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Unified Integrals of Generalized Mittag–Leffler Functions and Their Graphical Numerical Investigation

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Abstract: In this article, we obtain certain finite integrals concerning generalized Mittag–Leffler functions, which are evaluated in terms of the generalized Fox–Wright function. The integrals of concern are unified in nature and thereby yield some new integral formulas as special cases. Moreover, we numerically compute some integrals using the Gaussian quadrature formula and draw a comparison with the main integrals by using graphical numerical investigation.

Keywords: Fox–Wright function; generalized hypergeometric function; Mittag–Leffler function

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1. Introduction

In mathematics, functions and symmetric functions are very common in theory and applications. They have been applied to various fields including group theory, Lie algebras, and algebraic geometry, to mention but a few. In applied mathematics, many functions are defined via integrals or series (or infinite products), which are usually referred to as special functions [1–6]. One of them is the Mittag–Leffler function, which was introduced in connection with a method of summation of some divergent series. The Mittag–Leffler function has recently received the interest of scientists due to its wide applications in pure as well as applied mathematics. It is noted that the importance of the Mittag–Leffler function has been envisaged during the last two decades due to its entanglement in physics, chemistry, biology, engineering and applied sciences. The Mittag–Leffler function naturally occurs as a solution of fractional order differential equations or fractional order integral equations. Problems of physics and applied mathematics involve a notable numerical implementation of the Mittag-Leffler function in general and modified forms; therefore, it remains an engaging object of applied research. The implementation of Mittag-Leffler functions is required in a wide variety of problems of physics and mathematics. Because of their crucial requirement, many research works have been dedicated to them, and various representations and generalizations of Mittag-Leffler functions can be found in the literature. Among the most popular special function of fractional calculus is the simplest ${}_p\Psi_q$ function and p = 0, q = 1, called the Wright function or the Bessel–Maitland function or the Wright–Bessel function. From this point of view, the Mittag–Leffler function, expressible in terms of the Fox–Right function, is a special function of fractional calculus. Therefore the Mittag–Leffler function is called the queen function of fractional calculus. The results obtained in the manuscript, connected with a generalized Mittag=-Leffler function that will be used to solve a variety of problems of fractional calculus, for example, Riemann-–Liouville fractional integrals and derivatives, Laplace and Sumudu fractional and integral derivatives and Marichev–Saigo–Maeda fractional integrals and derivatives, etc. Recently, fractional calculus associated with some special functions has proved itself to be a useful tool for applications in many fields of research such as physical systems, biomedicine, nonlinear electronic circuits, chaos-based cryptography, and image encryption. Examples of systems that can be precisely described by fractional-order differential equations (FODEs) involve viscoelastic material models, electrical components, electronic circuits, diffusion waves, the propagation of waves in non-local elastic continua, hydro-logic systems, earthquakes' nonlinear oscillations, models of world economies, fractional viscoelastic models and continuous random walk and equations of muscular blood vessels (see [7–12]). In the past few years, several integral formulas having a variety of special functions have been achieved by many authors (see [13–30]). The present paper provides the study of finite integrals of the generalized Mittag-Leffler function and investigates some useful formulas. We have computed many new results involving integral transforms of the Mittag–Leffler function and plotted three graphs as the major novelty of our work. The results derived in this paper are of general character and likely to find certain applications in the theory of special functions. Additionally, the results provide unification and extension of known results given earlier by various researchers. We compare the results of analytically evaluated integrals with integrals evaluated numerically using the Gaussian quadrature formula. We conclude that the results obtained will provide a significant step in the theory of integral formulas and can yield some potential applications in the field of classical and applied mathematics. Motivated by the aforementioned research and success of the application of integral formulas, we evaluate a new type of integral formulas involving the generalized Mittag–Leffler function (GMLF) expressed in terms of the Fox–Wright function. The Mittag–Leffler function [31,32] is defined as

$$E_{\sigma}(w) = \sum_{n=0}^{\infty} \frac{w^n}{\Gamma(\sigma n + 1)}, \quad \sigma \in \mathbb{C},\ Re(\sigma) > 0 \tag{1}$$

where ω ia a complex variable and $\Gamma(.)$ is the gamma function [25].

In 1905, A. Wiman [33] established a generalization of $E_{\sigma}(w)$, as follows:

$$E_{\sigma,\mu}(w) = \sum_{n=0}^{\infty} \frac{w^n}{\Gamma(\sigma n + \mu)}, \quad (\sigma, \mu \in \mathbb{C},\ \ Re(\mu) > 0,\ Re(\sigma) > 0). \tag{2}$$

In 1971, Prabhakar [23] came up with a further generalization of $E_{\sigma,\mu}(w)$ in the form

$$E^{\gamma}_{\sigma,\mu}(w) = \sum_{n=0}^{\infty} \frac{(\gamma)_n}{\Gamma(\sigma n + \mu)} \frac{w^n}{n!} \quad (\gamma, \mu, \sigma \in \mathbb{C},\ Re(\sigma) > 0, Re(\gamma) > 0, Re(\mu) > 0), \tag{3}$$

where $(\gamma)_n$ is known as the Pochhammer symbol [25]. The underlying generalization of the Mittag–Leffler function is given by Shukla and Prajapati (2007) [29] as

$$E^{\gamma,b}_{\sigma,\mu}(w) = \sum_{n=0}^{\infty} \frac{(\gamma)_{bn}}{\Gamma(\sigma n + \mu)} \frac{w^n}{n!} \tag{4}$$

and expressed by Salim (2009) [26] in the form

$$E^{\gamma,\delta}_{\sigma,\mu}(w) = \sum_{n=0}^{\infty} \frac{(\gamma)_n\, w^n}{\Gamma(\sigma n + \mu)(\delta)_n}. \tag{5}$$

A certain further generalization of the Mittag–Leffler function was given by Salim and Faraj (2012) [27] as

$$E^{\gamma,\delta,b}_{\sigma,\mu,a}(w) = \sum_{n=0}^{\infty} \frac{(\gamma)_{bn}\, w^n}{\Gamma(\sigma n + \mu)(\delta)_{an}}. \tag{6}$$

On the other hand, Khan and Ahmad introduced a new generalization of the Mittag–Leffler function (2013) [34] as

$$E^{\gamma,b}_{\sigma,\mu,\delta}(w) = \sum_{n=0}^{\infty} \frac{(\gamma)_{bn}\, w^n}{\Gamma(\sigma n + \mu)\ (\delta)_n}, \tag{7}$$

where $\sigma,\ \mu,\ \gamma,\ \delta \in \mathbb{C};\ Re(\sigma) > 0, Re(\mu) > 0, Re(\gamma) > 0, Re(\delta) > 0;\ b \in (0,1) \cup \mathbb{N}$.

Consequently, they have introduced a generalization of (7) in the following form [34]

$$E^{\xi,\lambda,\gamma,b}_{\sigma,\mu,\nu,\phi,\delta,a}(w) = \sum_{n=0}^{\infty} \frac{(\xi)_{\lambda n}\,(\gamma)_{bn}\,w^n}{\Gamma(\sigma n+\mu)(\nu)_{\phi n}(\delta)_{an}}, \tag{8}$$

where $\sigma,\mu,\nu,\phi,\delta,\xi,\lambda,\gamma \in \mathbb{C}$; $\min\{Re(\sigma),\ Re(\mu),\ Re(\nu), Re(\phi), Re(\delta), Re(\xi),\ Re(\lambda), Re(\gamma)\} > 0; a, b > 0, b \leq Re(\sigma) + a$.

Above all, (8) is the most generalized definition of all the above formalizations introduced in (1)–(7). Upon substituting $\xi = \nu,\ \lambda = \phi$ and $a = 1$ in (8), it becomes (7), which has been established by Khan and Ahmad (2013) [34]. Upon substituting $\xi = \nu$ and $\lambda = \phi$, in (8), it becomes a special case (6), which has been established by Salim and Faraj (2012) [27]. Upon substituting $\xi = \nu,\ \lambda = \phi$ and $b = a = 1$ in (8), it becomes (5), which has been discussed by Salim (2009) [26]. Upon substituting $\xi = \nu,\ \lambda = \phi$ and $\delta = a = 1$ in equation (8), it is a special case (4); see Shukla and Prajapati (2007) [29]. If $b = 1$, it becomes a special case (3) of Prabhakar (1971) [23]. On substituting $\xi = \nu,\ \lambda = \phi$ and $\gamma = \delta = a = b = 1$ in (8), it becomes a special case (2) established by A. Wiman (1905) [33]. Furthermore, if $\mu = 1$, we get the Mittag–Leffler function $E_\sigma(w)$ defined in (1). Finally, on setting $\delta = a = b = 1$ in (8), we establish a new generalization of the Mittag–Leffler function in the form

$$E^{\xi,\lambda,\gamma}_{\sigma,\mu,\nu,\phi}(w) = \sum_{n=0}^{\infty} \frac{(\xi)_{\lambda n}\,(\gamma)_n}{\Gamma(\sigma n+\mu)(\nu)_{\phi n}}\,\frac{w^n}{n!}\,, \tag{9}$$

where $\sigma,\mu,\nu,\phi,\xi,\lambda,\gamma \in \mathbb{C};\ Re(\mu) > 0, Re(\sigma) > 0, Re(\nu) > 0, Re(\phi) > 0, Re(\xi) > 0, Re(\lambda) > 0$ *and* $Re(\gamma) > 0$.

The Fox–Wright function ${}_r\Psi_s[w]$ (see [35–42]), is defined by

$${}_r\Psi_s[w] = {}_r\Psi_s\left[\begin{array}{c} (\lambda_1,\acute{\lambda}_1),\ldots,(\lambda_r,\acute{\lambda}_r); \\ \\ (l_1,\acute{l}_1),\ldots,(l_s,\acute{l}_s); \end{array} w\right] \tag{10}$$

$$= \sum_{k=0}^{\infty} \frac{\Gamma(\lambda_1+\acute{\lambda}_1 k),\ldots,\Gamma(\lambda_r+\acute{\lambda}_r k)}{\Gamma(l_1+\acute{l}_1 k),\ldots,\Gamma(l_s+\acute{l}_s k)}\,\frac{w^k}{k!} \tag{11}$$

$$= H^{1,r}_{r,s+1}\left[-w\left|\begin{array}{c} (1-\lambda_1,\acute{\lambda}_1),\ldots,(1-\lambda_r,\acute{\lambda}_r) \\ \\ (0,1),(1-l_1,\acute{l}_1),\ldots,(1-l_s,\acute{l}_s) \end{array}\right.\right], \tag{12}$$

where $H^{1,r}_{r,s+1}[w]$ represents the Fox-H function [38]. When $\acute{\lambda}_1,\ldots,\acute{\lambda}_r = 1, \acute{l}_1,\ldots,\acute{l}_s = 1$ in (10), the Fox–Wright function reduces to the generalized hypergeometric function ${}_rF_s$[w] (see [41])

$${}_r\Psi_s\left[\begin{array}{c} (\lambda_1,1),\ldots,(\lambda_r,1); \\ (l_1,1),\ldots,(l_s,1); \end{array} w\right] = \frac{\Gamma(\lambda)_1,\ldots,\Gamma(\lambda)_r}{\Gamma(l)_1,\ldots,\Gamma(l)_s}\ {}_rF_s(\lambda_1,\ldots,\lambda_r;l_1,\ldots,l_s;w). \tag{13}$$

Here, we recall the result due to Prudnikov et al. [24] (see also [39], p. 250 (2.8)), by means of which we have established our main result in the present article

$$\int_p^q \frac{(x-p)^{\alpha-1}(q-x)^{\beta-1}}{[(q-p)+r_1(x-p)+r_2(q-x)]^{\alpha+\beta}}\,dx$$

$$= \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}\ (q-p)^{-1}(1+r_1)^{-\alpha}(1+r_2)^{-\beta}, \tag{14}$$

provided that $Re(\alpha) > 0,\ Re(\beta) > 0,\ q \neq p$ and the constants r_1 and r_2 are such that none of the expression $1+r_1, 1+r_2, [(q-p)+r_1(x-p)+r_2(q-x)],$ where $p \leq x \leq q$ is zero.

2. Main Results

Theorem 1. *Let α and β exist such that $Re(\alpha) > 0, Re(\beta) > 0, q \neq p$ and the constants r_1 and r_2 are such that none of the expressions $1 + r_1$, $1 + r_2$, $[(q-p) + r_1(x-p) + r_2(q-x)]$, where $p \leq x \leq q$ is zero. Let $\sigma, \mu, \nu, \phi, \delta, \xi, \lambda, \gamma \in \mathbb{C}$; if $\min\{Re(\sigma), Re(\mu), Re(\nu), Re(\phi), Re(\delta), Re(\xi), Re(\lambda), Re(\gamma)\} > 0$; $a, b > 0$, $b \leq Re(\sigma) + a$, then the following identity holds:*

$$\int_p^q \frac{(x-p)^{\alpha-1}(q-x)^{\beta-1}}{[(q-p)+r_1(x-p)+r_2(q-x)]^{\alpha+\beta}} E^{\xi,\lambda,\gamma,b}_{\sigma,\mu,\nu,\phi,\delta,a}\left[w\left\{\frac{(x-p)(q-x)}{[(q-p)+r_1(x-p)+r_2(q-x)]^2}\right\}^m\right]dx$$
$$= \frac{1}{(q-p)\,(1+r_1)^{\alpha}\,(1+r_2)^{\beta}}\,\frac{\Gamma(\nu)\Gamma(\delta)}{\Gamma(\xi)\Gamma(\gamma)}$$
$$\times\, {}_5\Psi_4\left[\begin{array}{ccccc} (\xi,\lambda), & (\gamma,b), & (\alpha,m), & (\beta,m), & (1,1); \\ (\mu,\sigma), & (\nu,\phi), & (\delta,a), & & (\alpha+\beta,2m); \end{array} \frac{w}{(1+r_1)^m(1+r_2)^m}\right], \tag{15}$$

where $E^{\xi,\lambda,\gamma,b}_{\sigma,\mu,\nu,\phi,\delta,a}(w)$ is a GMLF given by (8).

Proof. Denoting the left hand side of (15) by I, writing $E^{\xi,\lambda,\gamma,b}_{\sigma,\mu,\nu,\phi,\delta,a}(w)$ in its summation formula in the integrand with the help of (8), we obtain

$$I = \int_p^q \frac{(x-p)^{\alpha-1}(q-x)^{\beta-1}}{[(q-p)+r_1(x-p)+r_2(q-x)]^{\alpha+\beta}}$$
$$\times \sum_{n=0}^{\infty} \frac{(\xi)_{\lambda n}(\gamma)_{bn}\,(1)_n\, w^n\,(x-p)^{mn}(q-x)^{mn}}{\Gamma(\sigma n+\mu)\,(\nu)_{\phi n}\,(\delta)_{an}\,[(q-p)+r_1(x-p)+r_2(q-x)]^{2mn}\,n!}\,dx, \tag{16}$$

which, by further simplification, yields

$$I = \sum_{n=0}^{\infty} \frac{(\xi)_{\lambda n}\,(\gamma)_{bn}}{\Gamma(\sigma n+\mu)\,(\nu)_{\phi n}\,(\delta)_{an}}\frac{w^n}{n!}\int_p^q \frac{(x-p)^{mn+\alpha-1}\,(q-x)^{mn+\beta-1}}{[(q-p)+r_1(x-p)+r_2(q-x)]^{2mn+\alpha+\beta}}\,dx. \tag{17}$$

We apply the result of (14), and, through simplifying, this yields

$$I = \frac{1}{(q-p)\,(1+r_1)^{\alpha}\,(1+r_2)^{\beta}}\,\frac{\Gamma(\nu)\,\Gamma(\delta)}{\Gamma(\xi)\,\Gamma(\gamma)}$$
$$\times \sum_{n=0}^{\infty} \frac{\Gamma(\xi+\lambda n)\,\Gamma(\gamma+bn)\,\Gamma(\alpha+mn)\,\Gamma(\beta+mn)\,\Gamma(1+n)\left(\frac{w}{(1+r_1)^m\,(1+r_2)^m}\right)^n}{\Gamma(\mu+\sigma n)\,\Gamma(\nu+\lambda n)\,\Gamma(\delta+an)\,\Gamma(\alpha+\beta+2mn)\,n!}. \tag{18}$$

Finally, after summing up, with the help of (11), we arrive at (15). This completes the proof of Theorem 1. □

Corollary 1. *For $b = \delta = a = 1$ and all the conditions already stated in (15), the following identity holds:*

$$\int_p^q \frac{(x-p)^{\alpha-1}(q-x)^{\beta-1}}{[(q-p)+r_1(x-p)+r_2(q-x)]^{\alpha+\beta}} E^{\xi,\lambda,\gamma}_{\sigma,\mu,\nu,\phi}\left[w\left\{\frac{(x-p)(q-x)}{[(q-p)+r_1(x-p)+r_2(q-x)]^2}\right\}^m\right]dx$$
$$= \frac{1}{(q-p)\,(1+r_1)^{\alpha}\,(1+r_2)^{\beta}}\,\frac{\Gamma(\nu)}{\Gamma(\xi)\Gamma(\gamma)}$$
$$\times\, {}_4\Psi_3\left[\begin{array}{cccc} (\xi,\lambda), & (\gamma,1), & (\alpha,m), & (\beta,m); \\ (\mu,\sigma), & (\nu,\phi), & & (\alpha+\beta,2m); \end{array} \frac{w}{(1+r_1)^m(1+r_2)^m}\right]. \tag{19}$$

Theorem 2. *Let α and β be such that $Re(\alpha) > 0$, $Re(\beta) > 0$, $q \neq p$ and the constants r_1 and r_2 are such that none of the expressions $1 + r_1$, $1 + r_2$, $[(q-p) + r_1(x-p) + r_2(q-x)]$, where $p \leq x \leq q$ is zero. Let $\sigma, \mu, \nu, \phi, \delta, \xi, \lambda, \gamma \in \mathbb{C}$; if $\min\{Re(\sigma), Re(\mu), Re(\nu), Re(\phi), Re(\delta), Re(\xi), Re(\lambda), Re(\gamma)\} > 0$; $a, b > 0$, $b \leq Re(\sigma) + a$, the following identity holds:*

$$\int_p^q \frac{(x-p)^{\alpha-1}(q-x)^{\beta-1}}{[(q-p)+r_1(x-p)+r_2(q-x)]^{\alpha+\beta}} E^{\xi,\lambda,\gamma,b}_{\sigma,\mu,\nu,\phi,\delta,a}\left[\left\{\frac{(x-p)(q-x)}{[(q-p)+r_1(x-p)+r_2(q-x)]^2}\right\}^m w\right] dx$$

$$= \frac{1}{(q-p)\ (1+r_1)^\alpha\ (1+r_1)^\beta} \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)\Gamma(\mu)}\ {}_{2m+\lambda+b+1}F_{2m+\sigma+\phi+a}\left[\begin{matrix} \Delta(m;\alpha), & \Delta(m;\beta), & \Delta(\lambda;\xi), \\ \Delta(\sigma;\mu), & \Delta(\phi;\nu), & \Delta(a;\delta), \end{matrix}\right.$$

$$\left.\begin{matrix} \Delta(b;\gamma), & 1; \\ \Delta(2m;\alpha+\beta); & \end{matrix} \frac{w\,\lambda^\lambda b^b}{\sigma^\sigma \phi^\phi a^a 4^m (1+r_1)^m (1+r_2)^m}\right], \tag{20}$$

where $\Delta(m;\lambda)$ abbreviates the arrangement of m parameters $\frac{\lambda}{m}\ \frac{\lambda+1}{m} \ldots \frac{\lambda+m-1}{m}$ and $m \geq 1$.

Proof. By using the formulas

$$\Gamma(\lambda+n) = \Gamma(\lambda)(\lambda)_n \tag{21}$$

and

$$(\lambda)_{mn} = m^{mn}\left(\frac{\lambda}{m}\right)_n \left(\frac{\lambda+1}{m}\right)_n \cdots \left(\frac{\lambda+m-1}{m}\right)_n, \tag{22}$$

and after a little simplification, the required result (20) can be obtained. Therefore, we omit the proof. □

Corollary 2. *On putting $a = b = \delta = 1$ under the condition already set out in (20), the following identity holds:*

$$\int_p^q \frac{(x-p)^{\alpha-1}(q-x)^{\beta-1}}{[(q-p)+r_1(x-p)+r_2(q-x)]^{\alpha+\beta}} E^{\xi,\lambda,\gamma}_{\sigma,\mu,\nu,\phi}\left[w\left\{\frac{(x-p)(q-x)}{[(q-p)+r_1(x-p)+r_2(q-x)]^2}\right\}^m\right] dx$$

$$= \frac{1}{(q-p)\ (1+r_1)^\alpha\ (1+r_2)^\beta} \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)\Gamma(\mu)}$$

$$\times\ {}_{2m+\lambda+1}F_{2m+\sigma+\phi}\left[\begin{matrix} \Delta(m;\alpha), & \Delta(m;\beta), & \Delta(\lambda;\xi), & \gamma; \\ \Delta(\sigma;\mu), & \Delta(\phi;\nu), & & \Delta(2m;\alpha+\beta); \end{matrix} \frac{w\,\lambda^\lambda}{\sigma^\sigma \phi^\phi 4^m (1+r_1)^m (1+r_2)^m}\right]. \tag{23}$$

3. Special Cases

Here, we compute certain integral formulas as special cases of our key results.

(i) On setting $\xi = \nu$, $\lambda = \phi$ in (15), the following identity holds:

$$\int_p^q \frac{(x-p)^{\alpha-1}(q-x)^{\beta-1}}{[(q-p)+r_1(x-p)+r_2(q-x)]^{\alpha+\beta}} E^{\gamma,\delta,b}_{\sigma,\mu,a}\left[w\left\{\frac{(x-p)(q-x)}{[(q-p)+r_1(x-p)+r_2(q-x)]^2}\right\}^m\right] dx$$

$$= \frac{1}{(q-p)\ (1+r_1)^\alpha\ (1+r_2)^\beta} \frac{\Gamma(\delta)}{\Gamma(\gamma)}$$

$$\times\ {}_4\Psi_3\left[\begin{matrix} (\gamma,b), & (\alpha,\ m), & (\beta,\ m) & (1,1); \\ (\mu,\sigma), & (\delta,a), & & (\alpha+\beta,\ 2m); \end{matrix} \frac{w}{(1+r_1)^m(1+r_2)^m}\right]; \tag{24}$$

(ii) Setting $\xi = \nu$, $\lambda = \phi$ and $a = 1$ in (15), the following identity holds:

$$\int_p^q \frac{(x-p)^{\alpha-1}(q-x)^{\beta-1}}{[(q-p)+r_1(x-p)+r_2(q-x)]^{\alpha+\beta}} E^{\gamma,b}_{\sigma,\mu,\delta}\left[w\left\{\frac{(x-p)(q-x)}{[(q-p)+r_1(x-p)+r_2(q-x)+]^2}\right\}^m\right] dx$$

$$= \frac{1}{(q-p)\ (1+r_1)^{\alpha}\ (1+r_2)^{\beta}}\ \frac{\Gamma(\delta)}{\Gamma(\gamma)}$$

$$\times\ {}_4\Psi_3\left[\begin{array}{ccc} (\gamma,b),\ (\alpha,\ m),\ (\beta,\ m) & (1,1); & \\ & & \frac{w}{(1+r_1)^m(1+r_2)^m} \\ (\mu,\sigma),\ (\delta,1), & (\alpha+\beta,\ 2m); & \end{array}\right]; \tag{25}$$

(iii) Setting $\xi = \nu$, $\lambda = \phi$ and $a = b = 1$ in (15), the following identity holds:

$$\int_p^q \frac{(x-p)^{\alpha-1}(q-x)^{\beta-1}}{[(q-p)+r_1(x-p)+r_2(q-x)]^{\alpha+\beta}}\ E^{\gamma,\delta}_{\sigma,\mu}\left[w\left\{\frac{(x-p)(q-x)}{[(q-p)+r_1(x-p)+r_2(q-x)]^2}\right\}^m\right]dx$$

$$= \frac{1}{(q-p)\ (1+r_1)^{\alpha}\ (1+r_2)^{\beta}}\ \frac{\Gamma(\delta)}{\Gamma(\gamma)}$$

$$\times\ {}_4\Psi_3\left[\begin{array}{ccc} (\gamma,1),\ (\alpha,\ m),\ (\beta,\ m) & (1,1); & \\ & & \frac{w}{(1+r_1)^m(1+r_2)^m} \\ (\mu,\sigma),\ (\delta,1), & (\alpha+\beta,\ 2m); & \end{array}\right]; \tag{26}$$

(iv) Setting $\xi = \nu$, $\lambda = \phi$ and $a = \delta = 1$ in (15), the following identity holds:

$$\int_p^q \frac{(x-p)^{\alpha-1}(q-x)^{\beta-1}}{[(q-p)+r_1(x-p)+r_2(q-x)]^{\alpha+\beta}}\ E^{\gamma,b}_{\sigma,\mu}\left[w\left\{\frac{(x-p)(q-x)}{[(q-p)+r_1(x-p)+r_2(q-x)]^2}\right\}^m\right]dx$$

$$= \frac{1}{(q-p)\ (1+r_1)^{\alpha}\ (1+r_2)^{\beta}}\ \frac{1}{\Gamma(\gamma)}$$

$$\times\ {}_3\Psi_2\left[\begin{array}{ccc} (\gamma,b),\ (\alpha,\ m), & (\beta,\ m); & \\ & & \frac{w}{(1+r_1)^m(1+r_2)^m} \\ (\mu,\sigma), & (\alpha+\beta,\ 2m); & \end{array}\right]; \tag{27}$$

(v) Setting $\xi = \nu$, $\lambda = \phi$ and $a = b = \delta = 1$ in (15), the following identity holds:

$$\int_p^q \frac{(x-p)^{\alpha-1}(q-x)^{\beta-1}}{[(q-p)+r_1(x-p)+r_2(q-x)]^{\alpha+\beta}}\ E^{\gamma}_{\sigma,\mu}\left[w\left\{\frac{(x-p)(q-x)}{[(q-p)+r_1(x-p)+r_2(q-x)]^2}\right\}^m\right]dx$$

$$= \frac{1}{(q-p)\ (1+r_1)^{\alpha}\ (1+r_2)^{\beta}}\ \frac{1}{\Gamma(\gamma)}$$

$$\times\ {}_3\Psi_2\left[\begin{array}{ccc} (\gamma,1),\ (\alpha,\ m), & (\beta,\ m); & \\ & & \frac{w}{(1+r_1)^m(1+r_2)^m} \\ (\mu,\sigma), & (\alpha+\beta,\ 2m); & \end{array}\right]; \tag{28}$$

(vi) Setting $\xi = \nu$, $\lambda = \phi$ and $a = b = \gamma = \delta = 1$ in (15), the following identity holds:

$$\int_p^q \frac{(x-p)^{\alpha-1}(q-x)^{\beta-1}}{[(q-p)+r_1(x-p)+r_2(q-x)]^{\alpha+\beta}}\ E_{\sigma,\mu}\left[w\left\{\frac{(x-p)(q-x)}{[(q-p)+r_1(x-p)+r_2(q-x)]^2}\right\}^m\right]dx$$

$$= \frac{1}{(q-p)\ (1+r_1)^{\alpha}\ (1+r_2)^{\beta}}$$

$$\times\ {}_3\Psi_2\left[\begin{array}{ccc} (\alpha,\ m),\ (\beta,\ m),\ (1,1); & & \\ & & \frac{w}{(1+r_1)^m(1+r_2)^m} \\ (\mu,\sigma), & (\alpha+\beta,\ 2m); & \end{array}\right]; \tag{29}$$

(vii) Setting $\xi = \nu$, $\lambda = \phi$ and $a = b = \gamma = \delta = \mu = 1$ in (15), the following identity holds:

$$\int_p^q \frac{(x-p)^{\alpha-1}(q-x)^{\beta-1}}{[(q-p)+r_1(x-p)+r_2(q-x)]^{\alpha+\beta}}\ E_{\sigma}\left[w\left\{\frac{(x-p)(q-x)}{[(q-p)+r_1(x-p)+r_2(q-x)]^2}\right\}^m\right]dx$$

$$= \frac{1}{(q-p)\ (1+r_1)^{\alpha}\ (1+r_2)^{\beta}}$$

$$\times\ {}_3\Psi_2\left[\begin{matrix}(\alpha,m), & (\beta,m), & (1,1); & \\ & (1,\sigma), & & (\alpha+\beta,2m);\end{matrix}\ \frac{w}{(1+r_1)^m(1+r_2)^m}\right]; \tag{30}$$

(viii) Setting $\xi = \nu$, $\lambda = \phi$ and $a = b = \gamma = \delta = \mu = \sigma = 1$ in (15), the following identity holds:

$$\int_p^q \frac{(x-p)^{\alpha-1}(q-x)^{\beta-1}}{[(q-p)+r_1(x-p)+r_2(q-x)]^{\alpha+\beta}}\, e^{\left[w\left\{\frac{(x-p)(q-x)}{[(q-p)+r_1(x-p)+r_2(q-x)]^2}\right\}^m\right]}\, dx$$

$$= \frac{1}{(q-p)\ (1+r_1)^{\alpha}\ (1+r_2)^{\beta}}$$

$$\times\ {}_2\Psi_1\left[\begin{matrix}(\alpha,m), & (\beta,m), \\ & (\alpha+\beta,2m);\end{matrix}\ \frac{w}{(1+r_1)^m(1+r_2)^m}\right]; \tag{31}$$

(ix) Setting $\xi = \nu$, $\lambda = \phi$ and $a = b = \gamma = \delta = \mu = 1$, $\sigma = 0$ in (15), the following identity holds:

$$\int_p^q \frac{(x-p)^{\alpha-1}(q-x)^{\beta-1}}{[(q-p)+r_1(x-p)+r_2(q-x)]^{\alpha+\beta}}\, \frac{1}{\left[1-w\left\{\frac{(x-p)(q-x)}{[(q-p)+r_1(x-p)+r_2(q-x)]^2}\right\}^m\right]}\, dx$$

$$= \frac{1}{(q-p)\ (1+r_1)^{\alpha}\ (1+r_2)^{\beta}}$$

$$\times\ {}_3\Psi_2\left[\begin{matrix}(\alpha,m), & (\beta,m), & (1,1); & \\ & (1,0), & & (\alpha+\beta,2m);\end{matrix}\ \frac{w}{(1+r_1)^m(1+r_2)^m}\right]; \tag{32}$$

(x) Setting $\xi = \nu$, $\lambda = \phi$ in (20), the following identity holds:

$$\int_p^q \frac{(x-p)^{\alpha-1}(q-x)^{\beta-1}}{[(q-p)+r_1(x-p)+r_2(q-x)]^{\alpha+\beta}}\, E^{\gamma,\delta,b}_{\sigma,\mu,a}\left[w\left\{\frac{(x-p)(q-x)}{[(q-p)+r_1(x-p)+r_2(q-x)]^2}\right\}^m\right] dx$$

$$= \frac{1}{(q-p)\ (1+r_1)^{\alpha}\ (1+r_2)^{\beta}}\ \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)\Gamma(\mu)}$$

$$\times\ {}_{2m+b+1}F_{2m+\sigma+a}\left[\begin{matrix}\Delta(m;\alpha), & \Delta(m;\beta), & \Delta(b;\gamma), & 1; \\ \Delta(\sigma;\mu), & \Delta(a;\delta), & & \Delta(2m;\alpha+\beta);\end{matrix}\ \frac{w\, b^b}{4^m\sigma^\sigma a^a(1+r_1)^m(1+r_2)^m}\right]; \tag{33}$$

(xi) Setting $\xi = \nu$, $\lambda = \phi$ and $a = 1$ in (20), the following identity holds:

$$\int_p^q \frac{(x-p)^{\alpha-1}(q-x)^{\beta-1}}{[(q-p)+r_1(x-p)+r_2(q-x)]^{\alpha+\beta}}\, E^{\gamma,b}_{\sigma,\mu,\delta}\left[w\left\{\frac{(x-p)(q-x)}{[(q-p)+r_1(x-p)+r_2(q-x)]^2}\right\}^m\right] dx$$

$$= \frac{1}{(q-p)\ (1+r_1)^{\alpha}\ (1+r_2)^{\beta}}\ \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)\Gamma(\mu)}$$

$$\times\ {}_{2m+b+1}F_{2m+\sigma+1}\left[\begin{matrix}\Delta(m;\alpha), & \Delta(m;\beta), & \Delta(b;\gamma), & 1; \\ \Delta(\sigma;\mu), & \Delta(2m;\alpha+\beta), & & \delta;\end{matrix}\ \frac{w\, b^b}{4^m\sigma^\sigma(1+r_1)^m(1+r_2)^m}\right]; \tag{34}$$

(xii) Setting $\xi = \nu$, $\lambda = \phi$ and $a = b = 1$ in (20), the following identity holds:

$$\int_p^q \frac{(x-p)^{\alpha-1}(q-x)^{\beta-1}}{[(q-p)+r_1(x-p)+r_2(q-x)]^{\alpha+\beta}}\, E^{\gamma,\delta}_{\sigma,\mu}\left[w\left\{\frac{(x-p)(q-x)}{[(q-p)+r_1(x-p)+r_2(q-x)]^2}\right\}^m\right] dx$$

$$= \frac{1}{(q-p)\ (1+r_1)^{\alpha}\ (1+r_2)^{\beta}}\ \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)\Gamma(\mu)}$$

$$\times\ {}_{2m+2}F_{2m+\sigma+1}\left[\begin{array}{llll}\Delta(m;\alpha), & \Delta(m;\beta), \ \gamma, & 1; & \\ & & & \dfrac{w}{4^m\sigma^\sigma(1+r_1)^m(1+r_2)^m} \\ \Delta(\sigma;\mu), & \Delta(2m;\alpha+\beta), & \delta; & \end{array}\right]; \tag{35}$$

(xiii) Setting $\xi = \nu$, $\lambda = \phi$ and $a = \delta = 1$ in (20), the following identity holds:

$$\int_p^q \frac{(x-p)^{\alpha-1}(q-x)^{\beta-1}}{[(q-p)+r_1(x-p)+r_2(q-x)]^{\alpha+\beta}} E^{\gamma,b}_{\sigma,\mu}\left[w\left\{\frac{(x-p)(q-x)}{[+(q-p)r_1(x-p)+r_2(q-x)]^2}\right\}^m\right] dx$$
$$= \frac{1}{(q-p)\ (1+r_1)^\alpha\ (1+r_2)^\beta}\ \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)\Gamma(\mu)}$$
$$\times\ {}_{2m+b}F_{2m+\sigma}\left[\begin{array}{lll}\Delta(m;\alpha), & \Delta(m;\beta), \ \Delta(b;\gamma); & \\ & & \dfrac{w\ b^b}{4^m\sigma^\sigma(1+r_1)^m(1+r_2)^m} \\ \Delta(\sigma;\mu), & \Delta(2m;\alpha+\beta); & \end{array}\right]; \tag{36}$$

(xiv) Setting $\xi = \nu$, $\lambda = \phi$ and $a = b = \delta = 1$ in (20), the following identity holds:

$$\int_p^q \frac{(x-p)^{\alpha-1}(q-x)^{\beta-1}}{[(q-p)+r_1(x-p)+r_2(q-x)]^{\alpha+\beta}} E^{\gamma}_{\sigma,\mu}\left[w\left\{\frac{(x-p)(q-x)}{[(q-p)+r_1(x-p)+r_2(q-x)]^2}\right\}^m\right] dx$$
$$= \frac{1}{(q-p)\ (1+r_1)^\alpha\ (1+r_2)^\beta}\ \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)\Gamma(\mu)}$$
$$\times\ {}_{2m+1}F_{2m+\sigma}\left[\begin{array}{lll}\Delta(m;\alpha), & \Delta(m;\beta), \ \gamma; & \\ & & \dfrac{w}{4^m\sigma^\sigma(1+r_1)^m(1+r_2)^m} \\ \Delta(\sigma;\mu), & \Delta(2m;\alpha+\beta); & \end{array}\right]; \tag{37}$$

(xv) Setting $\xi = \nu$, $\lambda = \phi$ and $a = b = \gamma = \delta = 1$ in (20), the following identity holds:

$$\int_p^q \frac{(x-p)^{\alpha-1}(q-x)^{\beta-1}}{[(q-p)+r_1(x-p)+r_2(q-x)]^{\alpha+\beta}} E_{\sigma,\mu}\left[w\left\{\frac{(x-p)(q-x)}{[(q-p)+r_1(x-p)+r_2(q-x)]^2}\right\}^m\right] dx$$
$$= \frac{1}{(q-p)\ (1+r_1)^\alpha\ (1+r_2)^\beta}\ \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)\Gamma(\mu)}$$
$$\times\ {}_{2m+1}F_{2m+\sigma}\left[\begin{array}{lll}\Delta(m;\alpha), & \Delta(m;\beta), \ 1; & \\ & & \dfrac{w}{4^m\sigma^\sigma(1+r_1)^m(1+r_2)^m} \\ \Delta(\sigma;\mu), & \Delta(2m;\alpha+\beta); & \end{array}\right]; \tag{38}$$

(xvi) Setting $\xi = \nu$, $\lambda = \phi$ and $a = b = \gamma = \delta = \mu = 1$ in (20), the following identity holds:

$$\int_p^q \frac{(x-p)^{\alpha-1}(q-x)^{\beta-1}}{[(q-p)+r_1(x-p)+r_2(q-x)]^{\alpha+\beta}} E_{\sigma}\left[w\left\{\frac{(x-p)(q-x)}{[(q-p)+r_1(x-p)+r_2(q-x)]^2}\right\}^m\right] dx$$
$$= \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}\ \frac{1}{(q-p)\ (1+r_1)^\alpha\ (1+r_2)^\beta}$$
$$\times\ {}_{2m+1}F_{2m+\sigma}\left[\begin{array}{lll}\Delta(m;\alpha), & \Delta(m;\beta), \ 1; & \\ & & \dfrac{w}{4^m\sigma^\sigma(1+r_1)^m(1+r_2)^m} \\ \Delta(\sigma;1), & \Delta(2m;\alpha+\beta); & \end{array}\right]; \tag{39}$$

(xvii) Setting $\xi = \nu$, $\lambda = \phi$ and $a = b = \gamma = \delta = \mu = \sigma = 1$ in (20), the following identity holds:

$$\int_p^q \frac{(x-p)^{\alpha-1}(q-x)^{\beta-1}}{[(q-p)+r_1(x-p)+r_2(q-x)]^{\alpha+\beta}}\ e^{\left[w\left\{\frac{(x-p)(q-x)}{[(q-p)+r_1(x-p)+r_2(q-x)]^2}\right\}^m\right]} dx$$
$$= \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}\ \frac{1}{(q-p)\ (1+r_1)^\alpha\ (1+r_2)^\beta}$$

$$\times {}_{2m}F_{2m}\left[\begin{array}{ll}\Delta(m;\alpha), \quad \Delta(m;\beta); & \\ \Delta(2m;\alpha+\beta); & \dfrac{w}{4^m\sigma^\sigma(1+r_1)^m(1+r_2)^m}\end{array}\right]. \tag{40}$$

4. Graphical Representation

Here, in terms of the parameter β, we illustrate Equations (14) and (15) using graphical simulations. For this, we evaluate the integrals numerically using the Gaussian quadrature Method (see [37]) and compare this with the main results. We choose $k = 5$ and $n = 8$ to get more precise results.

5. Conclusions

It is worth stressing that the generalized Mittag–Leffler function obtained and the integral formulas computed are amenable to further generalizations and future investigation. We have attempted to exploit the close connection of the generalized Mittag–Leffler functions with several important special functions and compute the integrals of the functions mentioned above in the form of the generalized Mittag–Leffler, linking different families of special functions. Our main results (15) and (20) and some special cases (24)–(32) can yield several new integrals in terms of Fox-H functions obtained from Equations (11) and (13). For instance, we write

$$\int_p^q \frac{(x-p)^{\alpha-1}(q-x)^{\beta-1}}{[(q-p)+r_1(x-p)+r_2(q-x)]^{\alpha+\beta}}\, E^{\xi,\lambda,\gamma,b}_{\sigma,\mu,\nu,\phi,\delta,a}\left[w\left\{\frac{(x-p)(q-x)}{[(q-p)+r_1(x-p)+r_2(q-x)]^2}\right\}^m\right]dx$$
$$= \frac{1}{(q-p)\,(1+r_1)^\alpha\,(1+r_2)^\beta}\,\frac{\Gamma(\nu)\,\Gamma(\delta)}{\Gamma(\xi)\,\Gamma(\gamma)}$$
$$\times {}_5\Psi_4\left[\begin{array}{ll}(\xi,\lambda),\ (\gamma,b),\ (\alpha,m),\ (\beta,m),\quad (1,1); & \\ (\mu,\sigma),\ (\nu,\phi),\ (\delta,a),\quad (\alpha+\beta,2m); & \dfrac{w}{(1+r_1)^m(1+r_2)^m}\end{array}\right] \tag{41}$$
$$= \frac{1}{(q-p)\,(1+r_1)^\alpha\,(1+r_2)^\beta}\,\frac{\Gamma(\nu)\,\Gamma(\delta)}{\Gamma(\xi)\,\Gamma(\gamma)}$$
$$\times H^{1,5}_{5,5}\left[\frac{-w}{(1+r_1)^m(1+r_2)^m}\,\middle|\,\begin{array}{l}(1-\xi,\lambda),\ (1-\gamma,b),\ (1-\alpha,m),\ (1-\beta,m),\quad (0,1) \\ (0,1),\ (1-\mu,\sigma),\ (1-\nu,\phi),\ (1-\delta,a),\quad \{1-(\alpha+\beta,2m)]\}\end{array}\right], \tag{42}$$

with all the conditions prescribed in Theorem 1. We have also proved that Figures 1–3 show a good compatibility of the numerical solution obtained by the Gaussian quadrature method and the analytic expression. We conclude that the results obtained will provide a significant step in the theory of integral transforms and can yield some potential applications in the field of the classical and applied mathematics.

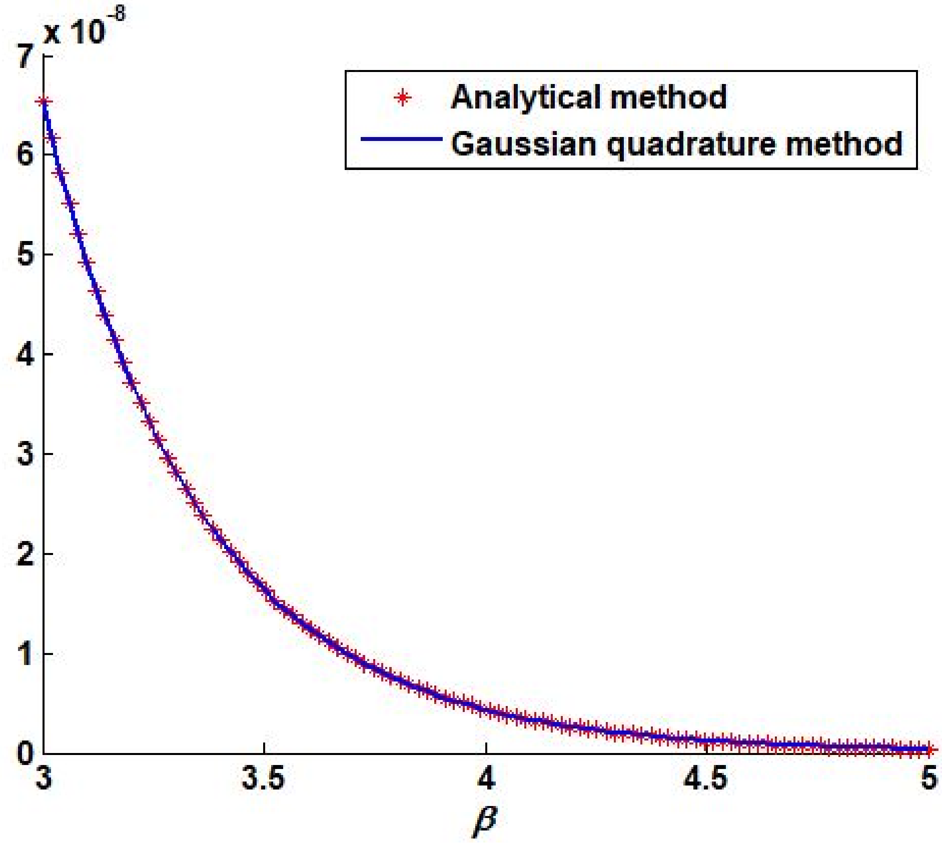

Figure 1. Solution of (14) for $\alpha = 6$, $r_1 = 2$, $r_2 = 2$, $p = 0$ and $q = 1$.

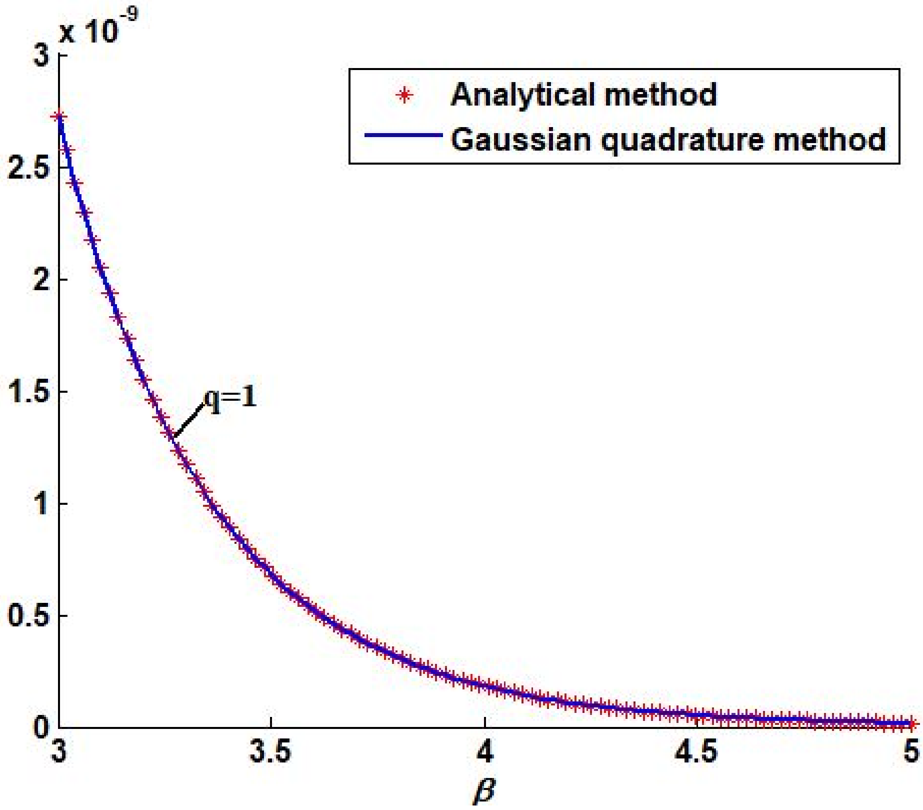

Figure 2. Solution of (15) (for $q = 1$) for $\alpha = 6$, $r_1 = 2$, $r_2 = 4$, $p = 0$, $\xi = 1$, $\lambda = 2$, $\gamma = 2$ $b = 3$, $\sigma = 2$, $\mu = 5$, $v = 2$, $\phi = 3$, $\delta = 2$, $a = 4$, $w = 3$ and $m = 2$.

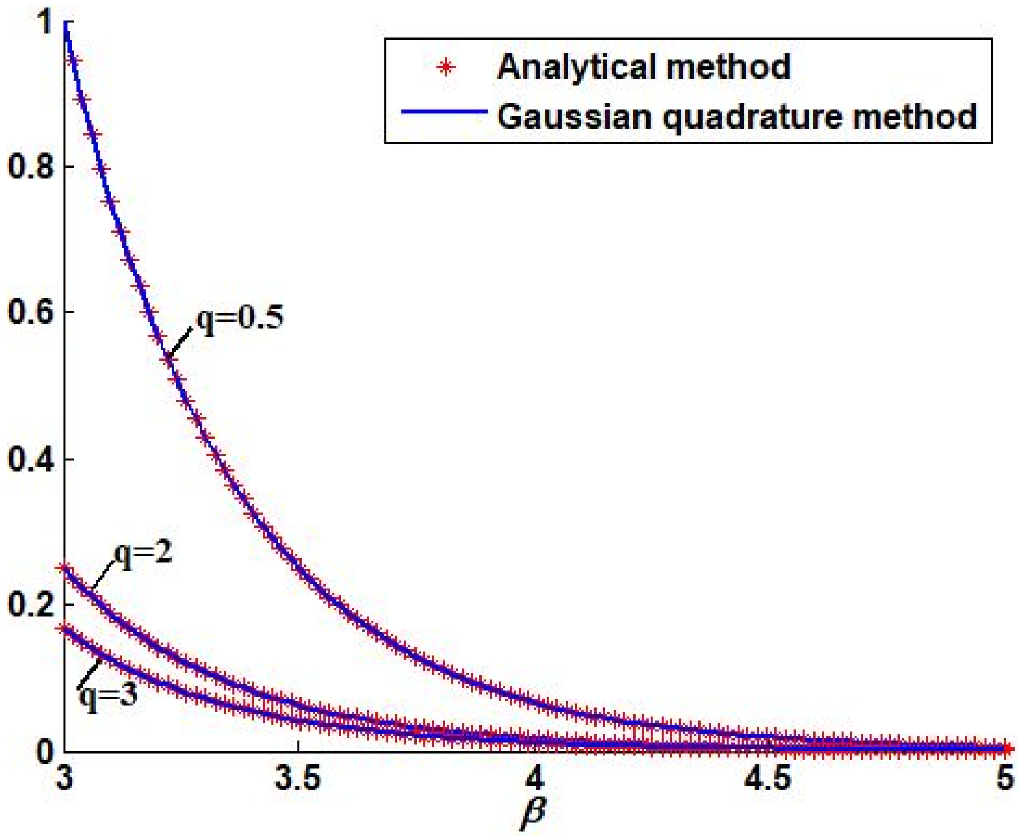

Figure 3. Solution of (15) (for all q) for $\alpha = 6$, $r_1 = 2$, $r_2 = 4$, $p = 0$, $\xi = 1$, $\lambda = 2$, $\gamma = 2$ $b = 3$, $\sigma = 2$, $\mu = 5$, $v = 2$, $\phi = 3$, $\delta = 2$, $a = 4$, $w = 3$ and $m = 2$.

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