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A Note on Modified Degenerate Changhee–Genocchi Polynomials of the Second Kind

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Abstract: In this study, we introduce modified degenerate Changhee–Genocchi polynomials of the second kind, and analyze some properties by providing several relations and applications. We first attain diverse relations and formulas covering addition formulas, recurrence rules, implicit summation formulas, and relations with the earlier polynomials in the literature. By using their generating function, we derive some new relations, including the Stirling numbers of the first and second kinds. Moreover, we introduce modified higher-order degenerate Changhee–Genocchi polynomials of the second kind. We also derive some new identities and properties of this type of polynomials.

Keywords: Genocchi polynomials and numbers; modified degenerate Changhee–Genocchi polynomials; higher-order modified degenerate Changhee–Genocchi polynomials and numbers

MSC: 11B83; 11B73; 05A19

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1. Introduction

Many researchers [1–5] defined and constructed generating maps for novel families of special polynomials, such as Bernoulli, Euler, and Genocchi by utilizing Changhee and Changhee–Genocchi polynomials. These studies provided fundamental properties and diverse applications for these polynomials. For instance, not only several explicit and implicit summation formulas, recurrence formulas, symmetric properties, and many correlations with the well-known polynomials in the literature have been derived intensely, but we also derived some beautiful correlations between some special polynomials. Additionally, the aforementioned polynomials allow for the derivation of utility properties in a quite basic procedure and assist in defining the novel families of special polynomials. By motivating the above, here, we introduce modified degenerate Changhee–Genocchi polynomials of the second kind, and analyze some properties by providing several relations and applications. We first attain diverse relations and formulas covering addition formulas, recurrence rules, implicit summation formulas, and relations with the earlier polynomials in the literature.

The ordinary Bernoulli, Euler and Genocchi polynomials are defined by (see [6–8]):

$$\frac{\omega}{e^{\omega}-1}e^{\xi\omega}=\sum_{v=0}^{\infty}\mathbb{B}_{v}(\xi)\frac{\omega^{v}}{v!}\quad |\,\omega\,|<2\pi, \tag{1}$$

$$\frac{2}{e^{\omega}+1}e^{\xi\omega}=\sum_{v=0}^{\infty}\mathbb{E}_{v}(\xi)\frac{\omega^{v}}{v!}\quad |\,\omega\,|<\pi, \tag{2}$$

and

$$\frac{2\omega}{e^{\omega}+1}e^{\xi\omega}=\sum_{v=0}^{\infty}\mathbb{G}_{v}(\xi)\frac{\omega^{v}}{v!}\quad |\,\omega\,|<\pi. \tag{3}$$

In the case when $\xi = 0$, $\mathbb{B}_v = \mathbb{B}_v(0)$, $\mathbb{E}_v = \mathbb{E}_v(0)$ and $\mathbb{G}_v = \mathbb{G}_v(0)$, $(v \in \mathbb{N}_0)$ are called the ordinary Bernoulli Euler and Genocchi numbers.

We note that

$$\mathbb{G}_0(\xi) = 0, \quad \mathbb{E}_v(\xi) = \frac{\mathbb{G}_{v+1}(\xi)}{v+1} \ (v \geq 0).$$

Stirling numbers of the first kind are given by (see [1,6,9–13]):

$$\frac{1}{k!}(\log(1+\omega))^k = \sum_{v=k}^{\infty} S_1(v,k)\frac{\omega^v}{v!} \ (k \geq 0). \tag{4}$$

Stirling numbers of the second kind are given by (see [2,3,14–20]):

$$\frac{1}{k!}(e^{\omega}-1)^k = \sum_{v=k}^{\infty} S_2(v,k)\frac{\omega^v}{v!} \ (k \geq 0). \tag{5}$$

The Daehee polynomials are defined by (see [18]):

$$\frac{\log(1+\omega)}{\omega}(1+\omega)^{\xi} = \sum_{v=0}^{\infty} \mathbb{D}_v(\xi)\frac{\omega^v}{v!}. \tag{6}$$

When $\xi = 0$, $\mathbb{D}_v = \mathbb{D}_v(0)$ are called the Daehee numbers. We find that

$$\mathbb{D}_v = (-1)^v \frac{v!}{v+1} \ (v \in \mathbb{N}_0).$$

The first few are

$$\mathbb{D}_0 = 1, \mathbb{D}_1 = -\frac{1}{2}, \mathbb{D}_2 = \frac{2}{3}, \mathbb{D}_3 = -\frac{3}{2}, \cdots.$$

The Changhee polynomials are defined by (see [14]):

$$\frac{2}{2+\omega}(1+\omega)^{\xi} = \sum_{v=0}^{\infty} Ch_v(\xi)\frac{\omega^v}{v!}. \tag{7}$$

When $\xi = 0$, $Ch_v = Ch_v(0)$, $(v \in \mathbb{N}_0)$ are called the Changhee numbers.

Changhee–Genocchi polynomials are defined via generating function (see [10])

$$\frac{2\log(1+\omega)}{2+\omega}(1+\omega)^{\xi} = \sum_{v=0}^{\infty} CG_v(\xi)\frac{\omega^v}{v!}. \tag{8}$$

When $\xi = 0$, $CG_v = CG_v(0)$ are called the Changhee–Genocchi numbers.

Recently, Kim et al. [16] introduced modified Changhee–Genocchi polynomials defined by

$$\frac{2\omega}{2+\omega}(1+\omega)^{\xi} = \sum_{v=0}^{\infty} CG^*_v(\xi)\frac{\omega^v}{v!}. \tag{9}$$

When $\xi = 0$, $CG^*_v = CG^*_v(0)$ are called the modified Changhee–Genocchi numbers.

The Bernoulli numbers of the second kind are defined by (see [11]):

$$\frac{\omega}{\log(1+\omega)} = \sum_{v=0}^{\infty} b_v \frac{\omega^v}{v!} \ (v \in \mathbb{N}_0). \tag{10}$$

Via (10), we see that

$$\left(\frac{\omega}{\log(1+\omega)}\right)^{r}(1+\omega)^{\xi-1}=\sum_{v=0}^{\infty}\mathbb{B}_{v}^{(v-r+1)}(\xi)\frac{\omega^{v}}{v!},\tag{11}$$

where $\mathbb{B}_{v}^{(r)}(\xi)$ are the higher-order Bernoulli polynomials defined by

$$\left(\frac{\omega}{e^{\omega}-1}\right)^{r}e^{\xi\omega}=\sum_{v=0}^{\infty}\mathbb{B}_{v}^{(r)}(\xi)\frac{\omega^{v}}{v!}.\tag{12}$$

For $\xi=1$ and $r=1$ in (11) and (12), we obtain

$$b_{v}=\mathbb{B}_{v}^{(v)}(1).$$

The degenerate Changhee–Genocchi polynomials are defined by (see [15]):

$$\frac{2\lambda\log\left(1+\frac{1}{\lambda}\log(1+\lambda\omega)\right)}{2\lambda+\log(1+\lambda\omega)}\left(1+\frac{1}{\lambda}\log(1+\lambda\omega)\right)^{\xi}=\sum_{v=0}^{\infty}CG_{v,\lambda}(\xi)\frac{\omega^{v}}{v!}.\tag{13}$$

Via (8) and (13), we see that

$$\lim_{\lambda\to0}CG_{v,\lambda}(\xi)=CG_{v}(\xi)\ (v\geq0).$$

The modified degenerate Changhee–Genocchi polynomials are defined by (see [10]):

$$\frac{2\lambda\omega}{2\lambda+\log(1+\lambda\omega)}\left(1+\frac{1}{\lambda}\log(1+\lambda\omega)\right)^{\xi}=\sum_{v=0}^{\infty}CG_{v,\lambda}^{*}(\xi)\frac{\omega^{v}}{v!}.\tag{14}$$

From (9) and (14), we have

$$\lim_{\lambda\to0}CG_{v,\lambda}^{*}(\xi)=CG_{v}^{*}(\xi)\ (v\geq0).$$

Replacing ω by $\frac{1}{\lambda}(e^{\lambda\omega}-1)$ in (13), we obtain

$$\begin{aligned}\frac{2\log(1+\omega)}{2+\omega}(1+\omega)^{\xi}&=\sum_{\sigma=0}^{\infty}CG_{\sigma,\lambda}(\xi)\lambda^{-\sigma}\frac{1}{\sigma!}(e^{\lambda\omega}-1)^{\sigma}\\&=\sum_{\sigma=0}^{\infty}CG_{\sigma,\lambda}\lambda^{-\sigma}\sum_{v=\sigma}^{\infty}S_{2}(v,\sigma)\lambda^{v}\frac{\omega^{v}}{v!}\\&=\sum_{v=0}^{\infty}\left(\sum_{\sigma=0}^{v}CG_{\sigma,\lambda}\lambda^{v-\sigma}S_{2}(v,\sigma)\right)\frac{\omega^{v}}{v!}.\end{aligned}\tag{15}$$

Thus, from (8) and (15), we obtain

$$CG_{v}(\xi)=\sum_{\sigma=0}^{v}CG_{\sigma,\lambda}\lambda^{v-\sigma}S_{2}(v,\sigma)\ \ (v\geq0).\tag{16}$$

The degenerate Changhee polynomials (or λ-Changhee polynomials) are defined by (see [19]):

$$\frac{2\lambda}{2\lambda+\log(1+\lambda\omega)}\left(1+\frac{1}{\lambda}\log(1+\lambda\omega)\right)^{\xi}=\sum_{v=0}^{\infty}Ch_{v,\lambda}(\xi)\frac{\omega^{v}}{v!}.\tag{17}$$

When $\xi=0$, $Ch_{v,\lambda}=Ch_{v,\lambda}(0)$, $(v\in\mathbb{N}_{0})$ are called the degenerate Changhee numbers.

The higher-order partially degenerate Changhee–Genocchi polynomials are defined by (see [17]):

$$\left(\frac{2\log(1+\omega)}{2+\log(1+\lambda\omega)^{\frac{1}{\lambda}}}\right)^{k}\left(1+\log(1+\lambda\omega)^{\frac{1}{\lambda}}\right)^{\xi}=\sum_{v=0}^{\infty}\widehat{CG}^{(k)}_{v,\lambda}(\xi)\frac{\omega^{v}}{v!}. \tag{18}$$

When $\xi=0$, $\widehat{CG}^{(k)}_{v,\lambda}=\widehat{CG}^{(k)}_{v,\lambda}(0)$ are called the higher-order partially degenerate Changhee–Genocchi numbers.

Inspired by the works of Kim and Kim [10,17], in this paper, we define modified degenerate Changhee–Genocchi numbers and polynomials of the second kind, investigate some new properties of these numbers and polynomials, and derive some new identities and relations between the modified degenerate Changhee–Genocchi numbers and polynomials of the second kind. We also derive higher-order modified degenerate Changhee–Genocchi polynomials and construct relations between some beautiful special polynomials and numbers.

2. Modified Degenerate Changhee–Genocchi Polynomials of the Second Kind

In this section, we introduce modified degenerate Changhee–Genocchi polynomials of the second kind, and investigate some explicit expressions for degenerate Changhee–Genocchi polynomials and numbers of the second kind. We begin with the following definition.

For $\lambda\in\mathbb{R}$, we consider the modified degenerate Changhee–Genocchi polynomials of the second kind, defined by means of the following generating function:

$$\frac{2\log(1+\lambda\omega)^{\frac{1}{\lambda}}}{2+\log(1+\lambda\omega)^{\frac{1}{\lambda}}}(1+\log(1+\lambda\omega)^{\frac{1}{\lambda}})^{\xi}=\sum_{v=0}^{\infty}CG^{*}_{v,\lambda,2}(\xi)\frac{\omega^{v}}{v!}. \tag{19}$$

At point $\xi=0, CG^{*}_{v,\lambda}=CG^{*}_{v,\lambda}(0)$, $(v\in\mathbb{N}_0)$ are called the modified degenerate Changhee–Genocchi numbers of the second kind. Here, the function $\log(1+\lambda\omega)^{\frac{1}{\lambda}}$ is called the degenerate function of ω.

We note that

$$\begin{aligned}\sum_{v=0}^{\infty}\lim_{\lambda\to 0}CG^{*}_{v,\lambda,2}(\xi)\frac{\omega^{v}}{v!}&=\lim_{\lambda\to 0}\frac{2\log(1+\lambda\omega)^{\frac{1}{\lambda}}}{2+\log(1+\lambda\omega)^{\frac{1}{\lambda}}}(1+\log(1+\lambda\omega)^{\frac{1}{\lambda}})^{\xi}\\&=\frac{2\omega}{2+\omega}(1+\omega)^{\xi}=\sum_{v=0}^{\infty}CG^{*}_{v}(\xi)\frac{\omega^{v}}{v!}.\end{aligned} \tag{20}$$

From (19) and (20), we have

$$\lim_{\lambda\to 0}CG^{*}_{v,\lambda,2}(\xi)=CG^{*}_{v}(\xi)\ \ (v\geq 0).$$

Theorem 1. *For $v\geq 0$, we have*

$$CG^{*}_{v}(\xi)=\sum_{\sigma=0}^{v}CG^{*}_{\sigma,\lambda,2}(\xi)\lambda^{v-\sigma}S_2(v,\sigma). \tag{21}$$

Proof. Replacing ω by $\frac{1}{\lambda}(e^{\lambda\omega}-1)$ in (19) and using (5), we obtain

$$\frac{2\omega}{2+\omega}(1+\omega)^{\xi}=\sum_{\sigma=0}^{\infty}CG^{*}_{\sigma,\lambda}(\xi)\lambda^{-\sigma}\frac{1}{\sigma!}(e^{\lambda\omega}-1)^{m}$$

$$= \sum_{\sigma=0}^{\infty} CG^{*}_{\sigma,\lambda,2}(\xi)\lambda^{-\sigma} \sum_{v=\sigma}^{\infty} S_2(v,\sigma)\lambda^{v}\frac{\omega^v}{v!}$$

$$= \sum_{v=0}^{\infty}\left(\sum_{\sigma=0}^{v} CG^{*}_{\sigma,\lambda,2}(\xi)\lambda^{v-\sigma}S_2(v,\sigma)\right)\frac{\omega^v}{v!}. \tag{22}$$

Therefore, via (22), we obtain the result. □

Theorem 2. *For $v \geq 0$, we have*

$$CG^{*}_{v,\lambda,2}(\xi) = \sum_{\sigma=0}^{v} CG^{*}_{\sigma,\lambda,2}(\xi)S_1(v,\sigma)\lambda^{v-\sigma}. \tag{23}$$

Proof. By using (4) and (19), we see that

$$\frac{2\log(1+\lambda\omega)^{\frac{1}{\lambda}}}{2+\log(1+\lambda\omega)^{\frac{1}{\lambda}}}(1+\log(1+\lambda\omega)^{\frac{1}{\lambda}})^{\xi} = \sum_{\sigma=0}^{\infty} CG^{*}_{\sigma,\lambda}(\xi)\frac{1}{\sigma!}\left(\frac{1}{\lambda}\log(1+\lambda\omega)\right)^{\sigma}$$

$$= \sum_{\sigma=0}^{\infty} CG^{*}_{\sigma,\lambda}(\xi)\sum_{v=\sigma}^{\infty} S_1(v,\sigma)\lambda^{v-\sigma}\frac{\omega^v}{v!}$$

$$= \sum_{v=0}^{\infty}\left(\sum_{\sigma=0}^{v} CG^{*}_{\sigma,\lambda}(\xi)S_1(v,\sigma)\lambda^{v-\sigma}\right)\frac{\omega^v}{v!}. \tag{24}$$

Therefore, via (19) and (24), we obtain the result. □

Theorem 3. *For $v \geq 0$, we have*

$$CG^{*}_{v,\lambda}(\xi) = \sum_{\sigma=0}^{v}\sum_{\rho=0}^{\sigma}\binom{v}{\sigma}(\xi)_{\rho}\lambda^{\sigma-\rho}S_1(\sigma,\rho)CG^{*}_{v-\sigma,\lambda}. \tag{25}$$

Proof. Through (4) and (19), we obtain

$$\sum_{v=0}^{\infty} CG^{*}_{v,\lambda,2}(\xi)\frac{\omega^v}{v!} = \frac{2\log(1+\lambda\omega)^{\frac{1}{\lambda}}}{2+\log(1+\lambda\omega)^{\frac{1}{\lambda}}}(1+\log(1+\lambda\omega)^{\frac{1}{\lambda}})^{\xi}$$

$$= \sum_{v=0}^{\infty} CG^{*}_{v,\lambda,2}\frac{\omega^v}{v!}\sum_{\sigma=0}^{\infty}\binom{\xi}{\sigma}\left(\log(1+\lambda\omega)^{\frac{1}{\lambda}}\right)^{\sigma}$$

$$= \sum_{v=0}^{\infty} CG^{*}_{v,\lambda,2}\frac{\omega^v}{v!}\sum_{\sigma=0}^{\infty}(\xi)_{\sigma}\frac{1}{\sigma!}\lambda^{-\sigma}(\log(1+\lambda\omega))^{\sigma}$$

$$= \sum_{v=0}^{\infty} CG^{*}_{v,\lambda,2}\frac{\omega^v}{v!}\sum_{\sigma=0}^{\infty}(\xi)_{\sigma}\lambda^{-\sigma}\sum_{\sigma=\rho}^{\infty} S_1(\sigma,\rho)\lambda^{\sigma}\frac{\omega^{\sigma}}{\sigma!}$$

$$= \left(\sum_{v=0}^{\infty} CG^{*}_{v,\lambda,2}\frac{\omega^v}{v!}\right)\left(\sum_{\sigma=0}^{\infty}\sum_{\rho=0}^{\sigma}(\xi)_{\rho}\lambda^{\sigma-\rho}S_1(\sigma,\rho)\frac{\omega^{\sigma}}{\sigma!}\right)$$

$$= \sum_{v=0}^{\infty}\left(\sum_{\sigma=0}^{v}\sum_{\rho=0}^{\sigma}\binom{v}{\sigma}(\xi)_{\rho}\lambda^{\sigma-\rho}S_1(\sigma,\rho)CG^{*}_{v-\sigma,\lambda,2}\right)\frac{\omega^v}{v!}. \tag{26}$$

Therefore, via (19) and (27), we obtain at the required result. □

Theorem 4. *For $v \geq 0$, we have*

$$CG^{*}_{v,\lambda,2}(\xi) = \sum_{\sigma=0}^{v} CG^{*}_{\sigma}(\xi)\lambda^{v-\sigma}S_1(v,\sigma). \tag{27}$$

Proof. Replacing ω by $\log(1+\lambda\omega)^{\frac{1}{\lambda}}$ in (9) and applying (4), we obtain

$$\frac{2\log(1+\lambda\omega)^{\frac{1}{\lambda}}}{2+\log(1+\lambda\omega)^{\frac{1}{\lambda}}}(1+\log(1+\lambda\omega)^{\frac{1}{\lambda}})^{\omega}\xi = \sum_{\sigma=0}^{\infty} CG^{*}_{\sigma}(\xi)\frac{1}{\sigma!}(\log(1+\lambda\omega)^{\frac{1}{\lambda}})^{\sigma}$$

$$= \sum_{\sigma=0}^{\infty} CG^{*}_{\sigma}(\xi)\lambda^{v-\sigma}\sum_{v=\sigma}^{\infty} S_1(v,\sigma)\frac{\omega^{v}}{v!}$$

$$= \sum_{v=0}^{\infty}\left(\sum_{\sigma=0}^{v} CG^{*}_{\sigma}(\xi)\lambda^{v-\sigma}S_1(v,\sigma)\right)\frac{\omega^{v}}{v!}. \tag{28}$$

By using (19) and (28), we acquire at the desired result. □

Theorem 5. *For $v \geq 0$, we have*

$$CG^{*}_{v,\lambda,2}(\xi) = \sum_{\sigma=0}^{v}\binom{v}{\sigma}CG_{v-\sigma,\lambda}(\xi)\mathbb{D}_{\sigma}\lambda^{\sigma}. \tag{29}$$

Proof. From (14) and (19), we note that

$$\sum_{v=0}^{\infty} CG^{*}_{v,\lambda,2}(\xi)\frac{\omega^{v}}{v!} = \frac{2\log(1+\lambda\omega)^{\frac{1}{\lambda}}}{2+\log(1+\lambda\omega)^{\frac{1}{\lambda}}}(1+\log(1+\lambda\omega)^{\frac{1}{\lambda}})^{\xi}$$

$$= \frac{2\omega}{2+\log(1+\lambda\omega)^{\frac{1}{\lambda}}}(1+\log(1+\lambda\omega)^{\frac{1}{\lambda}})^{\xi}\frac{\log(1+\lambda\omega)}{\lambda\omega}$$

$$= \sum_{v=0}^{\infty} CG^{*}_{v,\lambda}(\xi)\frac{\omega^{v}}{v!}\sum_{\sigma=0}^{\infty}\mathbb{D}_{\sigma}\lambda^{\sigma}\frac{\omega^{\sigma}}{\sigma!}$$

$$= \sum_{v=0}^{\infty}\left(\sum_{\sigma=0}^{v}\binom{v}{\sigma}CG^{*}_{v-\sigma,\lambda}(\xi)\mathbb{D}_{\sigma}\lambda^{\sigma}\right)\frac{\omega^{v}}{v!}. \tag{30}$$

Therefore, via (19) and (30), we obtain the result. □

Theorem 6. *For $v \geq 0$, we have*

$$\mathbb{G}_{v}(\xi) = \sum_{r=0}^{v}\sum_{l=0}^{r}\sum_{\sigma=0}^{l}\binom{v}{r}CG^{*}_{\sigma,\lambda,2}(\xi)\lambda^{l-\sigma}S_2(l,\sigma)S_2(r,l)\mathbb{B}_{v-r}. \tag{31}$$

Proof. Replacing ω by $e^{\omega}-1$ in (22) and using Equation (1), we obtain

$$\frac{2(e^{\omega}-1)}{e^{\omega}+1}e^{\xi\omega} = \sum_{l=0}^{\infty}\sum_{\sigma=0}^{l} CG^{*}_{\sigma,\lambda,2}(\xi)\lambda^{l-\sigma}S_2(l,\sigma)\frac{1}{l!}(e^{\omega}-1)^{l}$$

$$\frac{2\omega}{e^{\omega}+1}e^{\xi\omega} = \frac{\omega}{e^{\omega}-1}\sum_{r=0}^{\infty}\sum_{l=0}^{r}\sum_{\sigma=0}^{l} CG^{*}_{\sigma,\lambda}(\xi)\lambda^{l-\sigma}S_2(l,\sigma)S_2(r,l)\frac{\omega^{r}}{r!}$$

$$= \sum_{v=0}^{\infty}\left(\sum_{r=0}^{v}\sum_{l=0}^{r}\sum_{\sigma=0}^{l}\binom{v}{r}CG^{*}_{\sigma,\lambda,2}(\xi)\lambda^{l-\sigma}S_2(l,\sigma)S_2(r,l)\mathbb{B}_{v-r}\right)\frac{\omega^{v}}{v!}. \tag{32}$$

Through (3) and (32), we obtain the result. □

Theorem 7. *For $v \geq 0$, we have*

$$CG_v^*(\xi) = \sum_{l=0}^{v} \sum_{\sigma=0}^{l} CG_{\sigma,\lambda}^*(\xi) S_1(l,\sigma) \lambda^{v-\sigma} S_2(v,l). \tag{33}$$

Proof. Replacing ω by $\frac{1}{\lambda}(e^{\lambda\omega} - 1)$ in (24), we obtain

$$\frac{2\omega}{2+\omega}(1+\omega)^{\xi}$$

$$= \sum_{l=0}^{\infty} \sum_{\sigma=0}^{l} CG_{\sigma,\lambda}^*(\xi) S_1(l,\sigma) \lambda^{l-\sigma} \lambda^{-l} \frac{1}{l!}(e^{\lambda\omega} - 1)^l$$

$$= \sum_{l=0}^{\infty} \sum_{\sigma=0}^{l} CG_{\sigma,\lambda}^*(\xi) S_1(l,\sigma) \lambda^{l-\sigma} \lambda^{-l} \sum_{v=l}^{\infty} S_2(v,l) \lambda^v \frac{\omega^v}{v!}$$

$$= \sum_{v=0}^{\infty} \left(\sum_{l=0}^{v} \sum_{\sigma=0}^{l} CG_{\sigma,\lambda}^*(\xi) S_1(l,m) \lambda^{v-\sigma} S_2(v,l) \right) \frac{\omega^v}{v!}. \tag{34}$$

Therefore, via (9) and (28), we obtain the result. □

Theorem 8. *For $v \geq 0$, we have*

$$CG_{v,\lambda}^*(\xi) = \sum_{l=0}^{v} \binom{v}{l} b_{v-l} \lambda^{v-l} CG_{l,\lambda,2}^*(\xi), \tag{35}$$

where b_v are called the Bernoulli polynomials of the second kind (see Equation (10)).

Proof. Using (14) and (19), we have

$$\sum_{v=0}^{\infty} CG_{v,\lambda}^*(\xi) \frac{\omega^v}{v!} = \frac{2\omega}{2 + \log(1+\lambda\omega)^{\frac{1}{\lambda}}} (1 + \log(1+\lambda\omega)^{\frac{1}{\lambda}})^{\xi}$$

$$= \frac{\lambda\omega}{\log(1+\lambda\omega)} \frac{2\log(1+\lambda\omega)^{\frac{1}{\lambda}}}{2 + \log(1+\lambda\omega)^{\frac{1}{\lambda}}} (1 + \log(1+\lambda\omega)^{\frac{1}{\lambda}})^{\xi}$$

$$= \left(\sum_{v=0}^{\infty} b_v \lambda^v \frac{\omega^v}{v!} \right) \left(\sum_{l=0}^{\infty} CG_{l,\lambda,2}^*(\xi) \frac{\omega^l}{l!} \right)$$

$$= \left(\sum_{v=0}^{\infty} b_v \lambda^v \frac{\omega^v}{v!} \right) \left(\sum_{l=0}^{\infty} CG_{l,\lambda,2}^*(\xi) \frac{\omega^l}{l!} \right)$$

$$= \sum_{v=0}^{\infty} \left(\sum_{l=0}^{v} \binom{v}{l} b_{v-l} \lambda^{v-l} CG_{l,\lambda,2}^*(\xi) \right) \frac{\omega^v}{v!}. \tag{36}$$

Via (19) and (36), we obtain the result. □

Theorem 9. *For $v \geq 0$, we have*

$$2\mathbb{D}_v \lambda^v = CG_{v+1,\lambda}^* \frac{2}{v+1} + \sum_{\sigma=0}^{v} \binom{v}{\sigma} \mathbb{D}_{\sigma} \lambda^{\sigma} CG_{v-\sigma,\lambda,2}^*. \tag{37}$$

Proof. From (19), we have

$$2\log(1+\lambda\omega)^{\frac{1}{\lambda}} = (2+\log(1+\lambda\omega)^{\frac{1}{\lambda}})\sum_{\omega=0}^{\infty} CG^{*}_{v,\lambda,2}\frac{\omega^{v}}{v!}$$

$$2\sum_{v=0}^{\infty}\mathbb{D}_{v}\lambda^{v}\frac{\omega^{v}}{v!} = 2\sum_{v=1}^{\infty} CG^{*}_{v+1,\lambda,2}\frac{1}{v+1}\frac{\omega^{v}}{v!} + \frac{\log(1+\lambda\omega)^{\frac{1}{\lambda}}}{\omega}\sum_{v=0}^{\infty} CG^{*}_{v,\lambda,2}\frac{\omega^{v}}{v!}$$

$$2\sum_{v=1}^{\infty}\mathbb{D}_{v}\lambda^{v}\frac{\omega^{v}}{v!} = 2\sum_{v=1}^{\infty} CG^{*}_{v+1,\lambda,2}\frac{1}{v+1}\frac{\omega^{v}}{v!} + \sum_{v=1}^{\infty}\sum_{\sigma=0}^{v}\binom{v}{\sigma}\mathbb{D}_{\sigma}\lambda^{\sigma} CG^{*}_{v-\sigma,\lambda,2}\frac{\omega^{v}}{v!}. \tag{38}$$

Via (38), we obtain (37). □

Theorem 10. *For $v \geq 0$, we have*

$$CG_{v,\lambda}(\xi) = \sum_{\rho=0}^{v}\sum_{\sigma=0}^{\rho}\binom{v}{\rho}\frac{(-1)^{\sigma}}{\sigma+1}\lambda^{\rho-\sigma-1}\sigma! S_{1}(\rho,\sigma) CG^{*}_{v-\rho,\lambda}(\xi). \tag{39}$$

Proof. From (4) and (19), we have

$$\sum_{v=0}^{\infty} CG_{v,\lambda}(\xi)\frac{\omega^{v}}{v!} = \frac{2\log(1+\log(1+\lambda\omega)^{\frac{1}{\lambda}})}{2+\log(1+\lambda\omega)^{\frac{1}{\lambda}}}(1+\log(1+\lambda\omega)^{\frac{1}{\lambda}})^{\xi}$$

$$= \frac{2\log(1+\lambda\omega)^{\frac{1}{\lambda}}}{2+\log(1+\lambda\omega)^{\frac{1}{\lambda}}}(1+\log(1+\lambda\omega)^{\frac{1}{\lambda}})^{\xi}\frac{\log(1+\log(1+\lambda\omega)^{\frac{1}{\lambda}})}{\log(1+\lambda\omega)^{\frac{1}{\lambda}}}$$

$$= \sum_{v=0}^{\infty} CG^{*}_{v,\lambda,2}(\xi)\frac{\omega^{v}}{v!}\frac{1}{\log(1+\lambda\omega)^{\frac{1}{\lambda}}}\sum_{\sigma=1}^{\infty}\frac{(-1)^{\sigma-1}}{\sigma}\lambda^{-\sigma}(\log(1+\lambda\omega))^{\sigma}$$

$$= \sum_{v=0}^{\infty} CG^{*}_{v,\lambda,2}(\xi)\frac{\omega^{v}}{v!}\sum_{\sigma=1}^{\infty}\frac{(-1)^{\sigma-1}}{\sigma}\lambda^{-\sigma}(\log(1+\lambda\omega))^{\sigma-1}$$

$$= \sum_{v=0}^{\infty} CG^{*}_{v,\lambda,2}(\xi)\frac{\omega^{v}}{v!}\sum_{\sigma=0}^{\infty}\frac{(-1)^{\sigma}}{\sigma+1}\lambda^{-\sigma-1}\sigma!\frac{(\log(1+\lambda\omega))^{\sigma}}{\sigma!}$$

$$= \sum_{v=0}^{\infty} CG^{*}_{v,\lambda,2}(\xi)\frac{\omega^{v}}{v!}\sum_{\sigma=0}^{\infty}\frac{(-1)^{\sigma}}{\sigma+1}\lambda^{-\sigma-1}\sigma!\sum_{k=\sigma}^{\infty} S_{1}(k,\sigma)\lambda^{k}\frac{\omega^{k}}{k!}$$

$$= \sum_{v=0}^{\infty} CG^{*}_{v,\lambda,2}(\xi)\frac{\omega^{v}}{v!}\sum_{k=0}^{\infty}\sum_{\sigma=0}^{k}\frac{(-1)^{\sigma}}{\sigma+1}\lambda^{k-\sigma-1}\sigma! S_{1}(k,\sigma)\frac{\omega^{k}}{k!}$$

$$= \sum_{v=0}^{\infty}\left(\sum_{k=0}^{v}\sum_{\sigma=0}^{k}\binom{v}{k}\frac{(-1)^{\sigma}}{\sigma+1}\lambda^{k-\sigma-1}\sigma! S_{1}(k,\sigma) CG^{*}_{v-k,\lambda,2}(\xi)\right)\frac{\omega^{v}}{v!}. \tag{40}$$

Therefore, via (40), we obtain the result. □

Theorem 11. *For $v \geq 0$, we have*

$$CG_{v,\lambda}(\xi) = \sum_{\sigma=0}^{v} S_{1}(v,\sigma)\lambda^{v-\sigma} CG_{\sigma}(\xi). \tag{41}$$

Proof. Replacing ω by $\log(1+\lambda\omega)^{\frac{1}{\lambda}}$ in (8), we obtain

$$\frac{2\log(1+\log(1+\lambda\omega)^{\frac{1}{\lambda}})}{2+\log(1+\lambda\omega)^{\frac{1}{\lambda}}}(1+\log(1+\lambda\omega)^{\frac{1}{\lambda}})^{\xi} = \sum_{\sigma=0}^{\infty} CG_{\sigma,\lambda}(\xi)\frac{(\log(1+\lambda\omega)^{\frac{1}{\lambda}})^{\sigma}}{\sigma!}$$

$$= \sum_{\sigma=0}^{\infty} CG_{\sigma,\lambda}(\xi)\lambda^{-\sigma} \sum_{v=\sigma}^{\infty} S_1(v,\sigma)\lambda^{v}\frac{\omega^{v}}{v!}$$

$$= \sum_{v=0}^{\infty}\left(\sum_{\sigma=0}^{v} S_1(v,\sigma)\lambda^{v-\sigma}CG_{\sigma,\lambda}(\xi)\right)\frac{\omega^{v}}{v!}. \quad (42)$$

On the other hand,

$$\frac{2\log(1+\log(1+\lambda\omega)^{\frac{1}{\lambda}})}{2+\log(1+\lambda\omega)^{\frac{1}{\lambda}}}(1+\log(1+\lambda\omega)^{\frac{1}{\lambda}})^{\xi} = \sum_{v=0}^{\infty} CG_{v,\lambda}(\xi)\frac{\omega^{v}}{v!}. \quad (43)$$

Through (42) and (43), we obtain (41). □

Here, we consider the higher-order modified degenerate Changhee–Genocchi polynomials by the following definition.

Let $r \in \mathbb{N}$; we consider the higher-order modified degenerate Changhee–Genocchi polynomials of the second kind given by the following generating function:

$$\left(\frac{2\log(1+\lambda\omega)^{\frac{1}{\lambda}}}{2+\log(1+\lambda\omega)^{\frac{1}{\lambda}}}\right)^{r}(1+\log(1+\lambda\omega)^{\frac{1}{\lambda}})^{\xi} = \sum_{v=0}^{\infty} CG_{v,\lambda,2}^{(*,r)}(\xi)\frac{\omega^{v}}{v!}. \quad (44)$$

When $\xi = 0, CG_{v,\lambda,2}^{(*,r)} = CG_{v,\lambda,2}^{(*,r)}(0)$ are called the higher-order modified degenerate Changhee–Genocchi numbers of the second kind.

$$\lim_{\lambda\to 0} CG_{v,\lambda,2}^{(*,r)}(\xi) = CG_{v}^{(*,r)}(\xi)\ (v \geq 0),$$

are called the higher-order modified Changhee–Genocchi polynomials.

Theorem 12. *For $v \geq 0$, we have*

$$CG_{v,\lambda,2}^{(*,r)}(\xi) = \sum_{\sigma=0}^{v}\binom{v}{\sigma}\lambda^{\sigma}\mathbb{D}_{\sigma}^{(r)}CG_{v-\sigma,\lambda}^{(*,r)}(\xi). \quad (45)$$

Proof. From (6), (14) and (44), we note that

$$\sum_{v=0}^{\infty} CG_{v,\lambda,2}^{(*,r)}(\xi)\frac{\omega^{v}}{v!} = \left(\frac{\log(1+\lambda\omega)}{\lambda\omega}\right)^{r}\left(\frac{2\omega}{2+\log(1+\lambda\omega)^{\frac{1}{\lambda}}}\right)^{r}(1+\log(1+\lambda\omega)^{\frac{1}{\lambda}})^{\xi}$$

$$= \left(\sum_{\sigma=0}^{\infty}\mathbb{D}_{\sigma}^{(r)}\lambda^{\sigma}\frac{\omega^{\sigma}}{\sigma!}\right)\left(\sum_{v=0}^{\infty} CG_{v,\lambda}^{(*,r)}(\xi)\frac{\omega^{v}}{v!}\right)$$

$$= \sum_{v=0}^{\infty}\left(\sum_{\sigma=0}^{v}\binom{v}{\sigma}\lambda^{\sigma}\mathbb{D}_{\sigma}^{(r)}CG_{v-\sigma,\lambda}^{(*,r)}(\xi)\right)\frac{\omega^{v}}{v!}. \quad (46)$$

Therefore, via (44) and (46), we obtain the result. □

Theorem 13. *For $v \geq 0$, we have*

$$CG_{v,\lambda,2}^{(*,r)}(\xi) = \sum_{\sigma=0}^{v}\binom{v}{\sigma}S_1(\sigma+r,r)\frac{\lambda^{\sigma}}{\binom{\sigma+r}{r}}CG_{v-\sigma,\lambda}^{(*,r)}(\xi). \quad (47)$$

Proof. From (44), we note that

$$\sum_{v=0}^{\infty} CG_{v,\lambda,2}^{(*,r)}(\xi)\frac{\omega^v}{v!} = \left(\frac{\log(1+\lambda\omega)}{\lambda\omega}\right)^r \left(\frac{2\omega}{2+\log(1+\lambda\omega)^{\frac{1}{\lambda}}}\right)^r (1+\log(1+\lambda\omega)^{\frac{1}{\lambda}})^{\xi}$$

$$= \left(\sum_{\sigma=0}^{\infty} S_1(\sigma+r,r)\frac{\lambda^\sigma}{\binom{\sigma+r}{r}}\frac{\omega^\sigma}{\sigma!}\right)\left(\sum_{v=0}^{\infty} CG_{v,\lambda}^{(*,r)}(\xi)\frac{\omega^v}{v!}\right)$$

$$= \sum_{v=0}^{\infty}\left(\sum_{\sigma=0}^{v}\binom{v}{\sigma} S_1(\sigma+r,r)\frac{\lambda^\sigma}{\binom{\sigma+r}{r}} CG_{n-m,\lambda}^{(*,r)}(\xi)\right)\frac{\omega^v}{v!}. \tag{48}$$

Therefore, via (44) and (48), we obtain the result. □

Theorem 14. *For $v \geq 0$, we have*

$$CG_{v,\lambda,2}^{(*,r)}(\xi) = \sum_{l=0}^{v}\sum_{\sigma=0}^{l}\binom{v}{l}\binom{l}{\sigma} S_1(\sigma+r,r)\frac{\lambda^\sigma}{\binom{\sigma+r}{r}}\mathbb{B}_{l-\sigma}^{(l-r+1)}(1)\widehat{CG_{v-l,\lambda}}(\xi). \tag{49}$$

Proof. From (18) and (44), we note that

$$\sum_{v=0}^{\infty} CG_{v,\lambda,2}^{(*,r)}(\xi)\frac{\omega^v}{v!} = \left(\frac{2\log(1+\omega)}{2+\log(1+\lambda\omega)^{\frac{1}{\lambda}}}\right)^r (1+\log(1+\lambda\omega)^{\frac{1}{\lambda}})^{\xi}\left(\frac{\log(1+\lambda\omega)}{\lambda\omega}\right)^r\left(\frac{\omega}{\log(1+\omega)}\right)^r$$

$$= \left(\sum_{v=0}^{\infty}\widehat{CG_{v,\lambda}}(\xi)\frac{\omega^v}{v!}\right)\left(\sum_{\sigma=0}^{\infty} S_1(\sigma+r,r)\frac{\lambda^\sigma}{\binom{\sigma+r}{r}}\frac{\omega^\sigma}{\sigma!}\right)\left(\sum_{l=0}^{\infty}\mathbb{B}_l^{(l-r+1)}(1)\frac{\omega^l}{l!}\right)$$

$$= \left(\sum_{v=0}^{\infty}\widehat{CG_{v,\lambda}}(\xi)\frac{\omega^v}{v!}\right)\sum_{l=0}^{\infty}\left(\sum_{m=0}^{l}\binom{l}{m} S_1(m+r,r)\frac{\lambda^m}{\binom{m+r}{r}}\mathbb{B}_{l-\sigma}^{(l-r+1)}(1)\right)\frac{\omega^l}{l!}$$

$$= \sum_{v=0}^{\infty}\left(\sum_{l=0}^{v}\sum_{\sigma=0}^{l}\binom{v}{l}\binom{l}{\sigma} S_1(\sigma+r,r)\frac{\lambda^\sigma}{\binom{\sigma+r}{r}}\mathbb{B}_{l-\sigma}^{(l-r+1)}(1)\widehat{CG_{v-l,\lambda}}(\xi)\right)\frac{\omega^v}{v!}. \tag{50}$$

Therefore, via (44) and (50), we obtain the result. □

Theorem 15. *For $r, k \in \mathbb{N}$, with $r > k$ and $v \geq 0$, we have*

$$CG_{v,\lambda,2}^{(*,r)}(\xi) = \sum_{l=0}^{v}\binom{v}{l} CG_{l,\lambda,2}^{(*,r-k)} CG_{n-l,\lambda,2}^{(*,k)}(\xi). \tag{51}$$

Proof. Through (44), we see that

$$\left(\frac{2\log(1+\lambda\omega)^{\frac{1}{\lambda}}}{2+\log(1+\lambda\omega)^{\frac{1}{\lambda}}}\right)^r (1+\log(1+\lambda\omega)^{\frac{1}{\lambda}})^{\xi}$$

$$= \left(\frac{2\log(1+\lambda\omega)^{\frac{1}{\lambda}}}{2+\log(1+\lambda\omega)^{\frac{1}{\lambda}}}\right)^{r-k}\left(\frac{2\log(1+\lambda\omega)^{\frac{1}{\lambda}}}{2+\log(1+\lambda\omega)^{\frac{1}{\lambda}}}\right)^k (1+\log(1+\lambda\omega)^{\frac{1}{\lambda}})^{\xi}$$

$$= \left(\sum_{l=0}^{\infty} CG_{l,\lambda,2}^{(*,r-k)}\frac{\omega^l}{l!}\right)\left(\sum_{\sigma=0}^{\infty} CG_{\sigma,\lambda,2}^{(*,k)}(\xi)\frac{\omega^\sigma}{\sigma!}\right)$$

$$= \sum_{v=0}^{\infty}\left(\sum_{l=0}^{v}\binom{v}{l} CG_{l,\lambda,2}^{(*,r-k)} CG_{n-l,\lambda,2}^{(*,k)}(\xi)\right)\frac{\omega^v}{v!}. \tag{52}$$

Therefore, via (44) and (52), we obtain the result. □

Theorem 16. *For $v \geq 0$, we have*

$$CG^{(*,r)}_{v,\lambda,2}(\xi+\eta) = \sum_{k=0}^{v}\sum_{\sigma=0}^{k}\binom{v}{k}CG^{(*,r)}_{v-k,\lambda,2}(\xi)(\eta)_\sigma\lambda^{k-\sigma}S_1(k,\sigma). \tag{53}$$

Proof. Now, we observe that

$$\begin{aligned}\sum_{v=0}^{\infty}CG^{(*,r)}_{v,\lambda,2}(\xi+\eta)\frac{\omega^v}{v!} &= \left(\frac{2\log(1+\lambda\omega)^{\frac{1}{\lambda}}}{2+\log(1+\lambda\omega)^{\frac{1}{\lambda}}}\right)^r(1+\log(1+\lambda\omega)^{\frac{1}{\lambda}})^{\xi+\eta} \\ &= \left(\sum_{l=0}^{\infty}CG^{(*,r)}_{l,\lambda,2}(\xi)\frac{\omega^l}{l!}\right)\left(\sum_{\sigma=0}^{\infty}(\eta)_\sigma\lambda^{-\sigma}\frac{(\log(1+\lambda\omega))^\sigma}{\sigma!}\right) \\ &= \left(\sum_{v=0}^{\infty}CG^{(*,r)}_{v,\lambda,2}(\xi)\frac{\omega^v}{v!}\right)\left(\sum_{k=0}^{\infty}\sum_{\sigma=0}^{k}(\eta)_\sigma\lambda^{k-\sigma}S_1(k,\sigma)\frac{\omega^k}{k!}\right) \\ &= \sum_{v=0}^{\infty}\left(\sum_{k=0}^{v}\sum_{\sigma=0}^{k}\binom{v}{k}CG^{(*,r)}_{v-k,\lambda,2}(\xi)(\eta)_\sigma\lambda^{k-\sigma}S_1(k,\sigma)\right)\frac{\omega^v}{v!}. \end{aligned} \tag{54}$$

coefficients of ω^v on both sides, we obtain the result. □

Theorem 17. *For $v \geq 0$, we have*

$$CG^{(*,r)}_{v,\lambda,2} = \sum_{\sigma=0}^{v}\binom{v}{\sigma}CG^{(*,r)}_{v-\sigma,\lambda,2}b^{(r)}_\sigma\lambda^\sigma. \tag{55}$$

Proof. By using of (14) and (44), we have

$$\begin{aligned}\left(\frac{2\omega}{2+\log(1+\lambda\omega)^{\frac{1}{\lambda}}}\right)^r &= \left(\frac{\lambda\omega}{\log(1+\lambda\omega)}\right)^r\sum_{v=0}^{\infty}CG^{(*,r)}_{v,\lambda,2}\frac{\omega^v}{v!} \\ &= \left(\sum_{\sigma=0}^{\infty}b^{(r)}_\sigma\lambda^\sigma\frac{\omega^\sigma}{\sigma!}\right)\left(\sum_{v=0}^{\infty}CG^{(*,r)}_{v,\lambda,2}\frac{\omega^v}{v!}\right) \\ &= \sum_{v=0}^{\infty}\left(\sum_{\sigma=0}^{v}\binom{v}{\sigma}CG^{(*,r)}_{v-\sigma,\lambda,2}b^{(r)}_\sigma\lambda^\sigma\right)\frac{\omega^v}{v!}. \end{aligned} \tag{56}$$

On the other hand,

$$\left(\frac{2\omega}{2+\log(1+\lambda\omega)^{\frac{1}{\lambda}}}\right)^r = \sum_{v=0}^{\infty}CG^{(*,r)}_{v,\lambda}\frac{\omega^v}{v!}. \tag{57}$$

Therefore, via (56) and (57), we obtain the result. □

3. Conclusions

In the present paper, we introduced modified degenerate Changhee–Genocchi polynomials of the second kind, and analyzed some properties and relations by using the generating function. We also acquired several properties and formulas covering addition formulas, recurrence relations, implicit summation formulas, and relations with the earlier polynomials in the literature. Moreover, we derived the higher-order degenerate Changhee–Genocchi polynomials of the second kind, and constructed relations between some special polynomials and numbers. In addition, for advancing the purpose of this article, we will proceed with this idea in several directions in our next research studies.

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