symmetry

MDPI

Editorial

Special Issue Editorial "Chaotic Systems and Nonlinear Dynamics"

Giuseppe Grassi

Dipartimento Ingegneria Innovazione, Università del Salento, 73100 Lecce, Italy; giuseppe.grassi@unisalento.it

Referring to chaotic systems, it is well-known that they are nonlinear dynamical systems, which are distinguished by sensitive dependence on initial conditions and by having evolution through phase space that appears to be quite random [1]. More precisely, chaotic systems are characterized by having a dense collection of points with periodic orbits and by being sensitive to the initial conditions, so that initially nearby points can evolve quickly into very different states [2]. Referring to nonlinear dynamics, they represent an interdisciplinary research field that may involve physics, applied mathematics, engineering and other applied disciplines [1]. When a dynamical system contains at least one nonlinearity in its equations, the system may display very complex behaviours, including bifurcations, multi-stability, chimera states, synchronization phenomena and fractal structures [3].

Although recent years have witnessed remarkable developments in the area of chaotic systems and nonlinear dynamics, many theoretical problems and practical applications remain to be further explored. This Special Issue is devoted to analysing recent developments regarding chaotic systems and nonlinear dynamics in different fields of science and engineering. The results presented in this Special Issue are related to dynamical systems described by differential equations or difference equations, as well as by integer-order or fractional-order operators. The topics covered herein include applications of chaos to communications, to random number generation, to game theory and to hardware implementation of encryption systems. The following manuscripts were selected for publication. The articles were prepared by scientists working in leading universities and research centres in China, France, Germany, Iraq, Italy, Lebanon, Poland, Romania, Saudi Arabia, Ukraine and Vietnam.

S. Haliuk et al., in the paper "Memristive Structure-Based Chaotic System for PRNG" [4], have proposed an approach to generate pseudo-random sequences based on a discrete memristive chaotic system. Preliminarily, it is worth recalling that a Random Number Generator (RNG) is a basic element in cryptographic applications. There are two types of generators, i.e., TRNG (truly random number generator) and PRNG (pseudo random number generator), where the latter can be only implemented digitally. The authors of reference [4], who have focused their attention on PRNG, have proposed a criterion to separate any binary representation of the chaotic time sequences into random and non-random parts. All the obtained sequences have successfully passed the NIST SP 800-22 statistical tests [4]. The conceived chaos-based PRNG has been implemented on an FPGA able to reach the speed of 1.2 Gbits/s at a clock frequency of 50 MHz. Finally, an application of the proposed chaotic memristive system to image encryption has been illustrated [4].

N. Abdoun et al., in the paper "Authenticated Encryption Based on Chaotic Neural Networks and Duplex Construction" [5], have illustrated an encryption scheme that contains a chaotic compression function based on a chaotic neural network. In particular, the compression function exploits two chaotic maps, i.e., the skew-tent map and the PWLCM map [5]. Note that authenticated encryption (AE) is a term used to describe encryption systems that simultaneously protect the confidentiality, integrity, and authenticity of the data transmitted over insecure channels. The AE scheme proposed in [5] has been tested against different cryptanalytic attacks, including NIST and correlation analysis. Simulation results

Citation: Grassi, G. Special Issue Editorial "Chaotic Systems and Nonlinear Dynamics". *Symmetry* **2022**, *14*, 1137. https://doi.org/10.3390/sym14061137

Received: 9 May 2022
Accepted: 18 May 2022
Published: 31 May 2022

have shown that the proposed scheme ensures the confidentiality of data transmitted over unsecured channels as well as the authenticity of the data source [5].

M. Berezowski and M. Lawnik, in the paper "Homotopic Parametric Continuation Method for Determining Stationary States of Chemical Reactors with Dispersion" [6], have illustrated an approach that combines the homotopy method and the parametric continuation method to solve a system of nonlinear differential equations. Preliminarily, it is worth noting that some physical processes occurring in devices with distributed variables and turbulent tides with dispersion of mass/heat are often modelled using systems of nonlinear equations. Solving such a system is sometimes impossible in an analytical manner or ineffective with iterative methods such as Newton's method. By virtue of their approach, the authors of reference [6] have shown that it is possible to find all the system solutions, even if they are close to each other. The conceived method has been exploited to determine the stationary states of a non-adiabatic catalytic tubular chemical reactor with longitudinal dispersion, where the gas temperature is equal to the catalyst temperature [6]. Numerical results have clearly shown the effectiveness of the proposed method [6].

Q. Lu et al., in the paper "A New Conservative Hyperchaotic System-Based Image Symmetric Encryption Scheme with DNA Coding" [7], have first proposed a complexity analysis of a new hyperchaotic system using phase diagrams, bifurcation diagrams, Lyapunov exponents and Kaplan–Yorke dimension. In addition, a hyperchaos-based image encryption scheme with dynamic DNA coding has been developed. Specifically, the DNA coding mechanism has been introduced by using the chaotic sequence [7]. Note that DNA (deoxyribonucleic acid) computing is a new generation of computing technology that exploits biological DNA molecules as information carriers. Compared with traditional computing, DNA computing has some potential advantages, such as massive parallelism and ultra-low power consumption. This is the reason why the authors of reference [7] have introduced the concept of DNA computing into cryptography and have used it to develop the encryption algorithm. In particular, Q. Lu et al. have shown that their chaos-based approach involving DNA coding has good cryptographic effects, given that the encrypted information can effectively resist brute force attacks, statistical attacks, chosen-plaintext attacks and differential attacks [7].

S. Askar, in the paper "On Dynamic Investigations of Cournot Duopoly Game: When Firms Want to Maximize Their Relative Profits" [8], has investigated a Cournot duopoly game in which firms produce homogeneous goods and adopt a bounded rationality rule for updating productions. Note that herein the two competing firms seek the optimal quantities of their production by maximizing their relative profits [8]. In particular, the author shows that the model describing the game's evolution is a two-dimensional nonlinear discrete map, where the unique equilibrium point is a Nash point [8]. By using local analysis, it is also shown that this point loses its stability through two types of bifurcations, flip and Neimark–Sacker. Finally, a number of chaotic attractors have been displayed, with the aim to highlight all the dynamic properties of the conceived duopoly game [8].

T. Lei et al., in the paper "Dynamics Analysis and Synchronous Control of Fractional-Order Entanglement Symmetrical Chaotic Systems" [9], have proposed the Adomian decomposition method to solve a fractional-order entanglement symmetrical chaotic system. Preliminarily, it is worth recalling that fractional order is an extension of integer order, and the dynamic behaviour of the system is related to the parameters of the system itself as well as to the fractional order operators. This leads to an increased complexity, which makes fractional chaotic systems suitable for applications in the field of information security. Based on these considerations, the authors of reference [9] have carefully analysed the dynamics of the proposed fractional system via bifurcation diagrams, Lyapunov exponents and Poincaré diagrams. Moreover, the synchronization of fractional-order chaotic systems has been investigated theoretically and numerically [9]. The results have shown that the synchronization algorithm can provide a theoretical basis for the application of the conceived approach in communication security of images, sounds and videos [9].

Y-Y. Lee, in the paper "Frequency–Amplitude Relationship of a Nonlinear Symmetric Panel Absorber Mounted on a Flexible Wall" [10], has investigated the frequency–amplitude relationship of a nonlinear symmetric panel absorber mounted on a flexible wall. It is worth recalling that panel absorbers are employed to control sound reflection, absorption, echo and reverberation in a room. Since the panel absorbers, which are made of thin metal or plastic sheets backed by a symmetrical air gap, vibrate nonlinearly, linear techniques are inappropriate for their design [10]. This is why a nonlinear dynamic approach is requested to take into account the properties of nonlinearly vibrating structures. Based on these considerations, the author of reference [10] has exploited the weighted residual elliptic integral method to analyse the issue involving nonlinear multi-mode equations of two flexible panels coupled with a cavity. The results, obtained using the proposed method, have proved to be in good agreement with finite element methods [10]. Thus, the frequency–amplitude approach in [10] can be considered as a reference work for effectively designing nonlinear panel absorbers.

X. Zhou et al., in the paper "A 2D Hyperchaotic Map: Amplitude Control, Coexisting Symmetrical Attractors and Circuit Implementation" [11], have investigated the dynamical properties of a two-dimensional hyperchaotic map with absolute value function. By analysing the stability of its fixed points, the authors have shown that the map can realize the global amplitude control via a single controller as well as the partial amplitude control via a single knob [11]. These dynamic characteristics have been confirmed by the conceived hardware platform, which mainly includes the STM32F103 chip and the 12-bit digital-to-analogue conversion module TLV5618. The authors have also found a special symmetry mode, which represents a step toward the generation of coexisting oscillations with inverse polarity of the chaotic signal [11]. Finally, potential applications of the obtained results to secure communications have been discussed [11].

S. Kong et al., in the paper "Asymmetry Evolvement and Controllability of a Symmetric Hyperchaotic Map" [12], have analysed the use of trigonometric functions to construct a 2D symmetrical hyperchaotic map with infinitely many attractors. The authors have shown that the initial conditions of the conceived map trigger a specific multi-stability evolvement, in which infinitely countless symmetric and asymmetric attractors are generated [12]. Moreover, S. Kong et al. have experimentally proved that the polarity balance is maintained by the initial conditions and by a negative coefficient related to the trigonometric functions. The experimental set-up has been realized using the core board of the single-chip microcomputer STM32F103VE-EK, whereas the ARM Cortex-M3 has been used as the main control chip for the overall system [12].

S. He et al., in the paper "Complexity and Chimera States in a Network of Fractional-Order Laser Systems" [13], have utilized the Adams–Bashforth–Moulton method to study complexity and synchronization of a fractional-order laser dynamical model. It is worth recalling that "chimera states" indicate a dynamic behaviour according to which a population of identical coupled oscillators can attain a state where one part synchronizes and the other oscillates incoherently. In order to explore the appearance of chimera states, the authors of reference [13] have constructed a ring network of fractional-order laser systems and have tuned the network coupling strength, the connection number of each node and the order of the fractional derivative [13]. Additionally, by varying the value of the fractional order, S. He et al. have investigated the presence of symmetric chaotic attractors as well as the appearance of high complexity regions in the parameter planes [13].

G. Grassi, in the paper "Chaos in the Real World: Recent Applications to Communications, Computing, Distributed Sensing, Robotic Motion, Bio-Impedance Modelling and Encryption Systems" [14], has reviewed the applications of chaos as reported in the literature published during the years 2018 to 2020. The author got started by the observation that most of the papers in literature had historically focused on the theoretical phenomena underlying the formation of chaos, rather than on the investigation of potential applications of chaos to the real world. Based on these considerations, the author of reference [14] has focused his attention on reviewing the applications of chaos to communications (including the use of chaos in radar-based systems and underwater acoustic channels) as well as on some applications of chaos to random number generation (including the use of chaos for secure transfer of medical images). Additionally, G. Grassi has reviewed the applications of chaos in robotic motion, in chaos-based logic gates for computing and in the hardware implementations of encryption systems [14].

In conclusion, by considering the interdisciplinary nature of the topics discussed herein, this volume will be surely of interest to mathematicians, physicists and engineers who are interested in fostering theory and applications of chaotic systems and nonlinear dynamics.

Funding: This research received no external funding.

Conflicts of Interest: The authors declare no conflict of interest.

References

1. Strogatz, S.H. *Nonlinear Dynamics and Chaos with Applications to Physics, Biology, Chemistry, and Engineering*, 2nd ed; CRC Press: Boca Raton, FL, USA, 2015.
2. Skiadas, C.H.; Skiadas, C. *Handbook of Applications of Chaos Theory*; CRC Press: Boca Raton, FL, USA, 2016.
3. Wiggins, S. *Introduction to Applied Nonlinear Dynamical Systems and Chaos*; Springer: Berlin/Heidelberg, Germany, 1996.
4. Haliuk, S.; Krulikovskyi, O.; Vovchuk, D.; Corinto, F. Memristive Structure-Based Chaotic System for PRNG. *Symmetry* **2022**, *14*, 68. [CrossRef]
5. Abdoun, N.; El Assad, S.; Manh Hoang, T.; Deforges, O.; Assaf, R.; Khalil, M. Authenticated Encryption Based on Chaotic Neural Networks and Duplex Construction. *Symmetry* **2021**, *13*, 2432. [CrossRef]
6. Berezowski, M.; Lawnik, M. Homotopic Parametric Continuation Method for Determining Stationary States of Chemical Reactors with Dispersion. *Symmetry* **2021**, *13*, 2324. [CrossRef]
7. Lu, Q.; Yu, L.; Zhu, C.A. New Conservative Hyperchaotic System-Based Image Symmetric Encryption Scheme with DNA Coding. *Symmetry* **2021**, *13*, 2317. [CrossRef]
8. Askar, S. On Dynamic Investigations of Cournot Duopoly Game: When Firms Want to Maximize Their Relative Profits. *Symmetry* **2021**, *13*, 2235. [CrossRef]
9. Lei, T.; Mao, B.; Zhou, X.; Fu, H. Dynamics Analysis and Synchronous Control of Fractional-Order Entanglement Symmetrical Chaotic Systems. *Symmetry* **2021**, *13*, 1996. [CrossRef]
10. Lee, Y.-Y. Frequency–Amplitude Relationship of a Nonlinear Symmetric Panel Absorber Mounted on a Flexible Wall. *Symmetry* **2021**, *13*, 1188. [CrossRef]
11. Zhou, X.; Li, C.; Lu, X.; Lei, T.; Zhao, Y. A 2D Hyperchaotic Map: Amplitude Control, Coexisting Symmetrical Attractors and Circuit Implementation. *Symmetry* **2021**, *13*, 1047. [CrossRef]
12. Kong, S.; Li, C.; Jiang, H.; Zhao, Y.; Wang, Y. Asymmetry Evolvement and Controllability of a Symmetric Hyperchaotic Map. *Symmetry* **2021**, *13*, 1039. [CrossRef]
13. He, S.; Natiq, H.; Banerjee, S.; Sun, K. Complexity and Chimera States in a Network of Fractional-Order Laser Systems. *Symmetry* **2021**, *13*, 341. [CrossRef]
14. Grassi, G. Chaos in the Real World: Recent Applications to Communications, Computing, Distributed Sensing, Robotic Motion, Bio-Impedance Modelling and Encryption Systems. *Symmetry* **2021**, *13*, 2151. [CrossRef]