symmetry

MDPI

Article

Results on Univalent Functions Defined by q-Analogues of Salagean and Ruscheweh Operators

Ebrahim Amini [1], Mojtaba Fardi [2], Shrideh Al-Omari [3] and Kamsing Nonlaopon [4,*]

[1] Department of Mathematics, Payme Noor University, Tehran P.O. Box 19395-4697, Iran
[2] Department of Mathematics, Faculty of Mathematical Science, Shahrekord University, Shahrekord P.O. Box 115, Iran
[3] Department of Scientific Basic Sciences, Faculty of Engineering Technology, Al-Balqa Applied University, Amman 11134, Jordan
[4] Department of Mathematics, Faculty of Science, Khon Kaen University, Khon Kaen 40002, Thailand
* Correspondence: nkamsi@kku.ac.th

Abstract: In this paper, we define and discuss properties of various classes of analytic univalent functions by using modified q-Sigmoid functions. We make use of an idea of Salagean to introduce the q-analogue of the Salagean differential operator. In addition, we derive families of analytic univalent functions associated with new q-Salagean and q-Ruscheweh differential operators. In addition, we obtain coefficient bounds for the functions in such new subclasses of analytic functions and establish certain growth and distortion theorems. By using the concept of the (q, δ)-neighbourhood, we provide several inclusion symmetric relations for certain (q, δ)-neighbourhoods of analytic univalent functions of negative coefficients. Various q-inequalities are also discussed in more details.

Keywords: univalent functions; q-analogues; Salagean operators; q-calculus; Ruscheweh operators

MSC: 05A30; 26D10; 26D15; 26A33

check for updates

Citation: Amini, E.; Fardi, M.; Al-Omari, S.; Nonlaopon, K. Results on Univalent Functions Defined by q-Analogues of Salagean and Ruscheweh Operators. *Symmetry* **2022**, *14*, 1725. https://doi.org/10.3390/sym14081725

Academic Editor: Erhan Deniz

Received: 17 July 2022
Accepted: 14 August 2022
Published: 18 August 2022

Publisher's Note: MDPI stays neutral with regard to jurisdictional claims in published maps and institutional affiliations.

Copyright: © 2022 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (https://creativecommons.org/licenses/by/4.0/).

1. Introduction

In recent decades, the q-derivative has experienced accelerated developments in various fields of science due to its numerous applications in mathematical analysis and physical sciences including q-difference operators, fractional and q-symmetric fractional q-calculus, optimal control, q-symmetric functions and q-integral equations, to mention but a few (see, e.g., [1–9]). In [10], Jackson introduces the q-difference operator and discusses some applications of the q-derivative and the q-integral (see, also, [11] for more details and concepts). In [12], Srivastava introduces a connection between the geometric function theory of the complex analysis and the theory of the q-calculus. In [13], Arif et al. describe important applications of the q-calculus concept. In [14], Ismail et al. describe starlike functions by using q-difference operators. In [15], Sokol et al. investigate a subclass of analytic functions with a Ruscheweyh q-differential operator. In [16], Kanas and Raducanu introduce q-analogues of the Ruscheweyh differential operators and establish some convolution properties of some normalized analytic functions. In [17], Darus et al. study a q-analogue of some operator by using q-hypergeometric functions. Moreover, authors of [18–21] apply properties of the q-difference operator to discuss subclasses of complex analytic functions. In this paper, we define q-analogues of the Salagean and Ruscheweyh differential operators for certain univalent functions and study some interesting properties of the obtained results.

Let $\mathcal{H}$ be the set of all analytic functions on the unite open disc $D = \{z \in \mathbb{C} : |z| < 1$. Let $\mathcal{A}_0$ be the subset of $\mathcal{H}$ of all functions normalized by $f(0) = 0$ and $f'(0) = 1$ and S be the set of univalent functions (consult, for details, [22]). For a function f in the class S, Salagean in [23] introduced a differential operator and studied some of its applications on a certain subclass of univalent functions. Ruscheweyh in [24] defined the differential operator and investigated some properties of the univalent functions. Recently, in various papers, several authors have introduced generalizations to the Salagean differential operator (see, e.g., [25]). The q-analogue of the Ruscheweyh operator was also studied in [26–28]. For more details on this theory, we refer to [29–42] and references cited therein.

For any real number $q, 0 < q < 1$, the q-difference operator for a complex valued function f is defined by

$$D_q f(z) = \frac{f(z) - f(qz)}{z - qz}, \quad z \in D.$$

It is clear that $D_q f \to f'$ when $q \to 1$. Let $\mathcal{U}$ denote the subclass of all normalized functions f in $\mathcal{H}$ such that

$$f(z) = z - \sum_{k=1}^{\infty} a_k z^k, \qquad a_k \geq 0. \tag{1}$$

Then, for every function f furnished by (1), we assert that

$$D_q f(z) = 1 - \sum_{k=2}^{\infty} [k]_q a_k z^k, \qquad a_k \geq 0,$$

where $[k]_q = \frac{1-q^n}{1-q}, 0 \leq q < 1$ and $n \in \mathbb{N}$. The q-real number for $a \in \mathbb{C}$ is defined by [11]

$$(a;q)_n = \begin{cases} 1, & n = 0, \\ \prod_{i=0}^{n-1}(1 - aq^i), & n \in \mathbb{N}. \end{cases} \tag{2}$$

Therefore, for a complex number α, we use the following notation to denote the q-binomial coefficients [43]

$$B_q^k(\alpha) = \binom{\alpha}{k}_q = \begin{cases} 1, & k = 0, \\ \frac{(1-q^{\alpha})(1-q^{\alpha-1})\ldots(1-q^{\alpha-k+1})}{(q;q)_k}, & k \in \mathbb{N}. \end{cases}$$

The q-analogue exponential function is defined in [43] by

$$e_q^s = 1 + \sum_{k=1}^{\infty} \frac{s^k}{[k]_q!},$$

where the q-factorial $[k]_q!$ is given by

$$[k]_q! = \begin{cases} 1, & k = 0, \\ [k]_q [k-1]_q \ldots [1]_q, & k = 1, 2, \ldots \end{cases}$$

The modified q-Sigmoid function is defined in [43] by

$$\gamma_q(s) = \frac{1}{1 + e_q^s}. \tag{3}$$

Alternatively, it can be expressed as

$$\gamma_q(s) = 1 + \frac{1}{2[1]_q!} s + \left(\frac{1}{4([1]_q!)^2} - \frac{1}{2[2]_q!} \right) s^2 + \left(\frac{1}{2[3]_q!} - \frac{1}{2[1]_q![2]_q!} + \frac{1}{(2[1]_q!)^3} \right) s^3 + \ldots,$$

where the q-factorial $[k]_q!$ is given by

$$[k]_q! = \begin{cases} 1, & k = 0, \\ [k]_q [k-1]_q \ldots [1]_q, & k = 1, 2, \ldots \end{cases}$$

By keeping track of the definition of the q-Sigmoid function $\gamma_q(s)$, we define

$$f_{\gamma_q}(z) = z - \sum_{k=2}^{\infty} \gamma_q(s) a_k z^k, \qquad a_k \geq 0. \tag{4}$$

Hence, we introduce the (q, λ)-Salagean differential operator for the function $f_{\gamma_q}(z)$ to be

$$\begin{aligned} D^0_{q,\lambda} f_{\gamma_q}(z) &= f_{\gamma_q}(z); \\ D_{q,\lambda} f_{\gamma_q}(z) &= \gamma_q(s)\Big((1-\lambda) f_{\gamma_q}(z) + \lambda z D_q f_{\gamma_q}(z)\Big); \\ &\vdots \\ D^n_{q,\lambda} f_{\gamma_q}(z) &= D_{q,\lambda}(D^{n-1}_{q,\lambda} f_{\gamma_q}(z)), \end{aligned}$$

where $n \in \mathbb{N}_0 = \mathbb{N} \bigcup \{0\}$.

If f is satisfies (4), then we have

$$D^n_{q,\lambda} f_{\gamma_q}(z) = \gamma^n_q(s) z - \sum_{n=k}^{\infty} \gamma^{n+1}_q(s) \big[(1-\lambda) + \lambda [k]_q\big]^n a_k z^k. \tag{5}$$

From the preceding definition, we observe that if $s = 0$ and $q \to 1^-$, then we have

$$D^n_\lambda f(z) = z - \sum_{k=2}^{\infty} [1 + (k-1)\lambda]^n a_k z^k,$$

where the operator $D_\lambda f$ is the generalized Salagean differential operator defined by [25]. Let f be given by (1), then the q-analogue involving a modified q-Sigmoid function of the Ruscheweyh operator is defined by

$$R^n_q f_{\gamma_q}(z) = z - \sum_{n=2}^{\infty} \gamma_q(s) B^k_q(n) a_k z^k, \tag{6}$$

where $B^k_q(n)$ has a significance of (2).

Definition 1. *Let $f \in \mathcal{U}$ and $0 \leq q \leq 1$. Then, we define $S^{n,R}_{\gamma_q}(\alpha)$ to be the subclass of $\mathcal{U}$ defined as*

$$S^{n,R}_{\gamma_q}(\alpha) = \left\{ f \in \mathcal{U} : Re\left(\frac{z D_q R^n_q f_{\gamma_q}(z)}{R^n_q f_{\gamma_q}(z)} \right) > \alpha, z \in D \right\}.$$

Definition 2. *Let $f \in \mathcal{U}$ and $0 \leq q \leq 1$. Then, by $C^{n,R}_{\gamma_q}(\alpha)$ we denote the subclass of $\mathcal{U}$ such that*

$$C^{n,R}_{\gamma_q}(\alpha) = \left\{ f \in \mathcal{U} : Re\left(1 + \frac{z D_q(z D_q R^n_q f_{\gamma_q}(z))}{z D_q R^n_q f_{\gamma_q}(z)} \right) > \alpha, z \in D \right\}.$$

In [44,45], the authors raised a definition of the (n, δ)-neighbourhood of a function f in $\mathcal{U}$. In [46], they introduced the (δ, q)-neighbourhood of a function $f \in \mathcal{U}$ in the form

$$N_{\delta,q}(f) = \left\{ g \in \mathcal{U}_p : g(z) = z - \sum_{k=n}^{\infty} b_k z^k \ \ and \ \ \sum_{k=n}^{\infty} [k]_q |a_k - b_k| \leq \delta \right\}, \quad (\delta > 0). \tag{7}$$

Therefore, it can be inferred from (7) that if $h(z) = z$, then

$$N_{\delta,q}(h) = \left\{ g \in \mathcal{U} : g(z) = z - \sum_{k=n}^{\infty} b_k z^k \ \ and \ \ \sum_{k=n}^{\infty} [k]_q |b_k| \leq \delta \right\}, \ \ (\delta > 0). \tag{8}$$

However, the aim of the present paper is to discuss various characteristics of an analytic univalent function in the class $T_{\gamma_q}^{n,R}(\mu,\alpha)$ of those functions f possessing the inequality

$$Re\left\{ \frac{zD_q R_q^n f_{\gamma_q}(z) + \mu z D_q(zD_q R_q^n f_{\gamma_q}(z))}{(1-\mu) R_q^n f_{\gamma_q}(z) + \mu z D_q R_q^n f_{\gamma_q}(z)} \right\} > \alpha, (z \in D, 0 \leq \mu \leq 1 0 \leq \alpha < 1). \tag{9}$$

Clearly, in terms of the simpler classes $S_{\gamma_q}^{n,R}(\alpha)$ and $C_{\gamma_q}^{n,R}(\alpha)$, we, respectively, have

$$T_{\gamma_q}^{n,R}(0,\alpha) = S_{\gamma_q,\lambda}^{n,R}(\alpha) \qquad and \qquad T_{\gamma_q}^{n,R}(1,\alpha) = C_{\gamma_q}^{n,R}(\alpha).$$

For further demonstration, we denote by $\mathcal{K}_{\gamma_q}^{n,R}(\mu,\alpha)$ the subclass of $\mathcal{U}$ of all functions f possessing the inequality

$$Re\left\{ R_q^n f_{\gamma_q}(z) + \mu z D_q R_q^n f_{\gamma_q}(z) \right\} > \alpha.$$

Now, we combine the q-analogue of the generalized Salagean differential operator involving the modified q-Sigmoid function defined by (5) and the Ruscheweyh operator involving the modified q-Sigmoid function expressed in (6) to obtain a new operator as follows:

$$\Psi_{q,\lambda}^n f_{\gamma_q}(z) = \beta D_{\gamma_q,\lambda}^n f_{\gamma_q}(z) + (1-\beta) R_q^n f_{\gamma_q}(z), \qquad 0 \leq \lambda \leq 1, 0 \leq \beta \leq 1, z \in U.$$

Thus, we write

$$\Psi_{q,\lambda}^n f_{\gamma_q}(z) = \left(\beta \gamma_q^n(s) - \beta + 1\right) z - \sum_{k=2}^{\infty} \gamma_q(s) \left[\beta \gamma_q^n(s) \left[(1-\lambda) + \lambda [k]_q\right]^n + (1-\beta) B_q^k(n)\right] a_k z^k.$$

Definition 3. *Let $f \in \mathcal{U}$, $0 \leq q \leq 1$ and $0 \leq \lambda \leq 1$. Then, we define the subclass $S_{\gamma_q,\lambda}^{n,\Psi}(\alpha)$ of the class $\mathcal{U}$ by*

$$S_{\gamma_q,\lambda}^{n,\Psi}(\alpha) = \left\{ f \in \mathcal{U} : Re\left(\frac{zD_q \Psi_{q,\lambda}^n f_{\gamma_q}(z)}{\Psi_{q,\lambda}^n f_{\gamma_q}(z)} \right) > \alpha, z \in D \right\}.$$

Definition 4. *Let $f \in \mathcal{U}$, $0 \leq q \leq 1$ and $0 \leq \lambda \leq 1$. Then, we define the subclass $C_{\gamma_q,\lambda}^{n,\Psi}(\alpha)$ of the class $\mathcal{U}$ by*

$$C_{\gamma_q,\lambda}^{n,\Psi}(\alpha) = \left\{ f \in \mathcal{U} : Re\left(1 + \frac{zD_q(zD_q \Psi_{q,\lambda}^n f_{\gamma_q}(z))}{zD_q \Psi_{q,\lambda}^n f_{\gamma_q}(z)} \right) > \alpha, z \in D \right\}.$$

Finally, let $T_{\gamma_q,\lambda}^{n,\Psi}(\mu,\alpha)$ denote the subclass of $\mathcal{U}$ of functions such that the following inequality holds

$$Re\left\{ \frac{zD_q \Psi_{q,\lambda}^n f_{\gamma_q}(z) + \mu z D_q(zD_q \Psi_{q,\lambda}^n f_{\gamma_q}(z))}{(1-\mu) \Psi_{q,\lambda}^n f_{\gamma_q}(z) + \mu z D_q \Psi_{q,\lambda}^n f_{\gamma_q}(z)} \right\} > \alpha, (z \in D, 0 \leq \mu \leq 1 0 \leq \alpha < 1). \tag{10}$$

Then, in terms of the simpler classes $S^{n,\Psi}_{\gamma_q,\lambda}(\alpha)$ and $C^{n,\Psi}_{\gamma_q,\lambda}(\alpha)$, we, respectively, have

$$T^{n,\Psi}_{\gamma_q,\lambda}(0,\alpha) = S^{n,\Psi}_{\gamma_q,\lambda}(\alpha) \qquad \textit{and} \qquad T^{n,\Psi}_{\gamma_q,\lambda}(1,\alpha) = C^{n,\Psi}_{\gamma_q,\lambda}(\alpha).$$

Furthermore, let $\mathcal{K}^{n,\Psi}_{\gamma_q,\lambda}(\mu,\alpha)$ denote the subclass of $\mathcal{U}$ of functions f that satisfy the inequality

$$Re\Big\{\Psi^n_{q,\lambda} f_{\gamma_q}(z) + \mu z D_q \Psi^n_{q,\lambda} f_{\gamma_q}(z)\Big\} > \alpha.$$

One part of deriving a set of coefficient bounds for each of such function classes is to establish several inclusion relationships associated with the (q,δ)-neighbourhoods of analytic univalent functions of negative missing coefficients in the same subclasses.

2. A Set of Coefficient q-Inequalities

In this section, we establish the following result, which gives a coefficient inequality for functions in the subclass $T^{n,\Psi}_{\gamma_q,\lambda}(\mu,\alpha)$.

Theorem 1. *Let $n \in \mathbb{N}\bigcup\{0\}$ be a fixed number. A function $f \in \mathcal{U}$ is in the class $T^{n,\Psi}_{\gamma_q,\lambda}(\mu,\alpha)$ if and only if*

$$\sum_{k=2}^{\infty} \phi_k \Big[\alpha\mu - \alpha + (1-\alpha\mu)[k]_q + \mu[k]_q^2\Big] a_k, \leq (1+\mu-\alpha)(\beta\gamma_q^n(s) - \beta + 1) \tag{11}$$

$$(0 \leq q < 1, 0 \leq \beta \leq 1, 0 \leq \mu < 1, 0 \leq \alpha < 1, z \in D),$$

where

$$\phi_k = \gamma_q(s)\Big[\beta\gamma_q^n(s)\big[(1-\lambda) + \lambda[k]_q\big]^n + (1-\beta)B_q^k(n)\Big], k = 2,3,\ldots. \tag{12}$$

This result is sharp.

Proof. Assume that $f \in T^{n,\Psi}_{\gamma_q,\lambda}(\mu,\alpha)$. Then, we have

$$Re\left\{\frac{(1+\mu)(\beta\gamma_q^n(s) - \beta + 1) - \sum_{k=2}^{\infty}\phi_k\Big[[k]_q + \mu[k]_q^2\Big]a_k z^{k-1}}{\beta\gamma_q^n(s) - \beta + 1 - \sum_{k=2}^{\infty}\phi_k\Big[1-\mu+\mu[k]_q\Big]a_k z^{k-1}}\right\} > \alpha,$$

$$(0 \leq q < 1, 0 \leq \beta \leq 1, 0 \leq \mu < 1, 0 \leq \alpha < 1, z \in D).$$

Choose z to be real and let $z \to 1^-$. Then, we obtain

$$\frac{(1+\mu)(\beta\gamma_q^n(s) - \beta + 1) - \sum_{k=2}^{\infty}\phi_k\Big[[k]_q + \mu[k]_q^2\Big]a_k}{\gamma_q^n(s) - \beta + 1 - \sum_{k=2}^{\infty}\phi_k\Big[1-\mu+\mu[k]_q\Big]a_k} > \alpha,$$

$$(0 \leq q < 1, 0 \leq \beta \leq 1, 0 \leq \mu < 1, 0 \leq \alpha < 1, z \in D).$$

Or, alternatively, we write

$$(1+\mu)(\beta\gamma_q^n(s) - \beta + 1) - \sum_{k=2}^{\infty}\phi_k\Big[[k]_q + \mu[k]_q^2\Big]a_k \geq \alpha\Big(\gamma_q^n(s) - \beta + 1\Big) - \sum_{k=2}^{\infty}\alpha\phi_k\Big[1-\mu+\mu[k]_q\Big]a_k,$$

$$(0 \leq q < 1, 0 \leq \beta \leq 1, 0 \leq \mu < 1, 0 \leq \alpha < 1, z \in D).$$

Thus, it follows

$$-\sum_{k=2}^{\infty}\phi_k\left[\alpha\mu-\alpha+(1-\alpha\mu)[k]_q+\mu[k]_q^2\right]a_k \geq (\alpha-1-\mu)(\beta\gamma_q^n(s)-\beta+1)$$

$$(0 \leq q < 1, 0 \leq \beta \leq 1, 0 \leq \mu < 1, 0 \leq \alpha < 1, z \in D),$$

which is precisely the assertion (11) of Theorem 1.

Conversely, assume that the inequality (11) truly holds and let

$$z \in \partial D = \{z \in \mathbb{C} : |z| = 1\}.$$

Then, from the assertion (10) and the assumption that the inequality (11) holds, we find that

$$\begin{aligned}
&\left|\frac{zD_q\Psi_{q,\lambda}^n f_{\gamma_q}(z)+\mu zD_q(zD_q\Psi_{q,\lambda}^n f_{\gamma_q}(z))}{(1-\mu)\Psi_{q,\lambda}^n f_{\gamma_q}(z)+\mu zD_q\Psi_{q,\lambda}^n f_{\gamma_q}(z)}-(1+\mu-\alpha)\right| \\
= &\left|\frac{\alpha(\beta\gamma_q^n(s)-\beta+1)-\sum_{k=2}^{\infty}\phi_k\left[\mu[k]_q^2+(1-\mu(1+\mu-\alpha))[k]_q-(1-\mu)(1+\mu-\alpha)\right]a_k z^{k-1}}{\beta\gamma_q^n(s)-\beta+1-\sum_{k=2}^{\infty}\phi_k\left[1-\mu+\mu[k]_q\right]a_k z^{k-1}}\right| \\
\leq &\frac{\alpha(\beta\gamma_q^n(s)-\beta+1)+\sum_{k=2}^{\infty}\phi_k\left[\mu[k]_q^2+(1-\mu(1+\mu-\alpha))[k]_q-(1-\mu)(1+\mu-\alpha)\right]a_k|z|^{k-1}}{\beta\gamma_q^n(s)-\beta+1-\sum_{k=2}^{\infty}\phi_k\left[1-\mu+\mu[k]_q\right]a_k|z|^{k-1}} \\
= &\frac{\alpha(\beta\gamma_q^n(s)-\beta+1)+\sum_{k=2}^{\infty}\phi_k\left[\mu[k]_q^2+[k]_q\right]a_k|z|^{k-1}-(1+\mu-\alpha)\sum_{k=2}^{\infty}\phi_k\left[\mu[k]_q+(1-\mu)\right]a_k|z|^{k-1}}{\beta\gamma_q^n(s)-\beta+1-\sum_{k=2}^{\infty}\phi_k\left[1-\mu+\mu[k]_q\right]a_k|z|^{k-1}} \\
\leq &\ 1+\mu-2\alpha.
\end{aligned}$$

Thus, the maximum modulus theorem reveals $f \in T_{q,\lambda}^{n,\Psi}(\mu,\alpha)$. Finally, it is clear that the assertion (11) of Theorem 1 is sharp, where the extremal function is given by

$$f(z) = z - \frac{(1+\mu-\alpha)(\beta\gamma_q^n(s)-\beta+1)}{\phi_k\left[\alpha\mu-\alpha+(1-\alpha\mu)[k]_q+\mu[k]_q^2\right]}z^k$$

$$(0 \leq q < 1, 0 \leq \beta \leq 1, 0 \leq \mu < 1, 0 \leq \alpha < 1, z \in D).$$

Hence, we have the required result. □

Putting $\beta = 0$ in Theorem 1, one may derive the following corollary.

Corollary 1. *Let $n \in \mathbb{N}\bigcup\{0\}$ be a fixed number. A function $f \in \mathcal{U}$ is in the class $T_{q,\lambda}^{n,R}(\mu,\alpha)$ if and only if*

$$\sum_{k=2}^{\infty}B_q^k(n)\left[\alpha\mu-\alpha+(1-\alpha\mu)[k]_q+\mu[k]_q^2\right]a_k \leq \frac{1+\mu-\alpha}{\gamma_q(s)}$$

$$(0 \leq q < 1,, 0 \leq \mu < 1, 0 \leq \alpha < 1, z \in D).$$

This result is indeed sharp for a function f in the form

$$f(z) = z - \frac{1+\mu-\alpha}{\gamma_q(s)(B_q^k(n)\left[\alpha\mu-\alpha+(1-\alpha\mu)[k]_q+\mu[k]_q^2\right])}z^k, \quad (0 \leq q < 1, 0 \leq \mu < 1, 0 \leq \alpha < 1, z \in D).$$

Similarly, we can prove the following theorem.

Theorem 2. *Let $n \in \mathbb{N}\bigcup\{0\}$ be a fixed number. A function $f \in \mathcal{U}$ is a member in the class $\mathcal{K}^{n\Psi}_{q,\lambda}(\mu,\alpha)$ if and only if*

$$\sum_{k=2}^{\infty} \phi_k(1+\mu[k]_q)a_k \le (1+\mu)(\beta\gamma_q^n(s)-\beta+1)-\alpha, \tag{13}$$
$$(0 \le q < 1, 0 \le \beta \le 1, 0 \le \mu < 1, 0 \le \alpha < 1, z \in D),$$

where ϕ_k is given by (12).

This result is sharp for a function f given by

$$f(z) = z - \frac{(1+\mu)(\beta\gamma_q^n(s)-\beta+1)-\alpha}{\phi_k(1+\mu[k]_q)}z^k, (0 \le q < 1, 0 \le \beta \le 1, 0 \le \mu < 1, 0 \le \alpha < 1, z \in D).$$

Putting $\beta = 0$ in Theorem 2, we derive a corollary as follows.

Corollary 2. *Let $n \in \mathbb{N}\bigcup\{0\}$ be a fixed number. A function $f \in \mathcal{U}$ is in the class $\mathcal{N}_q^{n,R}(\mu,\alpha)$ if and only if*

$$\sum_{k=2}^{\infty} B_q^k(n)a_k \le \frac{1+\mu-\alpha}{\gamma_q(s)}, \quad 0 \le q < 1, 0 \le \mu < 1, 0 \le \alpha < 1, z \in D.$$
$$(0 \le q < 1, , 0 \le \mu < 1, 0 \le \alpha < 1, z \in D).$$

This result is indeed sharp for a function f in the form

$$f(z) = z - \frac{1+\mu-\alpha}{\gamma_q(s)B_q^k(n)a_k}, \quad (0 \le q < 1, , 0 \le \mu < 1, 0 \le \alpha < 1, z \in D).$$

Theorem 3. *Let the function f be given by (1) and ϕ_k be given by (12). If f is in the class $T^{n,\Psi}_{q,\lambda}(\mu,\alpha)$, then we have*

$$\sum_{k=2}^{\infty} [k]_q a_k \le \frac{(1+\mu-\alpha)(\beta\gamma_q^n(s)-\beta+1)}{\phi_k\left[1-\alpha\mu+\mu[k]_q\right]}\left(1+\frac{\alpha\mu-\alpha}{\alpha\mu-\alpha+(1-\alpha\mu)[k]_q+\mu[k]_q^2}\right). \tag{14}$$

Proof. By using the assertion (11) in Theorem 1, we obtain

$$\begin{aligned}\phi_k\left(\alpha\mu-\alpha+(1-\alpha\mu[k]_q+\mu[k]_q^2\right)\sum_{k=2}^{\infty} a_k z^k &\le \sum_{k=2}^{\infty} \phi_k\left(\alpha\mu-\alpha+(1-\alpha\mu)[k]_q+\mu[k]_q^2\right)a_k z^k \\ &\le (1+\mu-\alpha)(\beta\gamma_q^n(s)-\beta+1),\end{aligned}$$

which immediately yields

$$\sum_{k=2}^{\infty} a_k z^k \le \frac{(1+\mu-\alpha)(\beta\gamma_q^n(s)-\beta+1)}{\phi_k\left(\alpha\mu-\alpha+(1-\alpha\mu)[k]_q+\mu[k]_q^2\right)}. \tag{15}$$

Furthermore, by using the inequality (11), we have

$$\begin{aligned}&\phi_k(\alpha\mu-\alpha)\sum_{k=2}^{\infty} a_k + \phi_k\left(1-\alpha\mu+\mu[k]_q\right)\sum_{k=1}^{\infty}[k]_q a_k \\ &\le \sum_{k=2}^{\infty} \phi_k\left(\alpha-\alpha\mu+(1-\alpha\mu)[k]_q+\mu[k]_q^2\right)a_k \le (1+\mu-\alpha)(\beta\gamma_q^n(s)-\beta+1).\end{aligned}$$

It follows from the inequality (15) that

$$\begin{aligned}\phi_k\Big(1-\alpha\mu+\mu[k]_q\Big)\sum_{k=1}^{\infty}[k]_q a_k \quad &\le \quad (\alpha-\alpha\mu)\frac{(1+\mu-\alpha)(\beta\gamma_q^n(s)-\beta+1)}{\alpha\mu-\alpha+(1-\alpha\mu)[k]_q+\mu[k]_q^2}\\ &+ \quad (1+\mu-\alpha)(\beta\gamma_q^n(s)-\beta+1).\end{aligned}$$

Hence, we derived the desired inequality as presented in (14). □

Putting $\beta=0$ in Theorem 3, we state without proof a corollary as follows.

Corollary 3. *Let f be a function furnished by (1). If f is in the class $T_{q,\lambda}^{n,R}(\mu,\alpha)$, then we have*

$$\sum_{k=2}^{\infty}[k]_q a_k = \frac{1+\mu-\alpha}{\gamma_q(s)B_q^k(n)\Big[1-\alpha\mu+\mu[k]_q\Big]}\left(1+\frac{\alpha\mu-\alpha}{\alpha\mu-\alpha+(1-\alpha\mu)[k]_q+\mu[k]_q^2}\right). \tag{16}$$

Similarly, by invoking inequality (13) in Theorem 2, we prove a new theorem as follows.

Theorem 4. *Let f be a function given by (1) and ϕ_k be given by (12). If f is in the class $\mathcal{N}_{q,\lambda}^{n,\Psi}(\mu,\alpha)$, then we have*

$$\sum_{k=2}^{\infty}a_k \le \frac{(1+\mu)(\beta\gamma_q^n(s)-\beta+1)-\alpha}{\phi_k\Big(1+\mu[k]_q\Big)}$$

and

$$\sum_{k=2}^{\infty}[k]_q a_k = \frac{(1+\mu)(\beta\gamma_q^n(s)-\beta+1)}{\mu\phi_k}\left(1-\frac{1}{1+\mu[k]_q}\right)+\frac{\alpha}{\mu\phi_k(1+\mu[k]_q)}. \tag{17}$$

Putting $\beta=0$ in Theorem 4, we derive a new corollary as follows.

Corollary 4. *Let the function f be given by (1). If f is in the class $\mathcal{N}_{q,\lambda}^{n,R}(\mu,\alpha)$, then we have*

$$\sum_{k=2}^{\infty}[k]_q a_k = \frac{1+\mu}{\mu\gamma_q(s)B_q^k(n)}\left(1-\frac{1}{1+\mu[k]_q}\right)+\frac{\alpha}{\mu_q(s)B_q^k(n)(1+\mu[k]_q)}. \tag{18}$$

In the following, we have the following growth and distortion theorem for the defined subclasses of univalent functions.

Theorem 5. *Let a function f be given by (1). If $f\in T_{\gamma_q,\lambda}^{n,\Psi}(\mu,\alpha)$, then we have*

$$r-r^2\frac{(1+\mu-\alpha)(\beta\gamma_q^n(s)-\beta+1)}{\phi_k\Big(\alpha\mu-\alpha+(1-\alpha\mu)[k]_q+\mu[k]_q^2\Big)} \le |f(z)| \le r+r^2\frac{(1+\mu-\alpha)(\beta\gamma_q^n(s)-\beta+1)}{\phi_k\Big(\alpha\mu-\alpha+(1-\alpha\mu)[k]_q+\mu[k]_q^2\Big)}, \tag{19}$$

where ϕ_k is defined in assertion (12).

Proof. Since $f\in T_{q,\lambda}^{n,\Psi}(\mu,\alpha)$, let $z\to 1^-$ in the above inequality to have

$$\sum_{k=2}^{\infty}a_k \le \frac{(1+\mu-\alpha)(\beta\gamma_q^n(s)-\beta+1)}{\phi_k\Big(\alpha\mu-\alpha+(1-\alpha\mu[k]_q+\mu[k]_q^2\Big)}. \tag{20}$$

In addition, since the function f is given by (1), we obtain

$$|z| - |z|^2 \sum_{k=2}^{\infty} a_k |z|^{k-2} \leq |f(z)| \leq |z| + |z|^2 \sum_{k=2}^{\infty} a_k |z|^{k-2}.$$

Thus, it gives

$$|z| - |z|^2 \sum_{k=2}^{\infty} a_k \leq |f(z)| \leq |z| + |z|^2 \sum_{k=2}^{\infty} a_k.$$

Hence, by using the inequality (20) we obtain the desired inequality of (19). $\square$

Putting $\beta = 0$ in Theorem 5, we derive the following corollary.

Corollary 5. *Let f be a function given by (1). If $f \in T_{q,\lambda}^{n,R}(\mu, \alpha)$, then we have*

$$\begin{aligned} r - r^2 \frac{(1+\mu-\alpha)}{B_q^k(n)\gamma_q(s)\left(\alpha\mu - \alpha + (1-\alpha\mu)[k]_q + \mu[k]_q^2\right)} &\leq |f(z)| \\ \leq r + r^2 \frac{(1+\mu-\alpha)}{B_q^k(n)\gamma_q(s)\left(\alpha\mu - \alpha + (1-\alpha\mu)[k]_q + \mu[k]_q^2\right)}. \end{aligned}$$

By following a proof similar to the proof of Theorem 5, we prove the following theorem.

Theorem 6. *Let f be a function given by (1). If $f \in T_{\gamma_q,\lambda}^{n,\Psi}(\mu, \alpha)$, then we have*

$$|D_q f(z)| \leq 1 + r \frac{(1+\mu)(\beta\gamma_q^n(s) - \beta + 1)}{\mu\phi_k}\left(1 - \frac{1}{1+\mu[k]_q}\right) + \frac{\alpha}{\mu\phi_k(1+\mu[k]_q)},$$
$$|D_q f(z)| \geq 1 - r \frac{(1+\mu)(\beta\gamma_q^n(s) - \beta + 1)}{\mu\phi_k}\left(1 - \frac{1}{1+\mu[k]_q}\right) + \frac{\alpha}{\mu\phi_k(1+\mu[k]_q)}.$$

where ϕ_k is defined in assertion (12).

Putting $\beta = 0$ in the Theorem 5, we derive the following corollary.

Corollary 6. *Let the function f be given by (1). If $f(z) \in T_{\gamma_q,\lambda}^{n,R}(\mu, \alpha)$, then we have*

$$|D_q f(z)| \leq 1 + r \frac{1+\mu-\alpha}{\gamma_q(s)B_q^k(n)\left[1 - \alpha\mu + \mu[k]_q\right]}\left(1 + \frac{\alpha\mu - \alpha}{\alpha\mu - \alpha + (1-\alpha\mu)[k]_q + \mu[k]_q^2}\right),$$
$$|D_q f(z)| \geq 1 - r \frac{1+\mu-\alpha}{\gamma_q(s)B_q^k(n)\left[1 - \alpha\mu + \mu[k]_q\right]}\left(1 + \frac{\alpha\mu - \alpha}{\alpha\mu - \alpha + (1-\alpha\mu)[k]_q + \mu[k]_q^2}\right).$$

3. Inclusion Relations Involving the (n, δ)-Neighborhoods

In this section, we derive inclusion relations associated with the (n, δ)-neighbourhoods and properties for each of the following subclasses of univalent functions with negative coefficients as follows.

Theorem 7. *Let f be given by (1), ϕ_k be given by (12) and $h(z) = z$ for each $z \in D$. If f is in the class $T_{q,\lambda}^{n,\Psi}(\mu, \alpha)$, then we have*

$$T_{q,\lambda}^{n,\Psi}(\mu, \alpha) \subseteq N_{\delta,q}(h), \tag{21}$$

where

$$\delta := \frac{(1+\mu-\alpha)(\beta\gamma_q^n(s)-\beta+1)}{\phi_k\left[1-\alpha\mu+\mu[k]_q\right]}\left(1+\frac{\alpha\mu-\alpha}{\alpha\mu-\alpha+(1-\alpha\mu)[k]_q+\mu[k]_q^2}\right).$$

Proof. By using the definition of $N_{\delta,q}(h)$ in (8), which is given when g is replaced by f, and using inequality (14) in Theorem 3, we derive the assertion presented (22). □

Similarly, by using the inequalities (16)–(18) and the definition of $N_{\delta,q}(h)$ presented in (8), we derive the three following theorems.

Theorem 8. *Let f be a function be given by* (1) *and $h(z) = z$ for each $z \in D$. If f belongs to the class $T_{q,\lambda}^{n,R}(\mu,\alpha)$, then we have*

$$T_{q,\lambda}^{n,R}(\mu,\alpha) \subseteq N_{\delta,q}(h),$$

where

$$\delta := \frac{1+\mu-\alpha}{\gamma_q(s)B_q^k(n)\left[1-\alpha\mu+\mu[k]_q\right]}\left(1+\frac{\alpha\mu-\alpha}{\alpha\mu-\alpha+(1-\alpha\mu)[k]_q+\mu[k]_q^2}\right).$$

Theorem 9. *Let f be a function given by* (1)*, ϕ_k be given by* (12) *and $h(z) = z$ for each $z \in D$. If f falls in the class $\mathcal{N}_{q,\lambda}^{n,\Psi}(\mu,\alpha)$, then we have*

$$\mathcal{N}_{q,\lambda}^{n,\Psi}(\mu,\alpha) \subseteq N_{\delta,q}(h), \tag{22}$$

where

$$\delta := \frac{(1+\mu)(\beta\gamma_q^n(s)-\beta+1)}{\mu\phi_k}\left(1-\frac{1}{1+\mu[k]_q}\right)+\frac{\alpha}{\mu\phi_k(1+\mu[k]_q)}.$$

Theorem 10. *Let the function f be given by* (1) *and $h(z) = z$ for each $z \in D$. If f is in the class $\mathcal{N}_{q,\lambda}^{n,R}(\mu,\alpha)$, then we have*

$$\mathcal{N}_{q,\lambda}^{n,R}(\mu,\alpha) \subseteq N_{\delta,q}(h),$$

where

$$\delta := \frac{1+\mu}{\mu\gamma_q(s)B_q^k(n)}\left(1-\frac{1}{1+\mu[k]_q}\right)+\frac{\alpha}{\mu_q(s)B_q^k(n)(1+\mu[k]_q)}.$$

Definition 5. *The function $f \in \mathcal{U}$ is said to belong to the class $S_{q,\lambda}^{n,\Psi}(\mu,\zeta)$ if there exists a function $g \in \mathcal{N}_{q,\lambda}^{n,\Psi}(\mu,\alpha))$ such that*

$$\left|\frac{f(z)}{g(z)}-1\right| \leq \eta, \qquad (0 < \eta \leq 1),$$

where

$$g(z) = z + \sum_{k=2}^{\infty} b_n z^n, \qquad (z \in D). \tag{23}$$

Theorem 11. *Let the function f be given by (23), ϕ_k be given by (12) and $g \in \mathcal{N}^{n,\Psi}_{q,\lambda}(\mu,\alpha)$). If*

$$\zeta \leq 1 - \frac{2\delta\phi_k(1+\mu[k]_q)}{[2]_q\Big(\phi_k\Big(1+\mu[k]_q\Big) - (1+\mu)(\beta\gamma^n_q(s) - \beta + 1)\Big)},$$

then we have

$$N_{\delta,q}(h) \subseteq S^{n,\Psi}_{q,\lambda}(\mu,\zeta).$$

Proof. Suppose that $f \in N_{\delta,q}(g)$, then in view of the relation (7), we have

$$\sum_{k=2}^{\infty} [k]_q |a_k - b_k| \leq \delta. \tag{24}$$

Since $\{[n]_q\}$ is a non-decreasing sequence, we obtain

$$\sum_{k=2}^{\infty} [2]_q |a_k - b_k| \leq \sum_{k=2}^{\infty} [k]_q |a_k - b_k|.$$

This implies that

$$[2]_q \sum_{k=2}^{\infty} |a_k - b_k| \leq \sum_{k=2}^{\infty} [k]_q |a_k - b_k|,$$

which, in view of the inequality (24), reveals

$$\sum_{k=2}^{\infty} |a_k - b_k| \leq \frac{\delta}{[2]_q}, \qquad (0 \leq q < 1,\ \delta \geq 0).$$

Therefore, for a function g in the class $S^{n,\Psi}_{q,\lambda}(\mu,\zeta)$, expressed by (23), by using the inequality (17), we obtain

$$\sum_{k=2}^{\infty} |b_k| \leq \frac{(1+\mu)(\beta\gamma^n_q(s) - \beta + 1) - \alpha}{\phi_k(1+\mu[k]_q)}.$$

By applying the equations (1) and (23) and the fact that $|z| < 1$, we get

$$\begin{aligned} \left|\frac{f(z)}{g(z)} - 1\right| &= \left|\frac{\sum_{k=2}^{\infty}(a_k - b_k)z^{k-1}}{1 + \sum_{k=2}^{\infty} b_n z^{n-1}}\right| \leq \frac{\sum_{k=2}^{\infty}|a_k - b_k|}{1 - \sum_{k=2}^{\infty} b_k} && (25)\\ &\leq \frac{\delta}{[2]_q}\left(\frac{1}{1 - \sum_{k=2}^{\infty}|b_k|}\right) && (26)\\ &\leq \frac{\delta}{[2]_q}\left[\frac{\phi_k\Big(1+\mu[k]_q\Big)}{\phi_k\Big(1+\mu[k]_q\Big) - (1+\mu)(\beta\gamma^n_q(s) - \beta + 1)}\right]. && (27)\end{aligned}$$

If we set

$$\zeta \leq 1 - \frac{2\delta\phi_k(1+\mu[k]_q)}{[2]_q\Big(\phi_k\Big(1+\mu[k]_q\Big) - (1+\mu)(\beta\gamma^n_q(s) - \beta + 1)\Big)},$$

then, in view of the definition (5) and the inequality (25), we obtain that $f \in S^{n,\Psi}_{q,\lambda}(\mu,\zeta)$. Hence, the proof of the theorem is completed. □

Putting $\beta = 0$ in Theorem 11 leads to the following theorem.

Theorem 12. *Let the function f be given by* (23) *and $g \in \mathcal{N}_{q,\lambda}^{n,R}(\mu, \alpha)$). If*

$$\zeta \leq 1 - \frac{2\delta\gamma_q(s)B_q^k(n)(1+\mu[k]_q)}{[2]_q\left(\gamma_q(s)B_q^k(n)\left(1+\mu[k]_q\right) - (1+\mu)\right)},$$

then we have

$$N_{\delta,q}(h) \subseteq S_{q,\lambda}^{n,R}(\mu, \zeta).$$

4. Conclusions

In the present work, certain analytic functions and new q-analogues of the Salagean differential operator are, respectively, obtained by using a modified q-Sigmoid function and a recent idea of Salagean. In addition, certain classes of analytic univalent functions associated with new q-Salagean differential operators and q-Ruscheweh operators are obtained. Moreover, coefficient bounds for functions in the mentioned subclasses and the growth and distortion theorems are established. Following the concept of (q, δ)-neighbourhoods of analytic univalent functions, several inclusion relations for the (q, δ)-neighbourhood of these functions are discussed.

Author Contributions: Conceptualization, E.A. and S.A.-O.; methodology, M.F.; software, M.F.; validation, E.A.; investigation, K.N.; resources, E.A.; writing—original draft preparation, E.A.; writing—review and editing, E.A.; funding acquisition, K.N. All authors have read and agreed to the published version of the manuscript.

Funding: This research has received funding support from the National Science, 43 Research and Innovation Fund (NSRF), Thailand.

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: Not applicable.

Acknowledgments: The authors are thankful to the editors and reviewers for their valuable suggestions and comments.

Conflicts of Interest: The authors declare no conflict of interest.

References

1. Aral, A.; Gupta, V. Generalized q-Baskakov operators. *Math. Slovaca* **2011**, *61*, 619–634. [CrossRef]
2. Aral, A.; Gupta, V. On the Durrmeyer type modification of the q-Baskakov type operators. *Nonlinear Anal.* **2010**, *72*, 1171–1180. [CrossRef]
3. Abu-Risha, M.H.; Annaby, M.H.; Ismail, M.E.H.; Mansour, Z.S. Linear q-difference equations. *Anal. Anwend.* **2007**, *26*, 481–494. [CrossRef]
4. Kac, V.G.; Cheung, P. *Quantum Calculus. Universitext*; Springer: New York, NY, USA, 2002.
5. Gumah, G.; Naser, M.; Al-Smadi,M.; Al-Omari, S.A; Baleanu, D. Numerical solutions of hybrid fuzzy differential equations in a Hilbert space. *Appl. Numer. Math.* **2020**, *151*, 402–412. [CrossRef]
6. Ayasrah, M.; AL-Smadi,M.; Al-Omari, S.; Baleanu, D.; Momani, S. Structure of optical soliton solution for nonlinear resonant space-time Schrödinger equation in conformable sense with full nonlinearity term. *Phys. Scr.* **2020**, *95*, 105215.
7. Al-Smadi, M.; Djeddi, N.; Momani, S.; Al-Omari, S.; Araci, S. An attractive numerical algorithm for solving nonlinear Caputo-Fabrizio fractional Abel differential equation in a Hilbert space. *Adv. Differ. Equ.* **2021**, *2021*, 271. [CrossRef]
8. Momani, S.; Djeddi, N.; Al-Smadi, M.; Al-Omari, S. Numerical investigation for Caputo-Fabrizio fractional Riccati and Bernoulli equations using iterative reproducing kernel method. *Appl. Numer. Math.* **2021**, *170*, 418–434. [CrossRef]
9. Mohammed, S.; Alabedalhad, M.; Al-Omari, S.; Al-Smadi, S. Abundant exact travelling wave solutions for a fractional massive Thirring model using extended Jacobi elliptic function method. *Fractal Fract.* **2022**, *6*, 252.
10. Jackson, F.H. On q-definite integrals. *Quart. J. Pure Appl. Math.* **1910**, *41*, 193–203.
11. Gasper, G.; Rahman, M. *Basic Hypergeometric Series*, 2nd ed.; Cambridge University Press: Cambridge, UK, 2004.
12. Srivastava, H.M. Operators of basic (or q-) calculus and fractional q-calculus and their applications in geometric function theory of complex analysis. *Iran. J. Sci. Technol. Trans. A Sci.* **2020**, *44*, 327–344. [CrossRef]

13. Arif, M.; Srivastava, H.M.; Umar, S. Some application of a q-analogue of the Ruscheweyh type operator for multivalent functions. *Rev. Real Acad. Cienc. Exactas Fis. Natur. Ser. A Mat.* **2019**, *113*, 1121–1221. [CrossRef]
14. Ismail, M.E.H.; Merkes, E.; Styer, D. A generalization of starlike functions. *Complex Var. Theory Appl.* **1990**, *14*, 77–84. [CrossRef]
15. Mahmood, S.; Sokol, J. New subclass of analytic functions in conical domain associated with Ruscheweyh q-differential operator. *Results Math.* **2017**, *71*, 1345–1357. [CrossRef]
16. Kanas, S.; Răducanu, D. Some class of analytic functions related to conic domains. *Math. Slovaca* **2014**, *64*, 1183–1196. [CrossRef]
17. Chandak, S.; Suthar, D.L.; Al-Omari, S.; Gulyaz-Ozyurt, S. Estimates of classes of generalized special functions and their application in the fractional (k,s)-calculus theory. *J. Funct. Spaces* **2021**, *2021*, 9582879. [CrossRef]
18. Vijaya, K.; Murugusundaramoorthy, G.; Kasthuri, M. Starlike functions of complex order involving q-hypergeometric functions with fixed point. Annales Universitatis Paedagogicae Cracoviensis. *Stud. Math.* **2014**, *13*, 51–63.
19. Murugusundaramoorthy, G.; Vijaya, K.; Janani, T. Subclasses of Starlike functions with a fixed pPoint involving q-hypergeometric function. *J. Anal. Numb. Theory* **2016**, *4*, 41–47. [CrossRef]
20. Catas, A. On the Fekete–Szegö Problem for Meromorphic Functions Associated with p,q-Wright Type Hypergeometric Function. *Symmetry* **2021**, *13*, 2143. [CrossRef]
21. Amini, E.; Al-Omari, S.; Nonlaopon, K.; Baleanu, D. Estimates for coefficients of Bi-univalent functions associated with a fractional q-difference Operator. *Symmetry* **2022**, *14*, 879. [CrossRef]
22. Graham, I.; Kohr, G. *Geometric Function Theory in One and Higher Dimensions*, 1st ed.; Amsterdam University Press: Amsterdam, The Netherlands, 2003.
23. Salagean, G.S. Subclass of univalent functions. *Lect. Note Math.* **1983**, *1013*, 362–372.
24. Ruscheweyh, S. New criteria for univalent functions. *Proc. Am. Math. Soc.* **1975**, *49*, 109–115. [CrossRef]
25. Al-Oboudi, F.M. On univalent functions defined by a generalized Salagean operator. *Int. J. Math. Math. Sci.* **2009**, *27*, 1429–1436. [CrossRef]
26. Aldweby, H.; Darus, M. Some Subordination Results on q-analogue of Ruscheweyh Differential Operator. *Abstr. Appl. Anal.* **2014**, *2014*, 958563. [CrossRef]
27. Seoudy, T.M. Some subclasses of univalent functions associated with -Ruscheweyh derivative operator. *Ukr. Mat. Zhurnal* **2022**, *74*, 122–136. [CrossRef]
28. Khan, S.; Hussain, S.; Zaighum, M.A.; Khan, M.M. New Subclass of Analytic Functions in Conical Domain Associated with Ruscheweyh q-Differential Operator. *Inter. J. Anal. Appl.* **2018**, *18*, 239–253.
29. Al-Omari, S. On a q-Laplace-type integral operator and certain class of series expansion. *Math. Methods Appl. Sci.* **2020**, *44*, 8322–8332. [CrossRef]
30. Al-Omari, S. Estimates and properties of certain q-Mellin transform on generalized q-calculus theory. *Adv. Differ. Equ.* **2021**, *233*, 1–15. [CrossRef]
31. Al-Omari, S.Q. On q-analogues of the Natural transform of certain q-Bessel functions and some application. *Filomat* **2017**, *31*, 2587–2598. [CrossRef]
32. Hristov, J. Approximate solutions to fractional subdiffusion equations. *Eur. Phys. J. Spec. Top.* **2011**, *193*, 229–243. [CrossRef]
33. Lupas A. A q-analogue Bernstein operator, Seminar on Numerical and Statiscal Calculus. *Univ. Cluj-Napoca* **1987**, *9*, 85–92.
34. Al-qudah, Y.; Alaroud, M.; Qoqazeh, H.; Jaradat, A.; AlHazmi, S.; Al-Omari, S.K. Approximate Analytic-Numeric Fuzzy Solutions of Fuzzy Fractional Equations Using Residual Power Series Approach. *Symmetry* **2022**, *14*, 804. [CrossRef]
35. Al-Omari, S.; Baleanu, D.; Purohit, S. Some results for Laplace-type integral operator in quantum calculus. *Adv. Differ. Equ.* **2018**, *2018*, 124. [CrossRef]
36. Al-Omari, S. On q-analogues of Mangontarum transform of some polynomials and certain class of H-functions. *Nonlinear Stud.* **2016**, *23*, 51–61.
37. Al-Omari, S. q-analogues and properties of the Laplace-type integral operator in the quantum calculus theory. *J. Inequal. Appl.* **2020**, *2020*, 203. [CrossRef]
38. Srivastava, H.; Karlsson, P. *Multiple Gaussian hypergeometric Series*; Halsted Press, Ellis Horwood Limited: Chichester, UK; John Wiley and Sons: New York, NY, USA; Chichester, UK; Brisbane, Australia; Toronto, ON, Canada, 1985.
39. Al-Omari, S. The q-Sumudu transform and its certain properties in a generalized q-calculus theory. *Adv. Differ. Equ.* **2021**, *10*, 10. [CrossRef]
40. Al-Omari, S.; Almusawa, H.; Nisar, K. A new aspect of generalized integral operator and an estimation in a generalized function theory. *Adv. Contin. Discret. Model. Theory Appl.* **2021**, *2021*, 357.
41. Al-Omari, S.; Suthar, D.; Araci, S. A fractional q-integral operator associated with certain class of q-Bessel functions and q-generating series. *Adv. Contin. Discret. Model. Theory Appl.* **2022**, *in editing*.
42. Fardi, M.; Al-Omari, S.K.; Araci, S. A Pseudo-Spectral Method Based on Reproducing Kernel for Solving the Time-Fractional Diffusion-Wave Equation. *Adv. Contin. Discret. Model. Theory Appl.* **2022**, *in editing*.
43. Annaby, M.H.; Mansour, Z.S. *q-Fractional Calculus and Equations*; Springer Publishing: Berlin/Heidelberg, Germany, 2012.
44. Altintaş, O.; Irmak, H.; Srivastava, H. Fractional calculus and certain starlike functions with negative coefficients. *Comp. Math. Appl.* **1995**, *30*, 9–15. [CrossRef]

45. Goodman, A.W. Univalent functions and nonanalytic curves. *Proc. Am. Math. Soc.* **1957**, *8*, 598–601. [CrossRef]
46. Mahmood, S.; Raza, N.; AbuJarad, E.; Srivastava, G.; Srivastava, H.M.; Malik, S.N. Geometric Properties of Certain Classes of Analytic Functions Associated with a q-Integral Operator. *Symmetry* **2019**, *11*, 719. [CrossRef]