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On Prešić–Ćirić-Type α-ψ Contractions with an Application

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Abstract: In this paper, we extend the idea of α-ψ contraction mapping to the product spaces by introducing Prešić–Ćirić-type α-ψ contractions and utilize them to prove some coincidence and common fixed-point theorems in the context of ordered metric spaces using α-admissibility of the mapping. Our newly established results generalize a number of well-known fixed-point theorems from the literature. Moreover, we give some examples that attest to the credibility of our results. Further, we give an application to the nonlinear integral equations, which can be employed to study the existence and uniqueness of solutions to the integral equations.

Keywords: coincidence points; Prešić–Ćirić contractions; α-ψ contractions; ordered metric spaces

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1. Introduction

Fixed-point theory occupies a prominent position in both pure and applied mathematics due to its numerous applications in domains, such as differential and integral equations, variational inequalities, and approximation theory. Fixed-point results are useful in a variety of domains, including statistics, computer sciences, chemical sciences, physical sciences, economics, biological sciences, medical sciences, engineering, and game theory. The most fundamental theorem, the Banach contraction principle (BCP), was established by the Polish Mathematician S. Banach [1], which states that every contraction mapping in a complete metric space has a unique fixed point. Since then, several researchers have generalized the BCP in various directions.

In 1965, Prešić established the Banach contraction principle in the context of product spaces.

Theorem 1 ([2]). *Let (Ω, d) be a complete metric space and $f : \Omega^k \to \Omega$ be a mapping, where k is a positive integer. If there exist constants $a_1, a_2, ..., a_k \in (0,1)$ satisfying $a_1 + a_2 + ... + a_k < 1$ such that*

$$d(f(\zeta_1, \zeta_2, ..., \zeta_k), f(\zeta_2, \zeta_3, ..., \zeta_k, \zeta_{k+1})) \le \sum_{i=1}^{k} a_i d(\zeta_i, \zeta_{i+1})$$

for all $\zeta_1, \zeta_2, ..., \zeta_{k+1} \in \Omega$, then f has a unique fixed point.

Subsequently, there have been some generalizations of Theorem 1 (See [3–5] and references therein). In this sequel, Ćirić and Prešić amplified Theorem 1 by slightly modifying the Prešić contraction:

Theorem 2 ([3]). *Let (Ω, d) be a complete metric space and $f : \Omega^k \to \Omega$ a mapping. If there exists $a \in (0,1)$ such that*

$$d(f(\zeta_1, \zeta_2, ..., \zeta_k), f(\zeta_2, \zeta_3, ..., \zeta_k, \zeta_{k+1})) \le a \max_{1 \le i \le k} d(\zeta_i, \zeta_{i+1})$$

for all $\zeta_1, \zeta_2, ..., \zeta_{k+1} \in \Omega$, *then* f *has a fixed point.*

On the other hand, an important generalization of the Banach contraction principle in a partial ordered set was investigated by Ran and Reurings [6] in 2004 and then by Nieto and Lopez [7]. This generalization made a vital chapter in metric fixed-point theory because these results state that, to have a fixed point, only those elements that are related by the underlying partial ordering should be subjected to the contraction conditions. Several researchers worked on the fixed points in ordered metric spaces, see [8–12] and references therein.

Recently, Samet et al. [13] introduced a new generalization of Banach contraction mapping through α-ψ contraction mappings, which extends the results of Ran and Reurings.

Denote Ψ the family of nondecreasing functions $\psi : [0, \infty) \to [0, \infty)$ such that $\sum_{n=1}^{+\infty} \psi^n(t) < +\infty$ for each $t > 0$, where ψ^n is the nth iterate of ψ.

Definition 1 ([13])**.** *Let* (Ω, d) *be a metric space and* f *be a self-mapping on* Ω. *If there are two functions* $\alpha : \Omega \times \Omega \to [0, \infty)$ *and* $\psi \in \Psi$ *satisfying*

$$\alpha(\zeta, \vartheta) d(f\zeta, f\vartheta) \leq \psi(d(\zeta, \vartheta)) \text{ for all } \zeta, \vartheta \in \Omega,$$

then f *is said to be a* α-ψ *contraction mapping.*

Definition 2 ([13])**.** *Let* f *be a self-mapping on* Ω *and* $\alpha : \Omega \times \Omega \to [0, \infty)$. *Then,* f *is said to be* α*-admissible if*

$$\alpha(\zeta, \vartheta) \geq 1 \implies \alpha(f\zeta, f\vartheta) \geq 1 \text{ for all } \zeta, \vartheta \in \Omega.$$

Theorem 3 ([13])**.** *Let* (Ω, d) *be a complete metric space and the self-mapping* f *on* Ω *be* α-ψ *contraction mapping. Suppose the following conditions hold:*

(a) f *is* α*-admissible,*
(b) *there exists* $\zeta_0 \in \Omega$ *such that* $\alpha(\zeta_0, f\zeta_0) \geq 1$,
(c) f *is continuous.*

Then f *has a fixed point.*

In this paper, we extend the idea of α-ψ contraction mappings to the product spaces. To do so, we introduce Prešić–Ćirić-type α-ψ contraction mappings and utilize them to prove some coincidences and common fixed theorems in partial ordered metric spaces. Our results extend, enrich, and unify some existing fixed-point theorems in the literature. We also express an application of our recently proven results to an integral equation, in addition to offering some examples that substantiate the utility of our results.

2. Preliminaries

We will go over some basic definitions in this section that will help us to prove our primary results. In several instances, we refer to $\mathbb{N} \cup \{0\}$ as $\mathbb{N}_0$ and $g(\zeta)$ as $g\zeta$ throughout the paper.

Definition 3 ([14])**.** *Let* $f : \Omega^k \to \Omega$ *and* $g : \Omega \to \Omega$ *be two mappings. A point* $\zeta \in \Omega$ *is called the "coincidence point" (or in short, CP) of* f *and* g *if*

$$f(\zeta, \zeta, ..., \zeta) = g(\zeta).$$

Definition 4 ([14])**.** *Let* $f : \Omega^k \to \Omega$ *and* $g : \Omega \to \Omega$ *be two mappings. A point* $\zeta \in \Omega$ *is called the "common fixed point" (or in short, CFP) of* f *and* g *if*

$$f(\zeta, \zeta, ..., \zeta) = g(\zeta) = \zeta.$$

Definition 5 ([14]). *Two mappings $f : \Omega^k \to \Omega$ and $g : \Omega \to \Omega$ are said to be commuting if for $\zeta_1, \zeta_2, ..., \zeta_k \in \Omega$,*

$$f(g\zeta_1, g\zeta_2, ...g\zeta_k) = g(f(\zeta_1, \zeta_2, ..., \zeta_k)).$$

Definition 6 ([15]). *Let Ω be a non-empty set endowed with a partial order $\preceq$ and $f : \Omega^k \to \Omega$ and $g : \Omega \to \Omega$ be two mappings. Then,*

(*a*) *A sequence $\{\zeta_n\}_{n\in\mathbb{N}}$ is termed as "increasing with respect to $\preceq$" if*

$$\zeta_1 \preceq \zeta_2 \preceq ... \preceq \zeta_n \preceq ...,$$

(*b*) *f is termed as "increasing with respect to $\preceq$" if for any finite increasing sequence $\{\zeta_n\}_{n=1}^{k+1}$ we have,*

$$f(\zeta_1, \zeta_2, ..., \zeta_k) \preceq f(\zeta_2, \zeta_3, ..., \zeta_{k+1}),$$

(*c*) *f is termed as "g-increasing with respect to $\preceq$" if for any finite increasing sequence $\{g\zeta_n\}_{n=1}^{k+1}$ we have,*

$$f(\zeta_1, \zeta_2, ..., \zeta_k) \preceq f(\zeta_2, \zeta_3, ..., \zeta_{k+1}).$$

Definition 7 ([16]). *Let (Ω, d) be a metric space, $f : \Omega^k \to \Omega$ be a mapping, where k is a positive integer. Then, f is said to be α-admissible mapping if there exists $\alpha : \Omega \times \Omega \to [0, \infty)$ such that*

$$\min\{\alpha(\zeta_i, \zeta_{i+1}) : 1 \le i \le k\} \ge 1$$

$$\implies \alpha(f(\zeta_1, ..., \zeta_k), f(\zeta_2, ..., \zeta_{k+1})) \ge 1 \; \forall \zeta_1, \zeta_2, .., \zeta_{k+1} \in \Omega.$$

Inspired by the above definition of α-admissibility of f, we define α-g-admissible mappings.

Definition 8. *Let (Ω, d) be a metric space, $f : \Omega^k \to \Omega$ be a mapping, where k is a positive integer and g is a self-mapping on Ω. Then, f is said to be an α-g admissible mapping if there exists $\alpha : \Omega \times \Omega \to [0, \infty)$ such that*

$$\min\{\alpha(g\zeta_i, g\zeta_{i+1}) : 1 \le i \le k\} \ge 1$$

$$\implies \alpha(f(\zeta_1, ..., \zeta_k), f(\zeta_2, ..., \zeta_{k+1})) \ge 1 \; \forall \zeta_1, \zeta_2, .., \zeta_{k+1} \in \Omega.$$

Remark 1. *Every α-admissible mapping is an α-g admissible mapping, but the inverse need not be true. The following example attests to this fact.*

Example 1. *Consider (Ω, d) as a metric space with usual metric d and $\Omega = [0, 1]$. Now define the mappings:*

$$\alpha : \Omega \times \Omega \to [0, \infty) \text{ by } \alpha(\zeta, \vartheta) = \begin{cases} 1, & \text{when } \zeta, \vartheta \in [0, \frac{1}{2}] \\ 0, & \text{otherwise,} \end{cases}$$

$$g : \Omega \to \Omega \text{ by } g(\zeta) = \begin{cases} \frac{\zeta}{3}, & \text{when } \zeta \in [0, 0.3] \\ \frac{2}{3}, & \text{otherwise,} \end{cases}$$

$$f : \Omega^2 \to \Omega \text{ by } f(\zeta_1, \zeta_2) = \begin{cases} \frac{\zeta_1^2 + 2\zeta_2^2}{7}, & \text{when } \zeta_1, \zeta_2 \in [0, 0.3] \\ 1, & \text{when } \zeta_1, \zeta_2 \in (0.3, 1]. \end{cases}$$

Then, by routine calculation, it can be easily verified that f is not α-admissible, but it is α-g admissible.

3. Prešić–Ćirić-Type α-ψ Contractions

Motivated by Samet et al. [13], we introduce Prešić–Ćirić-type α-ψ contractions, which are indeed generalizations of some existing contraction mappings.

Definition 9. *Let (Ω, d) be a metric space and $f : \Omega^k \to \Omega$ be a mapping, where k is a positive integer. Then, a mapping f is said to be Prešić–Ćirić-type α-ψ contractions if there exists $\psi \in \Psi$ such that*

$$\min_{1\leq i\leq k}\{\alpha(\zeta_i,\zeta_{i+1})\}d(f(\zeta_1,\zeta_2,...,\zeta_k),f(\zeta_2,\zeta_3,...,\zeta_{k+1}))\leq\psi(\max_{1\leq i\leq k}\{d(\zeta_i,\zeta_{i+1})\})$$

for all $\zeta_1, \zeta_2, ..., \zeta_{k+1} \in \Omega$.

Example 2. *Consider $\Omega = [0, 1]$ be metric space with the standard metric d. Let $f : \Omega^2 \to \Omega$ be a mapping defined by*

$$f(\zeta,\vartheta)=\begin{cases}\frac{\zeta+2\vartheta}{7}, & \text{when } \zeta,\vartheta\in[0,\frac{1}{2}]\\ 1, & \text{when } \zeta,\vartheta\in(\frac{1}{2},1]\end{cases}.$$

and the mapping $\alpha : \Omega^2 \to [0, \infty)$ defined by

$$\alpha(\zeta,\vartheta)=\begin{cases}1, & \text{when } \zeta,\vartheta\in[0,\frac{1}{2}]\\ 0, & \text{otherwise.}\end{cases}$$

Then, f is Prešić–Ćirić-type α-ψ contraction mapping.

Remark 2. *For $k = 1$, a Prešić–Ćirić-type α-ψ contraction mapping becomes an α-ψ contraction mapping.*

Remark 3. *A Prešić–Ćirić-type α-ψ contraction mapping under $\psi(t) = at$, where $a \in (0, 1)$ and $\alpha(\zeta, \vartheta) = 1$ for all $\zeta, \vartheta \in \Omega$ a becomes Prešić–Ćirić contraction. A Prešić–Ćirić contraction is always a Prešić contraction. Indeed,*

$$a_1 d(\zeta_1,\zeta_2)+a_2 d(\zeta_2,\zeta_3)+...+a_k d(\zeta_k,\zeta_{k+1})$$

$$\leq(a_1+a_2+...+a_k)\max\{d(\zeta_1,\zeta_2),d(\zeta_2,\zeta_3),...,d(\zeta_k,\zeta_{k+1})\}$$

where $(a_1 + a_2 + ... + a_k) < 1$.

Remark 4. *A Prešić–Ćirić-type α-ψ contraction under the mapping $\psi \in \Psi$ defined by $\psi(t) = at$, where $a \in (0, 1)$ becomes*

$$\min_{1\leq i\leq k}\{\alpha(\zeta_i,\zeta_{i+1})\}d(f(\zeta_1,\zeta_2,...,\zeta_k),f(\zeta_2,\zeta_3,...,\zeta_{k+1}))\leq a\max_{1\leq i\leq k}\{d(\zeta_i,\zeta_{i+1})\}$$

for all $\zeta_1, \zeta_2, ..., \zeta_{k+1} \in \Omega$, which is an enriched version of an α-admissible Prešić operator defined by Shukla et al. [16].

Before we proceed further, we state an important lemma that will be required in the proof of our main results.

Lemma 1 ([17])**.** *Suppose that $\psi : [0, \infty) \to [0, \infty)$ is increasing. Then, for every $t > 0$, $\lim_{n\to\infty} \psi^n(t) = 0$ implies $\psi(t) < t$.*

4. Main Results

In this section, we prove some coincidence theorems for mappings satisfying Prešić–Ćirić-type α-ψ contraction in an ordered metric space.

Theorem 4. *Let $(\Omega, \preceq)$ be a complete metric space endowed with a partial order $\preceq$ and let the mapping $f : \Omega^k \to \Omega$ be g-increasing, where k is a positive integer. Suppose that the following conditions hold:*

(*a*) $f(\Omega^k) \subseteq g(\Omega)$,
(*b*) *f and g are a commuting pair,*
(*c*) *f is α-g admissible,*
(*d*) *f is continuous,*
(*e*) *g is continuous,*
(*f*) *there exist k elements* $\zeta_1, \zeta_2, ..., \zeta_k \in \Omega$ *such that*

$$g\zeta_1 \preceq g\zeta_2 \preceq ... \preceq g\zeta_k \text{ and } g\zeta_k \preceq f(\zeta_1, \zeta_2, ..., \zeta_k),$$

and

$$\min\{\alpha(g\zeta_i, g\zeta_{i+1}), \alpha(g\zeta_k, f(\zeta_1, \zeta_2, ..., \zeta_k)) : 1 \leq i \leq k-1\} \geq 1$$

(*g*) *there exists* $\psi \in \Psi$ *such that*

$$\min_{1 \leq i \leq k}\{\alpha(g\zeta_i, g\zeta_{i+1})\} d(f(\zeta_1, \zeta_2, ..., \zeta_k), f(\zeta_2, \zeta_3, ..., \zeta_{k+1})) \leq \psi(\max_{1 \leq i \leq k}\{d(g\zeta_i, g\zeta_{i+1})\}) \quad (1)$$

for all $\zeta_1, \zeta_2, ..., \zeta_{k+1} \in \Omega$ *with* $g\zeta_1 \preceq g\zeta_2 \preceq ... \preceq g\zeta_{k+1}$.

Then, f and g have a CP.

Proof. By assumption (f) there exist k elements $\zeta_1, \zeta_2, ..., \zeta_k \in \Omega$ such that

$$g\zeta_1 \preceq g\zeta_2 \preceq ... \preceq g\zeta_k \text{ and } g\zeta_k \preceq f(\zeta_1, \zeta_2, ..., \zeta_k).$$

Taking into account assumption (a), define a sequence $\{g\zeta_n\}_{n \in \mathbb{N}}$ satisfying

$$g(\zeta_{n+k}) = f(\zeta_n, \zeta_{n+1}, ..., \zeta_{n+k-1}). \quad (2)$$

Now

$$g\zeta_{k+1} = f(\zeta_1, \zeta_2, ..., \zeta_k) \succeq g\zeta_k$$
$$g\zeta_{k+2} = f(\zeta_2, \zeta_3, ..., \zeta_{k+1}) \succeq f(\zeta_1, \zeta_2, ..., \zeta_k) = g\zeta_{k+1}.$$

Continuing this process, we can show

$$g\zeta_1 \preceq g\zeta_2 \preceq ... \preceq g\zeta_n \preceq ... \quad (3)$$

Now, we will show that $\{g\zeta_n\}$ is a termwise α-sequence in Ω. Using assumption (f), we have $\alpha(g\zeta_i, g\zeta_{i+1}) \geq 1 : 1 \leq i \leq k$, which in view of the α-g admissibility of f, gives rise to

$$\alpha(f(\zeta_1, \zeta_2, ..., \zeta_k), f(\zeta_2, \zeta_3, ..., \zeta_{k+1})) \geq 1, \quad (4)$$

that is, $\alpha(g\zeta_{k+1}, g\zeta_{k+2}) \geq 1$. Again, using the α-g admissibility of f, we get $\alpha(g\zeta_{k+2}, g\zeta_{k+3}) \geq 1$. Continuing this process, we obtain that $\alpha(g\zeta_i, g\zeta_{i+1}) \geq 1$ for all $i = 1, 2, 3, ...$

Suppose $\alpha = \max\{d(g\zeta_1, g\zeta_2), d(g\zeta_2, g\zeta_3), ..., d(g\zeta_k, g\zeta_{k+1})\}$. Now if $g\zeta_1 = g\zeta_2 = ... = g\zeta_k = g\zeta_{k+1} = \zeta$, then we are successful. Otherwise, we may assume that $g\zeta_1, g\zeta_2, ..., g\zeta_k, g\zeta_{k+1}$ are not all equal, i.e., $\alpha > 0$.

In view of assumption (f) and employing (2) and Lemma 1, we have,

$$\begin{aligned} d(g\zeta_{k+1}, g\zeta_{k+2}) &\leq \min_{1 \leq i \leq k}\{\alpha(g\zeta_i, g\zeta_{i+1})\} d(f(\zeta_1, \zeta_2, ..., \zeta_k), f(\zeta_2, \zeta_3, .., \zeta_{k+1})) \\ &\leq \psi(\max\{d(g\zeta_1, g\zeta_2), d(g\zeta_2, g\zeta_3), ..., d(g\zeta_k, g\zeta_{k+1})\}) \\ &\leq \psi(\alpha) < \alpha. \end{aligned}$$

$$\begin{aligned} d(g\zeta_{k+2}, g\zeta_{k+3}) &\leq \min_{2 \leq i \leq k+1}\{\alpha(g\zeta_i, g\zeta_{i+1})\} d(f(\zeta_2, \zeta_3, ..., \zeta_{k+1}), f(\zeta_3, \zeta_4, ..., \zeta_{k+2})) \\ &\leq \psi(\max\{d(g\zeta_2, g\zeta_3), d(g\zeta_3, g\zeta_4), ..., d(g\zeta_{k+1}, \zeta_{k+2})\}) \\ &\leq \psi(\max\{\alpha, \psi(\alpha)\}) < \alpha. \end{aligned}$$

$$\begin{aligned} d(g\zeta_{2k}, g\zeta_{2k+1}) &\leq \min_{k\leq i\leq 2k-1}\{\alpha(g\zeta_i, g\zeta_{i+1})\}d(f(\zeta_k, \zeta_{k+1}, ..., \zeta_{2k-1}), f(\zeta_{k+1}, \zeta_{k+2}, ..., \zeta_{2k})) \\ &\leq \psi(\max\{d(g\zeta_k, g\zeta_{k+1}), d(g\zeta_{k+1}, g\zeta_{k+2}), ..., d(g\zeta_{2k-1}, g\zeta_{2k})\}) \\ &\leq \psi(\max\{\alpha, \psi(\alpha), ..., \psi(\alpha)\}) = \psi(\alpha) < \alpha. \end{aligned}$$

$$\begin{aligned} d(g\zeta_{2k+1}, g\zeta_{2k+2}) &\leq \min_{k+1\leq i\leq 2k}\{\alpha(g\zeta_i, g\zeta_{i+1})\}d(f(\zeta_{k+1}, \zeta_{k+2}, ..., \zeta_{2k}), f(\zeta_{k+2}, \zeta_{k+3}, ..., \zeta_{2k+1})) \\ &\leq \psi(\max\{d(g\zeta_{k+1}, g\zeta_{k+2}), d(g\zeta_{k+2}, g\zeta_{k+3}), ..., d(g\zeta_{2k}, g\zeta_{2k+1})\}) \\ &\leq \psi(\max\{\psi(\alpha), \psi(\alpha), ..., \psi(\alpha)\}) = \psi^2(\alpha) < \alpha \end{aligned}$$

and so on

$$d(g\zeta_{nk+1}, g\zeta_{nk+2}) \leq \psi^n(\alpha), \quad n \geq 1$$

$$d(g\zeta_{n+1}, g\zeta_{n+2}) \leq \psi^{[\frac{n}{k}]}(\alpha), \quad n \geq k. \tag{5}$$

Utilizing the property of ψ and (5), we have

$$\lim_{n\to\infty} d(g\zeta_{n+1}, g\zeta_{n+2}) = 0. \tag{6}$$

For any $n, m \in \mathbb{N}$, $n > k$, we have,

$$\begin{aligned} d(g\zeta_n, g\zeta_{n+m}) &\leq d(g\zeta_n, g\zeta_{n+1}) + d(g\zeta_{n+1}, g\zeta_{n+2}) + ... + d(g\zeta_{n+m-1}, g\zeta_{n+m}) \\ &\leq \psi^{[\frac{n-1}{k}]}(\alpha) + \psi^{[\frac{n}{k}]}(\alpha) + ... + \psi^{[\frac{n+m-2}{k}]}(\alpha). \end{aligned} \tag{7}$$

Assume $l = [\frac{n-1}{k}]$ and $m' = [\frac{n+m-2}{k}]$, then $l \leq m'$.
From (7) we have,

$$\begin{aligned} d(g\zeta_n, g\zeta_{n+m}) \leq{}& \underbrace{\psi^l(\alpha) + \psi^l(\alpha) + ... + \psi^l(\alpha)}_{\text{k times}} \\ &+ \underbrace{\psi^{l+1}(\alpha) + \psi^{l+1}(\alpha) + ... + \psi^{l+1}(\alpha)}_{\text{k times}} \\ &+ ... + \underbrace{\psi^{m'}(\alpha) + \psi^{m'}(\alpha) + ... + \psi^{m'}(\alpha)}_{\text{k times}}. \end{aligned}$$

Therefore,

$$d(g\zeta_n, g\zeta_{n+m}) \leq k\sum_{i=l}^{m'}\psi^i(\alpha), \tag{8}$$

which implies that

$$\lim_{l\to\infty}\sum_{i=l}^{\infty}\psi^i(t) = 0$$

and in view of (8), we have $d(g\zeta_n, g\zeta_{n+m}) \to 0$ as $n \to \infty$, which attests that $\{g\zeta_n\}_{n\in\mathbb{N}}$ is a Cauchy sequence. The completeness of Ω confirms the availability of a $\zeta \in \Omega$ such that

$$\lim_{n\to\infty} g(\zeta_n) = \zeta. \tag{9}$$

Using the continuity of g and (9), we have

$$\lim_{n\to\infty} g(g(\zeta_n)) = g(\zeta). \tag{10}$$

Using assumption (b) and (2), we obtain

$$\begin{aligned} g(g\zeta_{n+k}) &= g(f(\zeta_n,\zeta_{n+1},...,\zeta_{n+k-1})) \\ &= f(g\zeta_n,g\zeta_{n+1},...,g\zeta_{n+k-1}). \end{aligned} \tag{11}$$

Now, since f is continuous, using (9)–(11), we get

$$\begin{aligned} g(\zeta) &= \lim_{n\to\infty} g(g\zeta_{n+k}) \\ &= f(g(\zeta_n,g\zeta_{n+1},...,g\zeta_{n+k-1})) \\ &= f(\zeta,\zeta,...,\zeta). \end{aligned}$$

Hence, ζ is a CP of f and g. □

In the next theorem, we observe that the continuity requirement of f is not necessary to have a coincidence point.

Theorem 5. *In Theorem 4, if we consider $\psi \in \Psi$ is continuous in assumption (g) and replace condition (d) by the following condition:*

(a) *if $\{\zeta_n\}$ is an increasing sequence with $\zeta_n \to \zeta$ then $\zeta_n \preceq \zeta$, $\alpha(\zeta_n,\zeta) \geq 1$ and $\alpha(\zeta,\zeta) \geq 1$ for all n,*

then f has a CP.

Proof. Suppose assumption (a) holds. Since $\{g\zeta_n\}_{n\in\mathbb{N}}$ is increasing, we have

$$g\zeta_n \preceq \zeta, \alpha(g\zeta_n,\zeta) \geq 1 \text{ and } \alpha(\zeta,\zeta) \geq 1 \; \forall \, n \in \mathbb{N}. \tag{12}$$

Employing (11), we obtain

$$\begin{aligned} d(g\zeta,f(\zeta,\zeta,...,\zeta)) &\leq d(g\zeta,g(g\zeta_{n+k})) + d(g(g\zeta_{n+k}),f(\zeta,\zeta,...,\zeta)) \\ &= d(g\zeta,g(g\zeta_{n+k})) + d(f(g\zeta_n,g\zeta_{n+1},...,g\zeta_{n+k-1}),f(\zeta,\zeta,...,\zeta)), \end{aligned}$$

which gives

$$\begin{aligned} d(g\zeta,f(\zeta,\zeta,...,\zeta)) \leq\ & d(g\zeta,g(g\zeta_{n+k})) + d(f(g\zeta_n,g\zeta_{n+1},...,g\zeta_{n+k-1}),f(g\zeta_{n+1},...,g\zeta_{n+k-1},\zeta)) \\ &+ d(f(g\zeta_{n+1},...,g\zeta_{n+k-1},\zeta),f(g\zeta_{n+2},...,g\zeta_{n+k-1},\zeta,\zeta)) \\ &+ .... + d(f(g\zeta_{n+k-1},\zeta,...,\zeta),f(\zeta,\zeta,...,\zeta)). \end{aligned}$$

Hence, utilizing assumption (f) and (12), we get

$$\begin{aligned} d(g\zeta,f(\zeta,\zeta,...,\zeta)) \leq\ & d(g\zeta,g(g\zeta_{n+k})) + \psi(\max\{d(g(g\zeta_n),g(g\zeta_{n+1})),...,d(g(g\zeta_{n+k-1}),g(\zeta))\}) \\ &+ \psi(\max\{d(g(g\zeta_{n+1}),g(g\zeta_{n+2})),...,d(g(g\zeta_{n+k-1}),g(\zeta)),d(g(\zeta),g(\zeta))\}) \\ &+ .... + \psi(\max\{d(g(g\zeta_{n+k-1}),g(\zeta)),d(g(\zeta),g(\zeta)),...,d(g(\zeta),g(\zeta))\}). \end{aligned}$$

Now, letting $n \to \infty$ in the above equation and properties of ψ, we get

$$d(g\zeta,f(\zeta,\zeta,...,\zeta)) \leq 0,$$

which attests that ζ is a coincidence point of f and g. □

To establish the uniqueness of CP and CFP, we need an additional condition.

Definition 10. *Let Ω be a non-empty set and $\alpha : \Omega^2 \to [0,\infty)$ be a function. Let $A \subseteq \Omega$ and $A \neq \emptyset$. Then, A is said to be a well-ordered set if for all $\zeta,\vartheta \in A$, we have $\alpha(\zeta,\vartheta) \geq 1$ or $\alpha(\vartheta,\zeta) \geq 1$ or both. Note that if A is well-ordered, then $\alpha(\zeta,\zeta) \geq 1$ for all $\zeta \in A$.*

Theorem 6. *In addition to Theorem 4, if we add the following condition:*
(a) the set of all coincidence points of f is well-ordered,
then f has a unique CP and CFP.

Proof. Let ζ and ϑ be the two CP of f and g, then

$$f(\zeta,\zeta,...,\zeta) = g(\zeta) = \overline{\zeta}. \tag{13}$$

$$f(\vartheta,\vartheta,...,\vartheta) = g(\vartheta) = \overline{\vartheta}. \tag{14}$$

Then we shall show that

$$\overline{\zeta} = \overline{\vartheta}. \tag{15}$$

Now,

$$\begin{aligned} d(\bar{\zeta},\bar{\vartheta}) &= d(f(\zeta,\zeta..,\zeta),f(\vartheta,\vartheta...\vartheta)) \\ &\leq \alpha(\zeta,\vartheta)d(f(\zeta,\zeta..,\zeta),f(\vartheta,\vartheta...\vartheta)) \\ &\leq \phi(d(g\zeta,g\vartheta)) \\ &< d(g\zeta,g\vartheta) \\ &= d(\bar{\zeta},\bar{\vartheta}), \end{aligned}$$

which is absurd, thereby implying (15) holds. Since f and g commutes, utilizing (13), we obtain

$$\begin{aligned} g(\overline{\zeta}) &= g(f(\zeta,\zeta,...,\zeta)) \\ &= f(g\zeta,g\zeta,...,g\zeta) \\ &= f(\overline{\zeta},\overline{\zeta},...,\overline{\zeta}) \end{aligned}$$

so that

$$g(\overline{\zeta}) = f(\overline{\zeta},\overline{\zeta},...,\overline{\zeta}), \tag{16}$$

thereby implying $\overline{\zeta}$ is also a CP of f and g.

Using (15) and (16), we get

$$f(\overline{\zeta},\overline{\zeta},...,\overline{\zeta}) = g(\overline{\zeta}) = \overline{\zeta}.$$

Therefore, $\overline{\zeta}$ is a CFP of f and g. Let ζ^* be another CFT of f and g, (15) gives rise to

$$\zeta^* = g(\zeta^*) = g(\overline{\zeta}) = \overline{\zeta}.$$

Therefore, f and g have a unique CFP. □

In usual metric space, Theorem 4 reduces to the following coincidence point theorem.

Theorem 7. *Let (Ω, d) be a complete metric space, k is a positive integer. Consider the two mappings $f : \Omega^k \to \Omega$ and $g : \Omega \to \Omega$ satisfying the following conditions:*

(a) $f(\Omega^k) \subseteq g(\Omega)$,
(b) *f and g are a commuting pair,*
(c) *f is α-g admissible,*
(d) *f is continuous,*
(e) *g is continuous,*
(f) *there exists $\psi \in \Psi$ such that for all $\zeta_1, \zeta_2, ..., \zeta_{k+1} \in \Omega$,*

$$\min_{1\leq i\leq k}\{\alpha(g\zeta_i, g\zeta_{i+1})\}d(f(\zeta_1,\zeta_2,...,\zeta_k), f(\zeta_2,\zeta_3,...,\zeta_{k+1})) \leq \psi(\max_{1\leq i\leq k}\{d(g\zeta_i, g\zeta_{i+1})\}). \tag{17}$$

Then, f and g have a CP. Moreover, if $Coin(f)$ is α well-ordered, then f and g have a unique CP.

5. Related Fixed-Point Theorems

As a consequence of Theorem 4, we state some related fixed-point theorems in this section.

Under the mapping $g = I$, the identity map, we obtain a fixed-point theorem under Prešić–Ćirić-type α-ψ contraction in ordered metric space.

Theorem 8. *Let $(\Omega, \preceq, d)$ be a complete partially ordered metric space and $f : \Omega^k \to \Omega$ an increasing mapping, where k is a positive integer satisfyine the following conditions:*

(a) *f is α admissible,*

(b) *f is continuous or if $\{\zeta_n\}$ is an increasing sequence with $\zeta_n \to \zeta$ then $\zeta_n \preceq \zeta$, $\alpha(\zeta_n, \zeta) \geq 1$ and $\alpha(\zeta, \zeta) \geq 1$ for all n,*

(c) *there exist k elements $\zeta_1, \zeta_2, ..., \zeta_k \in \Omega$ such that*

$$\zeta_1 \preceq \zeta_2 \preceq ... \preceq \zeta_k \text{ and } \zeta_k \preceq f(\zeta_1, \zeta_2, ..., \zeta_k),$$

and

$$\min\{\alpha(\zeta_i, \zeta_{i+1}), \alpha(\zeta_k, f(\zeta_1, \zeta_2, ..., \zeta_k)) : 1 \leq i \leq k-1\} \geq 1$$

(d) *there exists $\psi \in \Psi$ such that ψ is continuous and*

$$\min_{1 \leq i \leq k}\{\alpha(\zeta_i, \zeta_{i+1})\} d(f(\zeta_1, \zeta_2, ..., \zeta_k), f(\zeta_2, \zeta_3, ..., \zeta_{k+1})) \leq \psi(\max_{1 \leq i \leq k}\{d(\zeta_i, \zeta_{i+1})\})$$

for all $\zeta_1, \zeta_2, ..., \zeta_{k+1} \in \Omega$ with $\zeta_1 \preceq \zeta_2 \preceq ... \preceq \zeta_{k+1}$.

Then f has a fixed point. Moreover, if $fix(f)$ = the set of all fixed points of f is α-well-ordered, then f has a unique fixed point.

Remark 5. *Under the mapping $g =$ the identity map and $k = 1$ and usual metric space, we obtain Theorem 3 of Samet et al. [13].*

Under the mapping $g =$ the identity map, $\psi(t) = at$, where $a \in (0, 1)$ and in view of Remark 3 , we obtain the following enriched version of Theorems 1 and 2.

Theorem 9. *Let $(\Omega, \preceq, d)$ be a complete partially ordered metric space and $f : \Omega^k \to \Omega$ an increasing mapping, where k is a positive integer satisfying the following conditions:*

(a) *f is α admissible,*

(b) *f is continuous or if $\{\zeta_n\}$ is an increasing sequence with $\zeta_n \to \zeta$ then $\zeta_n \preceq \zeta$, $\alpha(\zeta_n, \zeta) \geq 1$ and $\alpha(\zeta, \zeta) \geq 1$ for all n,*

(c) *there exist k elements $\zeta_1, \zeta_2, ..., \zeta_k \in \Omega$ such that*

$$\zeta_1 \preceq \zeta_2 \preceq ... \preceq \zeta_k \text{ and } \zeta_k \preceq f(\zeta_1, \zeta_2, ..., \zeta_k),$$

and

$$\min\{\alpha(\zeta_i, \zeta_{i+1}), \alpha(\zeta_k, f(\zeta_1, \zeta_2, ..., \zeta_k)) : 1 \leq i \leq k-1\} \geq 1$$

(d) *for all $\zeta_1, \zeta_2, ..., \zeta_{k+1} \in \Omega$ with $\zeta_1 \preceq \zeta_2 \preceq ... \preceq \zeta_{k+1}$*

$$\min_{1 \leq i \leq k}\{\alpha(\zeta_i, \zeta_{i+1})\} d(f(\zeta_1, \zeta_2, ..., \zeta_k), f(\zeta_2, \zeta_3, ..., \zeta_{k+1})) \leq a \max_{1 \leq i \leq k}\{d(\zeta_i, \zeta_{i+1})\}$$

Then f has a fixed point. Moreover, if $fix(f)$ is α-well-ordered, then f has a unique fixed point.

Now, we furnish an example to demonstrate the validity and utility of our new results.

Example 3. *Consider $\Omega = [0,1]$ with usual metric d. Now, endow the metric space with the following partial ordering $\preceq$ defined by*

$$\zeta \preceq \vartheta \iff \zeta \leq \vartheta \text{ and } \zeta, \vartheta \in [0, \frac{1}{2}].$$

Let $f : \Omega^2 \to \Omega$ and $g : \Omega \to \Omega$ be mappings given by

$$f(\zeta_1, \zeta_2) = \begin{cases} \frac{\zeta_1^2 + 2\zeta_2^2}{7}, & \text{if } \zeta_1, \zeta_2 \in [0, \frac{1}{2}] \\ \frac{2}{3}, & \text{if } \zeta_1, \zeta_2 \in (\frac{1}{2}, \frac{2}{3}) \\ 1, & \text{otherwise} \end{cases} \quad \text{and} \quad g(\zeta_1) = \zeta_1, \text{ the identity map.}$$

Further, define the mappings:

$$\alpha : \Omega \times \Omega \to [0, \infty) \text{ by } \alpha(\zeta, \vartheta) = \begin{cases} 1, & \text{when } \zeta, \vartheta \in [0, \frac{1}{2}] \\ 0, & \text{otherwise.} \end{cases}$$

Now we shall apply Theorem 4 to show that f and g have a CP. By simple observation, it is obvious that f is an α-g admissible function. To satisfy condition (f), it is enough to consider the case when $\zeta_1, \zeta_2, \zeta_3 \in [0, \frac{1}{2}]$. Now,

$$\begin{aligned} \min\{\alpha(\zeta_1, \zeta_2), \alpha(\zeta_2, \zeta_3)\} d(f(\zeta_1, \zeta_2), f(\zeta_2, \zeta_3)) &= \left| \frac{\zeta_1^2 + 2\zeta_2^2}{7} - \frac{\zeta_2^2 + 2\zeta_3^2}{7} \right| \\ &= \left| \frac{\zeta_1^2}{7} + \frac{2\zeta_2^2}{7} - \frac{\zeta_2^2}{7} - \frac{2\zeta_3^2}{7} \right| \\ &= \left| \frac{(\zeta_1^2 - \zeta_2^2)}{7} + \frac{2}{7}(\zeta_2^2 - \zeta_3^2) \right| \\ &\leq \frac{1}{7} \mid \zeta_1^2 - \zeta_2^2 \mid + \frac{2}{7} \mid \zeta_2^2 - \zeta_3^2 \mid \\ &\leq \frac{2}{7} \mid \zeta_1^2 - \zeta_2^2 \mid + \frac{2}{7} \mid \zeta_2^2 - \zeta_3^2 \mid \end{aligned}$$

which gives

$$\min\{\alpha(\zeta_1, \zeta_2), \alpha(\zeta_2, \zeta_3)\} d(f(\zeta_1, \zeta_2), f(\zeta_2, \zeta_3)) \leq \frac{2}{7} \mid (\zeta_1 + \zeta_2)(\zeta_1 - \zeta_2) \mid + \frac{2}{7} \mid (\zeta_2 + \zeta_3)(\zeta_2 - \zeta_3) \mid$$

thereby implying

$$\begin{aligned} \min\{\alpha(\zeta_1, \zeta_2), \alpha(\zeta_2, \zeta_3)\} d(f(\zeta_1, \zeta_2), f(\zeta_2, \zeta_3)) &\leq \frac{2}{7} \mid (\zeta_1 - \zeta_2) \mid + \frac{2}{7} \mid (\zeta_2 - \zeta_3) \mid \\ &= \frac{2}{7}[d(g\zeta_1, g\zeta_2) + d(g\zeta_2, g\zeta_3)] \\ &\leq \frac{4}{7} \max\{d(g\zeta_1, g\zeta_2), d(g\zeta_2, g\zeta_3)\}. \end{aligned}$$

Therefore, by routine calculation, it can be observed that all the conditions of Theorem 4 are satisfied with $\psi(t) = \frac{4}{7}t$. Hence, f and g have a unique CFP, i.e.,

$$f(0,0) = g(0) = 0.$$

Notice that this example cannot be covered by Theorems 1 and 2. For instance, take $\zeta_1 = 0$, $\zeta_2 = 0$ and $\zeta_3 = 1$. Then, $d(f(\zeta_1, \zeta_2), f(\zeta_2, \zeta_3)) = d(0,1) = 1$. Therefore, there exists no $a \in (0,1)$ satisfying

$$d(f(\zeta_1, \zeta_2), f(\zeta_2, \zeta_3)) \leq a \max\{d(\zeta_1, \zeta_2), d(\zeta_2, \zeta_3)\}.$$

6. Application

In this section, we provide an application to our main results. We will study the existence of nonlinear integral equations. Consider the following integral equation:

$$\zeta(t) = f\left(t, \int_0^{\varrho} g(t, \vartheta, \zeta(\rho(\vartheta)), ..., \zeta(\rho(\vartheta)))d\vartheta\right) \quad \text{where } t \in [0, \infty). \tag{18}$$

Let $BC[0,\infty)$ be the set of all real, bounded, and continuous functions of the interval $[0,\infty)$. Now, we endow the set with the following norm

$$||\zeta|| = \sup\{|\zeta(t)| : t \in [0,\infty)\}.$$

The metric defined on this space is given by

$$d(\zeta, \vartheta) = \sup\{|\zeta(t) - \vartheta(t)| : t \in [0,\infty)\}.$$

Now, we state the following theorem on the existence of Equation (18).

Theorem 10. *Suppose that the following conditions hold:*

(a) *Let* $\rho, \varrho : [0,\infty) \to [0,\infty)$ *be two continuous functions such that*

$$\Delta = \sup\{|\varrho(t)| : t \in [0,\infty)\} < 1,$$

(b) *let* $f : [0,\infty) \times \mathbb{R} \to \mathbb{R}$ *be a continuous function such that*

$$|f(t,\zeta) - f(t,v)| \le |\zeta - v| \quad \textit{for all } t \in [0,\infty) \textit{ and } \zeta, v \in \mathbb{R},$$

(c) *there exist* $\zeta_1, \zeta_2, ..., \zeta_k$ *such that*

$$\zeta_1 \le \zeta_2 \le \zeta_2 \le ... \le \zeta_k \le f\left(t, \int_0^{\varrho} g(t, \vartheta, \zeta_1(\rho(\vartheta)), ..., \zeta_k(\rho(\vartheta)))d\vartheta\right)$$

(d) *for all* $\zeta_1, \zeta_2, ..., \zeta_k \in BC[0,\infty)$ *with* $\zeta_1 \le \zeta_2 \le \zeta_2 \le ... \le \zeta_k$*, we have*

$$\zeta_k \le f\left(t, \int_0^{\varrho} g(t, \vartheta, \zeta_1(\rho(\vartheta)), ..., \zeta_k(\rho(\vartheta)))d\vartheta\right)$$

(e) *there exists a function* ψ *such that*

$$|g(t, \vartheta, \zeta_1(\rho(\vartheta)), ..., \zeta_k(\rho(\vartheta))) - g(t, \vartheta, \zeta_2(\rho(\vartheta)), ..., \zeta_{k+1}(\rho(\vartheta)))| \le \psi(\max_{1 \le i \le k}\{d(\zeta_i, \zeta_{i+1})\})$$

(f) $M = \max|f(t,0)) : t \in [0,\infty)| < \infty$ *and* $G = \sup\{g(t, \vartheta, 0, .., 0) : t \in [0,\infty)\} < \infty$.

Then, the integral equation has at least one solution in the space $BC[0,\infty)$.

Proof. Let us consider the operator: $\Upsilon : BC[0,\infty)^k \to BC[0,\infty)$ defined by

$$\Upsilon(\zeta_1, \zeta_2, ..., \zeta_k)(t) = f\left(t, \int_0^{\varrho} g(t, y, \zeta_1(\rho(y)), ..., \zeta_k(\rho(y)))dy\right). \tag{19}$$

In view of the given conditions, we show that function Υ is continuous and bounded.

$$\begin{aligned}
|\Upsilon(\zeta_1, \zeta_2, ..., \zeta_k)(t)| &= \left| f\left(t, \int_0^{\varrho} g(t, \vartheta, \zeta_1(\rho(\vartheta)), ..., \zeta_k(\rho(\vartheta)))d\vartheta\right)\right| \\
&\leq \left| f\left(t, \int_0^{\varrho} g(t, \vartheta, \zeta_1(\rho(\vartheta)), ..., \zeta_k(\rho(\vartheta)))d\vartheta\right) - f(t,0)\right| + \left|f(t,0)\right|, \\
&\leq \left| \int_0^{\varrho} g(t, \vartheta, \zeta_1(\rho(\vartheta)), ..., \zeta_k(\rho(\vartheta)))d\vartheta\right| + \left|f(t,0)\right| \\
&\leq \Delta \max\{||\zeta_1||, ||\zeta_2||, ..., ||\zeta_k||\} + \Delta G + M
\end{aligned}$$

thereby implying

$$||\Upsilon(\zeta_1, \zeta_2, ..., \zeta_k)(t)|| \leq \Delta \max\{||\zeta_1||, ||\zeta_2||, ..., ||\zeta_k||\} + \Delta G + M.$$

Hence, function Υ is bounded. Now, we will show that the function Υ satisfies all the conditions of Theorem 4.

Define the mapping $\alpha : BC[0,\infty)^2 \to [0,\infty)$ by

$$\alpha(\zeta_1, \zeta_2) = \begin{cases} 1, & \text{if } \zeta_1 \leq \zeta_2 \\ 0, & \text{otherwise.} \end{cases}$$

In view of assumption (d), f is an α-admissible mapping. Let $\zeta_1, \zeta_2, \zeta_3, .., \zeta_k$ be some elements of $BC[0,\infty)$ with $\zeta_1 \leq \zeta_2 \leq \zeta_3 \leq .. \leq \zeta_k$. Then, we have

$$\min_{1\leq i\leq k}\{\alpha(\zeta_i, \zeta_{i+1})\} d(\Upsilon(\zeta_1, \zeta_2, ..., \zeta_k), \Upsilon(\zeta_2, \zeta_3, ..., \zeta_{k+1}))$$

$$=|\Upsilon(\zeta_1, \zeta_2, ..., \zeta_k) - \Upsilon(\zeta_2, \zeta_3, ..., \zeta_{k+1})|$$

$$\leq \left| f\left(t, \textstyle\int_0^{\varrho} g(t, \vartheta, \zeta_1(\rho(\vartheta)), ..., \zeta_k(\rho(\vartheta)))d\vartheta\right) - f\left(t, \textstyle\int_0^{\varrho} g(t, \vartheta, \zeta_2(\rho(\vartheta)), ..., \zeta_{k+1}(\rho(\vartheta)))d\vartheta\right)\right|$$

$$\leq \left| \textstyle\int_0^{\varrho} |g(t, \vartheta, \zeta_1(\rho(\vartheta)), ..., \zeta_k(\rho(\vartheta))) - g(t, \vartheta, \zeta_2(\rho(\vartheta)), ..., \zeta_{k+1}(\rho(\vartheta)))| d\vartheta \right|$$

$$\leq \varrho(t)\psi(\max_{1\leq i\leq k}\{d(\zeta_i, \zeta_{i+1})\})$$

$$\leq \psi(\max_{1\leq i\leq k}\{d(\zeta_i, \zeta_{i+1})\}).$$

Hence, all the conditions of Theorem 4 are satisfied. Therefore, applying Theorem 4, we obtain that Υ has a CP, which implies that the integral equation has a solution in $BC[0,\infty)$. □

7. Conclusions

In this paper, we introduced the concept of α-ψ contraction mappings and utilized the same to establish some coincidence and common fixed-point theorems. Besides giving some examples pointing to the new development, we have also provided an application to solve a family of integral equations that certifies the importance of our newly proven results. Many well-known coincidence and fixed-point results in the literature are deduced from our results. By slightly changing the associated conditions, the approach presented in our study can be used for a variety of comparatively weaker contractions (e.g., Prešić–Reich type, Prešić–Hardy-Rogers type). As a consequence, similar outcomes are possible in the near future.

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