symmetry

MDPI

Article

A New Cosine-Originated Probability Distribution with Symmetrical and Asymmetrical Behaviors: Repetitive Acceptance Sampling with Reliability Application

Huda M. Alshanbari [1], Gadde Srinivasa Rao [2], Jin-Taek Seong [3,*], Sultan Salem [4] and Saima K. Khosa [5]

[1] Department of Mathematical Sciences, College of Science, Princess Nourah bint Abdulrahman University, P.O. Box 84428, Riyadh 11671, Saudi Arabia; hmalshanbari@pnu.edu.sa
[2] Department of Mathematics and Statistics, University of Dodoma, Dodoma P.O. Box 259, Tanzania; gadde.srinivasa@udom.ac.tz
[3] Graduate School of Data Science, Chonnam National University, Gwangju 61186, Republic of Korea
[4] Department of Economics, Birmingham Business School, College of Social Sciences, University of Birmingham, Edgbaston, Birmingham B15 2TT, UK; s.salem@bham.ac.uk
[5] Department of Mathematics and Statistics, University of Saskatchewan, Saskatoon, SK S7N 5A2, Canada; skk807@mail.usask.ca
* Correspondence: jtseong@jnu.ac.kr

Abstract: Several new acceptance sampling plans using various probability distribution methods have been developed in the literature. However, there is no published work on the design of new sampling plans using trigonometric-based probability distributions. In order to cover this amazing and fascinating research gap, we first introduce a novel probabilistic method called a new modified cosine-*G* method. A special member of the new modified cosine-*G* method, namely, a new modified cosine-Weibull distribution, is examined and implemented. The density function of the new model possesses symmetrical as well as asymmetrical behaviors. The usefulness and superior fitting power of the new modified cosine-Weibull distribution are demonstrated by analyzing an asymmetrical data set. Furthermore, based on the new modified cosine-Weibull distribution, we develop a new repetitive acceptance sampling strategy for attributes with specified shape parameters. Finally, a real-world application is presented to illustrate the proposed repetitive acceptance sampling strategy.

Keywords: cosine function; weibull model; acceptance sampling plan; producer's risk; consumer's risk; median life; repetitive sampling; statistical modeling

Citation: Alshanbari, H.M.; Rao, G.S.; Seong, J.-T.; Salem, S.; Khosa, S.K. A New Cosine-Originated Probability Distribution with Symmetrical and Asymmetrical Behaviors: Repetitive Acceptance Sampling with Reliability Application. *Symmetry* **2023**, *15*, 2187. https://doi.org/10.3390/sym15122187

Academic Editors: Pedro José Fernández de Córdoba Castellá, Juan Carlos Castro-Palacio, Shufei Wu and Miguel Enrique Iglesias Martínez

Received: 8 September 2023
Revised: 14 November 2023
Accepted: 15 November 2023
Published: 12 December 2023

1. Introduction

The acceptance sampling (AS) plan is a prominent and an efficient quality control mechanism. It is frequently implemented to ensure and verify the quality and conditions of services or products, etc. The AS plan is implemented to inspect the products of lots and to decide whether to keep or extract the incoming products of lots. Rather than investigating the entire lot, the AS technique allows us to decide about the entire lot via its portion (i.e., a sample of the entire lot) only. Thus, if the selected portion of the entire lot appears as good as expected, the entire lot as per the result will be accepted. On the other hand, if the selected portion of the entire lot does not appear as good as expected, the entire lot will be rejected accordingly. Henceforth, the products will be given back to the supplier [1–3].

The AS plan has great applicability on industrial scales and is implemented to decrease the expenditure of the inspection process. It provides a guideline to the inspection team on whether the specified products or services are reliable enough for marketing or not. In addition to the reduction of the inspection expenditure, it also helps to prevent both suppliers and buyers from future losses by minimizing:

(i) the supplier's risk to prevent the rejection of the good quality lot, and
(ii) the consumer's risk to prevent the buyers from accepting the poor quality lot. For more detail, see [4,5].

Due to the importance of the AS plan in the industry and other commercial sectors, researchers have been trying to develop more and more efficient AS plans using different probability distributions. For example,

(i) Singh and Tripathi [6] introduced an AS plan using the inverse Weibull distribution,
(ii) Abushal et al. [7] implemented the power inverted Topp–Leone distribution for developing a new AS plan,
(iii) Tripathi et al. [8] used the inverse log-logistic distribution to introduce a new AS plan,
(iv) Alyami et al. [9] created a new AS plan by adopting the Fréchet binomial distribution,
(v) Algarni [10] proposed a group acceptance sampling (GAS) plan with the three-parameter Weibull distribution,
(vi) Nassr et al. [11] used the inverted Topp-Leone distribution for studying a new AS plan,
(vii) Khan et al. [12] created the fuzzy AS plan using the transmuted Weibull distribution,
(viii) Fayomi and Khan [13] proposed a group AS plan using the generalized transmuted exponential distribution,
(ix) Al-Omari and Alomani [14] introduced a double AS plan by using the Xgamma distribution, and
(x) Yiğiter et al. [15] recommended a group AS plan for the compound Weibull-exponential distribution, among others.

From the above-mentioned literature, we can see the development of new AS plans has received great attention. Nonetheless, there is a lack of established research work and guidelines for constructing new AS plans through incorporating trigonometric-based probability distributions. In this ongoing part of our research, we aim to cover this curious and astonishing research gap. The core motivation of this work is therefore to design and implement a new AS plan by means of the cosine function.

In this paper, we first attempt to introduce a new probabilistic method by considering the cosine function. The suggested method is called a new modified cosine-*G* (NMC-*G*) family. The beauty of the NMC-*G* family is that it does not have any extra or additional parameters. In the second attempt, we incorporate the proposed model for generating a new AS plan.

Definition 1. *X follows a NMC-G model if its cumulative distribution function (CDF) $F(x;\boldsymbol{\eta})$ is expressed by*

$$F(x;\boldsymbol{\eta}) = 1 - \left(\frac{1-\cos\left[\frac{\pi}{2}\bar{G}(x;\boldsymbol{\eta})\right]}{1+\cos\left[\frac{\pi}{2}\bar{G}(x;\boldsymbol{\eta})\right]} \right)^2, \qquad x \in \mathbb{R}, \tag{1}$$

where $\bar{G}(x;\boldsymbol{\eta}) = 1 - G(x;\boldsymbol{\eta})$ and $G(x;\boldsymbol{\eta})$ is a valid CDF depending upon the parameter vector $\boldsymbol{\eta}$. The expression $F(x;\boldsymbol{\eta})$ defined by Equation (1) is a valid CDF as $\lim_{x\to-\infty} F(x;\boldsymbol{\eta}) = 0$ and $\lim_{x\to\infty} F(x;\boldsymbol{\eta}) = 1$, because $G(x;\boldsymbol{\eta})$ is a CDF. Moreover, $F(x;\boldsymbol{\eta})$ is an increasing differentiable function with a derivative provided in Equation (2).

The probability density function (PDF) $f(x;\boldsymbol{\eta})$ is

$$f(x;\boldsymbol{\eta}) = \frac{2\pi g(x;\boldsymbol{\eta}) \sin\left[\frac{\pi}{2}\bar{G}(x;\boldsymbol{\eta})\right]\left(1-\cos\left[\frac{\pi}{2}\bar{G}(x;\boldsymbol{\eta})\right]\right)}{\left(1+\cos\left[\frac{\pi}{2}\bar{G}(x;\boldsymbol{\eta})\right]\right)^3}, \qquad x \in \mathbb{R}, \tag{2}$$

where $g(x;\boldsymbol{\eta}) = \frac{d}{dx}G(x;\boldsymbol{\eta})$.

With a link to $G(x;\boldsymbol{\eta})$, the survival function (SF) $S(x;\boldsymbol{\eta})$ is

$$S(x;\boldsymbol{\eta}) = \left(\frac{1-\cos\left[\frac{\pi}{2}\bar{G}(x;\boldsymbol{\eta})\right]}{1+\cos\left[\frac{\pi}{2}\bar{G}(x;\boldsymbol{\eta})\right]} \right)^2, \qquad x \in \mathbb{R}.$$

The hazard function (HF) $h(x;\boldsymbol{\eta})$ is

$$h(x;\boldsymbol{\eta}) = \frac{2\pi g(x;\boldsymbol{\eta}) \sin\left[\frac{\pi}{2}\bar{G}(x;\boldsymbol{\eta})\right]}{\left(1+\cos\left[\frac{\pi}{2}\bar{G}(x;\boldsymbol{\eta})\right]\right)\left(1-\cos\left[\frac{\pi}{2}\bar{G}(x;\boldsymbol{\eta})\right]\right)}, \qquad x \in \mathbb{R}.$$

The cumulative hazard function (CHF) $H(x;\boldsymbol{\eta})$ is

$$H(x;\boldsymbol{\eta}) = -2\log\left(\frac{1-\cos\left[\frac{\pi}{2}\bar{G}(x;\boldsymbol{\eta})\right]}{1+\cos\left[\frac{\pi}{2}\bar{G}(x;\boldsymbol{\eta})\right]}\right), \qquad x \in \mathbb{R}.$$

In this paper, using the approach defined in Equation (1), we study a new trigonometric distribution. The new trigonometric distribution may be called a new modified cosine-Weibull (NMC-Weibull) distribution. Section 2 offers the basic key functions and visual illustrations of the NMC-Weibull distribution. The usefulness and optimality of the NMC-Weibull distribution are shown in the industrial sectors by analyzing two applications in Section 3. In addition to the practical illustrations, the important work based on the NMC-Weibull distribution is performed in Section 4. The work that is carried out in Section 4 deals with the construction of the NMC-Weibull distribution-based repeating acceptance sampling strategy for attributes with specified shape parameters. Finally, Section 5 summarizes the conclusions drawn from this research work.

2. Special Model

Assume that $X(\in \mathbb{R}^+)$ follows the Weibull distribution (chosen as a special case of the NMC-G method) with parameters $\alpha > 0$ and $\tau > 0$. Then, the CDF $G(x;\boldsymbol{\eta})$ of X is

$$G(x;\boldsymbol{\eta}) = 1 - e^{-\tau x^{\alpha}}, \tag{3}$$

and PDF $g(x;\boldsymbol{\eta})$

$$g(x;\boldsymbol{\eta}) = \alpha\tau x^{\alpha-1} e^{-\tau x^{\alpha}}, \tag{4}$$

where $\boldsymbol{\eta} = (\alpha, \tau)$.

Incorporating Equation (3) in Equation (1) gives the CDF of the proposed NMC Weibull distribution, expressed as

$$F(x;\boldsymbol{\eta}) = 1 - \left(\frac{1-\cos\left[\frac{\pi}{2}e^{-\tau x^{\alpha}}\right]}{1+\cos\left[\frac{\pi}{2}e^{-\tau x^{\alpha}}\right]}\right)^2, \qquad x \in \mathbb{R}^+, \tag{5}$$

and SF $S(x;\boldsymbol{\eta})$

$$S(x;\boldsymbol{\eta}) = \left(\frac{1-\cos\left[\frac{\pi}{2}e^{-\tau x^{\alpha}}\right]}{1+\cos\left[\frac{\pi}{2}e^{-\tau x^{\alpha}}\right]}\right)^2, \qquad x \in \mathbb{R}^+.$$

The PDF $f(x;\boldsymbol{\eta})$ corresponding to Equation (5) is

$$f(x;\boldsymbol{\eta}) = \frac{2\pi\alpha\tau x^{\alpha-1} e^{-\tau x^{\alpha}} \sin\left[\frac{\pi}{2}e^{-\tau x^{\alpha}}\right]\left(1-\cos\left[\frac{\pi}{2}e^{-\tau x^{\alpha}}\right]\right)}{\left(1+\cos\left[\frac{\pi}{2}e^{-\tau x^{\alpha}}\right]\right)^3}, \qquad x \in \mathbb{R}^+. \tag{6}$$

Furthermore, the expression of the HF $h(x;\boldsymbol{\eta})$ of the NMC-Weibull distribution is

$$h(x;\boldsymbol{\eta}) = \frac{2\pi\alpha\tau x^{\alpha-1} e^{-\tau x^{\alpha}} \sin\left[\frac{\pi}{2}e^{-\tau x^{\alpha}}\right]}{\left(1+\cos\left[\frac{\pi}{2}e^{-\tau x^{\alpha}}\right]\right)\left(1-\cos\left[\frac{\pi}{2}e^{-\tau x^{\alpha}}\right]\right)}, \qquad x \in \mathbb{R}^+,$$

and the expression of the CHF $H(x;\boldsymbol{\eta})$ of the NMC-Weibull distribution is

$$H(x;\boldsymbol{\eta}) = -2\log\left(\frac{1-\cos\left[\frac{\pi}{2}e^{-\tau x^{\alpha}}\right]}{1+\cos\left[\frac{\pi}{2}e^{-\tau x^{\alpha}}\right]}\right), \qquad x \in \mathbb{R}^{+}.$$

The visual representations for $F(x;\boldsymbol{\eta})$ and $S(x;\boldsymbol{\eta})$ of the NMC-Weibull distribution are shown in Figure 1. The plots of $F(x;\boldsymbol{\eta})$ and $S(x;\boldsymbol{\eta})$ are sketched for $\alpha = (1.2, 3.6, 4.5, 6.4)$ and $\tau = (0.3, 0.1, 0.02, 0.001)$.

The plots of $f(x;\boldsymbol{\eta})$ for $\alpha = (0.5, 3.6, 4.5, 6.4)$, $\tau = (0.2, 0.1, 0.02, 0.001)$ and the plots of $h(x;\boldsymbol{\eta})$ for $\alpha = (0.5, 1.4, 1.1)$, $\tau = (0.2, 0.4, 0.7)$ are, respectively, presented in Figures 2 and 3.

From Figure 2, we can see that when $\alpha < 1$, $f(x;\boldsymbol{\eta})$ of the NMC-Weibull model has a reverse-J shape (or decreasing shape). Furthermore, we can see that when the value of α increases and the value of τ decreases, $f(x;\boldsymbol{\eta})$ of the NMC-Weibull distribution captures the following:

(a) a right-skewed shape when $\alpha = 3.6$ and $\tau = 0.1$,
(b) a symmetrical shape when $\alpha = 4.5$ and $\tau = 0.02$, and
(c) a left-skewed shape when $\alpha = 6.4$ and $\tau = 0.001$.

Figure 3 shows that when $\alpha < 1$, $h(x;\boldsymbol{\eta})$ of the NMC-Weibull has a decreasing form. When $\alpha > 1$, $h(x;\boldsymbol{\eta})$ of the NMC-Weibull distribution has an increasing form. Moreover, when $\alpha > 1$ and the value of τ increases, $h(x;\boldsymbol{\eta})$ of the NMC-Weibull distribution tends to unimodal.

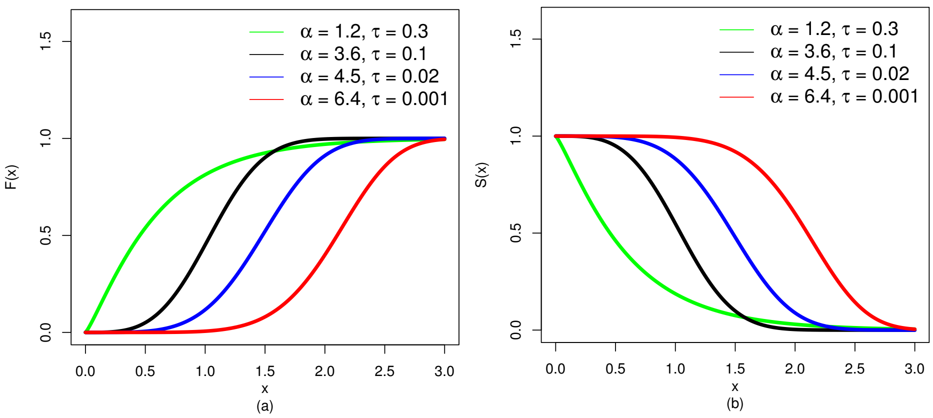

Figure 1. The visual representations for (**a**) $F(x;\boldsymbol{\eta})$ and (**b**) $S(x;\boldsymbol{\eta})$ of the NMC-Weibull distribution for $\alpha = (1.2, 3.6, 4.5, 6.4)$ and $\tau = (0.3, 0.1, 0.02, 0.001)$.

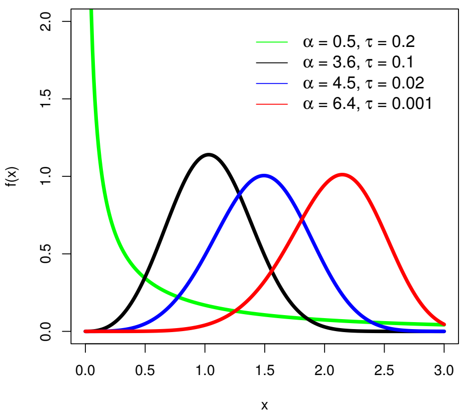

Figure 2. The PDF $f(x;\boldsymbol{\eta})$ plots for $\alpha = (0.5, 3.6, 4.5, 6.4)$ and $\tau = (0.2, 0.1, 0.02, 0.001)$.

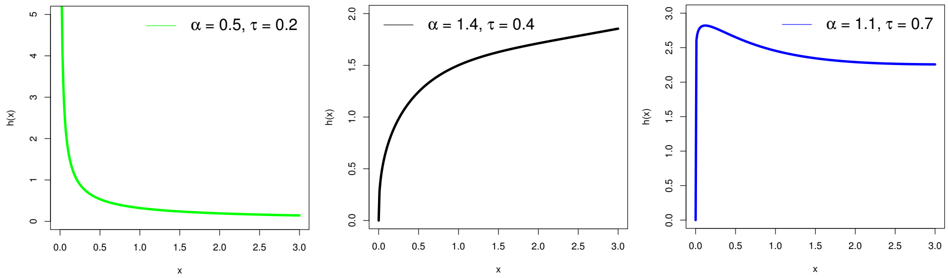

Figure 3. The HF $h(x;\boldsymbol{\eta})$ plots for $\alpha = (0.5, 1.4, 1.1)$ and $\tau = (0.2, 0.4, 0.7)$.

3. Data Analysis

Here, we practically show the implications of the NMS-Weibull distribution on an industrial scale. For this purpose, we consider a reliability engineering data set taken from [16]. The observations of the data set represent the waiting period between each successive failure when testing secondary reactor pumps, which are measured in thousands of hours. The observations of the considered reliability data sets are 6.560, 5.320, 4.992, 4.082, 3.465, 2.160, 1.921, 1.359, 1.060, 0.954, 0.746, 0.614, 0.605, 0.491, 0.402, 0.358, 0.347, 0.273, 0.199, 0.150, 0.101, 0.070, 0.062. Some descriptive measures of the data set are: $n = 23$, minimum = 0.062, maximum = 6.560, mean/$\bar{x}$ = 1.578, median/Q_2 = 0.614, variance = 3.7275, Q_1 = 0.310, standard deviation = 1.9306, Q_3 = 2.041, skewness = 1.3643, kurtosis = 3.54453, and range = 6.498. For information on researchers who have recently considered this data set, see [17,18]. Figure 4 sketches some baseline plots for the secondary reactor pump data set.

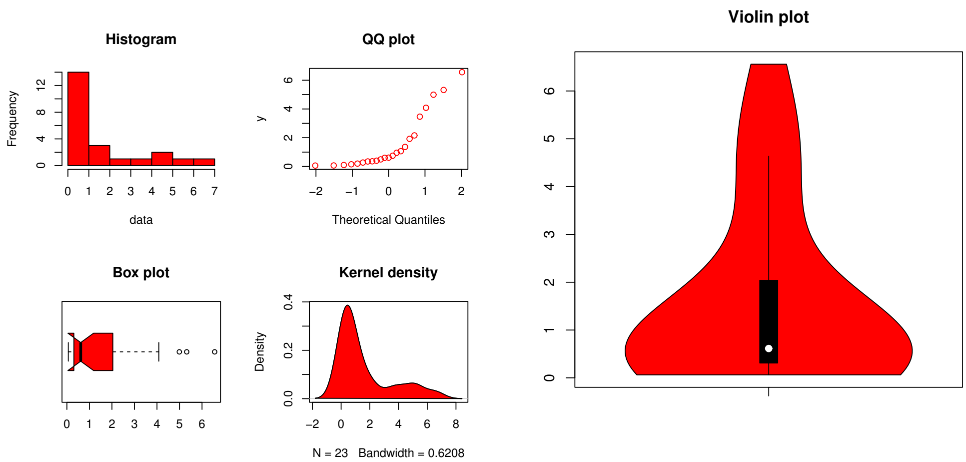

Figure 4. Some baseline plots for the secondary reactor pump data set.

By means of secondary reactor pump data, the performance of the NMC-Weibull distribution is compared numerically and visually with the baseline Weibull distribution. Additionally, the performance of the NMC-Weibull distribution is also compared numerically and visually with the (a) new modified Weibull (NM-Weibull) distribution with parameters $\alpha, \tau \in \mathbb{R}^+, \sigma \geq 1, \sigma \leq -1$, (b) exponential TX Weibull (ETX-Weibull) distribution with parameters $\alpha, \tau \in \mathbb{R}^+, \theta > 1$, and (c) new beta power transformed Weibull (NBPT-Weibull) distribution with parameter $\beta \in \mathbb{R}^+$.

For $X \in \mathbb{R}^+$, the SFs of the rival distributions are

- Weibull distribution

$$S(x) = e^{-\tau x^{\alpha}}.$$

- NM-Weibull distribution

$$S(x) = 1 - \left(\frac{1 - e^{-\tau x^{\alpha}}}{\sigma}\right)\left(\sigma - e^{-\tau x^{\alpha}}\right).$$

- ETX-Weibull distribution

$$S(x) = \frac{\theta e^{-\tau x^{\alpha}}}{\theta - 1 + e^{-\tau x^{\alpha}}}.$$

- NBPT-Weibull distribution

$$S(x) = \frac{\beta - \beta^{\left(1 - e^{-\tau x^{\alpha}}\right)} + e^{-\tau x^{\alpha}}}{\beta}.$$

The next step after selecting the competing models is to consider evaluation criteria to identify which distribution has the most optimal fit and best suited for the secondary reactor pump data. To figure out the most optimal model for the secondary reactor pumps data, we consider three well-known evaluation criteria. These assessment criteria are

- The Cramér–von Mises (CM)

$$\sum_{i=1}^{n}\left[\frac{2i-1}{2n} - G(x_i)\right]^2 + \frac{1}{12n}.$$

  Here, n and x_i indicate the data size (or number of observations) and the ith observations in the data, respectively.
- The Kolmogorov–Smirnov (KS)

$$sup_x\left|G(x) - G_n(x)\right|,$$

  where $G_n(x)$ represents the empirical CDF.
- The Anderson–Darling (AD)

$$-n - \sum_{i=1}^{n}\frac{(2i-1)}{n}\lambda_i,$$

  where the term λ_i is

$$[\log\{1 - G(x_{n+1-i})\} + \log G(x_i)].$$

In addition to the above three evaluation criteria, the p-value relating to the KS test is also taken into account to calculate when comparing the performances of the fitted distributions. The numerical values of the evaluation criteria as well as the p-value are obtained by implementing the optim() with R software using the SANN method.

After carrying out the analysis, the values of $(\hat{\alpha}_{MLE}, \hat{\tau}_{MLE}, \hat{\sigma}_{MLE}, \hat{\theta}_{MLE}, \hat{\beta}_{MLE})$ are provided in Table 1. In order to check the uniqueness of $\hat{\alpha}_{MLE}$ and $\hat{\tau}_{MLE}$ of the NMC-Weibull distribution, we obtain the log-likelihood profiles plots of $\hat{\alpha}_{MLE}$ and $\hat{\tau}_{MLE}$. Figure 5 illustrates and confirms the uniqueness of $\hat{\alpha}_{MLE}$ and $\hat{\tau}_{MLE}$.

With regard to the secondary reactor pump data, Table 2 reports the assessment criteria values as well as the p-value for the fitted distribution. Based on our analysis of the secondary reactor pumps data, we can conclude that the NMC-Weibull distribution may be the best choice to apply for the data sets on the industrial scale.

In addition to the numerical assessments in Table 2, we also carry out a visual illustration/comparison of the fitted distributions. For such comparison through the secondary reactor pumps data, we consider three graphical tools such as the (*i*) fitted PDF, (*ii*) empirical CDF, and (*iii*) estimated survival plots. Figure 6 presents the fitted plots, which graphically illustrate and demonstrate the fitting ability (or optimality) of the competing distributions. The given visual results in Figure 6 show that the secondary reactor pump data set is closely fitted by the NMC-Weibull distribution.

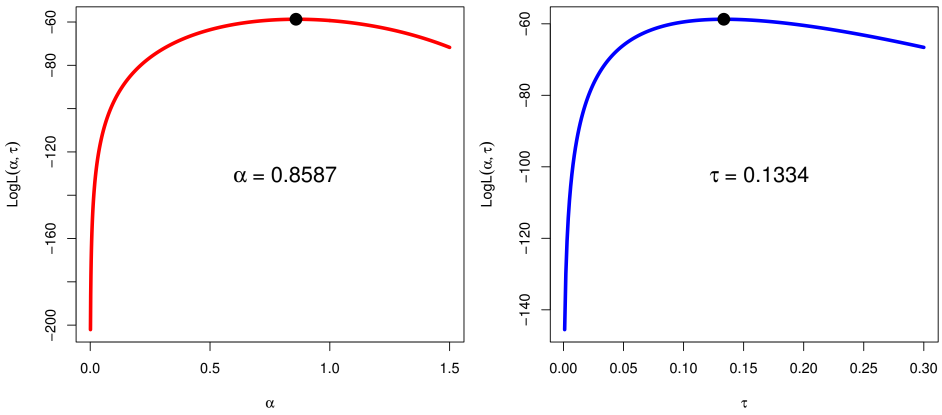

Figure 5. The log-likelihood profiles of $\hat{\alpha}_{MLE}$ and $\hat{\tau}_{MLE}$ of the NMC-Weibull model for the secondary reactor pumps data.

Table 1. Using the given secondary reactor pumps data, the values of $\hat{\alpha}_{MLE}, \hat{\tau}_{MLE}, \hat{\sigma}_{MLE}, \hat{\theta}_{MLE}$, and $\hat{\beta}_{MLE}$ of the fitted distributions.

Models	$\hat{\alpha}_{MLE}$	$\hat{\tau}_{MLE}$	$\hat{\sigma}_{MLE}$	$\hat{\theta}_{MLE}$	$\hat{\beta}_{MLE}$
NMC-Weibull	0.8587	0.1334	-	-	-
Weibull	0.8091	0.7642	-	-	-
ETX-Weibull	0.8009	26.772	-	0.7902	-
NM-Weibull	0.8000	0.7835	26.5708	-	-
NBPT-Weibull	0.8077	1.0008	-	-	0.7657

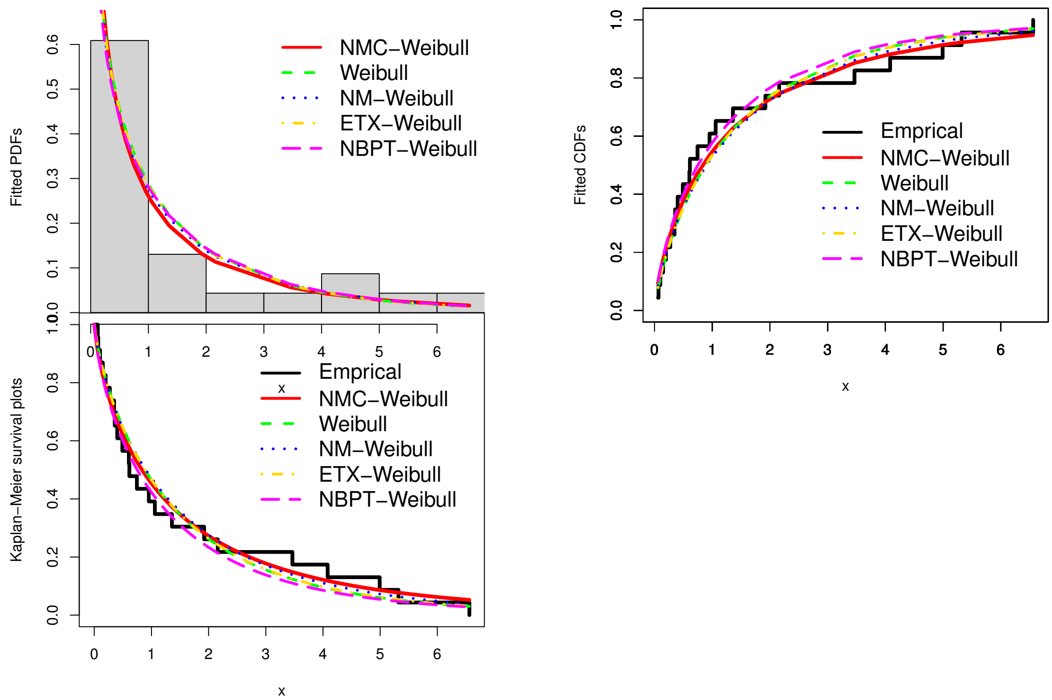

Figure 6. The optimal fitting comparison of the rival distributions for the data from the secondary reactor pumps.

Table 2. The assessment criteria values of the rival distributions for the data from the secondary reactor pumps.

Models	CVM	AD	KS	p-Value
NMC-Weibull	0.0575	0.3894	0.1105	0.9126
Weibull	0.0655	0.4315	0.1192	0.8615
ETX-Weibull	0.0664	0.4365	0.1168	0.8766
NM-Weibull	0.0662	0.4353	0.1192	0.8614
NBPT-Weibull	0.0654	0.4310	0.1183	0.8667

4. A New Repetitive Acceptance Sampling Plan

In medical research and life testing trials, type-I and type-II filtering are two censoring techniques that are frequently employed. The test duration is fixed by the type-I filtering system and the number of failures is fixed by the type-II filtering technique. Nonetheless, time-truncated schemes are now preferred in life testing. In life testing, the experiment is often terminated when the allotted amount of time has passed. To cut down on the time and expense of the experiment to obtain the ultimate conclusion, this plan is more practical than censorship systems.

Generally speaking, an attribute sampling plan is easier to apply than a variable sampling plan, but it does require more samples. A specified number of units from each lot are inspected during attribute sampling, and each unit is labeled as either conforming or nonconforming. Accept the lot if the sample's nonconforming unit count is less than or equal to the required minimum; if not, reject it. Depending on how many samples need to be taken from the lot, sampling plans can be further divided into single, double, multiple, sequential, repeated, and more categories. The cost of the inspection, which is directly correlated with sample size, is a concern for the producers during the product inspection process. Thus, to reduce the expense, duration, and effort of the inspection, the researchers would like to suggest a more effective sample approach. Industrial engineers love single sampling plans (SSP) for their simplicity, but in certain situations, deciding on lot sentencing solely based on a single sample might damage goodwill between producers and consumers.

In circumstances where sampling inspection products are harmful and extremely expensive, a repeated sampling plan is more suitable. Based on the excellent deal's repeated sample test findings, this plan allows for either acceptance or rejection of the lot. Comparing the repeated sampling plan to the single sampling plan, the latter can provide the smallest sample size with the appropriate protection. Regardless of the established acceptance sampling plans (ASP), the lot decision is always linked to the producer and consumer risk. This means that one may choose to accept a subpar lot or reject an excellent lot.

The risk to the consumer (β) is the possibility that a poor lot will be accepted, whereas the risk to the producer (α) is the possibility that a good lot will be rejected. Therefore, the sampling plan's objective is to collect as few samples as feasible to minimize these risks; see [19–23].

If a sampling plan was designed in a method that required a minimum sample size, it was deemed to be the best one out of the ones that were available for examination. These designs are referred to as inexpensive sampling plans because they have a minimal sample size, which reduces the inspection costs. It saves time as well. The main advantage of the recurring group acceptance sampling approach is a reduction in the ASN based on attribute repetitive group sampling. Sherman [24] proposed a repeating group acceptance sampling approach for a normal distribution. According to him, the repeated group acceptance sampling plan that was developed offers a sample size that is as near to the consumer risk as feasible. Many authors thought about repetitive acceptance sampling plans (RASP) for different distributions; for additional information, see [25–32].

The 100qth percentile of the NMC-Weibull is given as

$$t_q = \left(-\frac{1}{\lambda}\log\left[\frac{2}{\pi}\cos^{-1}\left(\frac{1-(1-q)^{1/2}}{1+(1-q)^{1/2}}\right)\right]\right)^{\frac{1}{\tau}}. \tag{7}$$

On simplifying Equation (7), we obtain

$$\lambda t_q^\tau = -\log\left[\frac{2}{\pi}\cos^{-1}\left(\frac{1-(1-q)^{1/2}}{1+(1-q)^{1/2}}\right)\right],$$

$$\lambda t_q^\tau = \frac{\varsigma_q}{t_q^\tau},$$

where

$$\varsigma_q = -\log\left[\frac{2}{\pi}\cos^{-1}\left(\frac{1-(1-q)^{1/2}}{1+(1-q)^{1/2}}\right)\right].$$

In order to obtain a lot with defective fraction p, we postulate the termination time t_0 as a multiple of the specified lifetime t_q^0. That is, $t_0 = at_q^0$, for a constant a which is known as the experiment termination ratio and the targeted 100qth lifetime percentile, t_q^0, thus,

$$\lambda t_0^\tau = \frac{\varsigma_q\left(at_q^0\right)^\tau}{t_q^\tau}.$$

4.1. Design of the Repetitive Acceptance Sampling Scheme

The following is a description of the RASP under the proposed plan's truncated life test:

- Step-1: select a sample at random of size n from the whole population, and subject them to a timed life test t_0
- Step-2: if the number of failures (D) is less than (or equal to) c_1, accept the lot (first acceptance number). As soon as the number of defectives surpasses c_2, the test and the lot should be terminated c_2 , where $c_2 \geq c_1$.
- Step-3: if $c_1 < D \leq c_2$, then move to Step-1. Continue the earlier experiment. The parameters of the suggested plan are n, c_1 and c_2. The single sample plan is generalized into the characteristics repetitive acceptance sampling plan, which reduces the above-mentioned strategy to a SSP. The operational characteristic (OC) function, from which the probability of acceptance lot is determined, is deduced to be:

$$P_A(p) = \frac{P_a}{P_a + P_r}, \qquad 0 < p < 1, \tag{8}$$

where P_a is the probability of acceptance of a submitted lot with a fraction of defective P based on a given sample.

$$P_a(p) = Pr(D \leq c_1|p) = \sum_{i=0}^{c_1}\left[\binom{n}{i}p^i(1-p)^{n-i}\right], \tag{9}$$

whereas, P_r is the corresponding probability of lot rejection.

$$P_r(p) = Pr(D > c_2|p) = 1 - \sum_{i=0}^{c_2}\left[\binom{n}{i}p^i(1-p)^{n-i}\right]. \tag{10}$$

The OC specified in Equation (8) can therefore be rewritten as

$$P_A(p) = \frac{\sum_{i=0}^{c_1}\left[\binom{n}{i}p^i(1-p)^{n-i}\right]}{\sum_{i=0}^{c_1}\left[\binom{n}{i}p^i(1-p)^{n-i}\right] + \left(1 - \sum_{i=0}^{c_2}\left[\binom{n}{i}p^i(1-p)^{n-i}\right]\right)}, \quad 0 < p < 1. \tag{11}$$

The design parameters of the proposed RASP are n, c_1 and c_2, as pointed out by Aslam and Jun [33]. It could be simplest to state the time of termination t_0, as a multiple of the specified length a. Accordingly, we will consider that $t_0 = at_q^0$ for a constant a and the targeted 100qth lifetime percentile, t_q^0, thus,

$$\lambda t_0^{\tau} = \frac{a^{\tau} \varsigma_q}{\left(\frac{t_q}{t_q^0}\right)^{\tau}}.$$

At two places on the OC curve, we select the strategy that accounts for both the producer's and the customer's risk. This viewpoint was adopted by many writers, such as Fertig and Mann [34], to create their sample plans. This approach calculates the quality level as a ratio of its life expectancy to the specified value $\frac{t_q}{t_q^0}$. A minimum probability of lot acceptance of $1-\alpha$ at the Acceptance Quality Level (AQL) is demanded from the producers; let p_1 be the probability of a failure corresponding to the producer's risk α at AQL, say $\frac{t_q}{t_q^0} = 1.5, 1.6, 1.8, 2.0, 2.2$. However, from the standpoint of the customer, the lot rejection probability should be at most β at the Restricted Quality Level. Let p_2 stand for the probability of a corresponding consumer's risk β at LQL, say $\frac{t_q}{t_q^0} = 1$. The following two inequality conditions must be met for the RASP parameters n, c_1, and c_2:

$$P_A(p_1) = \frac{\sum_{i=0}^{c_1}\left[\binom{n}{i} p_1^i (1-p_1)^{n-i}\right]}{\sum_{i=0}^{c_1}\left[\binom{n}{i} p_1^i (1-p_1)^{n-i}\right] + \left(1 - \sum_{i=0}^{c_2}\left[\binom{n}{i} p_1^i (1-p_1)^{n-i}\right]\right)} \geq 1-\alpha, \quad (12)$$

and

$$P_A(p_2) = \frac{\sum_{i=0}^{c_1}\left[\binom{n}{i} p_2^i (1-p_2)^{n-i}\right]}{\sum_{i=0}^{c_1}\left[\binom{n}{i} p_2^i (1-p_2)^{n-i}\right] + \left(1 - \sum_{i=0}^{c_2}\left[\binom{n}{i} p_2^i (1-p_2)^{n-i}\right]\right)} \leq \beta, \quad (13)$$

where p_1 and p_2 are given by

$$p_1 = 1 - \left(\frac{1 - \cos\left[\frac{\pi}{2}\left(1 - e^{-\frac{a^{\tau}\varsigma_q}{(t_q/t_q^0)}}\right)\right]}{1 + \cos\left[\frac{\pi}{2}\left(1 - e^{-\frac{a^{\tau}\varsigma_q}{(t_q/t_q^0)}}\right)\right]}\right)^2,$$

and

$$p_2 = 1 - \left(\frac{1 - \cos\left[\frac{\pi}{2}\left(1 - e^{-a^{\tau}\varsigma_q}\right)\right]}{1 + \cos\left[\frac{\pi}{2}\left(1 - e^{-a^{\tau}\varsigma_q}\right)\right]}\right)^2.$$

The minimum ASN at the LQL yields the recommended repetitive acceptance sampling plan's parameters. Here is the suggested plan's ASN when p is the true fraction faulty.

$$\text{ASN}(p) = \frac{n}{P_a + P_r}. \quad (14)$$

Thus, the proposed plan's n, c_1, and c_2 parameters can be found by resolving the following optimization issue:

Minimize $\text{ASN}(p)$, subject to

- $P_A(p_1) \geq 1-\alpha$,
- $P_A(p_2) \leq \beta$,

where n is an integer.

Given the producer's risk $\alpha = 0.05$ and its percentile ratio t_q/t_q^0 with $\alpha = 0.5$ to 1.0, there are three parameters n, c_1, and c_2 in this proposed RASP under the truncated life test at the specified time t_0, with $\tau = 1.0, 1.5$, and 2.0 obtained according to the consumer's confidence levels $\beta = 0.25, 0.10, 0.05$, and 0.01 for 50th percentile lifetime. The ASN is also reported and finally, the probability of acceptance is also reported.

Finally, the likelihood of acceptance is presented together with the ASN. Tables 3–6 present the findings. The NMC-Weibull distribution estimated parameters value $\hat{\tau} = 0.8587$, the sampling parameters for 50th percentile predicted lives are shown in Table 6.

The sample size n reduces as the constant increases from 0.5 to 1.0, as seen in Tables 3–6. Moreover, the sample size n is reduced as the shape parameter rises from 0.8587 to 2.0. Moreover, the sample size grows as consumer risk rises. The tables also include the ASN and acceptance probability p_a for the best acceptance strategies.

Table 3. The proposed plan's design criteria for the NMC-Weibull distribution at $\tau = 2.0$.

		$a = 0.5$					$a = 1.0$				
β	t_q/t_q^0	c_1	c_2	n	$P_A(p_1)$	ASN	c_1	c_2	n	$P_A(p_1)$	ASN
	1.5	4	6	41	0.9558	58.93	4	6	13	0.9515	20.52
	1.6	2	4	27	0.9545	43.51	3	5	11	0.9600	17.94
0.25	1.8	1	3	20	0.9706	35.76	3	4	10	0.9650	12.58
	2.0	0	2	13	0.9685	28.20	2	3	8	0.9672	10.24
	2.2	1	2	18	0.9694	23.47	1	2	6	0.9544	7.84
	1.5	2	6	37	0.9531	81.48	5	8	18	0.9515	28.09
	1.6	3	6	45	0.9512	66.59	5	7	17	0.9575	22.45
0.1	1.8	0	3	19	0.9509	45.57	2	4	10	0.9552	14.76
	2.0	1	3	25	0.9673	37.06	1	3	8	0.9539	11.91
	2.2	0	2	16	0.9654	28.52	0	2	5	0.9568	9.41
	1.5	5	9	66	0.9518	95.10	4	8	17	0.9583	32.41
	1.6	2	6	42	0.9510	73.25	5	8	19	0.9658	26.84
0.05	1.8	1	4	31	0.9513	49.80	1	4	9	0.9584	17.32
	2.0	0	3	23	0.9523	41.44	2	4	11	0.9690	14.51
	2.2	1	3	29	0.9711	37.99	1	3	9	0.9609	11.76
	1.5	5	11	80	0.9529	115.84	7	12	28	0.9567	38.86
	1.6	4	9	71	0.9532	93.54	3	8	18	0.9525	30.18
0.01	1.8	2	6	52	0.9532	66.62	2	6	15	0.9584	21.43
	2.0	0	4	31	0.9560	51.72	2	5	15	0.9541	17.59
	2.2	1	4	39	0.9764	48.71	1	4	12	0.9658	14.83

Table 4. The proposed plan's design criteria for the NMC-Weibull distribution at $\tau = 1.5$.

		$a = 0.5$					$a = 1.0$				
β	t_q/t_q^0	c_1	c_2	n	$P_A(p_1)$	ASN	c_1	c_2	n	$P_A(p_1)$	ASN
	1.5	7	10	47	0.9542	74.21	6	9	18	0.9519	34.21
	1.6	6	8	40	0.9509	54.70	6	8	17	0.9548	25.52
0.25	1.8	3	5	25	0.9568	38.07	6	7	17	0.9506	19.96
	2.0	2	4	20	0.9677	32.44	1	3	6	0.9592	13.24
	2.2	0	2	9	0.9504	21.01	3	4	10	0.9659	12.58
	1.5	6	11	50	0.9561	101.15	8	12	25	0.9518	45.14
	1.6	8	11	58	0.9523	77.92	4	8	16	0.9515	36.35
0.1	1.8	3	6	31	0.9516	50.04	3	6	13	0.9535	23.80
	2.0	2	5	27	0.9578	43.35	4	6	15	0.9595	19.85
	2.2	2	4	24	0.9588	33.30	2	4	10	0.9565	14.76
	1.5	6	12	55	0.9505	113.01	8	13	27	0.9502	51.32
	1.6	6	11	55	0.9524	89.38	9	13	29	0.9613	42.95
0.05	1.8	3	7	36	0.9539	60.09	5	8	19	0.9510	26.84
	2.0	3	6	35	0.9574	48.21	3	6	14	0.9664	22.10
	2.2	1	4	22	0.9576	37.00	1	4	9	0.9598	17.32
	1.5	10	18	89	0.9529	139.57	10	17	36	0.9510	62.97
	1.6	6	13	64	0.9550	108.05	8	14	31	0.9500	48.04
0.01	1.8	5	10	57	0.9502	74.55	4	9	20	0.9598	33.67
	2.0	1	6	30	0.9527	57.11	3	7	17	0.9548	24.57
	2.2	2	6	38	0.9521	48.58	2	6	15	0.9602	21.43

Table 5. The proposed plan's design criteria for the NMC-Weibull distribution at $\tau = 1.0$.

		$a = 0.5$					$a = 1.0$				
β	t_q/t_q^0	c_1	c_2	n	$P_A(p_1)$	**ASN**	c_1	c_2	n	$P_A(p_1)$	**ASN**
	1.5	12	17	55	0.9502	111.68	17	21	42	0.9535	72.57
	1.6	12	16	54	0.9571	91.72	9	13	25	0.9523	54.37
0.25	1.8	10	12	44	0.9503	57.18	7	10	20	0.9561	36.80
	2.0	4	7	24	0.9504	44.10	6	8	17	0.9517	25.52
	2.2	4	6	22	0.9526	32.89	5	7	15	0.9615	23.05
	1.5	16	23	77	0.9507	155.44	16	23	45	0.9545	107.34
	1.6	17	22	79	0.9505	118.64	17	22	46	0.9515	75.36
0.1	1.8	7	12	41	0.9545	81.09	14	17	38	0.9502	50.13
	2.0	5	9	32	0.9515	57.92	9	12	27	0.9542	38.00
	2.2	6	9	36	0.9502	49.65	6	9	20	0.9612	30.97
	1.5	17	26	87	0.9503	185.49	19	27	54	0.9505	115.86
	1.6	15	22	77	0.9505	134.45	20	26	55	0.9553	86.59
0.05	1.8	12	17	64	0.9509	91.66	11	16	34	0.9567	56.98
	2.0	6	11	40	0.9543	69.12	9	13	29	0.9574	42.95
	2.2	6	10	39	0.9595	57.32	4	8	17	0.9514	32.41
	1.5	37	47	174	0.9504	223.37	33	43	91	0.9533	136.20
	1.6	18	28	99	0.9524	166.05	26	34	74	0.9514	101.88
0.01	1.8	15	22	85	0.9513	111.84	14	21	46	0.9530	68.02
	2.0	6	13	47	0.9515	82.87	10	16	36	0.9568	51.66
	2.2	5	11	42	0.9521	65.90	9	14	33	0.9633	43.23

Table 6. The proposed plan's design criteria for the NMC-Weibull distribution at $\hat{\tau} = 0.8587$.

		$a = 0.5$					$a = 1.0$				
β	t_q/t_q^0	c_1	c_2	n	$P_A(p_1)$	**ASN**	c_1	c_2	n	$P_A(p_1)$	**ASN**
	1.5	16	22	65	0.9522	142.29	20	25	49	0.9516	95.30
	1.6	17	21	66	0.9503	104.18	17	21	42	0.9526	72.57
0.25	1.8	9	13	40	0.9546	74.31	13	16	33	0.9566	50.92
	2.0	7	10	32	0.9520	52.60	7	10	20	0.9583	36.80
	2.2	5	8	25	0.9624	45.19	8	10	22	0.9509	30.25
	1.5	31	38	122	0.9517	193.86	35	41	85	0.9524	130.77
	1.6	23	29	94	0.9532	147.83	21	27	55	0.9529	99.57
0.1	1.8	13	18	59	0.9503	95.92	17	21	45	0.9537	65.73
	2.0	6	11	34	0.9535	73.33	14	17	38	0.9533	50.13
	2.2	6	10	33	0.9558	57.65	10	13	29	0.9598	40.70
	1.5	33	42	135	0.9521	221.34	36	44	91	0.9521	148.63
	1.6	22	30	96	0.9536	167.93	27	34	71	0.9536	114.13
0.05	1.8	15	21	70	0.9508	109.30	15	21	44	0.9527	75.32
	2.0	11	16	55	0.9513	82.00	15	19	43	0.9501	56.44
	2.2	7	12	41	0.9511	66.55	9	13	29	0.9514	42.95
	1.5	-	-	-	-	-	-	-	-	-	-
	1.6	36	46	156	0.9514	206.10	33	43	91	0.9517	136.20
0.01	1.8	17	26	87	0.9512	134.20	21	29	63	0.9517	90.23
	2.0	10	18	60	0.9505	100.31	12	19	41	0.9510	65.27
	2.2	7	14	47	0.9541	80.25	14	19	45	0.9516	54.72

4.2. An Industrial Application of the Developed Plan

The waiting durations between successive failures when testing secondary reactor pumps are used to show the planned scheme for NMC-Weibull in this section using real lifetime data. Take the NMC-Weibull distribution for the data set as given. The MLEs of the parameters are $\hat{\tau} = 0.8587$ and $\hat{\lambda} = 0.1334$, respectively. The data set is reasonably suited for the NMC-Weibull distribution based on the results in Table 2 and Figure 6. Consider the NMC-Weibull distribution as the life cycle of the products with $\hat{\tau} = 0.8587$, $\alpha = 0.05$, $\beta = 0.1$, $t_q^0 = 0.15$, and $t_q = 0.300$; thus we obtain $t_q/t_q^0 = 2$. Using Table 6, we observe that $a = 0.5$, $n = 34$, $c_1 = 6$ and $c_2 = 11$ are the ideal design parameters. As a result, the sample plan can be put into practice as follows: choose a sample size, say $n = 34$, at random from the group. If there are more than 11 failures, reject the lot and stop the test. Accept the lot if six failures occur during the testing of secondary reactor pumps before 0.150 million hours. The experiments should be repeated if there are between 6 and 11 failures. Our data show that three failures occurred within 0.150 million hours. As a result, the provided lot is accepted as the best professional judgment.

4.3. Comparison Study

An attributes RASP based on truncated life tests is unquestionably superior to the corresponding SSP in terms of necessary sample size. The sample size for the RASP with the recommended features and the SSP when producer's risk $\alpha = 0.05$, $a = 0.5$, consumer's confidence levels $\beta = (0.25, 0.10, 0.05, 0.01)$, and $t_q/t_q^0 = (1.5, 1.6, 1.8, 2.0, 2.2)$ are reported in Tables 7 and 8. It is evident in Table 7 that the developed scheme required a small sample size than the SSP. For instance, when $\beta = 0.05$, $t_q/t_q^0 = 1.6$, $a = 0.5$, and $\tau = 2.0$, the sample size of the proposed plan is 42, whereas the SSP is 122. From Table 8, for example, $\beta = 0.10$, $t_q/t_q^0 = 1.6$, $a = 1.0$, and $\tau = 1.5$, the sample size of the proposed plan is 16, whereas the SSP is 53. According to this study's findings, the RASP approach offers greater benefits than the SSP.

Table 7. Comparison of sample sizes for the NMC-Weibull distribution between proposed and single sampling schemes when $\alpha = 0.05$ and $a = 0.5$.

		$\tau = 2.0$		$\tau = 1.5$		$\tau = 1.0$	
β	t_q/t_q^0	Proposed	Single	Proposed	Single	Proposed	Single
0.25	1.5	41	71	47	86 19	55	138
	1.6	27	58	40	67	54	107
	1.8	20	37	25	47	44	71
	2.0	13	30	20	37	24	53
	2.2	18	30	9	27	22	42
0.1	1.5	37	119	50	143	77	218
	1.6	45	91	58	113	79	168
	1.8	69	19	31	77	41	113
	2.0	25	55	27	61	32	83
	2.2	16	47	24	51	36	68
0.05	1.5	66	151	55	182	87	278
	1.6	42	122	55	141	77	212
	1.8	31	85	36	94	64	142
	2.0	23	69	35	73	40	108
	2.2	29	53	22	62	39	85
0.01	1.5	80	224	89	263	174	407
	1.6	71	172	64	206	99	311
	1.8	52	125	57	142	85	210
	2.0	31	101	30	108	47	160
	2.2	39	84	38	91	42	124

Table 8. Comparison of sample sizes for the NMC-Weibull distribution between proposed and single sampling schemes when $\alpha = 0.05$ and $a = 1.0$.

		$\tau = 2.0$		$\tau = 1.5$		$\tau = 1.0$	
β	t_q/t_q^0	Proposed	Single	Proposed	Single	Proposed	Single
0.25	1.5	13	27	18	44	42	90
	1.6	11	18	17	31	25	67
	1.8	10	14	17	23	20	46
	2.0	8	12	6	16	17	33
	2.2	6	10	10	14	15	27
0.1	1.5	18	39	25	66	45	136
	1.6	17	33	16	53	46	102
	1.8	10	21	13	35	38	70
	2.0	8	17	15	26	27	53
	2.2	5	14	10	21	20	39
0.05	1.5	17	51	27	82	54	173
	1.6	19	37	29	62	55	130
	1.8	9	28	19	44	34	89
	2.0	11	21	14	33	29	67
	2.2	9	18	9	28	17	51
0.01	1.5	28	73	36	116	91	250
	1.6	18	54	31	94	74	189
	1.8	15	40	20	64	46	125
	2.0	15	30	17	47	36	96
	2.2	12	25	15	40	33	78

5. Concluding Remarks

We attempted to fill the curious and astonishing research gap by constructing a new repetitive acceptance sample design using a trigonometric-based probability distribution. For this purpose, we proposed a new distributional method via the cosine function, called a NMC-G family. Using the NMC-G method, a new useful probability model called a NMC-Weibull distribution was studied. Based on the NMC-Weibull time-truncated life test, a new repetitive acceptance sample design was introduced. Producer and customer risk are taken into consideration simultaneously while determining the characteristics of the suggested sample plan, n, c_1, and c_2. Regarding sample size, a comparison is made between the SSP technique and the suggested recurrent group sampling plan. We discovered that the suggested method works better than the option of using the SSP.

In this paper, we considered the statistical modeling of the reliability data set using the NMC-Weibull distribution. Furthermore, we constructed an acceptance sampling strategy for the NMC-Weibull distribution. Motivated by [35–37], in the future, we intend to carry out Bayesian analysis using the NMC-Weibull distribution.

Author Contributions: Conceptualization, H.M.A., G.S.R., J.-T.S., S.S. and S.K.K.; Methodology, H.M.A., G.S.R., J.-T.S., S.S. and S.K.K.; Software, H.M.A., G.S.R., J.-T.S., S.S. and S.K.K.; Validation, H.M.A., G.S.R. and S.S.; Formal analysis, H.M.A., G.S.R., J.-T.S., S.S. and S.K.K.; Investigation, G.S.R. and S.K.K.; Resources, S.S.; Data curation, H.M.A., J.-T.S. and S.K.K.; Writing—original draft, H.M.A., G.S.R., J.-T.S., S.S. and S.K.K.; Visualization, H.M.A., G.S.R., J.-T.S. and S.K.K. All authors have read and agreed to the published version of the manuscript.

Funding: Princess Nourah bint Abdulrahman University Researchers Supporting Project number (PNURSP2023R 299), Princess Nourah bint Abdulrahman University, Riyadh, Saudi Arabia.

Data Availability Statement: Where requested, the data sets shall be made available by the corresponding author.

Conflicts of Interest: The authors declare no conflict of interest.

References

1. Butt, K.A.; Aslam, M.; Jun, C.H. Analysis of process yield in a cost-effective double acceptance sampling plan. *Commun. Stat.-Theory Methods* **2020**, *49*, 5975–5987. [CrossRef]
2. Liaqat, M.; Saeed, N.; Saleem, K.; Aslam, M.; Sherwani, R.A.K. A Modified Group Chain Sampling Plan for lifetime following Kumaraswamy Generalized Power Weibull Distribution with Minimum Angle Approach. *Sci. Iran.* **2023**. 10.24200/SCI.2023.59768.6413. [CrossRef]
3. Mahmood, Y.; Fatima, S.; Khan, H.; Amir, H.; Khoo, M.B.; Teh, S.Y. Acceptance sampling plans based on Topp-Leone Gompertz distribution. *Comput. Ind. Eng.* **2021**, *159*, 107526. [CrossRef]
4. Gharaibeh, N.G.; Liu, L.; Wani, S.S. Skip-lot acceptance sampling plans for highway construction and materials. *J. Constr. Eng. Manag.* **2012**, *138*, 791–796. [CrossRef]
5. Yen, C.H.; Lee, C.C.; Lo, K.H.; Shiue, Y.R.; Li, S.H. A rectifying acceptance sampling plan based on the process capability index. *Mathematics* **2020**, *8*, 141. [CrossRef]
6. Singh, S.; Tripathi, Y.M. Acceptance sampling plans for inverse Weibull distribution based on truncated life test. *Life Cycle Reliab. Saf. Eng.* **2017**, *6*, 169–178. [CrossRef]
7. Abushal, T.A.; Hassan, A.S.; El-Saeed, A.R.; Nassr, S.G. Power inverted Topp–Leone distribution in acceptance sampling plans. *Comput. Mater. Contin.* **2021**, *67*, 991–1011. [CrossRef]
8. Tripathi, H.; Dey, S.; Saha, M. Double and group acceptance sampling plan for truncated life test based on inverse log-logistic distribution. *J. Appl. Stat.* **2021**, *48*, 1227–1242. [CrossRef]
9. Alyami, S.A.; Elgarhy, M.; Elbatal, I.; Almetwally, E.M.; Alotaibi, N.; El-Saeed, A.R. Fréchet Binomial Distribution: Statistical Properties, Acceptance Sampling Plan, Statistical Inference and Applications to Lifetime Data. *Axioms* **2022**, *11*, 389. [CrossRef]
10. Algarni, A. Group Acceptance Sampling Plan Based on New Compounded Three-Parameter Weibull Model. *Axioms* **2022**, *11*, 438. [CrossRef]
11. Nassr, S.G.; Hassan, A.S.; Alsultan, R.; El-Saeed, A.R. Acceptance sampling plans for the three-parameter inverted Topp–Leone model. *Math. Biosci. Eng.* **2022**, *19*, 13628–13659. [CrossRef] [PubMed]
12. Khan, M.Z.; Khan, M.F.; Aslam, M.; Mughal, A.R. Fuzzy acceptance sampling plan for transmuted Weibull distribution. *Complex Intell. Syst.* **2022**, *8*, 4783–4795. [CrossRef]
13. Fayomi, A.; Khan, K. A group acceptance sampling plan for 'Another Generalized Transmuted-Exponential Distribution'based on truncated lifetimes. *Qual. Reliab. Eng. Int.* **2022**. [CrossRef]
14. Al-Omari, A.I.; Alomani, G. Double acceptance sampling plan based on truncated life tests for two-parameter Xgamma distribution. *Electron. J. Appl. Stat. Anal.* **2022**, *15*, 290–321.
15. Yiğiter, A.; Hamurkaroğlu, C.; Danacıoğlu, N. Group acceptance sampling plans based on time truncated life tests for compound Weibull-exponential distribution. *Int. J. Qual. Reliab. Manag.* **2023**, *40*, 304–315. [CrossRef]

16. Suprawhardana, M.S.; Prayoto, S. Total time on test plot analysis for mechanical components of the RSG-GAS reactor. *At. Indones.* **1999**, *25*, 81–90.
17. Coşkun, K.U.Ş; Korkmaz, M.Ç.; Kinaci, İ.; Karakaya, K.; Akdogan, Y. Modified-Lindley distribution and its applications to the real data. *Commun. Fac. Sci. Univ. Ank. Ser. Math. Stat.* **2022**, *71*, 252–272.
18. Demirci, B.H.; Bicer, C.; Bakouch, H.S.H. A geometric process with Hjorth marginal: Estimation, discrimination, and reliability data modeling. *Qual. Reliab. Eng. Int.* **2022**, *38*, 2795–2819. [CrossRef]
19. Kantam, R.R.L.; Rosaiah, K.; Rao, G.S. Acceptance sampling based on life tests: Log-logistic models. *J. Appl. Stat.* **2001**, *28*, 121–128. [CrossRef]
20. Balakrishnan, N.; Leiva, V.; Lopez, J. Acceptance sampling plans from truncated life tests based on the generalized Birnbaum-Saunders distribution. *Commun. -Stat.-Theory Methods.* **2007**, *36*, 643–656. [CrossRef]
21. Lio, Y.L.; Tsai, T.R.; Wu, S.J. Acceptance sampling plans from truncated life tests based on the burr type XII percentiles. *J. Chin. Inst. Ind. Eng.* **2010**, *27*, 270–280. [CrossRef]
22. Lio, Y.L.; Tsai, T.R.; Wu, S.J. Acceptance sampling plans from truncated life tests based on the Birnbaum-Saunders distribution for percentiles. *Commun. Stat.-Theory Methods.* **2010**, *39*, 119–136. [CrossRef]
23. Tsai, T.R.; Wu, S.J. Acceptance sampling based on truncated life tests for generalized Rayleigh distribution. *J. Appl. Stat.* **2006**, *33*, 595–600. [CrossRef]
24. Sherman, R.E. Design and evaluation of repetitive group sampling plan. *Technometrics* **1965**, *7*, 11–21. [CrossRef]
25. Balamurali, S.; Jun, C.-H. Repetitive group sampling procedure for variables inspection. *J. Appl. Stat.* **2006**, *33*, 327–338. [CrossRef]
26. Balamurali, S.; Park, H.; Jun, C.-H.; Kim, K.J.; Lee, J. Designing of variable repetitive group sampling plan involving minimum average sample number. *J. Stat. Simul. Comput.* **2005**, *34*, 799–807. [CrossRef]
27. Lee, S.; Aslam, M.; Jun, C.-H. Repetitive group sampling plans with two specification limits. *Pak. J. Stat.* **2012**, *28*, 41–57.
28. Aslam, M.; Jun, C.-H.; Fernez, A.J.; Ahmad, M.; Rasool, M. Repetitive Group Sampling Plan Based on Truncated Tests for Weibull Models. *Res. J. Appl. Sci. Eng. Technol.* **2014**, *7*, 1917–1924. [CrossRef]
29. Singh, N.; Singh, N.; Kaur, K. Design of repetitive acceptance sampling plan for truncated life test using inverse Weibull distribution. *Int. J. Manag. Technol. Eng.* **2018**, *8*, 433–441.
30. Singh, N.; Singh, N.; Kaur, K. A repetitive acceptance sampling plan for generalized inverted exponential distribution based on truncated life test. *Int. J. Sci. Res. Math. Stat. Sci.* **2018**, *5*, 58–64. [CrossRef]
31. Kalyani, K.; Rao, G.S.; Rosaiah, K.; Sivakumar, D.C.U. Repetitive acceptance sampling plan for odds exponential log-logistic distribution based on truncated life test. *J. Ind. Prod. Eng.* **2021**, *38*, 395–400. [CrossRef]
32. Rao, G.S.; Jilani, S.; Rao, A.V. Design of repetitive acceptance sampling plan for truncated life test using type II generalized half-logistic distribution. *Songklanakarin J. Sci. Technol.* **2021**, *43*, 1491–1498.
33. Aslam, M.; Jun, C.H. A group acceptance sampling plans for truncated life tests based on the inverse Rayleigh and log-logistic distributions. *Pak. J. Stat.* **2009**, *25*, 107–119.
34. Fertig, F.W.; Mann, N.R. Life-test sampling plans for two-parameter Weibull populations. *Technometrics* **1980**, *22*, 165–177. [CrossRef]
35. Zhuang, L.; Xu, A.; Wang, X.L. A prognostic driven predictive maintenance framework based on Bayesian deep learning. *Reliab. Eng. Syst. Safety* **2023**, *234*, 109181. [CrossRef]
36. Luo, C.; Shen, L.; Xu, A. Modelling and estimation of system reliability under dynamic operating environments and lifetime ordering constraints. *Reliab. Eng. Syst. Safety* **2022**, *218*, 108136. [CrossRef]
37. Zhou, S.; Xu, A.; Tang, Y.; Shen, L. Fast Bayesian Inference of Reparameterized Gamma Process With Random Effects. *IEEE Trans. Reliab.* 2023, *in press*. [CrossRef]