 1.617 | 3.059 | 2.840 | 1.974 | 2.437 | 1.846 | 1.036 | 1.194 | 2.750 | 1.841 | 0 | 0 | 0 |
| 13 | 0 | 0 | 0 | 0 | 0.544 | 2.743 | 6.430 | 4.523 | 5.843 | 8.970 | **14.278** | **16.686** | **16.230** | 7.684 | 5.641 | 3.494 | 3.709 | 3.022 | 0.098 | 0 |
| 14 | 0 | 0 | 1.014 | 4.688 | 4.589 | 15.475 | 12.369 | 7.291 | 7.632 | 7.301 | 4.532 | 4.261 | 5.506 | 7.260 | 9.389 | 7.088 | 1.388 | 0.156 | 0 | 0 |
| 15 | 0 | 0 | 0 | 0 | 0 | 0 | 0.027 | 0.240 | 0.843 | 1.193 | 4.831 | 4.762 | 4.173 | 5.293 | 5.690 | 7.237 | 5.689 | 11.531 | 24.092 | **24.546** |
| 16 | 1.502 | 4.672 | 5.208 | 7.894 | 4.832 | 5.828 | 3.493 | 5.231 | 4.254 | 6.149 | 7.930 | 12.550 | 8.829 | 6.802 | 4.938 | 4.505 | 2.869 | 1.952 | 0 | 0 |
| 17 | 0 | 0.007 | 0.051 | 0.853 | 2.513 | 1.548 | 3.029 | 2.234 | 2.208 | 2.407 | 2.768 | 3.267 | 3.224 | 4.242 | 4.574 | 4.537 | 7.554 | 8.834 | 20.365 | 25.946 |
| 18 | 0 | 0.911 | 1.917 | 6.350 | 6.670 | 5.378 | 2.969 | 2.716 | 2.671 | 4.124 | 5.455 | 5.803 | 7.250 | 6.261 | 6.216 | 5.267 | 17.145 | 10.214 | 2.750 | 0 |
| 19 | 9.287 | 8.994 | 11.505 | 8.588 | 7.298 | 10.529 | 4.830 | 7.371 | 9.033 | 7.724 | 3.773 | 3.027 | 2.540 | 1.491 | 0.987 | 1.441 | 0.936 | 0.389 | 0.040 | 0 |
| 20 | 1.253 | 2.283 | 3.272 | 2.965 | 5.357 | 4.690 | 3.484 | 3.260 | 4.433 | 3.657 | 4.278 | 5.034 | 6.449 | 7.494 | 7.612 | 9.610 | 8.725 | 10.571 | 5.929 | 0 |

Note: The numbers in bold indicate the biggest acceptability indices.

**Table A9.** The central weights obtained by SMAA-DEA and CH-SMAA-DEA.

| DMU | SMAA-DEA | | | | | CH-SMAA-DEA | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $\bar{w}_1$ | $\bar{w}_2$ | $\bar{w}_3$ | $\bar{w}_4$ | $\bar{w}_5$ | $w_1$ | $w_2$ | $w_3$ | $w_{12}$ | $w_{13}$ | $w_{23}$ | $w_{123}$ | $w_4$ | $w_5$ | $w_{45}$ |
| 1 | 0.3824 | 0.6127 | 0.0049 | 0.2387 | 0.7613 | 0.6090 | 0.7660 | 0.0072 | −0.4909 | 0.3417 | 0.0203 | −0.2532 | 0.2317 | 0.9810 | −0.2127 |
| 2 | 0.1198 | 0.7245 | 0.1557 | 0.1611 | 0.8389 | 0.2010 | 0.4717 | 0.1391 | 0.0827 | 0.0540 | 0.2546 | −0.2031 | 0.5091 | 0.8770 | −0.3861 |
| 3 | 0.0048 | 0.9699 | 0.0253 | 0.7861 | 0.2139 | 0.0100 | 0.4229 | 0.0389 | 0.2755 | 0.0202 | 0.4294 | −0.1969 | 0.3154 | 0.0464 | 0.6382 |
| 4 | 0.1629 | 0.8363 | 0.0008 | 0.8335 | 0.1665 | 0.0210 | 0.3010 | 0.0011 | 0.1304 | 0.3594 | 0.6329 | −0.4459 | 0.0280 | 0.0004 | 0.9716 |
| 5 | 0.0028 | 0.9908 | 0.0064 | 0.6752 | 0.3248 | 0.0210 | 0.2351 | 0.0040 | 0.5242 | 0.0158 | 0.7300 | −0.5301 | 0.0527 | 0.3558 | 0.5914 |
| 6 | 0.1827 | 0.1115 | 0.7057 | 0.2555 | 0.7445 | 0.2593 | 0.0997 | 0.3365 | 0.2216 | 0.1154 | 0.3822 | −0.4147 | 0.5011 | 0.7739 | −0.2750 |
| 7 | 0.7268 | 0.2706 | 0.0026 | 0.8236 | 0.1764 | 0.4069 | 0.2585 | 0.0109 | −0.0212 | 0.5657 | −0.0040 | −0.2166 | 0.1438 | 0.0147 | 0.8415 |
| 8 | 0.3852 | 0.6138 | 0.0009 | 0.2791 | 0.7209 | 0.2292 | 0.4964 | 0.0002 | 0.0882 | 0.2233 | 0.0086 | −0.0460 | 0.3687 | 0.5577 | 0.0735 |
| 9 | 0.0150 | 0.0251 | 0.9599 | 0.7924 | 0.2076 | 0.0582 | 0.0144 | 0.3592 | 0.1016 | 0.3525 | 0.5819 | −0.4679 | 0.1606 | 0.0651 | 0.7743 |
| 10 | 0.2374 | 0.4186 | 0.3439 | 0.5960 | 0.4040 | 0.3143 | 0.3694 | 0.3275 | 0.0493 | 0.0802 | 0.0702 | −0.2109 | 0.4837 | 0.4063 | 0.1100 |
| 11 | 0.7637 | 0.2169 | 0.0194 | 0.8029 | 0.1971 | 0.1263 | 0.1092 | 0.0887 | 0.2556 | 0.3966 | −0.0773 | 0.1009 | 0.0314 | 0.0962 | 0.8724 |
| 12 | 0.6077 | 0.2030 | 0.1893 | 0.2919 | 0.7081 | 0.2820 | 0.2191 | 0.2119 | 0.1841 | 0.1813 | 0.0168 | −0.0951 | 0.4968 | 0.7353 | −0.2322 |
| 13 | 0.3267 | 0.3888 | 0.2846 | 0.9997 | 0.0003 | 0.2096 | 0.4402 | 0.2622 | 0.1665 | 0.3581 | −0.2117 | −0.2249 | 0.9931 | 0.0062 | 0.0007 |
| 14 | 0.3688 | 0.6265 | 0.0047 | 0.1999 | 0.8001 | 0.2093 | 0.5564 | 0.0038 | 0.1629 | 0.7254 | 0.4282 | −1.0860 | 0.9839 | 0.9804 | −0.9642 |
| 15 | 0.0110 | 0.0178 | 0.9712 | 0.7891 | 0.2109 | 0.0834 | 0.0097 | 0.2056 | 0.0143 | 0.6752 | 0.7097 | −0.6979 | 0.1647 | 0.0517 | 0.7835 |
| 16 | 0.4388 | 0.4981 | 0.0631 | 0.9438 | 0.0562 | 0.3190 | 0.4148 | 0.0504 | −0.0088 | 0.3261 | 0.2701 | −0.3715 | 0.9081 | 0.0689 | 0.0230 |
| 17 | 0.2576 | 0.7378 | 0.0046 | 0.9935 | 0.0065 | 0.1539 | 0.2304 | 0.0011 | 0.1887 | 0.7439 | 0.7556 | −1.0735 | 0.9612 | 0.0565 | −0.0178 |
| 18 | 0.0308 | 0.6399 | 0.3293 | 0.9937 | 0.0063 | 0.1718 | 0.4313 | 0.2516 | 0.0772 | −0.1624 | 0.2031 | 0.0275 | 0.9927 | 0.0656 | −0.0583 |
| 19 | 0.4727 | 0.1135 | 0.4138 | 0.8955 | 0.1045 | 0.2744 | 0.0938 | 0.2530 | 0.2780 | 0.1519 | 0.2172 | −0.2683 | 0.6543 | 0.0989 | 0.2468 |
| 20 | 0.0858 | 0.8077 | 0.1064 | 0.9360 | 0.0640 | 0.1145 | 0.3961 | 0.0814 | 0.1243 | 0.0032 | 0.3914 | −0.1108 | 0.9228 | 0.0895 | −0.0123 |

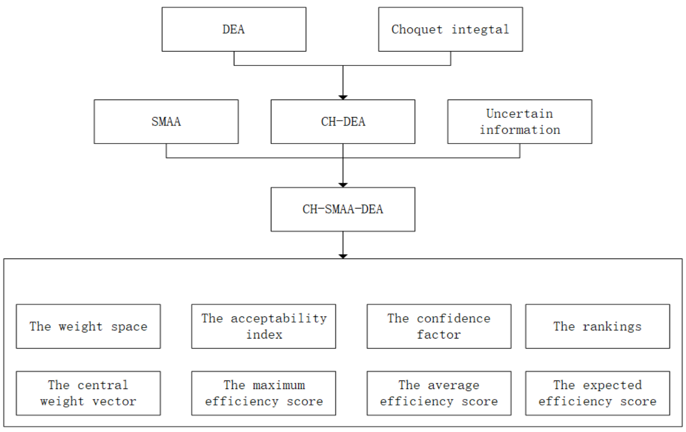


**Figure A1.** The flowchart of the proposed method.

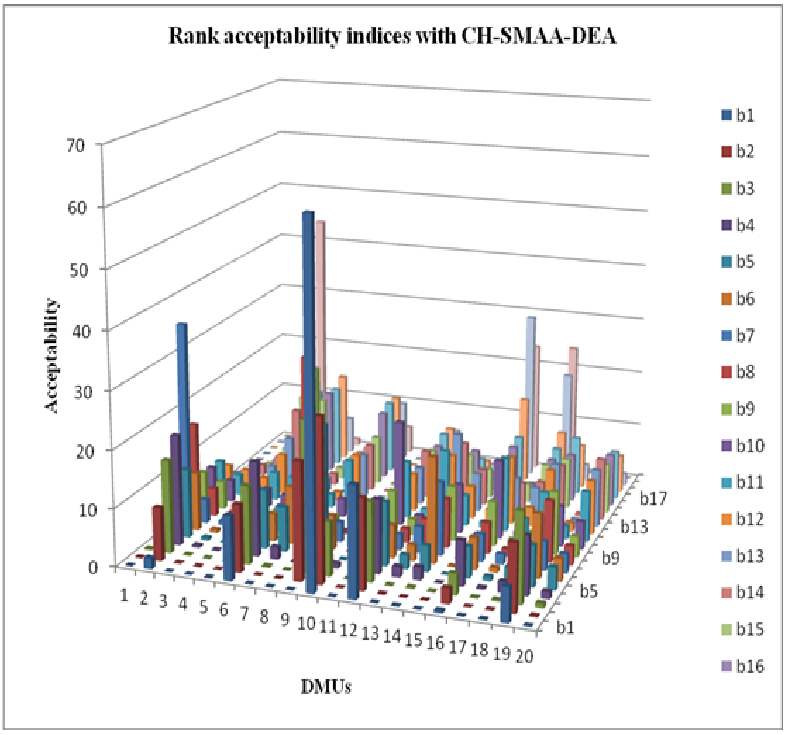


**Figure A2.** Rank acceptability indices with CH-SMAA-DEA.

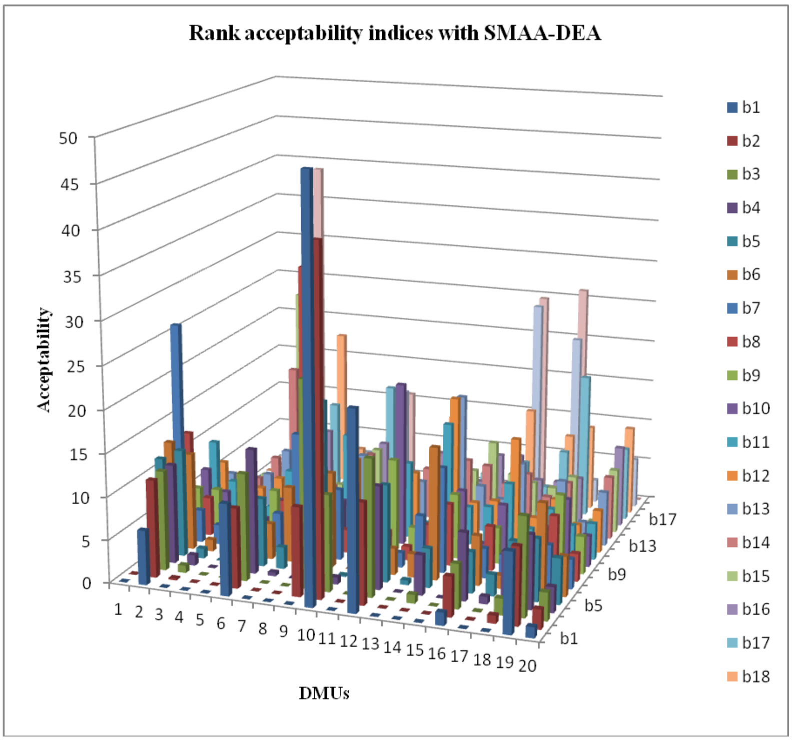


**Figure A3.** Rank acceptability indices with SMAA-DEA.

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