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A Fuzzy Parameterized Multiattribute Decision-Making Framework for Supplier Chain Management Based on Picture Fuzzy Soft Information

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Abstract: Supplier selection as a multiattribute decision-making (MADM) problem has various inherent uncertainties due to a number of symmetrical variables. In order to handle such information-based uncertainties, rational models like intuitionistic fuzzy sets have already been introduced in the literature. However, a picture fuzzy set (PiFS) with four dimensions of positive, neutral, negative, and rejection is better at capturing and interpreting such kinds of ambiguous information. Additionally, fuzzy parameterization (FPara) is helpful for evaluating the degree of uncertainty in the parameters. This study aims to develop a fuzzy parameterized picture fuzzy soft set (FpPiFSS) by integrating the ideas of PiFS and FPara. This integration is more adaptable and practical since it helps decision makers manage approximation depending on their objectivity and parameterization uncertainty. With the assistance of instructive examples, some of the set-theoretic operations are examined. A decision support framework is constructed using matrix manipulation, preferential weighting, fuzzy parameterized grades based on Pythagorean means, and the approximations of decision makers. This framework proposes a reliable algorithm to evaluate four timber suppliers (initially scrutinized by perusal process) based on eight categorical parameters for real estate projects. In order to accomplish suppliers evaluation, crucial validation outcomes are taken into account, including delivery level, purchase cost, capacity, product quality, lead time, green degree, location, and flexibility. To assess the advantages, dependability, and flexibility of the recommended strategy, comparisons in terms of computation and structure are provided. Consequently, the results are found to be reliable, analog, and consistent despite the use of fuzzy parameterization and picture fuzzy setting.

Keywords: fuzzy parameterization; picture fuzzy set; soft set; multiattribute decision making; supply chain management; degree of symmetry with uncertainty; optimization

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1. Introduction

The management disciplines with the highest growth have been supply chain management (SuCM) and strategic sourcing. These currently form a crucial component of business operations and fall under the broader category of logistics. They include areas like purchasing management, transportation management, warehouse management, and inventory management. Supply chains and logistics have grown more intricate and dynamic as a result of the sophistication of technology. Increased adaptability is required to maintain competitiveness and adjust to markets that are changing quickly [1,2]. By taking into account appropriate attributes and their subattribute values, the multiattribute decision making (MADM) project evaluates multiple options (elements under investigation) in the initial universe. It is a particular form of multicriteria decision making (MCDM), which accomplishes the same job by taking into account multiple factors. Using several traditional analytical and empirical methodologies, MADM has been effectively applied to SuCM in the classical literature [3–6]. Managing the flow of goods and services is what SuCM does. It includes all processes that turn raw resources into finished products. It involves a dynamic reorganization of a company's delivery-side operations to maximize the value of the customer and obtain an upper hand in the market.

It describes suppliers' efforts to develop and implement supply chains that are as creative and sensible as possible. The production, product development, and information systems needed to manage these operations are all covered by supply chains. SuCM often aims to centrally coordinate or control the manufacturing process, including delivery and supply. Firms can cut excess costs and speed up product delivery to the user by managing the supply chain. This is performed by maintaining strict control over domestic records, domestic production, supply, sales, and the wholesalers' inventories. With time, it has been noticed that a variety of uncertainties play a role in choosing the best suppliers to manage supply chains [7,8].

1.1. Literature Review

In order to deal with uncertain data and information, Zadeh and Atanassov, respectively, developed fuzzy sets (FuS) [9] and intuitionistic fuzzy sets (InFS) [10]. By introducing the term "real-valued membership grade", which is acquired by the membership function and expresses any entity's belongingness in sample space by real value within [0, 1], the essential requirement, "well-defined", of the classical set is relaxed in FuS. The terms "real-valued membership grade" and "real-valued nonmembership grade", on the other hand, are used by the InFS to accomplish the same objective. Both the membership and nonmembership grades can be used to earn these ratings. These grades serve as actual values between [0, 1] representations of any entity's belongingness and nonbelongingness in the sample space. Being dependent on each other, the sum of these grades must lie within [0, 1], and there exists a particular degree of hesitancy. Later on, Cuong and Kreinovich developed a picture fuzzy set (PiFS) [11] that is the direct generalization of FuS and InFS, as it considers the expected hesitancy grade as a refusal grade. It implies that three functions: membership, nonmembership, and neutral functions, are used to yield the respective real-valued grades within [0, 1] while defining the existence of any entity in sample space. In general, PiFS-based models might be sufficient in cases when we deal with human perceptions, including more responses like yes, abstain, no, and rejection. The illustration of polling can be used to truly grasp such a circumstance. Human voters can be categorized into four groups: those who vote for, abstain from voting, vote against, or refuse to cast a ballot.

Molodtsov developed the soft set (SoS) [12] to provide FuS, InFS, and PiFS a parameterization context, which improved their applicability in real-world vague situations. To characterize the key characteristics and operations of SoS, such as basic deductive properties, set-theoretic operations, relations, functions, and ordering are investigated by scholars [13–17]. For the first time in published works, Maji et al. [18] discussed the use of SoS in decision making (DMG). The fuzzy soft set (FuSS) [19,20], intuitionistic fuzzy soft set (InFSS) [21], and picture fuzzy soft set (PiFSS) [22,23] were developed in order to examine the properties of FuS, InFS, PiFS, and SoS collectively. Recently, Memiş [24] put forward a different perception regarding the basic notions of PiFSS and the relevant product operations for their use in DMG. Khan et al. [25,26] discussed tower construction and concept selection problems by using the generalized notions of PiFSS. The contributions of these researchers [27–31] to FuSs, SoSs, and their hybrids are commendable and worthy of emulation.

The significance of insightful SuCM has grown as a result of the modernization and commercialization of worldwide financial sectors. SuCM delivers item management and data that create an inducement for customers and integrate important business operations from the last client to the initial supplier. All businesses must be familiar with a select group of reliable suppliers in this way. The choice of trustworthy suppliers has a significant impact on an organization's success. So, a key component of SuCM is the supplier selection problem (SuSP). Hiring reliable suppliers significantly lowers the cost of purchasing materials and improves organizational integrity. The SuSP allows switching between a variety of qualitative and quantitative criteria. As a result, SuSP becomes an MCDM problem, and in order to select the best suppliers, it is crucial to strike a balance between inconsistent material and immaterial criteria. Due to the industry's anticipated side risks and deviations, construction supply chain management (CSuCM) is a unique and difficult challenge. Due to the increased complexity and variety of construction schemes, several researchers have emphasized the need for SuCM in construction-based operations [32]. A competent CSuCM can improve a strategy's effectiveness and reduce inefficiency caused by inadequate substance monitoring and organization [33]. Construction supply chains (CSuC) are complex systems that require the management and organization of construction-based materials throughout the construction-based procedures. They are not simple sequences or procedures. This increases the risk and complexity associated with CSuC. Construction-based plans with complexity and uniqueness frequently result in numerous changes and unforeseen conditions during the supply progression, where interferences might happen on both internal and external resources. Suppliers serve as an inevitable foundation for a variety of peripheral risks. In CSuC, the selection of suppliers is regarded as an MCDM issue that necessitates consideration of all quality-based factors. Suppliers in the CSuC must be able to respond to any diversions proficiently and effectively. Traditionally, managers primarily focus on making purchases from suppliers that can deliver goods at a lower cost, a higher standard, and quickly. With the use of several techniques, including the gray relational analysis approach, multiobjective programming approach, gray combined compromise solution method, hybrid fuzzy-based approach, and analytic hierarchy process approaches, among others, several researchers [34–44] made substantial improvements for the most effective supplier selection. The theory of fuzzy sets and associated contexts have been employed by different authors [45–48] to fuzzy and SoS-like environments to address SuSP through DMG with the goal to deal with anticipated inconsistencies in SuSP.

1.2. Research Gap and Motivation

Decision makers (DMS) may encounter circumstances that make it challenging for them to choose which criteria to accept and which to reject, as well as which ones to prioritize more highly and which ones less. In other words, when choosing, testing, and assessing parameters, they deal with some degree of uncertainty and ambiguity. In a similar vein, they require a DMG environment that respects both their positive and negative comments while still taking into account their objectivity while evaluating alternatives based on parameters. After a careful study of the literature, it can be concluded that there is a need in the literature for a mathematical framework that addresses these key features. In view of this literary need, a new mathematical framework, the fuzzy parameterized picture fuzzy soft set (FpPiFSS), is developed that is more flexible and adaptable, as it is capable of managing the following situations collectively:

1. Uncertainty and vagueness appeared while choosing the appropriate parameters.
2. Flexible opinions of the DMS in terms of truth, falsity, and neutrality grades.
3. Approximate function for the assessment of alternatives.

Three settings make up the proposed structure, FpPiFSS: the fuzzy parameterization idea (FPara), PiFS, and soft setting. These options increase FpPiFSS's adaptability to handle the aforementioned literary constraints. The concept of fuzzy parameterization (FPara), which enables the estimation of parameter uncertainty with a specific degree from the interval [0, 1], helps it avoid the first situation. Employing PiFS's three-dimensional membership function, it manages the second situation. Similarly, it handles the third situation by employing the soft approximate function of the soft setting. In other words, it has no trouble managing all three circumstances simultaneously.

The hypotheses formulations for the proposed framework are as follows:

Hypothesis 1 (H1). *The integration of fuzzy parameterization (FPara), picture fuzzy sets (PiFS), and soft settings into the FpPiFSS framework significantly clarifies the DMS's uncertainty in parameter selection.*

Hypothesis 2 (H2). *The integration of fuzzy parameterization (FPara), picture fuzzy sets (PiFS), and soft settings into the FpPiFSS framework significantly controls the DMS's impartiality in estimating the options.*

Hypothesis 3 (H3). *The approximate function significantly affects the evaluation of diverse options within the FpPiFSS framework.*

1.3. Salient Questions and Contributions

The research questions are initially discussed before going over the key contributions.

a. How may the DMS's uncertainty over the selection of parameters be addressed?
b. How can the DMS's impartiality in estimating the options be effectively controlled?
c. What part does the approximate function play in the evaluation of different options?

Undoubtedly, the objectives of this research are to identify the appropriate solutions to these problems. The prominent contributions of this study are outlined as follows:

1. A novel mathematical context, i.e., FpPiFSS, is characterized as the combination of three significant concepts: FPara idea, picture fuzzy set, and SOS. Such a combination is trustworthy to address the limitations of the published literature (this is a theoretical aspect linked with the above-described research questions).
2. In order to assess the vague nature of parameters, their respective fuzzy parameterized grades (FPGs) are determined by using the assigned weights of DMS (this is specifically linked with the first research question).
3. Based on the set-theoretic properties of FpPiFSS, an intelligent decision framework is established, accompanied by an algorithm for the evaluation of timber suppliers (this is particularly linked with the third research question).

The remaining portion of this paper is organized so that Section 2 is recalls fundamental knowledge from published work. Section 3 provides the investigation of a novel mathematical context, i.e., FpPiFSS and its operations, the decision framework with an algorithm, and the validation of the proposed algorithm. Section 4 presents structural and computation comparisons of this study with some published works, and Section 5 finally concludes this study.

The Table 1 presents the explanations of abbreviations and acronyms used in the paper.

Table 1. Explanation of Abbreviations and Acronyms.

Abbreviations	Stand for	Abbreviations	Stand for
SuCM	supply chain management	MADM	multiattribute decision making
MCDM	multicriteria decision making	FuS	fuzzy sets
InFS	intuitionistic fuzzy sets	PiFS	picture fuzzy set
SoS	soft set	DMG	decision making
FuSS	fuzzy soft set	InFSS	intuitionistic fuzzy soft set
PiFSS	picture fuzzy soft set	SuSP	supplier selection problem
CSuCM	construction supply chain management	CSuC	construction supply chains
DMS	decision makers	FpPiFSS	fuzzy parameterized picture fuzzy soft set
FPara	fuzzy parameterization	FPGs	fuzzy parameterized grades

2. Fundamental Knowledge

Some necessary definitions are presented in this part to assist the readers in understanding the concepts.

Definition 1 ([11]). *Let $\tilde{\zeta}_T$, $\tilde{\zeta}_F$, and $\tilde{\zeta}_N$ be membership, nonmembership, and neutral membership functions, respectively, with the nonempty set $\hat{\triangle}$ as their domain and [0, 1] as their range; then, PiFS A is defined as*

$$A = \left\{ \frac{\tilde{x}}{\langle \tilde{\zeta}_T(\tilde{x}), \tilde{\zeta}_F(\tilde{x}), \tilde{\zeta}_N(\tilde{x}) \rangle} : \tilde{x} \in \hat{\triangle}, \tilde{\zeta}_T(\tilde{x}), \tilde{\zeta}_F(\tilde{x}), \tilde{\zeta}_N(\tilde{x}) \in [0,1] \right\}$$

satisfying the condition $0 \leq \tilde{\zeta}_T(\tilde{x}) + \tilde{\zeta}_F(\tilde{x}) + \tilde{\zeta}_N(\tilde{x}) \leq 1$. The hesitancy grade is $\tilde{\zeta}_H(\tilde{x}) = 1 - (\tilde{\zeta}_T(\tilde{x}) + \tilde{\zeta}_F(\tilde{x}) + \tilde{\zeta}_N(\tilde{x}))$. The family of all PiFSs is represented by Ω_{PiFS}

Definition 2 ([12]). *Let $P^{\hat{\triangle}}$ and $\hat{\Xi}$ be the set of all subsets of $\hat{\triangle}$ and the set of attributes, respectively; then, the SoS B is defined as*

$$B = \left\{ (\tilde{e}, \varpi(\tilde{e})) : \tilde{e} \in \hat{\Xi}, \varpi(\tilde{e}) \subseteq P^{\hat{\triangle}} \right\}$$

such that $\varpi(\tilde{e}) = \emptyset$ for all $e \notin \hat{\Xi}$, where $\varpi : \hat{\Xi} \to P^{\hat{\triangle}}$ is an approximate mapping with $\varpi(\tilde{e})$ being an e-approximate element of B.

Definition 3 ([22,23]). *Let $P^{\hat{\triangle}}$ and $\hat{\Xi}$ be the set of all subsets of $\hat{\triangle}$ and the set of attributes, respectively. Let $\tilde{\zeta}_T$, $\tilde{\zeta}_F$, and $\tilde{\zeta}_N$ be membership, nonmembership, and neutral membership functions, respectively, with universal set $\hat{\triangle}$ as their domain and [0, 1] as their range; then, PiFSS C is characterized by an approximate mapping $\tilde{\psi} : \hat{\Xi} \to \Omega_{PiFS}$ and defined as*

$$C = \left\{ (\tilde{e}, \tilde{\psi}(\tilde{e})) : \tilde{e} \in \hat{\Xi}, \tilde{\psi}(\tilde{e}) \subseteq \Omega_{PiFS} \right\}$$

such that $\tilde{\psi}(\tilde{e}) = \emptyset$ for all $e \notin \hat{\Xi}$, where

$$\tilde{\psi}(\tilde{e}) = \left\{ \frac{x}{\langle \tilde{\zeta}_T(\tilde{x}), \tilde{\zeta}_F(\tilde{x}), \tilde{\zeta}_N(\tilde{x}) \rangle} : \tilde{x} \in \hat{\triangle}, \tilde{\zeta}_T(\tilde{x}), \tilde{\zeta}_F(\tilde{x}), \tilde{\zeta}_N(\tilde{x}) \in [0,1] \right\}.$$

3. Materials and Methods

The key elements of the suggested methodology are laid out in this section. There are two rounds. The suggested mathematical framework and its associated set-theoretic operations are introduced in the first round. Additionally, this round covers the explanation of chosen parameters, their functional roles, and the technique for figuring out FPGs of parameters. An algorithm-based decision support framework for the most effective selection of suppliers in real estate projects is presented in the second round.

3.1. Characterization of Proposed Structure, i.e., FpPiFSS

Controlling the uncertainty and ambiguity found in information is no less of a challenge for researchers. However, in this regard, various DMG support models have been introduced keeping in mind different decision-making situations. One of these situations can be when DMS provide their opinions positively in some cases and negatively in others, but in some cases, they want to take the path of neutrality. Along with this, if they face uncertainty and ambiguity in the selection of parameters, then the combination of FPara, PiFS, and SoS will be useful. Therefore, in this section, FpPiFSS is introduced that is capable of handling such DMG situations.

Definition 4. *Let $P^{\hat{\triangle}}$ and $\hat{\Xi} = \{\tilde{e}_i, i = 1,2,3,\ldots,n\}$ be the set of all subsets of $\hat{\triangle} = \{\tilde{x}_j, j = 1,2,3,\ldots,m\}$ and the set of attributes, respectively. Let $\hat{F}_{\hat{\Xi}} = \{\frac{\tilde{e}_i}{\tilde{\mu}_T(\tilde{e}_i)} : \tilde{e}_i \in \hat{\Xi}, \tilde{\mu}_T(\tilde{e}_i) \in [0,1], i = 1,2,3,\ldots,n\}$ be a fuzzy set over $\hat{\Xi}$ consisting of FPGs for attributes $\tilde{e}_i$ and $\tilde{\zeta}_T : \hat{\triangle} \to [0,1], \tilde{\zeta}_F : \hat{\triangle} \to [0,1]$, and $\tilde{\zeta}_N : \hat{\triangle} \to [0,1]$ are membership, nonmembership, and neutral membership functions, respectively; then, FpPiFSS $\hat{\Pi}$ is characterized by an approximate mapping $\tilde{\psi}_F : \hat{F}_{\hat{\Xi}} \to \Omega_{PiFS}$ and defined as*

$$\hat{\Pi} = \left\{ \left(\frac{\tilde{e}_i}{\tilde{\mu}_T(\tilde{e}_i)}, \tilde{\psi}_F(\frac{\tilde{e}_i}{\tilde{\mu}_T(\tilde{e}_i)}) \right) : \frac{\tilde{e}_i}{\tilde{\mu}_T(\tilde{e}_i)} \in \hat{F}_{\hat{\Xi}}, \tilde{\psi}_F(\frac{\tilde{e}_i}{\tilde{\mu}_T(\tilde{e}_i)}) \subseteq \Omega_{PiFS} \right\}$$

such that $\tilde{\psi}_F(\frac{\tilde{e}_i}{\tilde{\mu}_T(\tilde{e}_i)}) = \emptyset$ for all $\frac{\tilde{e}_i}{\tilde{\mu}_T(\tilde{e}_i)} \notin \hat{F}_{\hat{\Xi}}$ where

$$\tilde{\psi}_F(\frac{e}{\tilde{\mu}_T(\tilde{e}_i)}) = \left\{ \frac{\tilde{x}_j}{\langle \tilde{\zeta}_T(\tilde{x}_j), \tilde{\zeta}_F(\tilde{x}_j), \tilde{\zeta}_N(\tilde{x}_j) \rangle} : \tilde{x}_j \in \hat{\triangle}, \tilde{\zeta}_T(\tilde{x}_j), \tilde{\zeta}_F(\tilde{x}_j), \tilde{\zeta}_N(\tilde{x}_j) \in [0,1] \right\}.$$

Now, the above definition is explained in the following lines with a numerical example in which all the aspects (universal set, FPGs, parameter set, approximations, etc.) mentioned in this definition are taken into consideration.

Example 1. *Consider $\hat{\triangle} = \{\tilde{x}_1, \tilde{x}_2, \tilde{x}_3, \tilde{x}_4\}$ and $\hat{\Xi} = \{\tilde{e}_1, \tilde{e}_2, \tilde{e}_3, \tilde{e}_4\}$ as the set of alternatives and set of attributes, respectively. Let the FPGs for attributes $\tilde{e}_1, \tilde{e}_2, \tilde{e}_3$, and $\tilde{e}_4$ be $\tilde{\mu}_T(\tilde{e}_1) = 0.2$, $\tilde{\mu}_T(\tilde{e}_2) = 0.5$, $\tilde{\mu}_T(\tilde{e}_3) = 0.3$, and $\tilde{\mu}_T(\tilde{e}_4) = 0.6$, respectively. Thus, by treating $\hat{\Xi}$ and FPGs collectively, a fuzzy set $\hat{F}_{\hat{\Xi}} = \{\frac{\tilde{e}_1}{0.2}, \frac{\tilde{e}_2}{0.5}, \frac{\tilde{e}_3}{0.3}, \frac{\tilde{e}_4}{0.6}\}$ over $\hat{\Xi}$ is formed. The respective approximations of fuzzy parameterized attributes are*

- $\tilde{\psi}_F(\frac{e}{0.2}) = \left\{ \frac{\tilde{x}_1}{\langle 0.21,0.33,0.32 \rangle}, \frac{\tilde{x}_2}{\langle 0.23,0.35,0.34 \rangle}, \frac{\tilde{x}_3}{\langle 0.25,0.37,0.36 \rangle}, \frac{\tilde{x}_4}{\langle 0.27,0.39,0.38 \rangle} \right\}$,
- $\tilde{\psi}_F(\frac{e}{0.5}) = \left\{ \frac{\tilde{x}_1}{\langle 0.31,0.21,0.11 \rangle}, \frac{\tilde{x}_2}{\langle 0.33,0.23,0.13 \rangle}, \frac{\tilde{x}_3}{\langle 0.35,0.25,0.15 \rangle}, \frac{\tilde{x}_4}{\langle 0.37,0.27,0.17 \rangle} \right\}$,
- $\tilde{\psi}_F(\frac{e}{0.3}) = \left\{ \frac{\tilde{x}_1}{\langle 0.11,0.31,0.21 \rangle}, \frac{\tilde{x}_2}{\langle 0.13,0.33,0.23 \rangle}, \frac{\tilde{x}_3}{\langle 0.15,0.35,0.25 \rangle}, \frac{\tilde{x}_4}{\langle 0.17,0.37,0.27 \rangle} \right\}$,
- $\tilde{\psi}_F(\frac{e}{0.6}) = \left\{ \frac{\tilde{x}_1}{\langle 0.41,0.11,0.31 \rangle}, \frac{\tilde{x}_2}{\langle 0.43,0.13,0.33 \rangle}, \frac{\tilde{x}_3}{\langle 0.45,0.15,0.35 \rangle}, \frac{\tilde{x}_4}{\langle 0.47,0.17,0.37 \rangle} \right\}$.

The FpPiFSS $\hat{\Pi}$ can be constructed as

$$\hat{\Pi} = \left\{ \begin{array}{l} \left(\frac{\tilde{e}_1}{0.2}, \left\{ \frac{\tilde{x}_1}{\langle 0.21,0.33,0.32 \rangle}, \frac{\tilde{x}_2}{\langle 0.23,0.35,0.34 \rangle}, \frac{\tilde{x}_3}{\langle 0.25,0.37,0.36 \rangle}, \frac{\tilde{x}_4}{\langle 0.27,0.39,0.38 \rangle} \right\} \right), \\ \left(\frac{\tilde{e}_2}{0.5}, \left\{ \frac{\tilde{x}_1}{\langle 0.31,0.21,0.11 \rangle}, \frac{\tilde{x}_2}{\langle 0.33,0.23,0.13 \rangle}, \frac{\tilde{x}_3}{\langle 0.35,0.25,0.15 \rangle}, \frac{\tilde{x}_4}{\langle 0.37,0.27,0.17 \rangle} \right\} \right), \\ \left(\frac{\tilde{e}_3}{0.3}, \left\{ \frac{\tilde{x}_1}{\langle 0.11,0.31,0.21 \rangle}, \frac{\tilde{x}_2}{\langle 0.13,0.33,0.23 \rangle}, \frac{\tilde{x}_3}{\langle 0.15,0.35,0.25 \rangle}, \frac{\tilde{x}_4}{\langle 0.17,0.37,0.27 \rangle} \right\} \right), \\ \left(\frac{\tilde{e}_4}{0.6}, \left\{ \frac{\tilde{x}_1}{\langle 0.41,0.11,0.31 \rangle}, \frac{\tilde{x}_2}{\langle 0.43,0.13,0.33 \rangle}, \frac{\tilde{x}_3}{\langle 0.45,0.15,0.35 \rangle}, \frac{\tilde{x}_4}{\langle 0.47,0.17,0.37 \rangle} \right\} \right) \end{array} \right\}.$$

Its matrix representation is presented in Table 2.

Table 2. Tabular form of FpPiFSS $\hat{\Pi}$.

$\hat{\Pi}$	$\tilde{x}_1$	$\tilde{x}_2$	$\tilde{x}_3$	$\tilde{x}_4$
$\frac{\tilde{e}_1}{0.2}$	$\langle 0.21, 0.33, 0.32 \rangle$	$\langle 0.23, 0.35, 0.34 \rangle$	$\langle 0.25, 0.37, 0.36 \rangle$	$\langle 0.27, 0.39, 0.38 \rangle$
$\frac{\tilde{e}_2}{0.5}$	$\langle 0.31, 0.21, 0.11 \rangle$	$\langle 0.33, 0.23, 0.13 \rangle$	$\langle 0.35, 0.25, 0.15 \rangle$	$\langle 0.37, 0.27, 0.17 \rangle$
$\frac{\tilde{e}_3}{0.3}$	$\langle 0.11, 0.31, 0.21 \rangle$	$\langle 0.13, 0.33, 0.23 \rangle$	$\langle 0.15, 0.35, 0.25 \rangle$	$\langle 0.17, 0.37, 0.27 \rangle$
$\frac{\tilde{e}_4}{0.6}$	$\langle 0.41, 0.11, 0.31 \rangle$	$\langle 0.43, 0.13, 0.33 \rangle$	$\langle 0.45, 0.15, 0.35 \rangle$	$\langle 0.47, 0.17, 0.37 \rangle$

In this table, the first entry $\langle 0.21, 0.33, 0.32 \rangle$ is the approximation of the alternative $\tilde{x}_1$ with respect to fuzzy parameter $\frac{\tilde{e}_1}{0.2}$. This means that the alternative $\tilde{x}_1$ has 0.21, 0.33, and 0.32 as membership, nonmembership, and neutral grades according to DMS with respect to parameter $\tilde{e}_1$ that are 20% uncertain.

The following lines illustrate some of FpPiFSS's set-theoretic operations in order to broaden its adaptability to a broad range of academic subjects. These set-theoretic operations are significant mathematical ideas that are often used in a variety of disciplines, including computer science, statistics, engineering, and many more to solve problems, carry out calculations, and characterize relationships between elements and data.

Definition 5. *Let* $\hat{\Pi}_1 = \left\{ \left(\frac{\tilde{e}_i}{\mu_T^1(\tilde{e}_i)}, \tilde{\psi}_F^1(\frac{\tilde{e}_i}{\mu_T^1(\tilde{e}_i)}) \right) : \frac{\tilde{e}_i}{\mu_T^1(\tilde{e}_i)} \in \hat{F}^1_{\hat{\Xi}}, \tilde{\psi}_F^1(\frac{\tilde{e}_i}{\mu_T^1(\tilde{e}_i)}) \subseteq \Omega_{PiFS} \right\}$ *and* $\hat{\Pi}_2 = \left\{ \left(\frac{\tilde{e}_i}{\mu_T^2(\tilde{e}_i)}, \tilde{\psi}_F^2(\frac{\tilde{e}_i}{\mu_T^2(\tilde{e}_i)}) \right) : \frac{\tilde{e}_i}{\mu_T^2(\tilde{e}_i)} \in \hat{F}^2_{\hat{\Xi}}, \tilde{\psi}_F^2(\frac{\tilde{e}_i}{\mu_T^2(\tilde{e}_i)}) \subseteq \Omega_{PiFS} \right\}$ *be FpPiFHSSs; then the following holds:*

1. *The FpPiFHSS* $\hat{\Pi}_3 = \left\{ \left(\frac{\tilde{e}_i}{\mu_T^3(\tilde{e}_i)}, \tilde{\psi}_F^3(\frac{\tilde{e}_i}{\mu_T^3(\tilde{e}_i)}) \right) : \frac{\tilde{e}_i}{\mu_T^3(\tilde{e}_i)} \in \hat{F}^3_{\hat{\Xi}}, \tilde{\psi}_F^3(\frac{\tilde{e}_i}{\mu_T^3(\tilde{e}_i)}) \subseteq \Omega_{PiFS} \right\}$ *is their union such that*

$$\tilde{\psi}_F^3(\frac{\tilde{e}_i}{\mu_T^3(\tilde{e}_i)}) = \begin{cases} \tilde{\psi}_F^1(\frac{\tilde{e}_i}{\mu_T^1(\tilde{e}_i)}) & ; \ \frac{\tilde{e}_i}{\mu_T^3(\tilde{e}_i)} \in \hat{F}^1_{\hat{\Xi}} \setminus \hat{F}^2_{\hat{\Xi}} \\ \tilde{\psi}_F^2(\frac{\tilde{e}_i}{\mu_T^2(\tilde{e}_i)}) & ; \ \frac{\tilde{e}_i}{\mu_T^3(\tilde{e}_i)} \in \hat{F}^2_{\hat{\Xi}} \setminus \hat{F}^1_{\hat{\Xi}} \\ \tilde{\psi}_F^1(\frac{\tilde{e}_i}{\mu_T^1(\tilde{e}_i)}) \cup \tilde{\psi}_F^2(\frac{\tilde{e}_i}{\mu_T^2(\tilde{e}_i)}) & ; \ \frac{\tilde{e}_i}{\mu_T^3(\tilde{e}_i)} \in \hat{F}^1_{\hat{\Xi}} \cap \hat{F}^2_{\hat{\Xi}} \end{cases}$$

   where $\tilde{\psi}_F^1(\frac{\tilde{e}_i}{\mu_T^1(\tilde{e}_i)}) \cup \tilde{\psi}_F^2(\frac{\tilde{e}_i}{\mu_T^2(\tilde{e}_i)}) =$
   $\left\{ \begin{array}{c} \tilde{x}_j / \langle max\{\zeta_T^1(\tilde{x}_j), \zeta_T^2(\tilde{x}_j)\}, min\{\zeta_F^1(\tilde{x}_j), \zeta_F^2(\tilde{x}_j)\}, min\{\zeta_N^1(\tilde{x}_j), \zeta_N^2(\tilde{x}_j)\}\rangle : \\ \tilde{x}_j \in \hat{\triangle}, \zeta_T^k(\tilde{x}_j), \zeta_F^k(\tilde{x}_j), \zeta_N^k(\tilde{x}_j) \in [0,1] \end{array} \right\}.$
2. *The FpPiFHSS* $\hat{\Pi}_4 = \left\{ \left(\frac{\tilde{e}_i}{\mu_T^4(\tilde{e}_i)}, \tilde{\psi}_F^4(\frac{\tilde{e}_i}{\mu_T^4(\tilde{e}_i)}) \right) : \frac{\tilde{e}_i}{\mu_T^4(\tilde{e}_i)} \in \hat{F}^4_{\hat{\Xi}}, \tilde{\psi}_F^4(\frac{\tilde{e}_i}{\mu_T^4(\tilde{e}_i)}) \subseteq \Omega_{PiFS} \right\}$ *is their union such that* $\tilde{\psi}_F^4(\frac{\tilde{e}_i}{\mu_T^4(\tilde{e}_i)}) = \tilde{\psi}_F^1(\frac{\tilde{e}_i}{\mu_T^1(\tilde{e}_i)}) \cap \tilde{\psi}_F^2(\frac{\tilde{e}_i}{\mu_T^2(\tilde{e}_i)})$ *when* $\frac{\tilde{e}_i}{\mu_T^4(\tilde{e}_i)} \in \hat{F}^1_{\hat{\Xi}} \cap \hat{F}^2_{\hat{\Xi}}$ *where* $\tilde{\psi}_F^1(\frac{\tilde{e}_i}{\mu_T^1(\tilde{e}_i)}) \cap \tilde{\psi}_F^2(\frac{\tilde{e}_i}{\mu_T^2(\tilde{e}_i)}) =$
   $\left\{ \begin{array}{c} \tilde{x}_j / \langle min\{\zeta_T^1(\tilde{x}_j), \zeta_T^2(\tilde{x}_j)\}, max\{\zeta_F^1(\tilde{x}_j), \zeta_F^2(\tilde{x}_j)\}, max\{\zeta_N^1(\tilde{x}_j), \zeta_N^2(\tilde{x}_j)\}\rangle : \\ \tilde{x}_j \in \hat{\triangle}, \zeta_T^k(\tilde{x}_j), \zeta_F^k(\tilde{x}_j), \zeta_N^k(\tilde{x}_j) \in [0,1] \end{array} \right\}.$

Example 2. *By following the assumptions of Example 1, we tabulated representations of FpPiFSSs* $\hat{\Pi}_1$, $\hat{\Pi}_2$, $\hat{\Pi}_3$, *and* $\hat{\Pi}_4$ *in Table 3, Table 4, Table 5, and Table 6, respectively.*

Table 3. Tabular form of FpPiFSS $\hat{\Pi}_1$.

$\hat{\Pi}_1$	$\tilde{x}_1$	$\tilde{x}_2$	$\tilde{x}_3$	$\tilde{x}_4$
$\frac{\tilde{e}_1}{0.2}$	$\langle 0.21, 0.33, 0.32\rangle$	$\langle 0.23, 0.35, 0.34\rangle$	$\langle 0.25, 0.37, 0.36\rangle$	$\langle 0.27, 0.39, 0.38\rangle$
$\frac{\tilde{e}_2}{0.5}$	$\langle 0.31, 0.21, 0.11\rangle$	$\langle 0.33, 0.23, 0.13\rangle$	$\langle 0.35, 0.25, 0.15\rangle$	$\langle 0.37, 0.27, 0.17\rangle$
$\frac{\tilde{e}_3}{0.3}$	$\langle 0.11, 0.31, 0.21\rangle$	$\langle 0.13, 0.33, 0.23\rangle$	$\langle 0.15, 0.35, 0.25\rangle$	$\langle 0.17, 0.37, 0.27\rangle$
$\frac{\tilde{e}_4}{0.6}$	$\langle 0.41, 0.11, 0.31\rangle$	$\langle 0.43, 0.13, 0.33\rangle$	$\langle 0.45, 0.15, 0.35\rangle$	$\langle 0.47, 0.17, 0.37\rangle$

Table 4. Tabular form of FpPiFSS $\hat{\Pi}_2$.

$\hat{\Pi}_2$	$\tilde{x}_1$	$\tilde{x}_2$	$\tilde{x}_3$	$\tilde{x}_4$
$\frac{\tilde{e}_1}{0.2}$	$\langle 0.22, 0.32, 0.31\rangle$	$\langle 0.24, 0.34, 0.33\rangle$	$\langle 0.26, 0.36, 0.35\rangle$	$\langle 0.28, 0.38, 0.37\rangle$
$\frac{\tilde{e}_2}{0.5}$	$\langle 0.32, 0.20, 0.10\rangle$	$\langle 0.34, 0.22, 0.12\rangle$	$\langle 0.36, 0.24, 0.14\rangle$	$\langle 0.38, 0.26, 0.16\rangle$
$\frac{\tilde{e}_3}{0.3}$	$\langle 0.12, 0.30, 0.20\rangle$	$\langle 0.14, 0.32, 0.22\rangle$	$\langle 0.16, 0.34, 0.24\rangle$	$\langle 0.18, 0.36, 0.26\rangle$
$\frac{\tilde{e}_4}{0.6}$	$\langle 0.42, 0.10, 0.30\rangle$	$\langle 0.44, 0.12, 0.32\rangle$	$\langle 0.46, 0.14, 0.34\rangle$	$\langle 0.48, 0.16, 0.36\rangle$

Table 5. Tabular form of FpPiFSS $\hat{\Pi}_3 = \hat{\Pi}_1 \cup \hat{\Pi}_2$.

$\hat{\Pi}_3$	$\tilde{x}_1$	$\tilde{x}_2$	$\tilde{x}_3$	$\tilde{x}_4$
$\frac{\tilde{e}_1}{0.2}$	$\langle 0.22, 0.32, 0.31 \rangle$	$\langle 0.24, 0.34, 0.33 \rangle$	$\langle 0.26, 0.36, 0.35 \rangle$	$\langle 0.28, 0.38, 0.37 \rangle$
$\frac{\tilde{e}_2}{0.5}$	$\langle 0.32, 0.20, 0.10 \rangle$	$\langle 0.34, 0.22, 0.12 \rangle$	$\langle 0.36, 0.24, 0.14 \rangle$	$\langle 0.38, 0.26, 0.16 \rangle$
$\frac{\tilde{e}_3}{0.3}$	$\langle 0.12, 0.30, 0.20 \rangle$	$\langle 0.14, 0.32, 0.22 \rangle$	$\langle 0.16, 0.34, 0.24 \rangle$	$\langle 0.18, 0.36, 0.26 \rangle$
$\frac{\tilde{e}_4}{0.6}$	$\langle 0.42, 0.10, 0.30 \rangle$	$\langle 0.44, 0.12, 0.32 \rangle$	$\langle 0.46, 0.14, 0.34 \rangle$	$\langle 0.48, 0.16, 0.36 \rangle$

Table 6. Tabular form of FpPiFSS $\hat{\Pi}_4 = \hat{\Pi}_1 \cap \hat{\Pi}_2$.

$\hat{\Pi}_4$	$\tilde{x}_1$	$\tilde{x}_2$	$\tilde{x}_3$	$\tilde{x}_4$
$\frac{\tilde{e}_1}{0.2}$	$\langle 0.21, 0.33, 0.32 \rangle$	$\langle 0.23, 0.35, 0.34 \rangle$	$\langle 0.25, 0.37, 0.36 \rangle$	$\langle 0.27, 0.39, 0.38 \rangle$
$\frac{\tilde{e}_2}{0.5}$	$\langle 0.31, 0.21, 0.11 \rangle$	$\langle 0.33, 0.23, 0.13 \rangle$	$\langle 0.35, 0.25, 0.15 \rangle$	$\langle 0.37, 0.27, 0.17 \rangle$
$\frac{\tilde{e}_3}{0.3}$	$\langle 0.11, 0.31, 0.21 \rangle$	$\langle 0.13, 0.33, 0.23 \rangle$	$\langle 0.15, 0.35, 0.25 \rangle$	$\langle 0.17, 0.37, 0.27 \rangle$
$\frac{\tilde{e}_4}{0.6}$	$\langle 0.41, 0.11, 0.31 \rangle$	$\langle 0.43, 0.13, 0.33 \rangle$	$\langle 0.45, 0.15, 0.35 \rangle$	$\langle 0.47, 0.17, 0.37 \rangle$

3.2. Criteria for Selection of Parameters

From personal assessments to commercial decisions, attributes play an essential role in the DMG process in a variety of contexts. When making decisions, individuals take into account the attributes of possibilities, alternatives, or objects. Information regarding many possibilities can be gathered using attributes as the foundation. When faced with a decision, people or organizations frequently gather information about numerous aspects to assess and contrast the options. Not all attributes weigh identically in every selection. Depending on how important they are, various features receive varying weights from individuals. Alternatives can be more easily compared because of their attributes. The options are compared based on how well they perform on each attribute to see which one best fits the decision maker's preferences and objectives. This generally involves the use of instruments like scorecards or decision matrices. Certain attributes could be essential in one circumstance but less important in another. Making effective decisions requires an understanding of the context. Therefore, while making the most beneficial selection, DMS must priorities certain attributes above others and be prepared to forego some privileges. The selection of attributes is both an intellectual endeavor and a form of artistry. It combines data-driven analysis with judgments depending on the particular decision situation. The idea is to select features that are pertinent, significant, and consistent with the decision's goals. For the proposed study, a peer-review analysis of the literature like Tan et al. [49], Liao et al. [50], and Quan et al. [51] is accomplished to adopt the attributes. By following these references, eight major categories of parameters are shortlisted and, consequently, eight parameters are adopted that are presented in Table 7 and Figure 1.

Table 7. Selected parameters and their respective categories.

Sr. No.	Category	Parameter	Adoptation
1	Reputation	Purchase Cost	Valid
2	Certifications	Product Quality	Valid
3	Financial Health	Capacity	Valid
4	Collaboration	Delivery Level	Valid
5	Product Development	Lead Time	Valid
6	Customer Base	Location	Valid
7	Social Responsibility	Flexibility	Valid
8	Sustainability	Green Degree	Valid

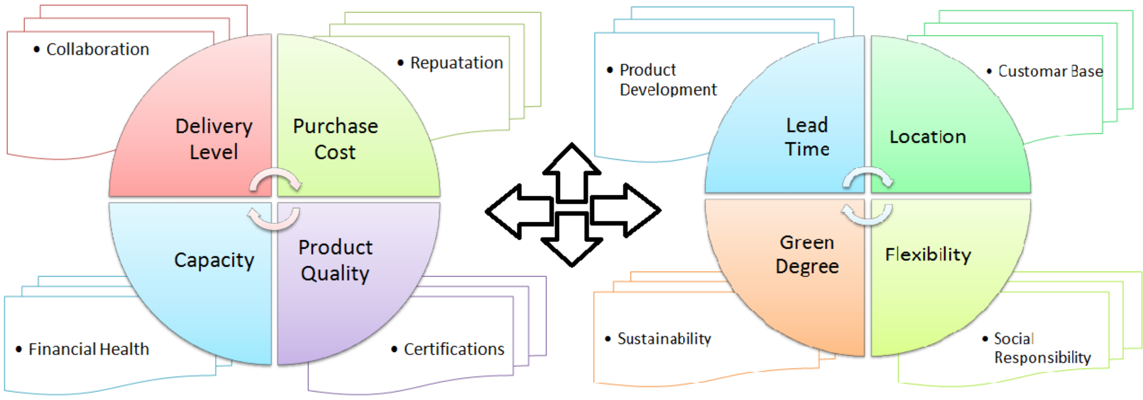

Figure 1. Selected parameters and their relevant categories.

3.3. Determination of Fuzzy Parameterized Grades

When choosing parameters, the DMS occasionally get into circumstances in which they are dubious. The FPGs are used to deal with such circumstances. The FPGs are specific values between [0, 1] that have parameters to determine how imprecise and ambiguous they are. The methodology for calculating FPGs is now presented.

After keen analysis of the published literature [52–59], it was observed that the concept of FPara is used in these references, but proper arithmetical criteria is not used for the determination of FPGs; instead, hypothetical grades are used.

Therefore, the following arithmetical criterion is presented to have preference over the published literature.

Let $\hat{\Xi} = \{\tilde{e}_1, \tilde{e}_2, \tilde{e}_3, \ldots, \tilde{e}_n\}$ be the set of attributes and $\mathbb{Z} = \{E_1, E_2, E_3, \ldots, E_p\}$ be the set of DMS. The DMS $E_j, (j = 1, 2, 3, \ldots, p)$ provide preferential values to parameters $\tilde{e}_i$, (i = 1, 2, 3, $\ldots, n$) in terms of weights $\varpi_{i,j}, (i = 1, 2, 3, \ldots, n; j = 1, 2, 3, \ldots, p)$ in order to settle their ambiguity levels about these parameters. The FPGs $\tilde{\mu}(\tilde{e}_i), (i = 1, 2, 3, \ldots, n)$ are determined by following the formulation:

$$\tilde{\mu}(\tilde{e}_i) = \frac{1}{3}\{\Omega_{AM}(\varpi_{i,j}) + \Omega_{GM}(\varpi_{i,j}) + \Omega_{HM}(\varpi_{i,j})\}. \quad (1)$$

In this equation, we utilize a combination of Pythagorean means (arithmetic mean = $\Omega_{AM}(\varpi_{i,j})$, geometric mean = $\Omega_{GM}(\varpi_{i,j})$, and harmonic mean = $\Omega_{HM}(\varpi_{i,j})$) of the assigned weights $\varpi_{i,j}$ to calculate the fuzzy preference grades $\tilde{\mu}(\tilde{e}_i)$ for each attribute $\tilde{e}_i$. These grades contribute to quantifying the DMS's preferences and uncertainties regarding the attributes.

3.4. Decision Support Framework

Suppliers act as an unavoidable beginning for several ancillary risk factors. In CSuC, choosing a supplier is seen as an MCDM problem that demands taking into account every aspect of quality. The CSuC's suppliers must be able to handle diversions expertly and successfully. Managers typically concentrate primarily on making purchases from suppliers that can supply things swiftly, at a cheaper cost, and to a higher standard. Due to the fact that choosing a supplier necessitates comparing and assessing potential providers based on a number of different criteria or aspects, it is known as a multicriteria decision-making problem. When businesses or organizations choose which suppliers to work with, they must take into account a number of characteristics and aspects that are pertinent to their unique demands and requirements. At the same time, choosing a supplier is seen as uncertain due to a number of variables, including insufficient information, fluctuating market conditions, supplier reliability, breakdowns in the supply chain, economical viability, etc., that can add ambiguity and uncertainty to the decision-making process. This section first proposes an algorithm to carve out the locus for optimum selection of timber suppliers, and then the algorithm is validated by a case-study-based scenario.

Problem Scenario

Real estate is a crucial industry with broad economic, social, and cultural ramifications. It is fundamental to the production of wealth, the provision of housing, and economic growth, making it an essential element of contemporary civilization. It is important to remember that the real estate industry's status can change over time and is influenced by a wide range of elements. The sector has to overcome obstacles such as increasing construction costs, the difficulty of finding affordable homes in many urban locations, and the requirement to adjust to shifting consumer demands. Timber is an adaptable and frequently employed resource in real estate projects, particularly in building and interior design. It is frequently seen in the form of timber or synthetic wood products. Its widespread use is a result of its usability, durability, and visual appeal. In the upcoming section, an algorithm is presented that will assist the real estate industry in selecting the most suitable timber suppliers.

3.5. Proposed Algorithm

A brief description of Algorithm 1's stages is presented in Figure 2. The suggested algorithm's significance and dependability can be inferred from its provenance, which it is interdisciplinary in nature, making it understandable even to researchers without a strong foundation in mathematics. Additionally, the computational complexity has been significantly reduced because of the usage of matrix computation. Pseudocodes for any software are not utilized, because the bulk of potential readers lack even the most fundamental understanding of computers and programming. However, with the aid of computer specialists and programs, they can be applied to machine learning. Microsoft Excel 2010 is used for all calculations and graphing.

Algorithm 1: This algorithm consists of three stages: (1) Input, (2) Construction and Computations, (3) Output. These stages are explained as below:

---

1. **Input**

1.1 Consider a set $\mathbb{Z} = \{E_1, E_2, E_3, \ldots, E_p\}$ consisting of experts (DMS) hired for the evaluation process.

1.2 Assume a set of alternatives $\hat{\triangle} = \{\tilde{x}_1, \tilde{x}_2, \tilde{x}_3, \ldots, \tilde{x}_m\}$ consisting of suppliers short listed by DMS through initial screening.

1.3 Assume a set $\hat{\tilde{\Xi}} = \{\tilde{e}_1, \tilde{e}_2, \tilde{e}_3, \ldots, \tilde{e}_n\}$ consisting of parameters selected by decision makers with mutual consensus.

1.4 Collect preferential weights $\varpi_{i,j}, (i = 1, 2, 3, \ldots, n; j = 1, 2, 3, \ldots, p)$ from decision makers for each parameter.

---

2. **Construction and Computations**

2.1 Determine FPGs for each parameter by using Equation (1).

2.2 Construct an FpPiFSS $\hat{\Pi}$ based on the opinions provided by DMS for the approximation of alternatives based on fuzzy parameters and represent it in tabular form.

2.3 Convert each picture fuzzy value into fuzzy value by using the criterion $|\tilde{\zeta}_T(\tilde{x}) - \tilde{\zeta}_F(\tilde{x}) - \tilde{\zeta}_N(\tilde{x})|$ and represent them in matrix $\mathbb{M}_1$.

2.4 Construct decision matrix $\mathbb{M}_2$ by multiplying each row entry with its respective FPG.

2.5 Compute the score values $\mathbb{S}(\tilde{x}_m)$ of each alternative by taking the sum of respective entries of the alternative column and represent them in matrix $\mathbb{M}_3$.

3. **Output**

3.1 Select the alternative with maximum score.

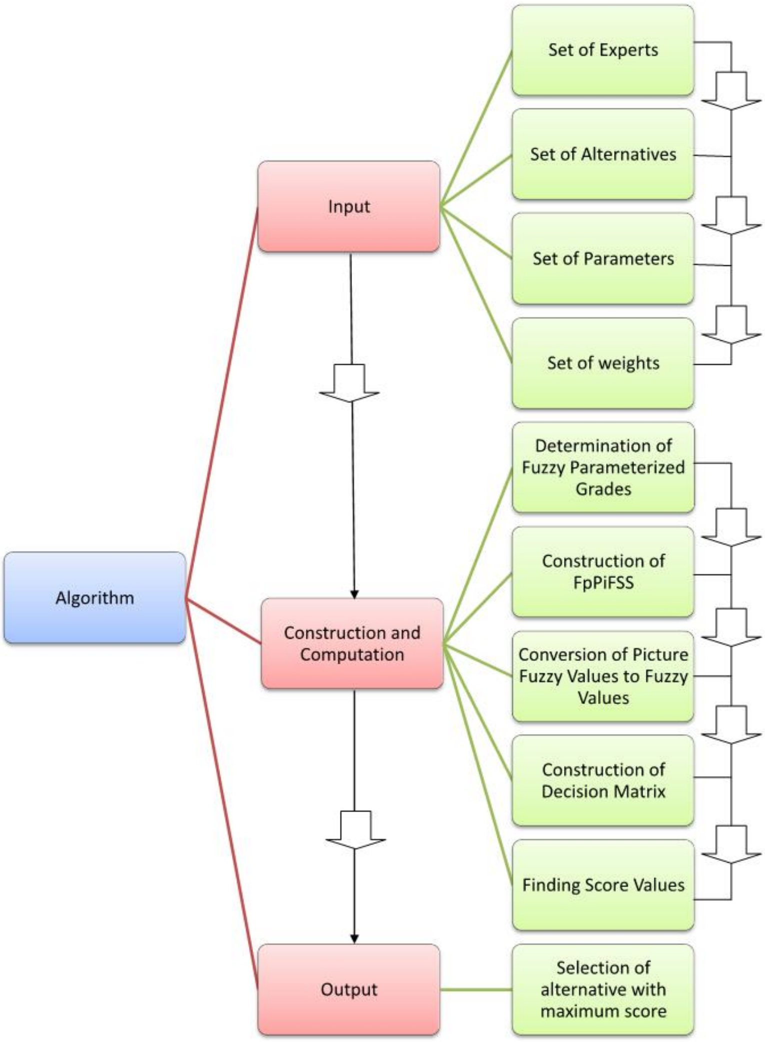

Figure 2. Steps of Algorithm 1.

3.6. Validation of Algorithm 1

In the following example, the steps of Algorithm 1 proposed here are explained.

Example 3. *"ZX Developers" is a real estate organization in Pakistan that deals with the selling and purchasing of residential property, as well as the construction of homes and commercial malls. It has established housing societies in different parts of the country. The administration is looking for timber suppliers for windows, doors, and other essentials for houses in its new residential project. Due to the availability of substandard timber in the market and mistrust of suppliers, the company has several concerns. Therefore, the administration advertises a "call for proposals" in reputed national and local newspapers. Consequently, the procurement department receives many proposals in response to the advertised calls. Therefore, a committee of four DMS (experts) (i.e., $\mathbb{Z} = \{E_1, E_2, E_3, E_4\}$) is constituted, consisting of two external and two internal experts with good procurement experience. After peer screening, the proposals of four suppliers (i.e., $\hat{\triangle} = \{\tilde{x}_1, \tilde{x}_2, \tilde{x}_3, \tilde{x}_4\}$) are selected for further assessment. In order to have an unbiased and reliable assessment, the committee has finalized eight parameters (i.e, $\hat{\Xi} = \{\tilde{e}_1, \tilde{e}_2, \tilde{e}_3, \ldots, \tilde{e}_8\}$) for the evaluation of suppliers with their mutual consensus, where $\tilde{e}_1 =$ purchase cost, $\tilde{e}_2 =$ product quality, $\tilde{e}_3 =$ capacity, $\tilde{e}_4 =$ delivery level, $\tilde{e}_5 =$ lead time, $\tilde{e}_6 =$ location, $\tilde{e}_7 =$ flexibility, and $\tilde{e}_8 =$ sustainability. The DMS are ambiguous regarding the preferential aspects the of selected parameters; therefore, they are allowed to provide preferential weights to each parameter that is presented in Table 8.*

Table 8. Preferential weights for selected parameters $\tilde{e}_i, i = 1,2,3,4$.

Parameters	DM E_1	DM E_2	DM E_3	DM E_4
$\tilde{e}_1$	0.21	0.32	0.29	0.18
$\tilde{e}_2$	0.11	0.42	0.39	0.08
$\tilde{e}_3$	0.15	0.35	0.25	0.25
$\tilde{e}_4$	0.28	0.22	0.23	0.27
$\tilde{e}_5$	0.41	0.19	0.26	0.14
$\tilde{e}_6$	0.42	0.18	0.22	0.18
$\tilde{e}_7$	0.22	0.22	0.16	0.40
$\tilde{e}_8$	0.14	0.16	0.25	0.45

Now, to assess the uncertain nature of parameters, the FPGs for eight parameters are determined by using Equation (1), which is presented in Table 9.

Table 9. The FPGs $\tilde{\mu}(\tilde{e}_i)$ for selected parameters $\tilde{e}_i, i = 1,2,3,4$.

$\tilde{e}_i$	$\tilde{\mu}(\tilde{e}_i)$	$\tilde{e}_i$	$\tilde{\mu}(\tilde{e}_i)$
$\tilde{e}_1$	$\frac{0.2500+0.2434+0.2368}{3} = 0.2434$	$\tilde{e}_5$	$\frac{0.2500+0.2308+0.2140}{3} = 0.2316$
$\tilde{e}_2$	$\frac{0.2500+0.1948+0.1507}{3} = 0.1985$	$\tilde{e}_6$	$\frac{0.2500+0.2339+0.2218}{3} = 0.2352$
$\tilde{e}_3$	$\frac{0.2500+0.2393+0.2283}{3} = 0.1559$	$\tilde{e}_7$	$\frac{0.2500+0.2359+0.2242}{3} = 0.2367$
$\tilde{e}_4$	$\frac{0.2500+0.2487+0.2474}{3} = 0.2487$	$\tilde{e}_8$	$\frac{0.2500+0.2241+0.2039}{3} = 0.2260$

In Table 9, the first entry, $\frac{0.2500+0.2434+0.2368}{3} = 0.2434$, *means that the values* 0.2500, 0.2434, *and* 0.2368 *are, respectively, the arithmetic, geometric, and harmonic means of the weights assigned by DMS to parameter* $\tilde{e}_1$. *Thus, using Equation (1), its FPG,* $\tilde{\mu}(\tilde{e}_1) = 0.2434$, *is calculated. After collecting the opinions of DMS, the FpPiFSS* $\hat{\Pi}$ *is constructed as*

$$\hat{\Pi} = \left\{ \begin{array}{l}
\left(\frac{\tilde{e}_1}{0.2434}, \left\{ \frac{\tilde{x}_1}{\langle 0.27,0.39,0.38 \rangle}, \frac{\tilde{x}_2}{\langle 0.25,0.37,0.36 \rangle}, \frac{\tilde{x}_3}{\langle 0.23,0.35,0.34 \rangle}, \frac{\tilde{x}_4}{\langle 0.21,0.33,0.32 \rangle} \right\} \right), \\
\left(\frac{\tilde{e}_2}{0.1985}, \left\{ \frac{\tilde{x}_1}{\langle 0.37,0.27,0.17 \rangle}, \frac{\tilde{x}_2}{\langle 0.35,0.25,0.15 \rangle}, \frac{\tilde{x}_3}{\langle 0.33,0.23,0.13 \rangle}, \frac{\tilde{x}_4}{\langle 0.31,0.21,0.11 \rangle} \right\} \right), \\
\left(\frac{\tilde{e}_3}{0.1559}, \left\{ \frac{\tilde{x}_1}{\langle 0.17,0.37,0.27 \rangle}, \frac{\tilde{x}_2}{\langle 0.15,0.35,0.25 \rangle}, \frac{\tilde{x}_3}{\langle 0.13,0.33,0.23 \rangle}, \frac{\tilde{x}_4}{\langle 0.11,0.31,0.21 \rangle} \right\} \right), \\
\left(\frac{\tilde{e}_4}{0.2487}, \left\{ \frac{\tilde{x}_1}{\langle 0.47,0.17,0.37 \rangle}, \frac{\tilde{x}_2}{\langle 0.45,0.15,0.35 \rangle}, \frac{\tilde{x}_3}{\langle 0.43,0.13,0.33 \rangle}, \frac{\tilde{x}_4}{\langle 0.41,0.11,0.31 \rangle} \right\} \right), \\
\left(\frac{\tilde{e}_5}{0.2316}, \left\{ \frac{\tilde{x}_1}{\langle 0.46,0.18,0.38 \rangle}, \frac{\tilde{x}_2}{\langle 0.35,0.25,0.25 \rangle}, \frac{\tilde{x}_3}{\langle 0.41,0.11,0.31 \rangle}, \frac{\tilde{x}_4}{\langle 0.43,0.13,0.33 \rangle} \right\} \right), \\
\left(\frac{\tilde{e}_6}{0.2352}, \left\{ \frac{\tilde{x}_1}{\langle 0.48,0.19,0.39 \rangle}, \frac{\tilde{x}_2}{\langle 0.25,0.25,0.45 \rangle}, \frac{\tilde{x}_3}{\langle 0.43,0.15,0.35 \rangle}, \frac{\tilde{x}_4}{\langle 0.42,0.17,0.37 \rangle} \right\} \right), \\
\left(\frac{\tilde{e}_7}{0.2367}, \left\{ \frac{\tilde{x}_1}{\langle 0.41,0.11,0.31 \rangle}, \frac{\tilde{x}_2}{\langle 0.45,0.25,0.15 \rangle}, \frac{\tilde{x}_3}{\langle 0.45,0.17,0.37 \rangle}, \frac{\tilde{x}_4}{\langle 0.47,0.19,0.36 \rangle} \right\} \right), \\
\left(\frac{\tilde{e}_8}{0.2260}, \left\{ \frac{\tilde{x}_1}{\langle 0.45,0.12,0.32 \rangle}, \frac{\tilde{x}_2}{\langle 0.47,0.24,0.34 \rangle}, \frac{\tilde{x}_3}{\langle 0.41,0.11,0.31 \rangle}, \frac{\tilde{x}_4}{\langle 0.48,0.21,0.21 \rangle} \right\} \right)
\end{array} \right\}.$$

Its matrix representation is presented in Table 10.

Real-world data might be unavailable or inaccessible due to concerns about confidentiality or other limitations; so, the weights and opinions of DMS are regarded as hypothetical. Making their experiments and methods more understandable for others through the use of fictitious information enables researchers to advance reproducibility and transparency in their work. In Table 10, every entry is in terms of picture fuzzy number $\langle \tilde{\zeta}_T(\tilde{x}), \tilde{\zeta}_F(\tilde{x}), \tilde{\zeta}_N(\tilde{x}) \rangle$, *which is reduced to fuzzy value by employing formulation* $|\tilde{\zeta}_T(\tilde{x}) - \tilde{\zeta}_F(\tilde{x}) - \tilde{\zeta}_N(\tilde{x})|$ *and expressed by matrix* $\mathbb{M}_1$ *in Table 11.*

Table 10. Tabular form of FpPiFSS $\hat{\Pi}$.

$\hat{\Pi}$	$\tilde{x}_1$	$\tilde{x}_2$	$\tilde{x}_3$	$\tilde{x}_4$
$\frac{\tilde{e}_1}{0.2434}$	$\langle 0.27, 0.39, 0.38\rangle$	$\langle 0.25, 0.37, 0.36\rangle$	$\langle 0.23, 0.35, 0.34\rangle$	$\langle 0.21, 0.33, 0.32\rangle$
$\frac{\tilde{e}_2}{0.1985}$	$\langle 0.37, 0.27, 0.17\rangle$	$\langle 0.35, 0.25, 0.15\rangle$	$\langle 0.33, 0.23, 0.13\rangle$	$\langle 0.31, 0.21, 0.11\rangle$
$\frac{\tilde{e}_3}{0.1559}$	$\langle 0.17, 0.37, 0.27\rangle$	$\langle 0.15, 0.35, 0.25\rangle$	$\langle 0.13, 0.33, 0.23\rangle$	$\langle 0.11, 0.31, 0.21\rangle$
$\frac{\tilde{e}_4}{0.2487}$	$\langle 0.47, 0.17, 0.37\rangle$	$\langle 0.45, 0.15, 0.35\rangle$	$\langle 0.43, 0.13, 0.33\rangle$	$\langle 0.41, 0.11, 0.31\rangle$
$\frac{\tilde{e}_5}{0.2316}$	$\langle 0.46, 0.18, 0.38\rangle$	$\langle 0.35, 0.25, 0.25\rangle$	$\langle 0.41, 0.11, 0.31\rangle$	$\langle 0.43, 0.13, 0.33\rangle$
$\frac{\tilde{e}_6}{0.2352}$	$\langle 0.48, 0.19, 0.39\rangle$	$\langle 0.25, 0.25, 0.45\rangle$	$\langle 0.43, 0.15, 0.35\rangle$	$\langle 0.42, 0.17, 0.37\rangle$
$\frac{\tilde{e}_7}{0.2367}$	$\langle 0.41, 0.11, 0.31\rangle$	$\langle 0.45, 0.25, 0.15\rangle$	$\langle 0.45, 0.17, 0.37\rangle$	$\langle 0.47, 0.19, 0.36\rangle$
$\frac{\tilde{e}_8}{0.2260}$	$\langle 0.45, 0.12, 0.32\rangle$	$\langle 0.47, 0.24, 0.34\rangle$	$\langle 0.41, 0.11, 0.31\rangle$	$\langle 0.48, 0.21, 0.21\rangle$

Table 11. Matrix with reduced fuzzy values.

$\mathbb{M}_1$	$\tilde{x}_1$	$\tilde{x}_2$	$\tilde{x}_3$	$\tilde{x}_4$
$\frac{\tilde{e}_1}{0.2434}$	0.50	0.48	0.46	0.44
$\frac{\tilde{e}_2}{0.1985}$	0.07	0.05	0.03	0.01
$\frac{\tilde{e}_3}{0.1559}$	0.47	0.45	0.43	0.41
$\frac{\tilde{e}_4}{0.2487}$	0.07	0.05	0.03	0.03
$\frac{\tilde{e}_5}{0.2316}$	0.10	0.15	0.01	0.03
$\frac{\tilde{e}_6}{0.2352}$	0.10	0.45	0.07	0.12
$\frac{\tilde{e}_7}{0.2367}$	0.01	0.05	0.09	0.08
$\frac{\tilde{e}_8}{0.2260}$	0.01	0.11	0.01	0.06

In order to obtain decision matrix $\mathbb{M}_2$, each FPG is multiplied by every entry in its respective row in Table 11, and thus, the obtained results are presented in Table 12.

Table 12. Decision Matrix.

$\mathbb{M}_2$	$\tilde{x}_1$	$\tilde{x}_2$	$\tilde{x}_3$	$\tilde{x}_4$
$\tilde{e}_1$	0.121700	0.116832	0.111964	0.107096
$\tilde{e}_2$	0.013895	0.009925	0.005955	0.001985
$\tilde{e}_3$	0.073273	0.070155	0.067037	0.063919
$\tilde{e}_4$	0.017409	0.012435	0.007461	0.007461
$\tilde{e}_5$	0.023160	0.034740	0.009264	0.006948
$\tilde{e}_6$	0.023520	0.105840	0.016464	0.028224
$\tilde{e}_7$	0.002367	0.011835	0.021303	0.018936
$\tilde{e}_8$	0.002260	0.024860	0.002260	0.013560

For the sake of supplier ranking, their score values $\mathbb{S}(\tilde{x}_m)$ *are determined by adding their approximations based on parameters* $\tilde{e}_i, (i = 1, 2, 3, \ldots, 8)$ *in the respective column of Table 12. The computed scores corresponding to suppliers are tabulated in Table 13.*

From Table 13 and Figure 3, it can be observed easily that the maximum score is obtained by supplier $\tilde{x}_2$*; therefore, the proposal of supplier* $\tilde{x}_2$ *is recommended for the contract to supply timber. The ranking of the suppliers is* $\tilde{x}_2 > \tilde{x}_1 > \tilde{x}_4 > \tilde{x}_3$.

Table 13. Scores of suppliers.

$\mathbb{M}_3$	$\tilde{x}_1$	$\tilde{x}_2$	$\tilde{x}_3$	$\tilde{x}_4$
$\mathbb{S}(\tilde{x}_m)$	0.277584	**0.386622**	0.241708	0.248129

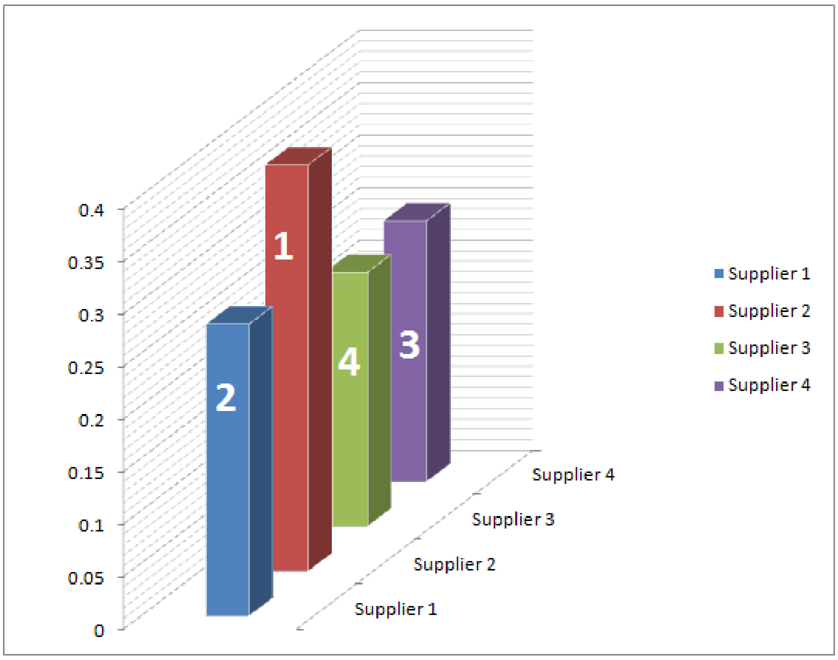

Figure 3. Ranking of supplier based on score values.

4. Discussion and Comparison Analysis

An exemplary explanation is offered for comparing the efficacy of the proposed strategy with certain predeveloped strategies before presenting an overview of the work. In this context, an appropriate measure is employed to determine whether the assessment procedure is successful, i.e., whether the implemented significant variables are realized in the current techniques or not. It is predicted that the suggested strategy is more appealing in terms of computing simplicity and logical inference.

1. Procurement has drawn a lot of attention because it has become crucial in determining the durability and efficacy of production teams. Purchaser–dealer correlations based solely on cost are insufficient to any further extent, as already discussed by Sarkis and Talluri [60]. Companies are being forced to reevaluate their strategies related to purchasing and evaluation as an effective procuring assessment directly depends on choosing the "right" supplier due to the increasing significance of supplier selection decisions.
2. The SuSP is an MCDM problem, as previously mentioned in the literature review section, and it is simple to see that the key component of each MCDM is the bias displayed by specialists for the objects under observation with reference to each decisive element. It is also possible to examine the fact that the primary source of study in many studies is the opinions of experts. However, the computational

process may be impacted if the opinions of experts show any flaws. Roughness in the computation and information is seen to be relevant in this situation.

3. The works from investigators Xiao et al. [46], Liu et al. [61], Mukherjee et al. [62], Tan et al. [49], Liao et al. [50], and Quan et al. [51] are regarded as the most significant and pertinent to the recommended strategy for SuSP when the aforementioned discussion is taken into consideration. In order to deal with ambiguous information and imperfect expert opinions, these approaches overlooked soft settings, the consideration of three-dimensional membership values $\langle \tilde{\zeta}_T(\tilde{x}), \tilde{\zeta}_F(\tilde{x}), \tilde{\zeta}_N(\tilde{x}) \rangle$, and the concept of FPara. The suggested strategy can manage all of the aforementioned factors simultaneously.
4. For the purpose of a favorable assessment, Tables 14 and 15 elaborate on its computation and structural comparison with the aforementioned methods. The subsequent assessment criteria are taken into account in this regard:
   (i). Three-dimensional membership-based opinions (3DMO) (i.e., provision of opinions based on dependent positive, negative, and neutral membership grades).
   (ii). Soft settings (SoS) (i.e., parameterization mode: the inclusion of parameters for the approximation of alternatives. This kind of setting provides an approximate function to accomplish this task).
   (iii). Fuzzy parameterization idea (FPI) (i.e., provision of fuzzy parameterized parameters to handle the uncertainties of DMS regarding the selection of parameters).
   (iv). Consideration of categorical criteria (CCC) (i.e., parameters with their relevant categories of criteria).

   The symbols ✓ and × are for YES and NO, respectively, in Table 15.

Table 14. Computation comparison.

References	Application	Ranking
Mukherjee et al. [62]	SuSP	$\tilde{x}_3 > \tilde{x}_2 > \tilde{x}_1 > \tilde{x}_4$
Liao et al. [50]	SuSP	$\tilde{x}_1 > \tilde{x}_2 > \tilde{x}_4 > \tilde{x}_3$
Quan et al. [51]	SuSP	$\tilde{x}_2 > \tilde{x}_1 > \tilde{x}_3 > \tilde{x}_4$
Proposed Approach	SuSP	$\tilde{x}_2 > \tilde{x}_1 > \tilde{x}_4 > \tilde{x}_3$

According to the rules of numerical computation, the smaller the values after the decimal point, the more reliable, convergent, and consistent the value is. In this sense, the results of the proposed algorithm are better than the previous ones that are presented in Table 15. In the same way, it is evident that the ranking of the outcomes in the suggested framework, despite the inclusion of fuzzy parameterized grades, is still consistent with the findings of Mukherjee et al. [62], Liao et al. [50], and Quan et al. [51]. These references covered the SuSP with fuzzy-set-like settings, which is closely linked to the subject of this study in terms of parameter selection, alternative options, and application. That is why we use these references for our computation-based comparison.

Table 15. Structural comparison.

References	3DMO	SoS	FPI	CCC
Xiao et al. [46]	×	✓	×	×
Liu et al. [61]	×	×	×	×
Mukherjee et al. [62]	×	×	×	✓
Tan et al. [49]	×	×	×	✓
Liao et al. [50]	×	×	×	✓
Quan et al. [51]	×	×	×	✓
Proposed Approach	✓	✓	✓	✓

Table 15 makes it evident that the proposed mathematical framework, FpPiFSS, is superior to previous ones, since it takes into account all of their characteristics and recognizes that they are all special cases.

5. Conclusions

SuSP is classified as an MADM problem, since selecting a supplier entails comparing and evaluating alternative suppliers based on a variety of distinct criteria or features. Businesses and organizations must consider a variety of traits and factors that are relevant to their particular needs and requirements when deciding which suppliers to deal with. Choosing a supplier is seen as uncertain due to a number of factors that can make decisions more ambiguous and uncertain. Therefore, the primary goal of this study is to address the restrictions of SuCM and CSuCM for choosing suppliers in the published literature. The technique of selection is typically biased and unreliable due to the categorical parameters' ambiguous nature. In order to introduce a more adaptable and generic framework, the FPara notion with the PiFSS environment is used in this study. Thus, the mathematical structure FpPiFSS is introduced, and some of its set-theoretic operations are presented with illustrated examples. As a part of the proposed methodology, simple Pythagorean means-based criteria are used to determine the FPGs for categorical parameters based on the weights assigned by DMS. Based on fuzzy parameterized categorical parameters, approximations of alternatives are obtained in terms of three-dimensional membership values $\langle \tilde{\zeta}_T(\tilde{x}), \tilde{\zeta}_F(\tilde{x}), \tilde{\zeta}_N(\tilde{x}) \rangle$ which facilitate the DMS in the evaluation of the objects by providing their opinions with yes, no, or neutral responses. In the decision support framework section, four timber suppliers are evaluated based on eight of the most significant categorical parameters by proposing an algorithm. An illustrative case is used to show the suggested algorithm's validity, and when compared with previously published similar work, the resulting findings have been determined to be trustworthy and consistent. The proposed approach is compared with published references on a computation and structure basis. As far as the advantages of the proposed framework are concerned, it can easily overcome the following genuine situations collectively:

- The DMS are sometimes faced with such situations that it becomes difficult for them to determine which parameters to select and which to reject, which to give more importance, and which to give less importance. In other words, they face some degree of uncertainty and ambiguity in selecting, testing, and evaluating features.
- The DMS sometimes need a DMG environment that not only reinforces their positive and negative opinions but also takes into account their impartiality to evaluate alternatives on the basis of parameters.
- A suitable mode of settings for approximating the alternatives based on parameters.

These are tackled using the FPGs, PiFSs, and approximate function of SoS, respectively. A stronger procurement process can result in greater quality, lower prices, reduced risk, and an all-around efficient operation through strategic decision making and careful supplier selection. Setting criteria to analyze the skills, dependability, goods, and services of potential suppliers makes all of this possible. However, it is sometimes possible to face such a situation in which the DMS make their uncertainty about parameters subject to the use of other degrees like intuitionistic fuzzy, neutrosophic, etc., instead of fuzzy degrees. Similarly, the DMS may insist that they express their opinions about alternatives in the form of a three-way membership function that contains independent positive, negative, and neutral degrees of membership. The proposed context shows inadequacy to deal with such a situation. So, researchers can further improve this framework based on these limitations and by using other degrees instead of fuzzy degrees, but this may lead to computational complications.

The suggested mathematical context may also be used in a number of other contexts, including the management of solid waste, choosing the right materials, parameter reduction, clinical diagnosis, etc., similar to how it can be used to solve pattern recognition and other associated issues if its information measures are developed.

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