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Effective Majorana Neutrino Mass for $\Delta L = 2$ Neutrino Oscillations

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Abstract: It is well known that the observations of neutrinoless double-beta decay prove the Majorana nature of the neutrino. However, with specific values of Majorana phases, the effective Majorana neutrino mass to be estimated from the observation of neutrinoless double-beta decay experiments is strongly suppressed if the neutrino mass pattern adheres to a normal ordering. In this case, double-beta decay might not be observed even though the neutrino is Majorana in nature. We show if neutrinos oscillate to antineutrinos in their propagation; then, the observation of this oscillation proves that neutrinos are Majorana and will provide a measurement of neutrino masses and Majorana phases.

Keywords: neutrino to antineutrino oscillation; Majorana phases; effective Majorana neutrino mass; leptonflavor violation; total lepton number violating process

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1. Introduction

The discovery of neutrino oscillation proves that at least two neutrinos should have nonzero mass and three of the flavors mix among each other [1]. Neutrino oscillation data are sensitive to the mass-squared differences of the neutrino (Δm^2_{21} and Δm^2_{31}), which allows us to set an upper limit on the absolute mass of two neutrinos. The current best fit value of $\Delta m^2_{21} = 7.42 \times 10^{-5}$ eV^2 and $\Delta m^2_{31} = 2.51 \times 10^{-3}$ eV^2 [2] leads to the upper limits $m_2 \geq 8.6$ meV and $m_3 \geq 50$ meV for normal ordering (NO, $m_1 < m_2 < m_3$). One of the the direct ways of measuring the absolute mass of a neutrino is by precisely measuring the energy spectrum of beta particles from single β-decay. In addition, the cosmological observation provides the upper limit on the sum of neutrino masses.

Another important way to probe the neutrino mass is by detecting neutrinoless double-beta decay, which would occur if neutrinos are Majorana particles, i.e., neutrinos are their own anti-particle, and the active light neutrinos are the mediator of the decay [3–6]. The Majorana nature of the neutrino allows the process of neutrino to antineutrino oscillation, which is studied in several works [7–18]. In this paper, we study the effective Majorana neutrino mass with neutrino to antineutrino oscillation.

This paper is organized in the following way. Section 2 describes the theory of neutrino to antineutrino oscillations. In Section 3, we derive the expressions for Majorana phases that result in the lowest possible effective Majorana neutrino mass. We show the enhancement in the lowest possible effective Majorana masses due to neutrino to antineutrino oscillation in Section 4. Finally, we summarize and conclude this work in Section 5.

2. Neutrino to Antineutrino Oscillation

The oscillation between a neutrino and antineutrino is an allowed process if the lepton number is not a good quantum number, which is true if the neutrino is a Majorana particle. The rate of this oscillation is suppressed due to the small mass of the neutrino. The amplitude of $\nu_\alpha \to \bar{\nu}_\gamma$ in propagation from the source to a detector at a distance L is given by [7–18]

$$A(\nu_\alpha \to \bar{\nu}_\gamma) = \sum_i [U^*_{\alpha i} U^*_{\gamma i} \frac{m_i}{E} exp(-i\frac{m_i^2}{2E}L)]\bar{K}. \quad (1)$$

The same for the CP-conjugate process $\bar{\nu}_\alpha \to \nu_\gamma$ is

$$A(\bar{\nu}_\alpha \to \nu_\gamma) = \sum_i [U_{\alpha i} U_{\gamma i} \frac{m_i}{E} exp(-i\frac{m_i^2}{2E}L)]K. \quad (2)$$

where K and $\bar{K}$ are functions of kinematical factors and nuclear matrix elements, and $K = \bar{K}$ due to the CP invariance in a strong interaction. The neutrino mass is denoted by m_i with $i \equiv 1,2,3$, and E stands for neutrino energy. The parameters $U_{\alpha/\gamma i}$ $(\alpha, \gamma \equiv e, \mu, \tau)$ are elements of the PMNS matrix, which is the lepton mixing matrix, and it is parameterized with three mixing angles, one Dirac CP phase, and two Majorana phases (if the neutrino is a Majorana particle):

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} e^{i\phi_1} & 0 & 0 \\ 0 & e^{i\phi_2} & 0 \\ 0 & 0 & e^{i\delta} \end{pmatrix}$$

where $c_{ij} \equiv \cos\theta_{ij}$, $s_{ij} \equiv \sin\theta_{ij}$. The neutrino oscillation in propagation depends on mixing angles θ_{12}, θ_{13}, θ_{23}, and one Dirac CP phase δ, along with neutrino mass-squared differences. On the other hand, the Majorana phases (without losing generality, Majorana phases can be restricted in $[0, \pi]$), ϕ_1 and ϕ_2, have no impact on neutrino oscillation; thus, the neutrino oscillation experiments are blind to these phases.

The probability of neutrino to antineutrino oscillation is given by

$$P(\nu_\alpha \to \bar{\nu}_\gamma) = \frac{|\bar{K}|^2}{E^2}(\bar{m}^L_{\alpha\gamma})^2 \quad (3)$$

with

$$\bar{m}^L_{\alpha\gamma} = \left| \sum_i [U^*_{\alpha i} U^*_{\gamma i} m_i exp(-i\frac{m_i^2}{2E}L) \right|. \quad (4)$$

We can write the same for the CP-conjugate process $\bar{\nu}_\alpha \to \nu_\gamma$ as

$$P(\bar{\nu}_\alpha \to \nu_\gamma) = \frac{|K|^2}{E^2}(m^L_{\alpha\gamma})^2 \quad (5)$$

with

$$m^L_{\alpha\gamma} = \left| \sum_i [U_{\alpha i} U_{\gamma i} m_i exp(-i\frac{m_i^2}{2E}L) \right|. \quad (6)$$

We call $m^L_{\alpha\gamma}$ and $\bar{m}^L_{\alpha\gamma}$ the "effective neutrino mass" in the presence of $\nu \rightleftharpoons \bar{\nu}$ oscillations. Equation (6) boils down to the effective Majorana neutrino mass ($m_{\beta\beta}$) with $L = 0$, and this can be measured in neutrinoless double-beta decay experiments. We discuss $m_{\beta\beta}$ in the next section.

3. Neutrinoless Double-Beta Decay

The observation of neutrinoless double-beta decay not only confirms the Majorana nature of the neutrino but provides the measurement of ethe ffective neutrino mass [3–6],

$$m_{\beta\beta} = |\sum_i U^2_{ei} m_i|. \quad (7)$$

where m_i is the neutrino mass for $i = 1, 2, 3$. The effective neutrino mass $m_{\beta\beta}$ depends on Majorana phases ϕ_1 and ϕ_2. Putting the elements of the PMNS matrix in Equation (7), one can write

$$m_{\beta\beta} = |\rho_1 e^{2i\phi_1} + \rho_2 e^{2i\phi_2} + \rho_3|,$$
$$\rho_1 = c_{12}^2 c_{13}^2 m_1, \quad \rho_2 = s_{12}^2 c_{13}^2 m_2, \quad \rho_3 = s_{13}^2 m_3. \tag{8}$$

The minimum value of $m_{\beta\beta}$ is [19]

$$\min_{\phi_1,\phi_2} m_{\beta\beta} = \begin{cases} |\rho_2 - \rho_3| - \rho_1, & \text{if } \rho_1 < |\rho_2 - \rho_3| & : \text{region I}, \\ 0, & \text{if } |\rho_2 - \rho_3| \le \rho_1 \le \rho_2 + \rho_3 & : \text{region II}, \\ \rho_1 - (\rho_2 + \rho_3), & \text{if } \rho_2 + \rho_3 < \rho_1 & : \text{region III}. \end{cases} \tag{9}$$

Comparing Equations (8) and (9), Majorana phases corresponding to a minimum $m_{\beta\beta}$ in different regions are estimated. In region I (we checked that in region I, the condition $\rho_2 > \rho_3$ is true over whole range of m_1 with the allowed range of m_1; therefore, $|\rho_2 - \rho_3| = \rho_2 - \rho_3$), the values of Majorana phases are $\phi_1 = 0$ and $\phi_2 = \pm\pi/2$, whereas in region III, $\phi_1 = \pm\pi/2$, and $\phi_2 = 0$ with the minimum value of $m_{\beta\beta}$. Therefore, Majorana phases responsible for minimum $m_{\beta\beta}$ are different in regions I to III. In region II, there is a continuous change of ϕ_1 and ϕ_2 as the functions of the lightest neutrino mass. This dependence of ϕ_1 and ϕ_2 on neutrino mixing angles and masses is given by

$$\phi_i^{\pm} = \frac{1}{2} \tan^{-1} \frac{Y_i^{\pm}}{X_i^{\pm}}, \tag{10}$$

where

$$X_1^{\pm} = \frac{-(\rho_1^2 - \rho_2^2 + \rho_3^2)}{\rho_1 \rho_3},$$
$$Y_1^{\pm} = \frac{\mp\sqrt{(2\rho_1^2\rho_2^2 + 2\rho_1^2\rho_3^2 + 2\rho_2^2\rho_3^2 - \rho_1^4 - \rho_2^4 - \rho_3^4)}}{\rho_1 \rho_3},$$
$$X_2^{\pm} = \frac{(\rho_1^2 - \rho_2^2 - \rho_3^2)}{\rho_2 \rho_3},$$
$$Y_2^{\pm} = \frac{\pm\sqrt{(2\rho_1^2\rho_2^2 + 2\rho_1^2\rho_3^2 + 2\rho_2^2\rho_3^2 - \rho_1^4 - \rho_2^4 - \rho_3^4)}}{\rho_2 \rho_3}. \tag{11}$$

With oscillation parameters $\theta_{12} = 34°$, $\theta_{13} = 8.5°$, $\Delta m_{21}^2 = 7.5 \times 10^{-5}$ eV^2, and $\Delta m_{21}^2 = 2.5 \times 10^{-3}$ eV^2, we find that the minimum value of $m_{\beta\beta}$ is zero in the range of 2.5 meV $< m_{\text{lightest}} <$ 6.5 meV. In Figure 1, we show these regions as functions of lightest mass (m_1 for NO).

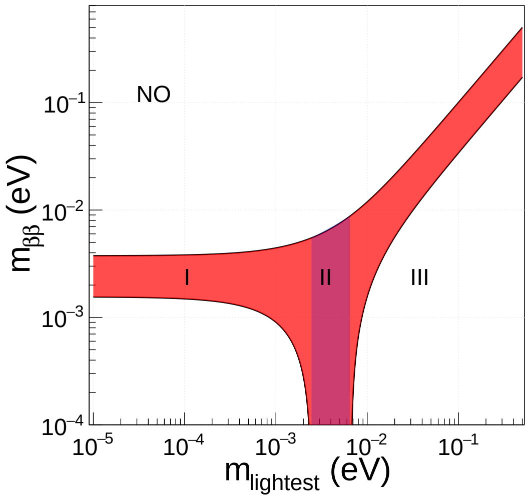

Figure 1. The effective neutrino mass as a function of the lightest neutrino mass assuming normal ordering ($m_1 < m_2 < m_3$). The blue shaded region depicts region II, where $m_{\rm lightest}$ is in the range of [2.5–6] meV and $m_{\beta\beta}$ is obtained as zero.

We note that in the case of a neutrino mass spectrum with normal ordering, one can have $m_{\beta\beta} = 0$ as a consequence of an "accidental" relation involving neutrino masses, mixing angles, and the Majorana phases. However, there does not exist a symmetry which forbids neutrinoless double-beta decay, although in this case, the neutrinoless double-beta decay will be allowed. The corresponding effective Majorana mass parameter is determined by [20]

$$\sum_{j=1}^{3} U_{ej}^2 \left(\frac{m_j}{q}\right)^3, \tag{12}$$

where q is the momentum of the virtual Majorana neutrino. For the average momentum $\langle q \rangle$, one typically has $|\langle q \rangle|^2 \approx (100~\text{MeV})^2$. Thus, if the region $m_{\beta\beta}$ is equal to zero, this contribution is nonzero, but negligible [20].

4. Results

This section is devoted to describing the results obtained in this study. First, we discuss the modification of the effective Majorana neutrino mass due to the propagation of antineutrinos and detect this as a neutrino in a detector at a distance L from the source. We rewrite Equation (6) in terms of mass-squared differences as

$$m_{\beta\beta}^L = \left| U_{e1}^2 m_1 + U_{e2}^2 m_2 exp(-i\frac{\Delta m_{21}^2}{2E}L) + U_{e3}^2 m_3 exp(-i\frac{\Delta m_{31}^2}{2E}L) \right|. \tag{13}$$

Figure 2 shows the L/E dependence of $m_{\beta\beta}^L$ with three choices of the lightest neutrino masses, 0.1 meV, 3 meV, and 20 meV, which are in region I, II, and III, respectively. In this paper, we describe all the results with L/E in units of m/MeV. These results are also valid for L/E of same value but in units of km/GeV. The amplitude of oscillation depends on the neutrino mixing angles as well as on the neutrino masses, whereas the frequency depends on the mass-squared difference and L/E. The imprint of two independent mass-squared differences are seen in the plot of $m_{\beta\beta}^L$.

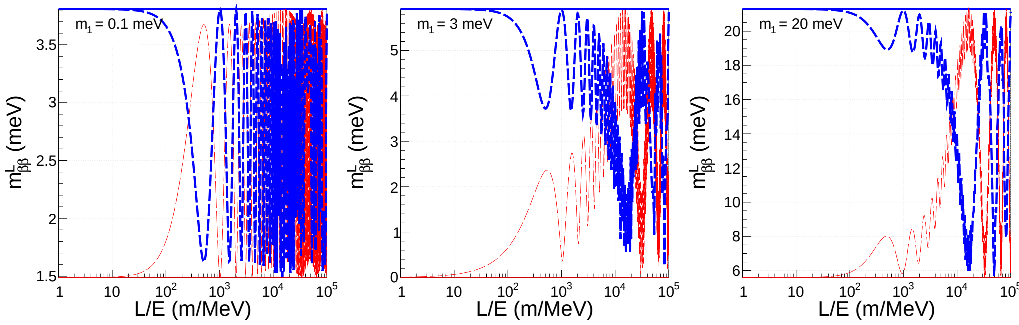

Figure 2. The dependence of the effective neutrino mass $m^L_{\beta\beta}$ with (dashed lines) and without (solid lines) $\nu \to \bar{\nu}$ oscillation. The red and blue lines are obtained with Majorana phases corresponding to the maximum ($\phi_1 = 0, \phi_2 = 0$) and minimum $m_{\beta\beta}$ (Equation (10)). Note that y-axis ranges are different in the three panels.

With $L/E < 2000$ m/MeV, the oscillation is predominantly due to $\Delta m^2_{31} L/2E$, and the amplitude is $m_3 \sin^2\theta_{13}$. For $L/E > 2000$ m/MeV, the second term starts to grow, and its effect is visible in the second and third panel of Figure 2. Since neutrino to antineutrino oscillation introduces an additional phase into the expression of effective neutrino mass, the allowed regions of $m_{\beta\beta}$ and $m^L_{\beta\beta}$ are same. However, an important point to note is that if the value of Majorana phases and the lightest neutrino mass in nature are such that the effective mass of the neutrino is suppressed, and $0\nu\beta\beta$ events are therefore suppressed, then the observation of neutrino to antineutrino oscillation can be a signature for measuring the Majorana phases as well as neutrino masses.

In Figure 3, we present the dependence of $m^L_{\beta\beta}$ on the lightest mass (m_1 for NO) for three different L/E values. With $L/E = 300$ m/MeV (or km/GeV), the major contributing factor to the difference between $m_{\beta\beta}$ and $m^L_{\beta\beta}$ arises due to the third term in Equation (13). For the lightest neutrino at around 0.1 meV, the oscillation due to Δm^2_{31} can change the effective mass from its maximum value to a small—close to minimum—value and vice versa. As the lightest neutrino mass increases, Δm^2_{31}-induced oscillation cannot cover the whole range between the highest and lowest limit of $m_{\beta\beta}$. In this case, with higher values of L/E, Δm^2_{21}-induced oscillation can change $m^L_{\beta\beta}$ significantly. This explains why we see a larger relative change in $m^L_{\beta\beta}$ at lower m_1 with L/E = 300 m/MeV, and almost no change $m_1 > 1$ meV. We find a relative change in $m^L_{\beta\beta}$ over the whole range of m_1 with $L/E = 10^4$ m/MeV staying almost the same. The discontinuity feature with a minimum $m_{\beta\beta}$ with a nonzero L/E arises due to the different behavior of Majorana phases in the three regions we describe in Section 3.

If the neutrino mass ordering is an inverted ordering (IO), $m_{\beta\beta}$ is nonzero over the whole range of m_{lightest}, suppressions of $0\nu\beta\beta$ events due to zero $m_{\beta\beta}$ do not arise. In addition, the bands of $m_{\beta\beta}$ are rather narrow for IO in the whole range of m_{lightest}, and $m^L_{\beta\beta}$ remains inside this band; therefore, the difference between $m_{\beta\beta}$ and $m^L_{\beta\beta}$ for IO will not be large.

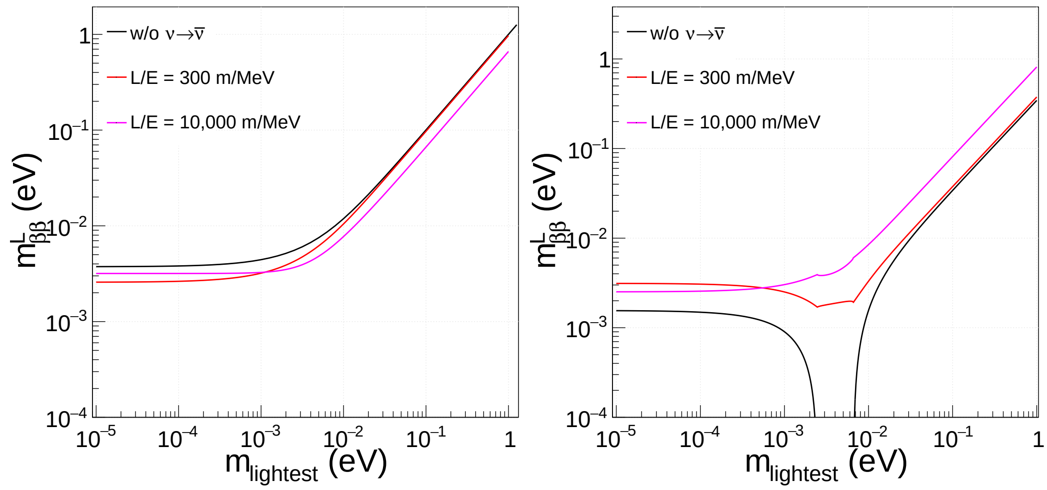

Figure 3. Dependence of $m^L_{\beta\beta}$ on lightest mass (m_1 for NO) with Majorana phases corresponding to maximum ($\phi_1 = 0, \phi_2 = 0$) and minimum $m_{\beta\beta}$ (Equation (10)) in left and right panels, respectively, with two choices of L/E.

5. Conclusions

The observation of neutrinoless double-beta decay confirms the Majorana nature of the neutrino. The rate of $0\nu\beta\beta$ events provides the measurement of the effective neutrino mass. The dependence of $m_{\beta\beta}$ on unknown Majorana phases results in a large uncertainty in the interpretation of the lightest neutrino mass from $m_{\beta\beta}$. In addition, it is important to note that if the neutrino mass ordering adheres to a normal order and if m_{lightest} is in range of [2.5–6] meV, $m_{\beta\beta}$ = 0 with a certain combination of Majorana phases.

In this study, we have derived the expressions for Majorana phases for which effective the Majorana mass becomes zero. We have distinguished three regions of neutrino mass for which the Majorana phases corresponding to the minimum $m_{\beta\beta}$ behave in a different way. These are region I: $m_1 \lesssim 2.5$ meV, II: 2.5 meV $< m_1 <$ 6.5 meV, and III: $m_1 \gtrsim$ 6.5 meV with NO. We have considered neutrino to antineutrino oscillation, which is a theoretically allowed process if a neutrino is a Majorana particle, and explored its role in the effective Majorana neutrino mass measurement. The neutrino to antineutrino oscillation provides an indirect measurement of the effective Majorana neutrino mass, which would be different than that measured in the $0\nu\beta\beta$ experiment due to the additional phases introduced in propagation. This oscillation phase depends on neutrino mass-squared differences and L/E-like neutrino flavor oscillation. However, the amplitude of $\nu \rightleftarrows \bar{\nu}$ oscillation not only depends on mixing angles but also on the absolute masses of neutrinos. This feature of neutrino to antineutrino oscillation will play an important role in neutrino mass measurement.

We have shown that the effective Majorana neutrino ($m^L_{\beta\beta}$) mass in neutrino to antineutrino oscillation is changed by a larger amount when the lightest neutrino mass is less than region I and II when L/E is as small as 300 m/MeV or km/GeV, where Δm^2_{31}-induced oscillation contributes to the additional phases. Therefore, reactor and accelerator-based neutrino experiment data can be used to search for the signature of $m^L_{\beta\beta}$ if m_1 lies in region I and II. If L/E becomes as large as 10^4 m/MeV or km/GeV, then $m^L_{\beta\beta}$ is different than $m_{\beta\beta}$ for the whole range of m_1 we consider this study, which is [0.01–10^3] meV. The experimental techniques and possible backgrounds for measuring $m^L_{\beta\beta}$ are within the scope of more detailed study.

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