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Editorial

Special Issue Editorial "Symmetric Distributions, Moments and Applications"

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1. Introduction

In 1933, Kolmogorov published his book, *Foundations of the Theory of Probability*, laying the modern axiomatic foundations of probability theory and establishing his reputation as the world's leading expert in this field. The concept of the probability distribution and the random variables they describe underlies the mathematical discipline of probability theory and the science of statistics. The probability distribution and the random variables serve as mathematical models in many branches of science, complex dynamical systems, population dynamics modeling, finance mathematics, insurance, physical sciences, and any field where stochastic modeling is used. In mathematics, the moments of a function are quantitative measures related to the shape of the function's graph. If the function is a probability distribution, then the first moment is the expected value, the second central moment is the variance, the third standardized moment is the skewness, and the fourth standardized moment is the kurtosis. That is, the moments describe the location (mean), size (variance), and shape (skewness and kurtosis) of a probability density function (PDF). The mathematical concept is closely related to the concept of moment in physics. The classical diffusion equation (heat equation) yields an approximation of the time evolution of the probability density function, associated with the position of the particle going under a Brownian movement under the physical definition.

The continuous time random walk (CTRW) method is the basis for a heuristic explanation of the physical behavior of normal and anomalous diffusion processes. The CTRW method can be characterized by the moments of the random mean motion. If the process is non-local or has a memory waiting time density, then the second moment $\mathbf{E}X^2$ of the random variable X of the jumps is proportional to a power t^α of order α of time, when the time is sufficiently large. This type of stochastic model lets us characterize sub-diffusion, normal diffusion, and super-diffusion, where $0 < \alpha < 1, \quad \alpha = 1$, and $1 < \alpha < 2$ respectively. A well-known method to calculate moments is using moment-generating functions or characteristic functions (CF). Since the coefficients of the Taylor expansion of the CF are related to the integer moments of a random variable, we usually state that the probabilistic description of a random variable may also be given in terms of integer moments. In the 1970s, the fractional moments of the type $\mathbf{E}X^\rho$, $\rho \in \mathbf{R}$, were studied, showing that the knowledge of some $\mathbf{E}X^\rho$ improves the convergence speed of the maximum entropy method. Fractional moments of a non-negative random variable are expressible by the Mellin transform of PDF, and this fact has been widely used in the literature principally in the field of the algebra of random variables [1]. That is, the Mellin transform is the principal mathematical tool to handle problems involving products and quotients of independent random variables. Another research direction on Mellin applications in probability is represented by the use of special functions such as the Mittag–Leffler, H-Fox, and Maijer's G-functions, due principally to A.M. Mathai and his co-workers. Such functions are indeed representable as Mellin–Barnes integrals of the product of gamma functions and are therefore suited to represent statistics of products and quotients of independent random variables whose fractional moments are expressible as gamma or gamma-related functions.

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The stable distributions are a fascinating and fruitful area of research in probability theory; furthermore, nowadays, they provide valuable models in physics, astronomy, economics, and communication theory. The general class of stable distributions was introduced and given this name by the French mathematician Paul Levy in the early 1920s; see Levy (1924, 1925). The inspiration for Levy was the desire to generalize the celebrated Central Limit Theorem, according to which any probability distribution with finite variance belongs to the domain of attraction of Gaussian distribution. Formerly, the topic attracted only moderate attention from leading experts, though there were also enthusiasts, of whom the Russian mathematician Alexander Yakovlevich Khintchine should be mentioned first of all. The concept of stable distributions took full shape in 1937 with the appearance of Levy's monograph (see Levy (1937–1954)), soon followed by Khintchine's monograph (1938). We can now cite the paper by Mainardi, Luchko, and Pagnini (2001), where the reader can find (convergent and asymptotic) representations and plots of the symmetric and non-symmetric stable densities generated by fractional diffusion equations.

This Special Issue includes seven papers with original results of symmetric random walks and their characterization, stochastic processes, computational number theory, stochastic integrals, probability inequalities, statistics parameter estimation, entropy, Stochastic differential equations, finance mathematics, optimization, information theory, Bayesian methods, Monte Carlo methods, etc.

In the paper "Convolutions for Bernoulli and Euler–Genocchi Polynomials of Order (r,m) and Their Probabilistic Interpretation", R. Frontczak and Z. Tomovski [2] introduced a new class of extended Bernoulli and Euler–Genocci polynomials of order (r,m) for which some convolutions, recurrence formulas, and combinatorial sums are presented. By using the concept of moment-generating functions, it is shown that the Bernoulli and Euler–Genocci polynomials can be expressed as moments of order n for some discrete random variables in the standard probability space. A new PDF associated with Bernoulli numbers is defined for which the mathematical expectation is calculated.

In the paper "A Flexible Extension to an Extreme Distribution", published by Mohamed S.Eliwa et al. [3], a new flexible extension of an extreme distribution with three parameters has been proposed, which generalizes the inverse exponential distribution. Furthermore, it can be utilized for modeling asymmetric "positive and negative" as well as symmetric datasets and can be used to model over- and under-dispersed data. Statistical and reliability properties of the extreme distribution, such as quantile function, skewness, kurtosis, incomplete moments, and entropy, are presented. The model parameters have been estimated utilizing the maximal likelihood approach. Finally, four data applications that illustrate the flexibility of the new extension and its excellence over other models have also been analyzed.

The paper "Taming Tail Risk: Regularized Multiple β Worst-Case CVaR Portfolio" published by Kei Nakagawa and Katsuya Ito [4] contains an optimization problem reduced to a linear programming problem, using mixture probability distributions as well as semi-nonparametric distribution. They performed experiments on well-known benchmarks in finance to evaluate the proposed portfolio. Their portfolio shows superior performance in terms of having both higher risk-adjusted returns and lower maximum drawdown despite the lower turnover rate.

The goal of the paper "Inventory Models for Non-Instantaneous Deteriorating Items with Expiration Dates and Imperfect Quality under Hybrid Payment Policy in the Three-Level Supply Chain" [5] is to determine an optimal replenishment cycle and the total annual cost function by exploring the functional properties of the total annual cost function and showing that the total annual cost function is convex. Theoretical analysis of the optimal properties shows the existence and uniqueness of the optimal solution. Then, the authors obtained simple and easy solution procedures for the inventory system. Moreover, numerical analysis of the inventory model was conducted, and the corresponding examples are considered with the ai of illustrating the application of the supply-chain model that is investigated in this article. The authors have established a sustainable inventory system in which the retailer sells the non-instantaneous deteriorating item that is fully deteriorated close to its expiry date and has imperfect quality such as those in seasonal products, food products, electronic components, and others. In order to manage the quality of the items, an inspection will occur during the state in which there is no deterioration. On the other hand, the supplier demands the retailer a distinct payment scheme, such as partial prepayment or cash and trade credit; in turn, the retailer grants customers partial cash and trade credit. The paper also presents convexity and monotonicity properties to develop efficient decision rules for the optimal replenishment cycle time T^*.

In the next paper, "An Alternate Generalized Odd Generalized Exponential Family with Applications to Premium Data" [6] an exponentiated odd generalized exponential (OGE2-G) class of distribution is proposed and studied with some mathematical properties such as ordinary and incomplete moments, mean deviations, Rényi entropy, and generating functions. The maximum likelihood (MLL) approach is used to estimate the model parameters. Then, the authors focussed their attention on one of the special members of the family defined with the Fréchet distribution, called the OGE2Fr distribution. They established the optimized the maximum likelihood methodology in particular, with the goal of effectively estimating model parameters, and validated their convergence by a simulation study, ensuring that the projections have asymptotic properties. The authors evaluated the sensitivity of the method of estimations using the MLL of OGE2Fr distribution parameters using the Monte Carlo simulation technique.

Stress-strength reliability, $R = P(X < Y)$, has been extensively investigated as a stress-strength model, and the research has also been extended to multi-component systems. For numerous statistical models, several scholars have examined the estimation of the stress-strength parameter. Several authors discussed Bayesian and maximum-likelihood estimation methods of reliability for point estimation of the parameter model. The authors Ehab M. Almetwally et al. in the paper "Optimal Plan of Multi-Stress–Strength Reliability Bayesian and Non-Bayesian Methods for the Alpha Power Exponential Model Using Progressive First Failure" [7] considered the inference for multi-reliability using unit alpha power exponential distributions for stress–strength variables based on the progressive first failure. The Fisher information and confidence intervals such as asymptotic, boot-p, and boot-t methods are also examined. Various optimal criteria were found. Monte Carlo simulations and real-world application examples were used to evaluate and compare the performance of the various proposed estimators.

The stochastic differential equation has been used to model various phenomena and investigate their properties, such as the moments, variance, and conditional moments, which are beneficial for estimating parameters that play significant roles in several practical applications. For example, financial derivative prices, such as moment swaps, can be obtained by calculating the conditional moments of their payoffs under the risk-neutral measure.

In "Simple Closed-Form Formulas for Conditional Moments of Inhomogeneous Nonlinear Drift Constant Elasticity of Variance Process" [8], the authors presented closed-form expressions for conditional moments of the inhomogeneous, nonlinear, drift-constant elasticity of variance (IND-CEV) process, without having a condition on eigenfunctions or the transition PDF. The analytical results are examined through Monte Carlo simulations.

This volume will be of interest to mathematicians, physicists, and engineers interested in probability theory, statistics, complex systems, finance mathematics, and insurance.

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References

1. Tomovski, Z.; Metzler, R.; Gerhold, S. Fractional characteristic functions, and a fractional calculus approach for moments of random variables. *Fract. Calc. Appl. Anal.* **2022**, *25*, 1307–1323. [CrossRef]
2. Frontczak, R.; Tomovski, Z. Convolutions for Bernoulli and Euler-Genocchi Polynomials of order (r,m) and Their Probabilistic Interpretation. *Symmetry* **2022**, *14*, 1220. [CrossRef]
3. Eliwa, M.; Alshammari, F.; Abualnaja, K.; El-Morshedy, M. A Flexible Extension to an Extreme Distribution. *Symmetry* **2021**, *13*, 745. [CrossRef]
4. Nakagawa, K.; Ito, K. Taming Tail Risk: Regularized Multiple b Worst-Case CVaR Portfolio. *Symmetry* **2021**, *13*, 922. [CrossRef]
5. Liao, J.-J.; Srivastava, H.M.; Chung, K.-J.; Lee, S.-F.; Huang, K.-N.; Lin, S.-D. Inventory models for Non-Instantaneous Deteriorating Items with Expiration Dates and Imperfect Quality under Hybrid Payment Policy in the Three-Level Supply Chain. *Symmetry* **2021**, *13*, 1695. [CrossRef]
6. Khan, S.; Balogun, O.S.; Tahir, M.H.; Almutiry, W.; Alahmadi, A.A. An alternate generalized odd generalized exponential family with applications to premium data. *Symmetry* **2021**, *13*, 2064. [CrossRef]
7. Almetwally, E.M.; Alotaibi, R.; Mutairi, A.A.; Park, C.; Rezk, H. Optimal plan of multi-stress-strength Reliability Bayesian and Non-Bayesian methods for the Alpha Power Exponential Model Using Progressive First Failure. *Symmetry* **2022**, *14*, 1306. [CrossRef]
8. Chumpong, K.; Tanadkithirun, R.; Tantiwattanapaibul, C. Simple Closed-Form Formulas for Conditional Moments of Inhomogeneous Nonlinear Drift Constant Elasticity of Variance Process. *Symmetry* **2022**, *14*, 1345. [CrossRef]