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Soliton Solutions and Sensitive Analysis of Modified Equal-Width Equation Using Fractional Operators

Muhammad Bilal Riaz ^{1,2,*} , Adam Wojciechowski ¹ , Georgia Irina Oros ³  and Riaz Ur Rahman ² 

¹ Faculty of Technical Physics, Information Technology and Applied Mathematics, Lodz University of Technology, 90-924 Lodz, Poland

² Department of Mathematics, University of Management and Technology, Lahore 54770, Pakistan

³ Department of Mathematics and Computer Science, University of Oradea, 410087 Oradea, Romania

* Correspondence: muhammad.riaz@p.lodz.pl or bilalsehole@gmail.com

Abstract: In this manuscript, the novel auxiliary equation methodology (NAEM) is employed to scrutinize various forms of solitary wave solutions for the modified equal-width wave (MEW) equation. M -truncated along with Atangana–Baleanu (AB)-fractional derivatives are employed to study the soliton solutions of the problem. The fractional MEW equations are important for describing hydro-magnetic waves in cold plasma. A comparative analysis is utilized to study the influence of the fractional parameter on the generated solutions. Secured solutions include bright, dark, singular, periodic and many other types of soliton solutions. In compared to other methods, the solutions demonstrate that the proposed technique is particularly effective, straightforward, and trustworthy that contains families of solutions. In addition, the symbolic soft computation is used to verify the obtained solutions. Finally, the system is subjected to a sensitive analysis. Integer-order results calculated by the symmetry method present in the literature can be addressed as limiting cases of the present study.



Citation: Riaz, M.B.; Wojciechowski, A.; Oros, G.I.; Rahman, R.U. Soliton Solutions and Sensitive Analysis of Modified Equal-Width Equation Using Fractional Operators.

Symmetry **2022**, *14*, 1731. <https://doi.org/10.3390/sym14081731>

Academic Editors: Haci Mehmet Baskonus and Yolanda Guerrero

Received: 18 July 2022

Accepted: 12 August 2022

Published: 19 August 2022

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Keywords: soliton; M -truncated derivative; Atangana–Baleanu fractional operator; modified equal-width equation; new auxiliary equation method; sensitivity analysis

1. Introduction

Fractional calculus is a rapidly developing topic of mathematics that has a variety of applications in physics, engineering and chemistry, such as signal processing, fluid dynamics, magnetism and electricity [1–5]. The study of fractional derivatives is fascinating, and numerous researchers have recently presented significant contributions on the subject. A fractional-order differential equation is a generalized form of an integer-order differential equation. In fact, these equations are regarded as a viable alternative to integer differential equations. Due to a remarkable memory fact, this mathematical technique is unique. The memory function's role is to demonstrate the correspondence between the fractional derivative kernel, which cannot be determined physically. The fractional order model is useful in many areas, e.g., in pure and applied sciences for the representation of a physical model of a variety of phenomena. For researchers and analysts, the resulting equations provide unimaginable possibilities [6–13].

The study of nonlinear wave propagation on the ocean's surface has caught the interest of scientists for decades. In many scientific disciplines, nonlinear wave phenomena have been observed such as ocean engineering, coastal engineering, fluid dynamics, plasma physics, communication industry, tsunami waves and theory of control, etc., [14–17]. In nature, linear or nonlinear evolution equations or evolution systems can be used to explain a wide range of real-world problems. The heat equation, wave equation, and Schrödinger equation are some well-known model examples of evolution equations that can be found in engineering and scientific applications [18–20]. Apart from these three well-known

paradigms, there are several other especially good evolution equations or systems, such as Boltzmann's equation and Navier–Stokes equations, to name a few [21–24].

Analytical and computational soliton solutions can clearly describe the occurrences, and mathematicians as well as scientists worked together to develop a number of techniques for studying nonlinear evolution equations (NLEEs) directly using these solutions [25–27]. Soliton or solitary wave solutions have acquired a lot of significance in light of its utilization in the field of applied physical science. Waves are created when a few unsettling influence happens in the peculiarity [28–30]. Soliton cooperation occurs when at least two solitons draw near enough to associate. Since solitons introduce themselves as minuscule, restricted energy groups, it is thus said that they show qualities akin to particles in a given framework. Solitons are administered by nonlinear Schrödinger equations, which address the physical phenomena as models utilizing NLEEs. The use of solitons in optical fibres to carry digital information is one of the most important technical applications [31–33].

There are numerous analytical strategies that have been developed to tackle such NLEEs. As for example, the new extended rational expansion scheme, the semi-inverse variational principle, the homogeneous extended balance technique, the Darboux transformation method, the Hirota bilinear method, and many others [34–39]. The main objective of this work is to explore an essential model known as the MEW equation using M -truncated and AB -fractional operators. In plasma physics and fluid dynamics, the mentioned equation plays a significant role.

Different analytical and numerical methods have been used to solve this equation, including: improved $(\frac{G'}{G})$ -expansion and the ansatz techniques, the tanh-function method, the Kudryashov's method and many more [40–48]. However, the NAEM has not been used to analyze the above-stated equation with a fractional M -truncated and AB -operators [49]. This strategy has likewise been utilized to research different models in different articles. In addition, by applying the NAEM, exact solutions to the WBBM equation have been found in [50]. Physical model equations involving the M -truncated and AB -derivatives have also been researched using various methodologies in a variety of applications. The goal of the current study is to expand previous literary efforts to address the MEW equation and its nonlinear variations along with the sensitive behavior of the system. It is identified here, waveform solutions for the MEW wave equation with time-fractional derivatives using the analytical approach [51–53].

The article is structured as follows: The basic concept of fractional calculus is found in Section 2. The proposed methodology is described in Section 3. The governing equation is mentioned in Section 4. Soliton's solutions have been extracted to the MEW equation in Section 5. The graphical representation of the solutions are depicted in Section 6. Section 7 represents the sensitive behavior of the given system. Finally, a conclusion is provided in Section 8.

2. Basic Preliminaries about Fractional Calculus

In this study, the M -Truncated and AB -fractional derivatives are employed, and some basic definitions are provided.

2.1. M -Truncated Fractional Operator

Definition 1. The Mittag–Leffler truncated function having a single parameter is stated as [54]:

$${}_i E_q(z) = \sum_{j=0}^i \frac{z^j}{\Gamma(qj+1)},$$

where $z \in \mathbb{C}$ and $q > 0$. It is defined as follows in terms of a non-fuzzy idea.

Definition 2. Suppose that

$$g : [0, \infty) \rightarrow \mathbb{R},$$

and $\delta \in (0, 1)$ the M -truncated fractional operator of g of order δ is given as:

$$\mathcal{D}_M^{\delta, \varrho} g(t) = \lim_{\varepsilon \rightarrow 0} \frac{g(t + {}_i E_\varrho(\varepsilon t^{-\delta})) - g(t)}{\varepsilon},$$

for $t > 0$ and ${}_i E_\varrho(\cdot), \varrho > 0$.

Theorem 1. Suppose that g is a differentiable function of δ order at $t_0 > 0$ with $\delta \in (0, 1]$ and $\varrho > 0$ then, g is continuous at t_0 .

Theorem 2. If $\delta \in (0, 1], \varrho > 0, m, n \in \mathbb{R}$ and g, h are δ -differentiable at $t > 0$, then:

1. ${}_i \mathcal{D}_M^{\delta, \varrho}(mg(t) + nh(t)) = m\mathcal{D}_M^{\delta, \varrho}(g(t)) + n\mathcal{D}_M^{\delta, \varrho}(h(t))$.
2. ${}_i \mathcal{D}_M^{\delta, \varrho}(g(t) \cdot h(t)) = g(t)\mathcal{D}_M^{\delta, \varrho}(h(t)) + h(t)\mathcal{D}_M^{\delta, \varrho}(g(t))$.
3. ${}_i \mathcal{D}_M^{\delta, \varrho}\left(\frac{g(t)}{h(t)}\right) = \frac{g(t)\mathcal{D}_M^{\delta, \varrho}(h(t)) - h(t)\mathcal{D}_M^{\delta, \varrho}(g(t))}{[h(t)]^2}$.
4. ${}_i \mathcal{D}_M^{\delta, \varrho}(c) = 0$, where $g(t) = c$ is a constant.
5. (Chain rule) If $g(t)$ is differentiable, then ${}_i \mathcal{D}_M^{\delta, \varrho}(g)(t) = \frac{t^{1-\delta}}{\Gamma(\varrho+1)} \frac{dg(t)}{dt}$.

2.2. AB-Fractional Operator

Definition 3. Let $g \in G'(a, b), \delta \in [0, 1], a < b$, then AB-fractional derivative is defined in Caputo sense as:

$${}_a^{ABC} D_t^\delta = \frac{AB(\delta)}{1-\delta} \int_a^t g'(\tau) E_\delta\left(-\delta \frac{(t-\tau)^\delta}{1-\delta}\right) d\tau.$$

Here, $AB(\delta)$ is a function of normalization with and $AB(0) = AB(1) = 1$.

Definition 4. Let $f \in G'(a, b), \delta \in [0, 1], a < b$, then in Riemann–Liouville, AB-operator is defined as:

$${}_a^{ABR} D_t^\delta = \frac{AB(\delta)}{1-\delta} \frac{d}{dt} \int_a^t g(\tau) E_\delta\left(-\delta \frac{(t-\tau)^\delta}{1-\delta}\right) d\tau.$$

3. General Methodology

We employ the proposed method to obtain all the solitary wave solutions of the MEW problem using M -Truncated and AB -derivatives in this section. By setting appropriate values to the fractional parameter δ , we display graphs of acquired results.

Overview of Analytical Technique

Consider the following statement that demonstrates how NLPDE is built in general:

$$S(U, U_x, UU_x, U_{tt}, U_t U_{xx}, \dots) = 0, \tag{1}$$

S is a polynomial function with respect to a given variable. Use the propagational transformation $U(x, t) = \mathcal{Q}(\eta)$ to convert Equation (1) into a basic form of NLODE where $\eta = x - vt$, then

$$T(\mathcal{Q}(\eta), \mathcal{Q}(\eta)', \mathcal{Q}(\eta)\mathcal{Q}(\eta)', \mathcal{Q}(\eta)'', \mathcal{Q}(\eta)' \mathcal{Q}(\eta)'', \dots) = 0. \tag{2}$$

The Q superscripts denote the derivative of Q with regard to η , and T is a function of a polynomial that includes both linear as well as nonlinear terms. The initial solution of Equation (2) can now be assumed employing the concept of NAEM as:

$$Q(\eta) = \sum_{i=0}^M \vartheta_i f^{i\varphi(\eta)}, \tag{3}$$

which satisfies the auxiliary equation

$$\varphi'(\eta) = \frac{1}{\ln(f)} \left(\beta f^{-\varphi(\eta)} + \chi + \gamma f^{\varphi(\eta)} \right), \tag{4}$$

where $\vartheta_0, \vartheta_1, \vartheta_2, \dots, \vartheta_M$ are the coefficients to be known in such a way that $\alpha_M \neq 0$. According to the balancing principle, one may compute the value of M by equating the largest nonlinear factor with the higher-order derivative in Equation (2). The different cases of possible solutions to Equation (4) are mentioned here.

Case 1: When $\gamma \neq 0$ and $\chi^2 - 4\beta\gamma < 0$,

$$f^{\varphi(\eta)} = \frac{-\chi}{2\gamma} + \frac{\sqrt{4\beta\gamma - \chi^2}}{2\gamma} \tan \left(\frac{\sqrt{4\beta\gamma - \chi^2}}{2} \eta \right), \tag{5}$$

$$f^{\varphi(\eta)} = \frac{-\chi}{2\gamma} - \frac{\sqrt{4\beta\gamma - \chi^2}}{2\gamma} \cot \left(\frac{\sqrt{4\beta\gamma - \chi^2}}{2} \eta \right). \tag{6}$$

Case 2: When $\chi^2 - 4\beta\gamma > 0$ and $\gamma \neq 0$,

$$f^{\varphi(\eta)} = \frac{-\chi}{2\gamma} - \frac{\sqrt{\chi^2 - 4\beta\gamma}}{2\gamma} \tanh \left(\frac{\sqrt{\chi^2 - 4\beta\gamma}}{2} \eta \right), \tag{7}$$

$$f^{\varphi(\eta)} = \frac{-\chi}{2\gamma} - \frac{\sqrt{\chi^2 - 4\beta\gamma}}{2\gamma} \coth \left(\frac{\sqrt{\chi^2 - 4\beta\gamma}}{2} \eta \right). \tag{8}$$

Case 3: When $\chi^2 + 4\beta^2 < 0$, $\gamma \neq 0$ and $\gamma = -\beta$,

$$f^{\varphi(\eta)} = \frac{\chi}{2\beta} - \frac{\sqrt{-4\beta^2 - \chi^2}}{2\beta} \tan \left(\frac{\sqrt{-4\beta^2 - \chi^2}}{2} \eta \right), \tag{9}$$

$$f^{\varphi(\eta)} = \frac{\chi}{2\beta} + \frac{\sqrt{-4\beta^2 - \chi^2}}{2\beta} \cot \left(\frac{\sqrt{-4\beta^2 - \chi^2}}{2} \eta \right). \tag{10}$$

Case 4: When $\chi^2 + 4\beta^2 > 0$, $\gamma \neq 0$ and $\gamma = -\beta$,

$$f^{\varphi(\eta)} = \frac{\chi}{2\beta} + \frac{\sqrt{4\beta^2 + \chi^2}}{2\beta} \tanh \left(\frac{\sqrt{4\beta^2 + \chi^2}}{2} \eta \right), \tag{11}$$

$$f^{\varphi(\eta)} = \frac{\chi}{2\beta} + \frac{\sqrt{4\beta^2 + \chi^2}}{2\beta} \coth \left(\frac{\sqrt{4\beta^2 + \chi^2}}{2} \eta \right). \tag{12}$$

Case 5: When $\chi^2 - 4\beta^2 < 0$ and $\gamma = \beta$,

$$f^{\varphi(\eta)} = \frac{-\chi}{2\beta} + \frac{\sqrt{4\beta^2 - \chi^2}}{2\beta} \tan \left(\frac{\sqrt{4\beta^2 - \chi^2}}{2} \eta \right), \tag{13}$$

$$f^{\varphi(\eta)} = \frac{-\chi}{2\beta} - \frac{\sqrt{4\beta^2 - \chi^2}}{2\beta} \cot \left(\frac{\sqrt{4\beta^2 - \chi^2}}{2} \eta \right). \tag{14}$$

Case 6: When $\chi^2 - 4\beta^2 > 0$ and $\gamma = \beta$,

$$f^{\varphi(\eta)} = \frac{-\chi}{2\beta} - \frac{\sqrt{-4\beta^2 + \chi^2}}{2\beta} \tanh\left(\frac{\sqrt{-4\beta^2 + \chi^2}}{2} \eta\right), \tag{15}$$

$$f^{\varphi(\eta)} = \frac{-\chi}{2\beta} - \frac{\sqrt{-4\beta^2 + \chi^2}}{2\beta} \coth\left(\frac{\sqrt{-4\beta^2 + \chi^2}}{2} \eta\right). \tag{16}$$

Case 7: When $\chi^2 = 4\beta\gamma$,

$$f^{\varphi(\eta)} = -\frac{2 + \chi \eta}{2\gamma\eta}. \tag{17}$$

Case 8: For $\beta\gamma < 0, \chi = 0$ and $\gamma \neq 0$,

$$f^{\varphi(\eta)} = -\sqrt{\frac{-\beta}{\gamma}} \tanh(\sqrt{-\beta\gamma} \eta), \tag{18}$$

$$f^{\varphi(\eta)} = -\sqrt{\frac{-\beta}{\gamma}} \coth(\sqrt{-\beta\gamma} \eta). \tag{19}$$

Case 9: When $\beta = -\gamma$ with $\chi = 0$,

$$f^{\varphi(\eta)} = -\left(\frac{1 + e^{-2\gamma \eta}}{1 - e^{-2\gamma \eta}}\right). \tag{20}$$

Case 10: For $\beta = \gamma = 0$,

$$f^{\varphi(\eta)} = \sinh(\chi \eta) + \cosh(\chi \eta). \tag{21}$$

Case 11: For $\beta = \chi = K, \gamma = 0$,

$$f^{\varphi(\eta)} = e^{K\eta} - 1. \tag{22}$$

Case 12: When $\gamma = \chi = K$ and $\beta = 0$,

$$f^{\varphi(\eta)} = \frac{e^{K\eta}}{1 - e^{K\eta}}. \tag{23}$$

Case 13: When $\chi = \beta + \gamma$,

$$f^{\varphi(\eta)} = -\frac{1 - \beta e^{(\beta-\gamma)\eta}}{1 - \gamma e^{(\beta-\gamma)\eta}}. \tag{24}$$

Case 14: When $\chi = -\beta - \gamma$,

$$f^{\varphi(\eta)} = \frac{e^{(\beta-\gamma)\eta} - \beta}{e^{(\beta-\gamma)\eta} - \gamma}. \tag{25}$$

Case 15: When $\beta = 0$,

$$f^{\varphi(\eta)} = \frac{\chi e^{\chi\eta}}{1 - \gamma e^{\chi\eta}}. \tag{26}$$

Case 16: When $\beta = \chi = \gamma \neq 0$,

$$f^{\varphi(\eta)} = \frac{1}{2} \left[\sqrt{3} \tan\left(\frac{\sqrt{3}}{2} \beta \eta\right) - 1 \right]. \tag{27}$$

Case 17: When $\chi = \gamma = 0$,

$$f^{\varphi(\eta)} = \beta \eta. \tag{28}$$

Case 18: When $\chi = \beta = 0$,

$$f^{\varphi(\eta)} = -\frac{1}{\gamma \eta}. \tag{29}$$

Case 19: For $\beta = \gamma$ and $\chi = 0$,

$$f^{\varphi(\eta)} = \tan(\beta \eta). \tag{30}$$

Case 20: For $\gamma = 0$,

$$f^{\varphi(\eta)} = e^{\chi \eta} - \frac{n}{l}. \tag{31}$$

4. Governing Equation

Consider the MEW Equation [45] with time fractional derivative is given as:

$$\frac{\partial^\delta q}{\partial t^\delta} + \theta \frac{\partial q^3}{\partial x} - \rho \frac{\partial^2}{\partial x^2} \left(\frac{\partial^\delta q}{\partial t^\delta} \right) = 0. \tag{32}$$

The wave profile is represented by $q = q(x, t)$, and the parameters are expressed by θ and ρ . The fractional-order derivative is described by a parameter in this expression. Fractional-order equations become classical equations when $\delta = 1$. Assume the following traveling wave transformation:

$$q(x, t) = Q(\eta). \tag{33}$$

$q(x, t)$ is the wave form of the solitons in this case, and η is categorized as:

i. For the M -Truncated operator, we have:

$$\eta = \left(\omega x - \frac{\Gamma(\varrho + 1)}{\delta} \nu t^\delta \right). \tag{34}$$

ii. By means of the AB fractional operator, we take:

$$\eta = \omega x - \frac{\nu (1 - \delta) t^{1-\delta n}}{AB(\delta) \sum_{n=0}^{\infty} \left(\frac{-\delta}{1-\delta} \right)^n \Gamma(1 - \delta n)}. \tag{35}$$

The fractional nonlinear MEW equation in terms of M -Truncated as well as AB -fractional operator is denoted as:

$$\begin{aligned} {}_0^A D_{M,t}^{\delta,\varrho} q + \theta \frac{\partial q^3}{\partial x} - \rho \frac{\partial^2}{\partial x^2} ({}_0^A D_{M,t}^{\delta,\varrho} q) &= 0, \\ {}_0^{ABC} D_t^\delta q + \theta \frac{\partial q^3}{\partial x} - \rho \frac{\partial^2}{\partial x^2} ({}_0^{ABC} D_t^\delta q) &= 0, \quad 0 < \delta \leq 1, \end{aligned}$$

where ${}_0^A D_{M,t}^{\delta,\varrho}$ and ${}_0^{ABC} D_x^\gamma$ are M -truncated and AB -fractional operators. We get the following NLODE when we apply the wave transformations in Equations (34) and (35):

$$- \nu Q' + \theta \omega (Q^3)' + \rho \nu \omega^2 Q''' = 0. \tag{36}$$

When we integrate Equation (36), it becomes:

$$- \nu Q + \theta \omega Q^3 + \rho \nu \omega^2 Q'' = 0. \tag{37}$$

5. Application to Fractional MEW Equation

This section aims to obtain the traveling wave solutions for the considered equation. To find M , we just apply the homogeneous balance principle to Equation (37), which gives $M = 1$. Equation (3) now has the following form:

$$Q(\eta) = \vartheta_0 + \vartheta_1 f^{\varphi(\eta)}. \tag{38}$$

By placing Equation (38) with Equation (4) into Equation (37), by matching all coefficients of various powers of $f^{\varphi(\eta)}$ to zero, a system of equations is created.

$$\begin{aligned} (f^{\varphi(\eta)})^0 &: \chi \beta \lambda \omega^2 \rho \vartheta_1 + \omega \theta \vartheta_0^3 - \lambda \vartheta_0, \\ (f^{\varphi(\eta)})^1 &: \chi^2 \lambda \omega^2 \rho \vartheta_1 + 2 \beta \gamma \lambda \omega^2 \rho \vartheta_1 + 3 \omega \theta \vartheta_0^2 - \lambda \vartheta_1, \\ (f^{\varphi(\eta)})^2 &: 3 \chi \gamma \lambda \omega^2 \rho \vartheta_1 + 3 \omega \theta \vartheta_0 \vartheta_1^2, \\ (f^{\varphi(\eta)})^3 &: 2 \gamma^2 \lambda \omega^2 \rho \vartheta_1 + \omega \theta \vartheta_1^3. \end{aligned}$$

The following feasible solution is obtained by solving the given system with Maple software:

$$\vartheta_0 = \chi \sqrt{-\frac{\omega \lambda \rho}{2\theta}}, \quad \vartheta_1 = 2 \gamma \sqrt{-\frac{\omega \lambda \rho}{2\theta}}, \tag{39}$$

where,

$$\omega = \sqrt{-\frac{2}{\rho(\chi^2 - 4\beta\gamma)}}. \tag{40}$$

The following is the result of inserting Equations (39) and (40) into Equation (38):

$$\mathcal{Q}(\eta) = \sqrt{-\frac{\omega \lambda \rho}{2\theta}} \left(\chi + 2 \gamma f^{\varphi(\eta)} \right). \tag{41}$$

Equation (41) yields a variety of surface waves solutions when the solutions identified by Equation (4) are substituted:

Case 1: When $\chi^2 - 4\beta\gamma < 0$ and $\gamma \neq 0$,

$$\mathcal{Q}_{1,1}(x, t) = \sqrt{-\frac{\omega \lambda \rho}{2\theta}} \left[\sqrt{4\beta\gamma - \chi^2} \tan \left(\frac{\sqrt{4\beta\gamma - \chi^2}}{2} \eta \right) \right], \tag{42}$$

$$\mathcal{Q}_{1,2}(x, t) = -\sqrt{-\frac{\omega \lambda \rho}{2\theta}} \left[\sqrt{4\beta\gamma - \chi^2} \cot \left(\frac{\sqrt{4\beta\gamma - \chi^2}}{2} \eta \right) \right]. \tag{43}$$

Case 2: When $\chi^2 - 4\beta\gamma > 0$ and $\gamma \neq 0$,

$$\mathcal{Q}_{2,1}(x, t) = -\sqrt{-\frac{\omega \lambda \rho}{2\theta}} \left[\sqrt{\chi^2 - 4\beta\gamma} \tanh \left(\frac{\sqrt{\chi^2 - 4\beta\gamma}}{2} \eta \right) \right], \tag{44}$$

$$\mathcal{Q}_{2,2}(x, t) = -\sqrt{-\frac{\omega \lambda \rho}{2\theta}} \left[\sqrt{\chi^2 - 4\beta\gamma} \coth \left(\frac{\sqrt{\chi^2 - 4\beta\gamma}}{2} \eta \right) \right]. \tag{45}$$

Case 3: When $\chi^2 + 4\beta\gamma < 0$, $\gamma \neq 0$ and $\gamma = -\beta$,

$$\mathcal{Q}_{3,1}(x, t) = -\sqrt{-\frac{\omega \lambda \rho}{2\theta}} \left[\sqrt{-4\beta^2 - \chi^2} \tan \left(\frac{\sqrt{-4\beta^2 - \chi^2}}{2} \eta \right) \right], \tag{46}$$

$$\mathcal{Q}_{3,2}(x, t) = \sqrt{-\frac{\omega \lambda \rho}{2\theta}} \left[\sqrt{-4\beta^2 - \chi^2} \cot \left(\frac{\sqrt{-4\beta^2 - \chi^2}}{2} \eta \right) \right]. \tag{47}$$

Case 4: When $\chi^2 + 4\beta\gamma > 0$, $\gamma \neq 0$ and $\gamma = -\beta$,

$$\mathcal{Q}_{4,1}(x, t) = -\sqrt{-\frac{\omega \lambda \rho}{2\theta}} \left[\sqrt{4\beta^2 + \chi^2} \tanh \left(\frac{\sqrt{4\beta^2 + \chi^2}}{2} \eta \right) \right], \tag{48}$$

$$\mathcal{Q}_{4,2}(x, t) = -\sqrt{-\frac{\omega \lambda \rho}{2\theta}} \left[\sqrt{4\beta^2 + \chi^2} \coth \left(\frac{\sqrt{4\beta^2 + \chi^2}}{2} \eta \right) \right]. \tag{49}$$

Case 5: When $\chi^2 - 4\beta^2 < 0$ and $\gamma = \beta$,

$$\mathcal{Q}_{5,1}(x, t) = \sqrt{-\frac{\omega \lambda \rho}{2\theta}} \left[\sqrt{4\beta^2 - \chi^2} \tan \left(\frac{\sqrt{4\beta^2 - \chi^2}}{2} \eta \right) \right], \quad (50)$$

$$\mathcal{Q}_{5,2}(x, t) = -\sqrt{-\frac{\omega \lambda \rho}{2\theta}} \left[\sqrt{4\beta^2 - \chi^2} \cot \left(\frac{\sqrt{4\beta^2 - \chi^2}}{2} \eta \right) \right]. \quad (51)$$

Case 6: When $\chi^2 - 4\beta^2 > 0$ and $\gamma = \beta$,

$$\mathcal{Q}_{6,1}(x, t) = -\sqrt{-\frac{\omega \lambda \rho}{2\theta}} \left[\sqrt{-4\beta^2 + \chi^2} \tanh \left(\frac{\sqrt{-4\beta^2 + \chi^2}}{2} \eta \right) \right], \quad (52)$$

$$\mathcal{Q}_{6,2}(x, t) = -\sqrt{-\frac{\omega \lambda \rho}{2\theta}} \left[\sqrt{-4\beta^2 + \chi^2} \coth \left(\frac{\sqrt{-4\beta^2 + \chi^2}}{2} \eta \right) \right]. \quad (53)$$

Case 7: When $\chi^2 = 4\beta\gamma$,

$$\mathcal{Q}_7(x, t) = \sqrt{-\frac{\omega \lambda \rho}{2\theta}} \left(\frac{-2 + 2\chi\eta}{\eta} \right). \quad (54)$$

Case 8: $\beta\gamma < 0$, $\chi = 0$ and $\gamma \neq 0$,

$$\mathcal{Q}_{8,1}(x, t) = -2\gamma \sqrt{-\frac{\omega \lambda \rho}{2\theta}} \left[\sqrt{-\frac{\beta}{\gamma}} \tanh(\sqrt{-\beta\gamma}\eta) \right], \quad (55)$$

$$\mathcal{Q}_{8,2}(x, t) = -2\gamma \sqrt{-\frac{\omega \lambda \rho}{2\theta}} \left[\sqrt{-\frac{\beta}{\gamma}} \coth(\sqrt{-\beta\gamma}\eta) \right]. \quad (56)$$

Case 9: When $\chi = 0$ and $\beta = -\gamma$,

$$\mathcal{Q}_{9,1}(x, t) = -2\gamma \sqrt{-\frac{\omega \lambda \rho}{2\theta}} \left(\frac{e^{-2\gamma\eta} + 1}{e^{-2\gamma\eta} - 1} \right). \quad (57)$$

Case 10: When $\gamma = \chi = K$ and $\beta = 0$,

$$\mathcal{Q}_{12}(x, t) = \sqrt{-\frac{\omega \lambda \rho}{2\theta}} \left[K + 2K \left(\frac{e^{K\eta}}{1 - e^{K\eta}} \right) \right]. \quad (58)$$

Case 11: When $\beta + \gamma = \chi$,

$$\mathcal{Q}_{13}(x, t) = \sqrt{-\frac{\omega \lambda \rho}{2\theta}} \left[(\beta + \gamma) + 2\gamma \left(\frac{\beta e^{(\beta-\gamma)\eta} - 1}{1 - \gamma e^{(\beta-\gamma)\eta}} \right) \right]. \quad (59)$$

Case 12: When $-(\beta + \gamma) = \chi$,

$$\mathcal{Q}_{14}(x, t) = \sqrt{-\frac{\omega \lambda \rho}{2\theta}} \left[-(\beta + \gamma) + 2\gamma \left(\frac{\beta - e^{(\beta-\gamma)\eta}}{e^{(\beta-\gamma)\eta} - \gamma} \right) \right]. \quad (60)$$

Case 13: When $\beta = 0$,

$$\mathcal{Q}_{15}(x, t) = \sqrt{-\frac{\omega \lambda \rho}{2\theta}} \left[\chi + 2\gamma \left(\frac{\chi e^{\chi\eta}}{1 - \gamma e^{\chi\eta}} \right) \right]. \quad (61)$$

Case 14: When $\chi = \beta = \gamma \neq 0$,

$$\mathcal{Q}_{16}(x, t) = \sqrt{-\frac{\omega \lambda \rho}{2\theta}} \left[\chi + \gamma \left(\sqrt{3} \tan \left(\frac{\sqrt{3}}{2} \beta \eta \right) - 1 \right) \right]. \quad (62)$$

Case 15: When $\chi = \beta = 0$,

$$Q_{18}(x, t) = \sqrt{-\frac{\omega \lambda \rho}{2\theta}} \left(\frac{-2}{\eta}\right). \quad (63)$$

Case 16: When $\beta = \gamma$ and $\chi = 0$,

$$Q_{19}(x, t) = 2\gamma \sqrt{-\frac{\omega \lambda \rho}{2\theta}} \tan(\gamma \eta). \quad (64)$$

6. Solutions in Graphical Layout via Fractional Operators

In this part, some of the analytical results of this research are represented graphically. The section mostly focuses on the background understanding of the specific findings investigated in this study. Using a current piece of professional tools of programming, graphs are constructed for clearer illustration. Additionally, each 2D and 3D graph is shown over a unique time frame. We utilize different colors to ensure that the wave's behavior will change, that distinct waves will overlap, or that different curves will appear at different points inside the same wave. Depending on the physical regions of the parameters, relevant quantities can be employed. As a key element of our inquiry, we can use changeable characteristic values to examine the distinctive dynamic features, shapes, and patterns of soliton solutions. However, it is important to keep in mind that the solutions also comprise dark functions, bright modules, and trigonometric functions. Here, the fractional MEW equation for *M*-truncated and *AB* fractional operators is investigated by NAEM. To analyze the efficacy of operators, we examine the solutions utilizing fractional data points. Figures 1–10 depict the graphical representation of two solutions $Q_{14}(x, t)$ and $Q_{16}(x, t)$ and explain the effects of fractional parameter δ on different values.

Figures 1–5:

One such graph represents a physical meaning of $Q_{14}(x, t)$. Plots show the results of applying fractional operators to the given solution while using various non-integer parametric values. Here is a graphical depiction of the acquired result using the parametric values, $\chi = -1$, $\beta = 0.7$, $\gamma = 0.3$, $\rho = -2$, $\nu = 1$, $\theta = \lambda = 1$ and $\varrho = 0.4$. (a,b) depict 3D profiles using *M*-Truncated and *AB*-fractional operators employing $\delta = 0.3$, $\delta = 0.5$, $\delta = 0.7$ and $\delta = 0.9$ at $t = 1$.

Figures 6–10:

Every such graph betrays a physical meaning of $Q_{16}(x, t)$. Plots display the outcomes of applying multiple non-integer parametric values to the provided solution while utilizing fractional operators. Here is a graphical exhibition of the obtained result using the parametric values, $\chi = 2$, $\beta = 0.5$, $\gamma = 1$, $\rho = -1$, $\nu = 2$, $\theta = 1.5$, $\lambda = 2$ and $\varrho = 0.3$. Figures (a,b) depict 3D plots using *M*-Truncated and *AB*-fractional operators assigning $\delta = 0.3$, $\delta = 0.5$, $\delta = 0.7$ and $\delta = 0.9$ at $t = 1$.

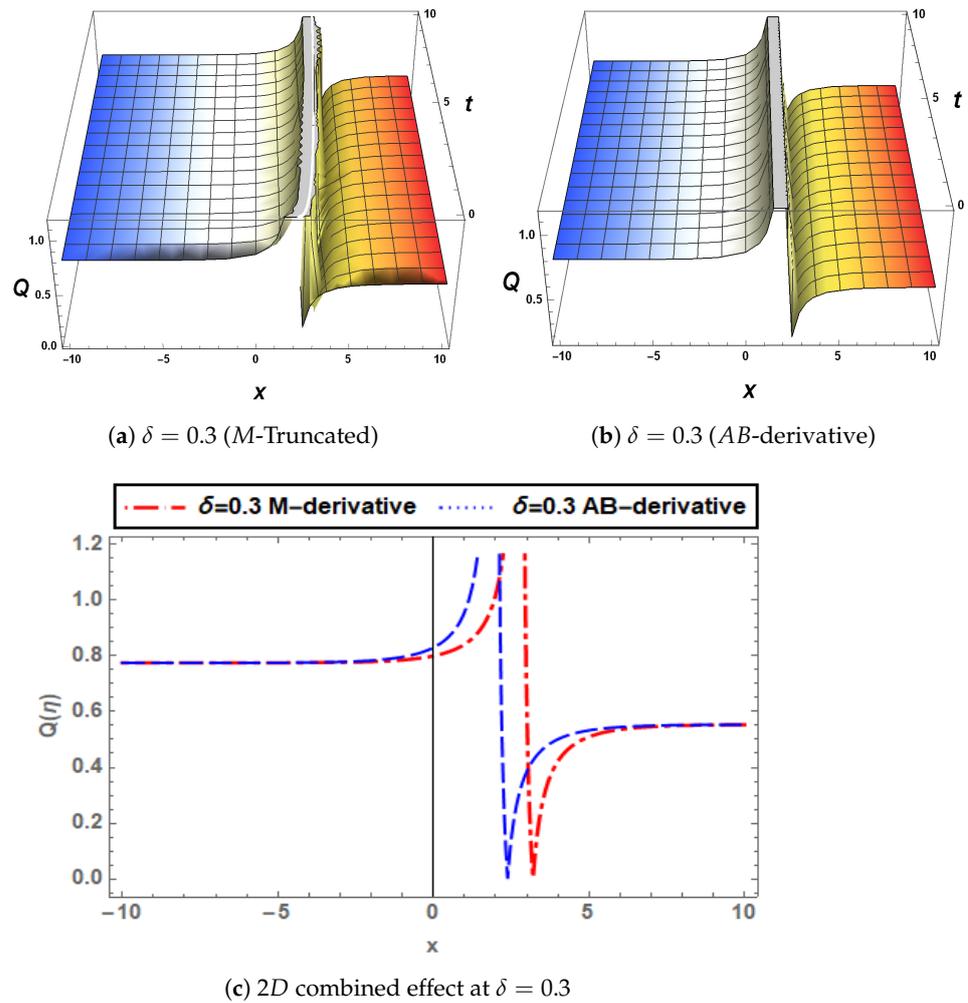


Figure 1. A graphical representation of $Q_{14}(x, t)$.

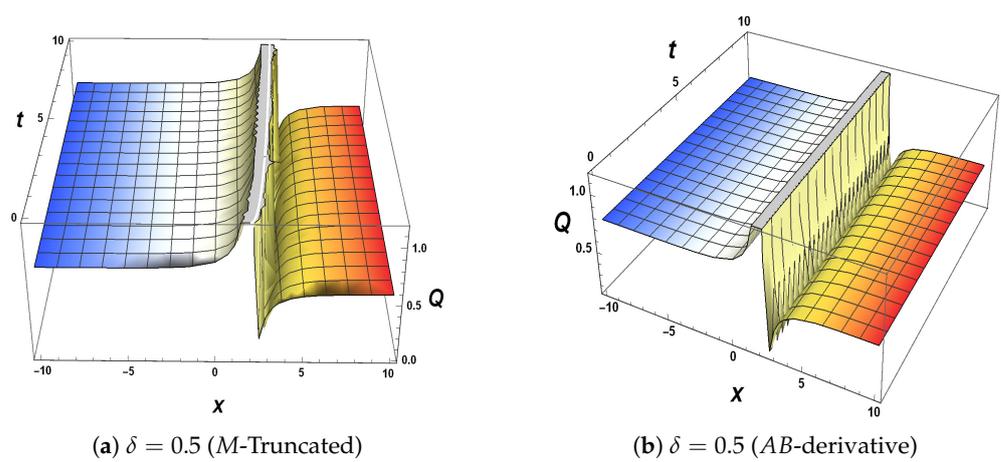
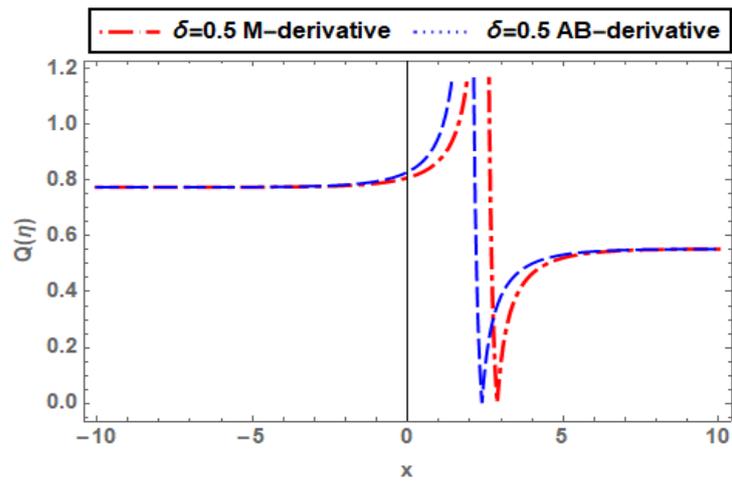
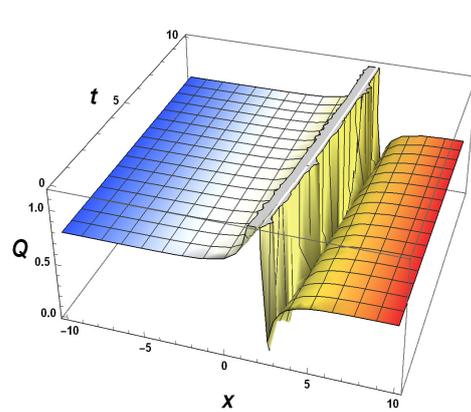


Figure 2. Cont.

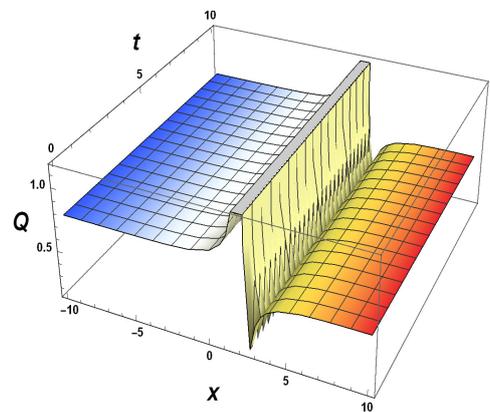


(c) 2D combined effect at $\delta = 0.5$

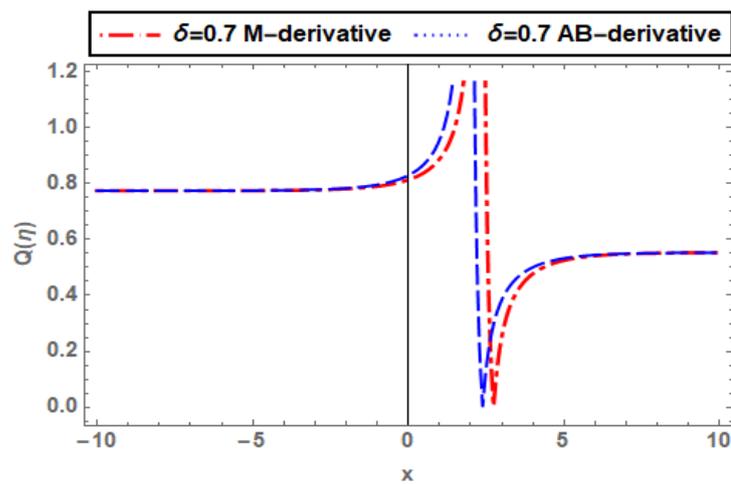
Figure 2. A graphical representation of $Q_{14}(x, t)$.



(a) $\delta = 0.7$ (M-Truncated)



(b) $\delta = 0.7$ (AB-derivative)



(c) 2D combined effect at $\delta = 0.7$

Figure 3. A graphical representation of $Q_{14}(x, t)$.

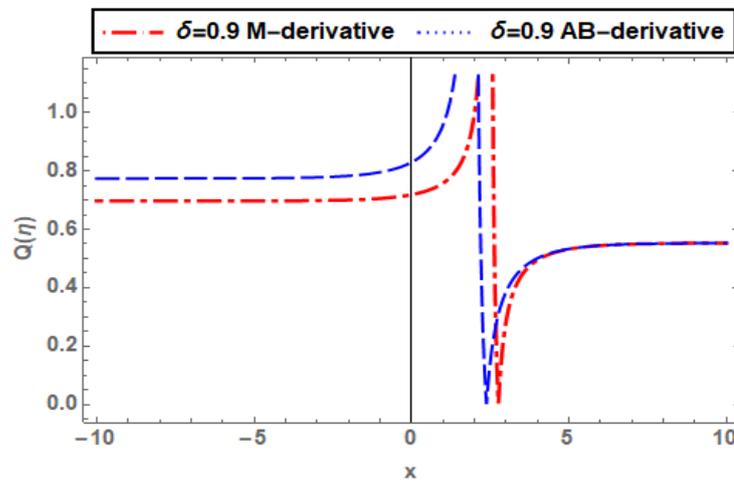
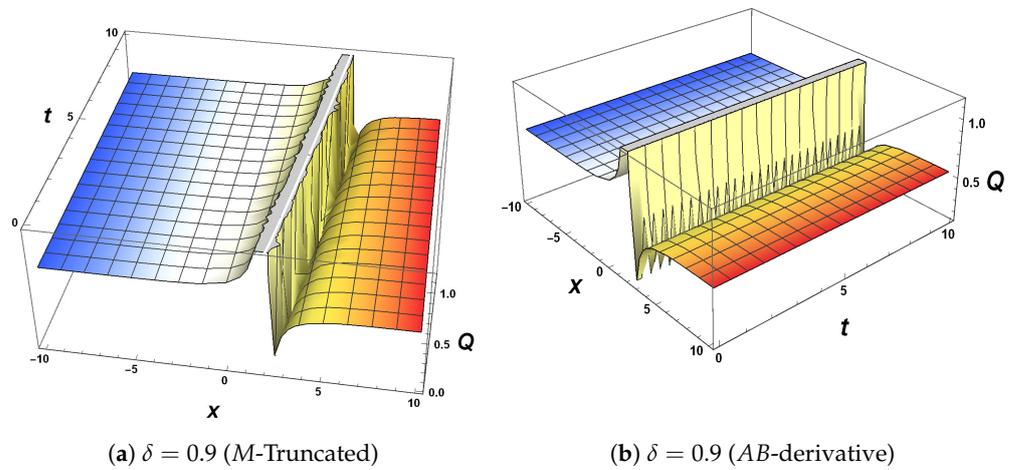


Figure 4. A graphical representation of $Q_{14}(x, t)$.

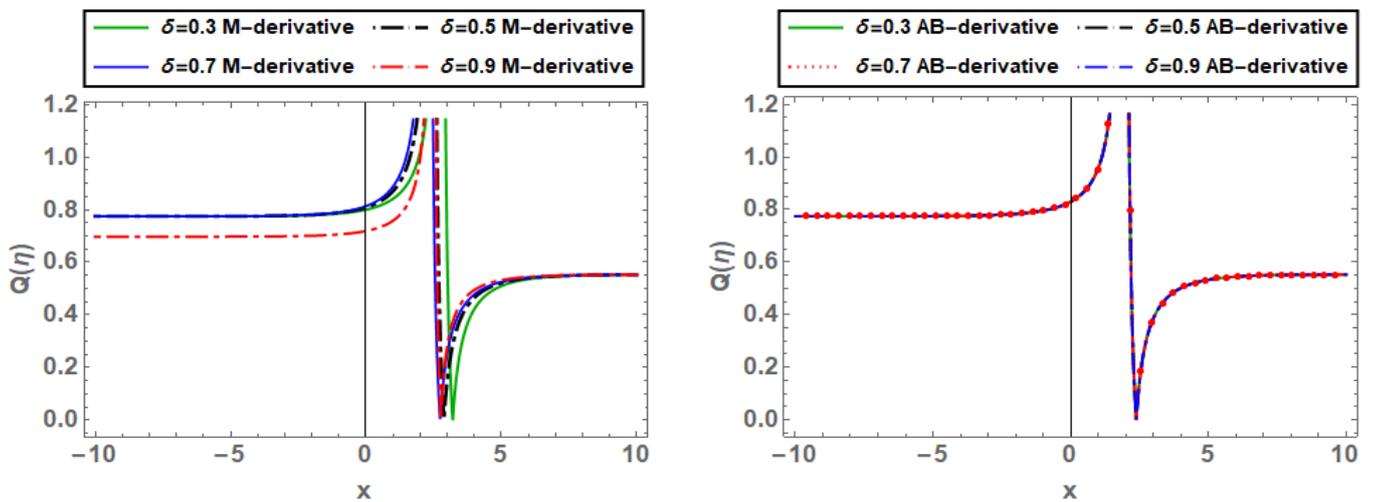
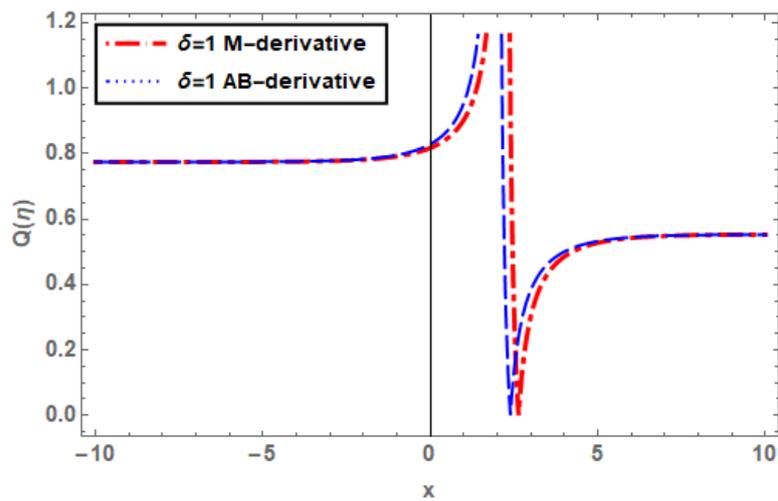
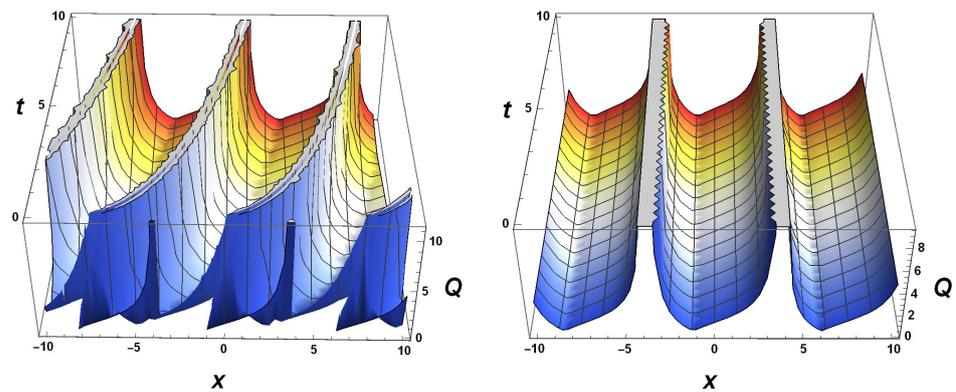


Figure 5. Cont.



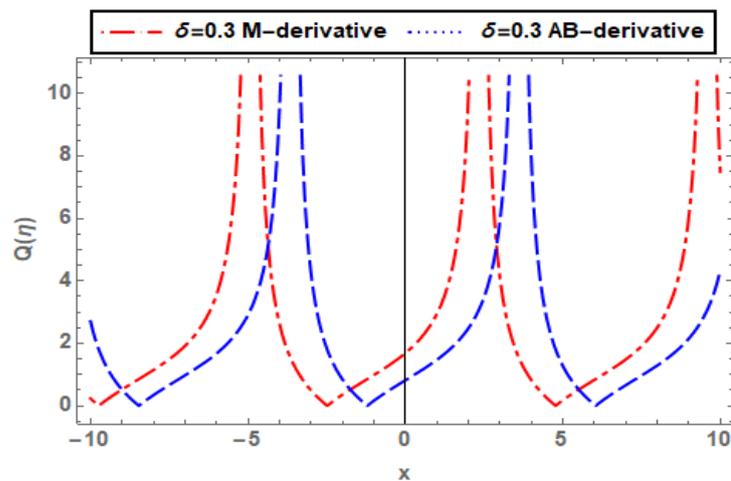
(c) Combined effect of $\delta=1$ for both operators

Figure 5. The 2D graphics of $Q_{14}(x, t)$.



(a) $\delta = 0.3$ (*M*-Truncated)

(b) $\delta = 0.3$ (*AB*-derivative)



(c) 2D combined effect at $\delta = 0.3$

Figure 6. A graphical representation of $Q_{16}(x, t)$.

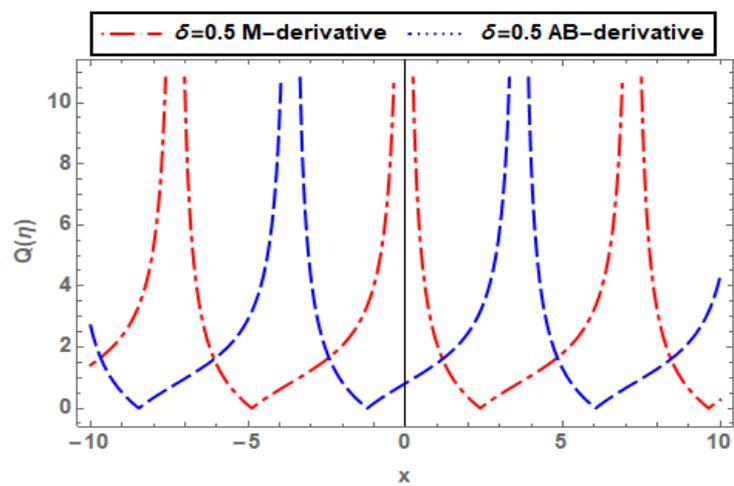
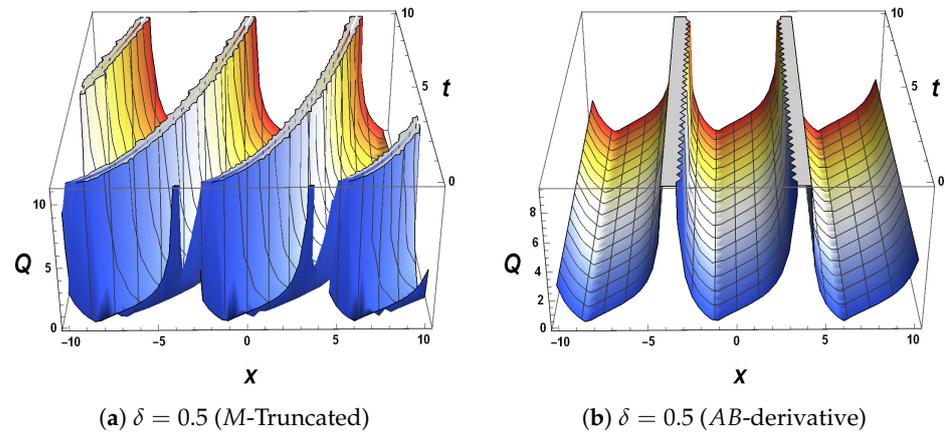


Figure 7. A graphical representation of $Q_{16}(x, t)$.

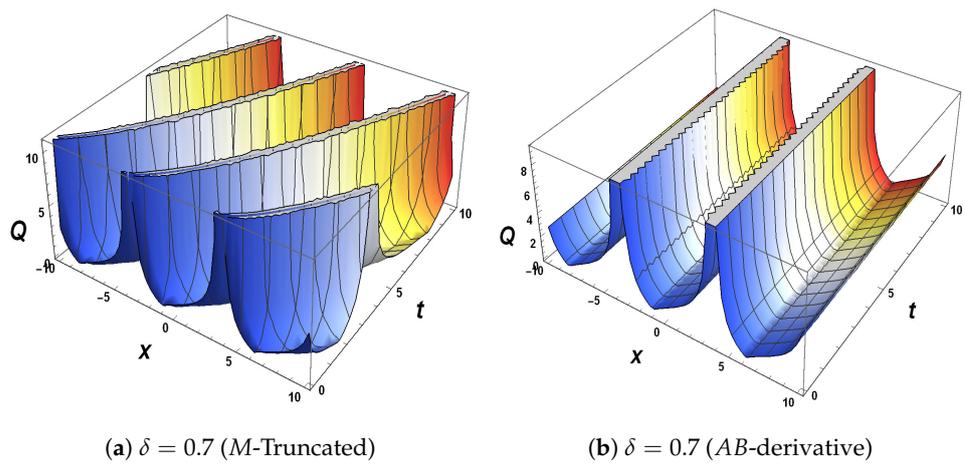
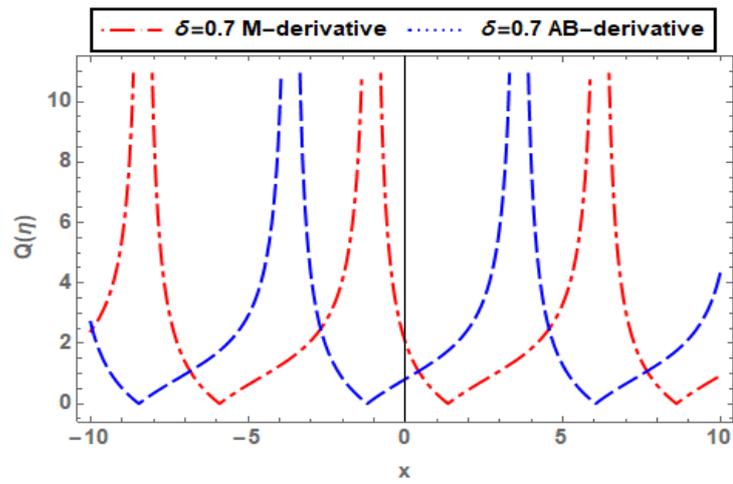
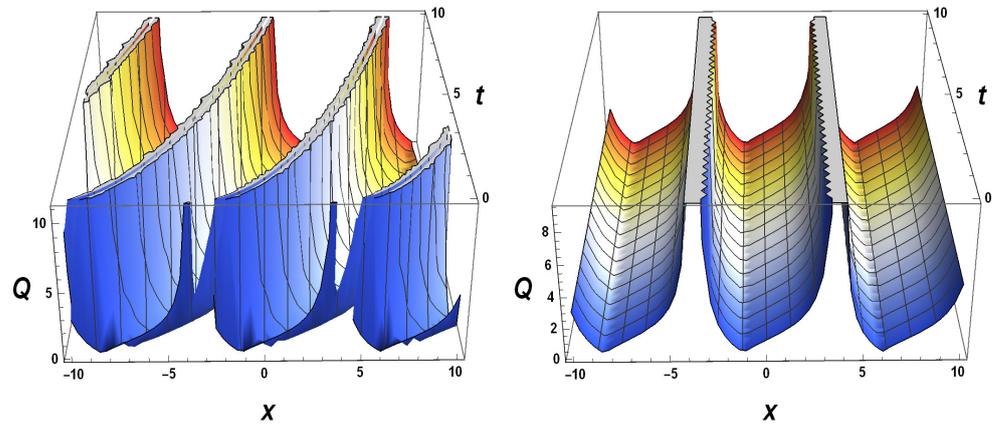


Figure 8. Cont.



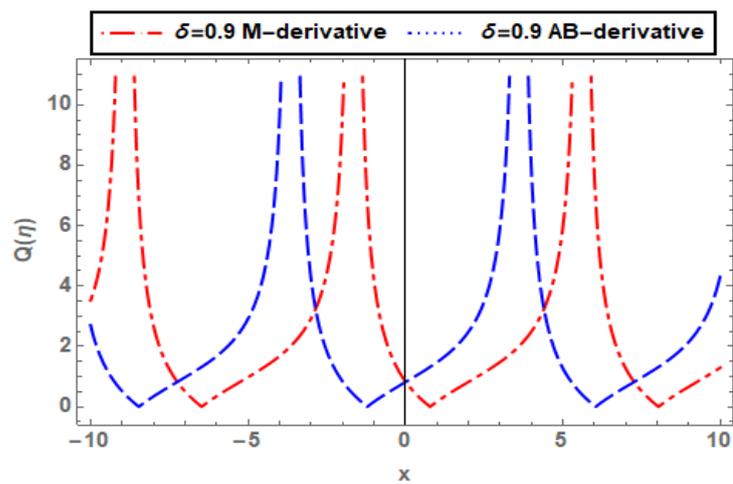
(c) 2D combined effect at $\delta = 0.7$

Figure 8. A graphical depiction of $Q_{16}(x, t)$.



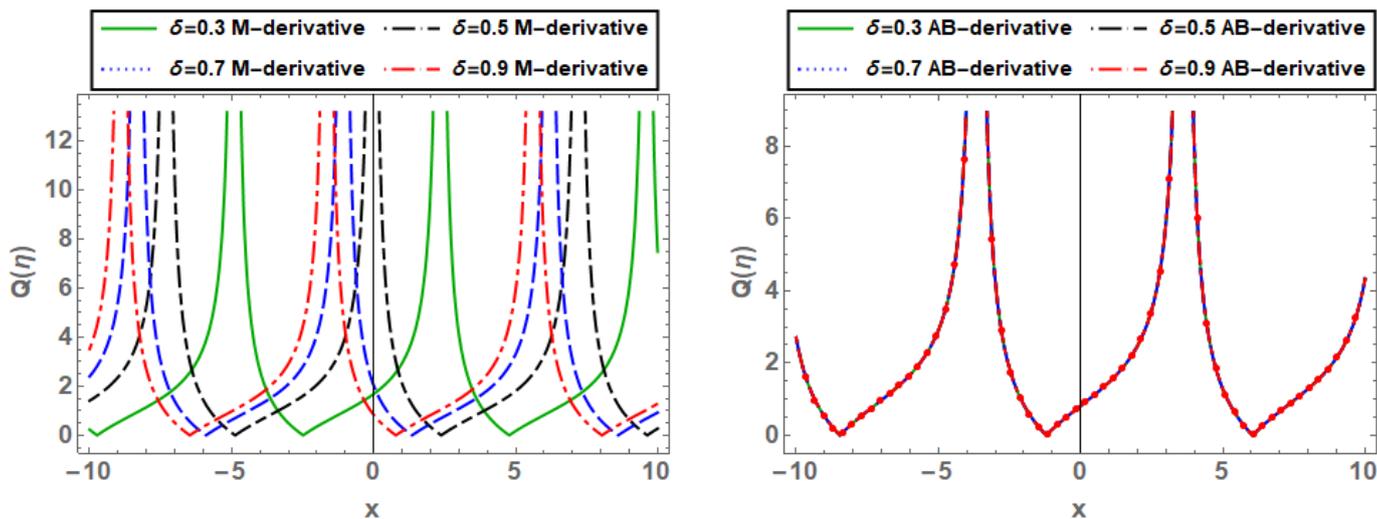
(a) $\delta = 0.9$ (M-Truncated)

(b) $\delta = 0.9$ (AB-derivative)



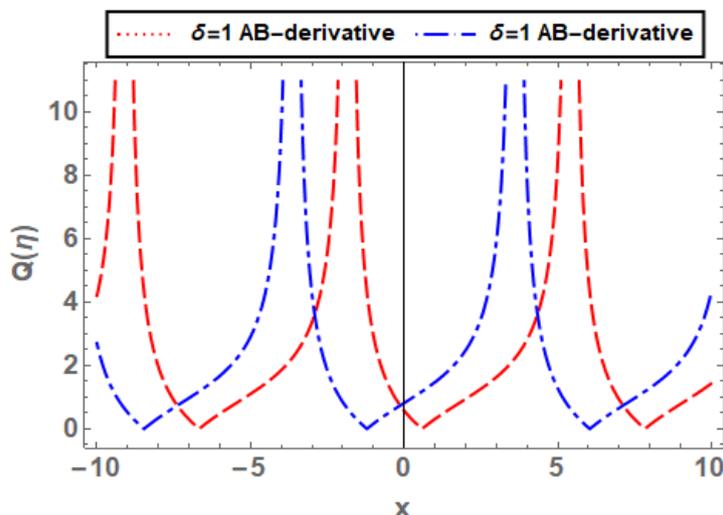
(c) 2D combined effect at $\delta = 0.9$

Figure 9. A graphical representation of $Q_{16}(x, t)$.



(a) Combined effect for different values of δ

(b) Combined effect for different values of δ



(c) Combined effect of $\delta = 1$ for both operators

Figure 10. The 2D graphics of $Q_{16}(x, t)$.

7. Sensitivity Behavior of Fractional MEW Equation

This section discusses the recommended model’s sensitive behavior after it has been converted into a system. Sensitivity is the determination of our system’s sensitivity. A system is lowly sensitive if a slight change in the initial conditions leads to a minor change in the system. As a result, the system is highly sensitive if it changes significantly as a result of a small change in the initial conditions. Accurately assessing the output disruption brought on by input changes is the primary goal of the current investigation. The analysis findings may be shown, which are displayed using a range of parametric values to demonstrate how little changes in input can result in big variances in the outcome. The following is a thorough analysis of the specified system (see Figure 11).

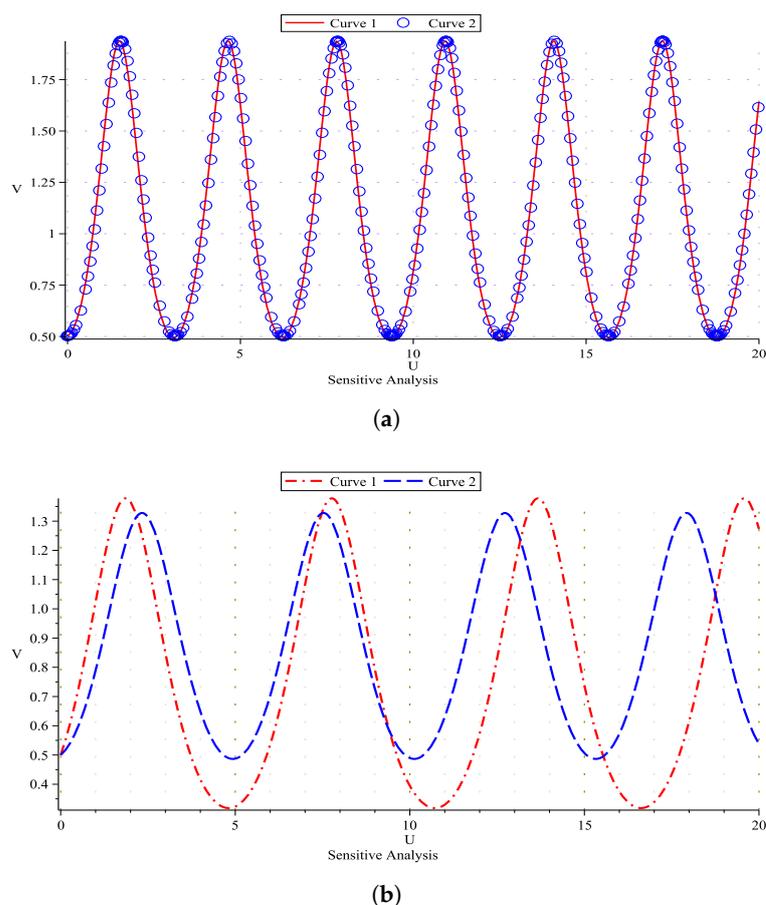


Figure 11. (a) represents the sensitivity analysis of the system by allowing the initial conditions $(U, V) = (0.5, 0.1)$ and $(U, V) = (0.5, 0.01)$ with $\nu = \omega = \rho = 1$, and $\theta = 0.5$. It is essential to note that overlapping between curves can be seen. (b) here now taking initial conditions $(U, V) = (0.5, 0.35)$ and $(U, V) = (0.5, 0.1)$ with $\nu = \omega = \rho = \theta = 1$ and it can be observed that by changing the small input in the initial conditions a large change in the output result is displayed.

8. Conclusions

In this research, soliton solutions of the fractional MEW equation have been explored using M -truncated and AB -derivatives. Several general soliton solutions to the MEW equation have been discovered using the NAEM in this work. Plotting 3D plots and 2D line graphs for various solutions have revealed the influence of fractional derivatives on the derived solutions graphically. It is also seen in the examined solutions that the wave solution obtains a stable shape substantially faster for the fractional derivatives. The pattern of the obtained wave becomes stable when the value of the fractional order of the derivative approaches unity for the fractional derivative according to a comparison of graphs employing fractional parameters. Furthermore, the achieved results demonstrate that the proposed scheme for extracting optical solitons of MEW equations using fractional operators is extremely simple, convenient, and effective. Dark, bright, periodic solitons, and other soliton solutions have been achieved. Further research in the disciplines of fractional calculus and NLEEs is expected to benefit from the results presented here.

Author Contributions: Conceptualization, M.B.R. and R.U.R.; methodology, A.W.; software, G.I.O.; validation, M.B.R., A.W. and G.I.O.; formal analysis, R.U.R.; investigation, M.B.R.; resources, G.I.O.; data curation, A.W.; writing—original draft preparation, R.U.R.; writing—review and editing, M.B.R., A.W., G.I.O. and R.U.R.; visualization, M.B.R. and R.U.R.; supervision, A.W.; project administration, G.I.O.; funding acquisition, G.I.O. All authors have read and agreed to the published version of the manuscript.

Funding: This research received no external funding.

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: Not applicable.

Acknowledgments: The Authors are highly thankful to the respective universities for their support.

Conflicts of Interest: The authors declare no conflict of interest.

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