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*Editorial*

# Metamaterials and Symmetry

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**Citation:** Lim, T.-C. Metamaterials and Symmetry. *Symmetry* **2022**, *14*, 1587. https://doi.org/10.3390/sym14081587

Received: 27 June 2022
Accepted: 18 July 2022
Published: 2 August 2022

How are metamaterials related to symmetry? Originating from the Greek word μετά (meta), for "beyond" and the Latin word "materia", which refers to "material", the term "metamaterials" has been coined to define materials that are micro-structured in a manner such that their equivalent physical behaviors are predominantly governed by the micro-architectural geometries instead of the properties of the base materials. Essentially, the metamaterials' representative units are repeated throughout the entire structure. Inevitably, the properties of symmetry play a salient role in determining the effective characteristics of the metamaterials. This Special Issue comprises papers on mechanical metamaterials with special attention paid to their symmetry properties.

The first paper, by Lim [1], explores the possibility of attaining perfect auxetic behavior of $v = -1$ by taking inspiration from an Islamic geometrical pattern. Specifically, this metamaterial consists of Y-element sub-units that are arranged and joined to form a circumference of eight rhombi, such that the four rhombi oriented to the axes are shared with neighboring units while the remaining four rhombi aligned to the diagonals are unshared. In the fully-closed configuration, each unit of the metamaterial forms a circumference of eight squares that constitutes an eight-pointed star; in the fully-opened configuration, each unit forms a regular octagon. In addition to exhibiting perfect auxeticity, the calculated results suggest that in-plane uniaxial stretching increases the overall Young's modulus exponentially until a complete extension is achieved.

In the second paper, Grima-Cornish et al. [2] evaluate the results of DFT-based simulations aimed at understanding the deformations that such crystals encounter when prescribed with shear loading to gain a deeper insight into the process in which this material responds to mechanical loads. The deformation mechanisms for shearing in the (001) plane are elucidated in terms of the 'rotating squares' model, which was used to expound the auxeticity in the same plane where it was shown that shear loading results predominantly in deformations that make the 'squares' become 'parallelogram-like' instead of rotating. The deficiency of the rigidity in the projected 'squares' was examined by observing the changes in their bond lengths and bond angles.

Bilski et al. [3] analyzed 2D crystalline structures with honeycomb geometry using the Monte Carlo method within the isobaric-isothermal group. The crystals that were selected for consideration were formed by hard discs (HD) of two different diameters that are in very close proximity. As opposed to equidiameter HD, which crystallize into a homogeneous solid that is elastically isotropic due to its sixfold symmetrical axis, the systems studied by Bilski et al. [3] contain artificial patterns and are (an)isotropic. It was found that the symmetry of the patterns acquired by a suitable arrangement of two types of discs strongly influences their elastic properties. The Poisson's ratio (PR) of each of the examined structures was analyzed with respect to two aspects: (a) their dependence on the external isotropic pressure and (b) concerning the function of the direction angle, in which the deformation of the system takes place, since some of the structures are anisotropic. To achieve the latter, the general analytic formula for the orientational dependence of PR in 2D systems was adopted. The PR analysis at very high pressures indicates that for the vast majority of the considered structures it is approximately isotropic and tends to the upper limit for isotropic 2D systems, which is $v = +1$. This is different from systems of equidiameter discs, for which their PR tends to $v = 0.13$.

Borcea and Streinu [4] investigate a relationship between the infinitesimal deformations of a periodic bar-and-joint framework with the periodic arrangements of quadrics. This intrinsic connection avails practical geometric characteristics. A direct result is a method for detecting auxetic deformations, identified by a pattern consisting of homothetic ellipsoids. Some examples include frameworks with higher crystallographic symmetry. In one of the examples, the authors illustrate the special case when the auxeticity criterion developed in the previous section yields circles for the homothetic ellipses. They applied a planar periodic framework introduced earlier [1], which has several distinctive features: (i) the framework has one degree of freedom, (ii) the framework's deformation is auxetic, and (iii) the framework maintains a planar crystallographic symmetry—throughout its deformation path—with point group $D_4$, the dihedral group of order eight that describes the symmetries of a square.

A generalized strain energy-based homogenization method for 2D and 3D cellular materials with and without periodicity constraints was suggested by Gad and Gao [5] using Hill's Lemma and the matrix method for spatial frames. In this novel method, the equilibrium equations are imposed at all boundary and interior nodes and each interior node is permitted to freely translate and rotate, in contrast to current approaches in which the equilibrium conditions are enforced at the boundary nodes only. The newly formulated homogenization approach can be prescribed to cellular materials with or without symmetry. To illustrate this new approach, four examples were analyzed: two for a 2D cellular material and two for a 3D pentamode metamaterial, with and without periodic constraints in each group. In the case of the 2D cellular material, an asymmetric microstructure with or without periodicity constraints was investigated and closed-form expressions of the effective stiffness components were obtained. In the case of the 3D pentamode metamaterial, a primitive diamond-shaped unit cell with or without periodicity constraints was considered; for each of these 3D cases, two different representative cells in two orientations were assessed. The homogenization study shows that the pentamode metamaterial demonstrates a cubic symmetry based on one representative cell, with the effective Poisson's ratio being nearly 0.5. In addition, it was shown that the pentamode metamaterial with the cubic symmetry can be tuned to be a rubber-like material ($\overline{v} \cong 0.5$) or an auxetic material ($\overline{v} < 0$).

Hinged tilings avail possibilities for the design of auxetic and equiauxetic frameworks in 2D, and generic auxetic behavior can normally be identified using a symmetry extension of the scalar counting rule for the mobility of periodic body–bar systems. The final paper, by Tarnai et al. [6], considers hinged frameworks based on Archimedean tilings. It is known that the regular hexagonal tiling, $\{6^3\}$, gives rise to an equiauxetic framework for both single-link and double-link connections between the tiles. In the case of the single-link connections in this work, three Archimedean tilings considered as hinged body–bar frameworks were found to be equiauxetic: $\{3.12^2\}$, $\{4.6.12\}$, and $\{4.8^2\}$. In the case of double-link connections, three Archimedean tilings considered as hinged body-bar frameworks were found to be equiauxetic: $\{3^4.6\}$, $\{3^2.4.3.4\}$, and $\{3.6.3.6\}$.

The collection of papers in this Special Issue is by no means complete but does offer a snapshot of various works on mechanical metamaterials and their connections to symmetry. It is hoped that through this Special Issue, further interest in metamaterials can be ignited from the perspectives of symmetry.

**Funding:** This research received no external funding.

**Conflicts of Interest:** The guest editor declares no conflict of interest.

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