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Editorial

Editorial of the Special Issue "Skewed (Asymmetrical) Probability Distributions and Applications across Disciplines"

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This Special Issue includes a total of 14 articles on skewed probability distributions and applications across disciplines. A range of distributions has been studied such as the Rayleigh distribution [1,2], the beta generalized inverse Rayleigh distribution (BGIRD) [3], the Burr X generator, the transmuted Burr X-G (TBX-G) family [4], Chen lifetime distribution [5], the Weibull distribution [6], and the half-logistic (DHLo-II) distribution [7]. Likewise, the Kuramoto–Sivashinsky (L-KS) SPDEs have also been studied in [8–10]. The use of the studied distributions has been exemplified in different relevant contexts, namely, pattern of development in perceptual Inhibition in adolescents [11], physics-inspired modelling of reaction time data using a Maxwell–Boltzmann-like distribution [12], lifetime performance index CL to evaluate the performance of lifetimes of products following the skewed Exponentiated Fréchet distribution in in manufacturing industries [13], lifetime data analysis [5], tail risk evaluation [14], reliability engineering and failure analyses [6], capability performance assessment of a manufacturing process, especially for Weibull products [6], and COVID-19 and kidney dysmorphogenesis [7].

In the following text, we comment on the main goals and results of these contributions:

The first one, *Multistage Estimation of the Scale Parameter of Rayleigh Distribution with Simulation* [1], analyses the sequential estimation of the scale parameter of the Rayleigh distribution using the three-stage sequential sampling procedure developed by Hall [15]. Both point estimation and confidence interval estimation are included through a unified optimal decision framework, allowing optimal use of the available data, and reducing sampling by using massive data. The asymptotic behaviour of the proposed procedure is analysed in relation to point estimation and confidence interval estimation. Likewise, Monte Carlo simulations [16–20] were performed to study the performance of small, moderate, and large sample sizes in typical contexts using Microsoft Developer Studio software. The procedure presents interesting asymptotic features that have been supported by the simulations.

In the second paper, *Multistage Estimation of the Rayleigh Distribution Variance* [2], the authors discuss the sequential multistage estimation of the variance of the Rayleigh distribution through the three-stage procedure developed by Hall [15]. Considering that the variance of the Rayleigh distribution is a linear function of the square of the scale parameter of the distribution, it is sufficient to estimate this parameter. The authors deal with two problems related to estimation. First is the problem of estimating the minimum risk point given a quadratic error loss function plus a linear sampling cost. The second problem is a fixed-width confidence interval estimation using a unified optimal stopping rule. The procedure achieves second-order asymptotic efficiency and asymptotic consistency. To support the asymptotic results, Monte Carlo simulations were performed to study the performance of the procedure with the optimal sample size.

In the third paper, *Asymptotic Distributions for Power Variations of the Solutions to Linearized Kuramoto–Sivashinsky SPDEs in One-to-Three Dimensions* [8], the authors studied power variations for the fourth order linearized Kuramoto–Sivashinsky (LKS) SPDEs and their gradient, driven by the space–time white noise in one-to-three dimensional spaces, in time, have infinite quadratic variation and dimension-dependent Gaussian asymptotic distributions. The authors used the relationship between LKS-SPDEs and the Houdré–Villaa bifractional Brownian motion (BBM), from which they obtain temporal central limit theorems for LKS-SPDEs and their gradient. The underlying explicit kernels and spectral/harmonic analysis are used to prove the results. This work builds on the recent works on the delicate analysis of variations of general Gaussian processes and stochastic heat equation driven by the space–time white noise and complements Allouba's earlier works on the LKS-SPDEs and their gradient [21–24].

The fourth paper, *Human Reaction Times: Linking Individual and Collective Behaviour Through Physics* Modeling [12], develops a model for reaction time data to visual stimuli in Experimental Psychology, which is alternative to the use of the ex-Gaussian function [25–27] to represent this type of data. The authors propose a Physics-inspired model to represent the reaction time data of a coetaneous group of individuals since reaction time data from different individuals appear to be correlated. A Maxwell–Boltzmann-like distribution, the same distribution as for the velocities of the molecules in an Ideal Gas model, arises to be a good proposal to model the correlated reaction time data of a group of individuals [28]. This work also provides a simple entropy-based methodology for the classification of the individuals within a correlated group with no need for an external reference, which can find suitable applications in diverse areas of social sciences. An entropy-based study of reaction time data has also been studied in [29].

The fifth article is titled *Development of Perceptual Inhibition in Adolescents—A Critical Period?* [11]. Authors base their work on recent studies suggesting that the developmental curves in adolescence, in relation to the development of executive functions, could be fitted to a non-linear trajectory of development with progressions and retrogressions. In this work, an analysis of the pattern of development in Perceptual Inhibition (PI) is proposed by considering all stages of adolescence (early, middle, and late) in intervals of one year. Ex-Gaussian functions [25–27] have been fitted to the probability distributions of the mean response times and combined a covariance analysis (ANCOVA). The results showed that the 10-to-13-year-old groups present a similar performance in the task and differ from the 14 to 19 year olds. Significant differences were found between the older group and all the rest of the groups. The relevant changes that can be observed in relation to the nonlinear trajectory of development that would show the PI during adolescence are discussed.

The sixth paper is about *The Flexible Burr X-G Family: Properties, Inference, and Applications in Engineering Science* [4]. The authors introduce a new flexible generator of continuous distributions called the transmuted Burr X-G (TBX-G) family to extend and increase the flexibility of the Burr X generator. The results include the calculation of the general statistical properties of the TBX-G family and the study of one special sub-model, the TBX-exponential distribution. Authors also discuss on eight estimation approaches to estimating the TBX-exponential parameters. Likewise, numerical simulations are developed to make a comparison among the suggested approaches. The results of this work recommend the Anderson–Darling estimators to estimate the TBX-exponential parameters. The importance and flexibility of the TBX-exponential model compared with other existing competing distributions is illustrated by using two skewed real data sets from the engineering sciences.

In the seventh article, *A New Generalization of the Generalized Inverse Rayleigh Distribution with Applications* [3], a new four-parameter lifetime model called the beta generalized inverse Rayleigh distribution (BGIRD) is introduced. The results include the derivation of the mixture representation of this model, the study of the curve's behaviour of probability density function, reliability function, and hazard function. Likewise, the quantile function, median, mode, moments, harmonic mean, skewness, and kurtosis are derived. The authors also investigate other important properties such as the entropy (Rényi and Shannon), which is a measure of the uncertainty for this distribution. The parameters are estimated by carrying out a simulation study. Four real-life data sets from difference fields were applied on this model. Furthermore, a comparison between the new model and some competitive models is carried out using information criteria. Their model shows the best fitting for the real data analysed.

In the eighth article, *The Evaluation on the Process Capability Index C_L for Exponentiated Fréchet Lifetime Product under Progressive Type I Interval Censoring* [13], the authors present the likelihood inferences on the lifetime performance index C_L to evaluate the performance of lifetimes of products following the skewed Exponentiated Fréchet distribution in many manufacturing industries as a generalization of some lifetime distributions. The results include the derivation of the maximum likelihood estimator for C_L for lifetimes with exponentiated Fréchet distribution which is used to develop a computational testing procedure that experimenters can implement to test whether the lifetime performance reached the pre-assigned level of significance with a given lower specification limit under progressive type I interval censoring. This work includes two examples to illustrate the implementation of the computational testing procedure.

In the nineth article, *Spatial Moduli of Non-Differentiability for Linearized Kuramoto–Sivashinsky SPDEs and Their Gradient* [9], the authors investigate spatial moduli of non-differentiability for the fourth-order linearized Kuramoto–Sivashinsky (L-KS) SPDEs and their gradient, driven by the space-time white noise in one-to-three dimensional spaces. For this purpose, the authors use underlying explicit kernels and symmetry analysis, yielding spatial moduli of non-differentiability for L-KS SPDEs and their gradient.

In the tenth article, *Temporal Moduli of Non-Differentiability for Linearized Kuramoto–Sivashinsky SPDEs and Their Gradient* [10], the authors consider that $\mathrm{U} = \mathrm{U}(\mathrm{t,x})$ for $(\mathrm{t,x}) \in \mathrm{R}_+ \times \mathrm{R}^{\mathrm{d}}$ and $\partial_{\mathrm{x}}\mathrm{U} = \partial_{\mathrm{x}}\mathrm{U}(\mathrm{t,x})$ for $(\mathrm{t,x}) \in \mathrm{R}_+ \times \mathrm{R}$ are the solution and gradient solution of the fourth order linearized Kuramoto–Sivashinsky (L-KS) SPDE driven by the space-time white noise in one-to-three dimensional spaces, respectively. They use the underlying explicit kernels and symmetry analysis, yielding exact, dimension-dependent, and temporal moduli of non-differentiability for $\mathrm{U}(\cdot,\mathrm{x})$ and $\partial_{\mathrm{x}}\mathrm{U}(\cdot,\mathrm{x})$. The authors therefore confirm that almost all sample paths of $\mathrm{U}(\cdot,\mathrm{x})$ and $\partial_{\mathrm{x}}\mathrm{U}(\cdot,\mathrm{x})$, in time, are nowhere differentiable.

The eleventh paper, *Bayesian Testing Procedure on the Lifetime Performance Index of Products Following Chen Lifetime Distribution Based on the Progressive Type-II Censored Sample* [5], focuses on the lifetime performance index C_L which is frequently used to monitor the larger-the-better lifetime performance of products. The authors derived the uniformly minimum variance unbiased estimator (UMVUE) for C_L, and they used this estimator to develop a hypothesis testing procedure of C_L under a lower specification limit based on the progressive type-II censored sample. The Bayesian estimator for C_L is also derived and used to develop another hypothesis testing procedure. Simulations are carried out to compare the average confidence levels for two procedures. This work also includes a practical example to illustrate the implementation of the proposed non-Bayesian and Bayesian testing procedure.

The twelfth article, *Hedging and Evaluating Tail Risks* via *Two Novel Options Based on Type II Extreme Value Distribution* [14], focused on tail risk, an important financial issue today, in which directly hedging tail risks with an ad hoc option is still an unresolved problem since it is not easy to specify a suitable and asymmetric pricing kernel. By defining two ad hoc underlying “assets”, two novel tail risk options (TROs) for hedging and evaluating short-term tail risks are designed in this paper. Under the Fréchet distribution assumption for maximum losses, the closed-form TRO pricing formulas are obtained. The accuracy of the pricing formulas is illustrated by a simulation procedure. The results show that, no matter whether at scale level (symmetric “normal” risk, with greater volatility) or shape level (asymmetric tail risk, with a smaller value in tail index), the greater the risk, the more expensive the TRO calls, and the cheaper the TRO puts. Using calibration, one can obtain the TRO-implied volatility and the TRO-implied tail index. From the newly proposed TRO and its implied tail index, economic implications can be offered to investors, portfolio managers, and policymakers.

In the thirteenth paper, *Experimental Design for the Lifetime Performance Index of Weibull Products Based on the Progressive Type I Interval Censored Sample* [6], an experimental design is developed based on the testing procedure for the lifetime performance index of products following Weibull lifetime distribution under progressive type I interval censoring. The asymptotic distribution of the maximum likelihood estimator of the lifetime performance index is used to develop the testing procedure. In order to reach the given power level, the minimum sample size is determined and tabulated. To minimize the total cost that occurred under progressive type I interval censoring, the sampling design is investigated to determine the minimum number of inspection intervals and equal interval lengths when the termination time of experiment is fixed or not fixed. This work includes one practical example for the implementation of the proposed sampling design to collect the progressive type I interval censored sample.

In the fourteenth paper, *A Probability Mass Function for Various Shapes of the Failure Rates, Asymmetric and Dispersed Data with Applications to Coronavirus and Kidney Dysmorphogenesis* [7], a discrete analogue of an extension to a two-parameter half-logistic model is proposed for modelling count data. The probability mass function of the new model can be expressed as a mixture representation of a geometric model. The main statistical properties of this distribution are derived, namely, the hazard rate function, moments, moment generating function, conditional moments, stress-strength analysis, residual entropy, cumulative residual entropy, and order statistics with its moments. The results indicate that this distribution can be used to model positive skewed data and to analyse equi- and over-dispersed data. Furthermore, the hazard rate function can be either decreasing, increasing or bathtub. The method of maximum likelihood has been used to carry out the parameter estimation through the classical point of view. A detailed simulation study is carried out to examine the outcomes of the estimators. This work includes the analysis of two real data sets to prove the flexibility of the proposed discrete distribution.

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References

1. Yousef, A.; Hassan, E.E.H.; Amin, A.A.; Hamdy, H.I. Multistage Estimation of the Scale Parameter of Rayleigh Distribution with Simulation. *Symmetry* **2020**, *12*, 1925. [CrossRef]
2. Yousef, A.; Amin, A.A.; Hassan, E.E.; Hamdy, H.I. Multistage Estimation of the Rayleigh Distribution Variance. *Symmetry* **2020**, *12*, 2084. [CrossRef]
3. Bakoban, R.A.; Al-Shehri, A.M. A New Generalization of the Generalized Inverse Rayleigh Distribution with Applications. *Symmetry* **2021**, *13*, 711. [CrossRef]
4. Al-Babtain, A.A.; Elbatal, I.; Al-Mofleh, H.; Gemeay, A.M.; Afify, A.Z.; Sarg, A.M. The Flexible Burr X-G Family: Properties, Inference, and Applications in Engineering Science. *Symmetry* **2021**, *13*, 474. [CrossRef]
5. Wu, S.-F.; Chang, W.-T. Bayesian Testing Procedure on the Lifetime Performance Index of Products following Chen Lifetime Distribution Based on the Progressive Type-II Censored Sample. *Symmetry* **2021**, *13*, 1322. [CrossRef]
6. Wu, S.-F.; Wu, Y.-C.; Wu, C.-H.; Chang, W.-T. Experimental Design for the Lifetime Performance Index of Weibull Products Based on the Progressive Type I Interval Censored Sample. *Symmetry* **2021**, *13*, 1691. [CrossRef]
7. El-Morshedy, M.; Alizadeh, M.; Al-Bossly, A.; Eliwa, M.S. A Probability Mass Function for Various Shapes of the Failure Rates, Asymmetric and Dispersed Data with Applications to Coronavirus and Kidney Dysmorphogenesis. *Symmetry* **2021**, *13*, 1790. [CrossRef]
8. Wang, W.; Wang, D. Asymptotic Distributions for Power Variations of the Solutions to Linearized Kuramoto–Sivashinsky SPDEs in One-to-Three Dimensions. *Symmetry* **2021**, *13*, 73. [CrossRef]
9. Wang, W. Spatial Moduli of Non-Differentiability for Linearized Kuramoto–Sivashinsky SPDEs and Their Gradient. *Symmetry* **2021**, *13*, 1251. [CrossRef]
10. Wang, W.; Zhou, C. Temporal Moduli of Non-Differentiability for Linearized Kuramoto–Sivashinsky SPDEs and Their Gradient. *Symmetry* **2021**, *13*, 1306. [CrossRef]
11. Introzzi, I.M.; Richard'S, M.M.; Aydmune, Y.; Zamora, E.V.; Stelzer, F.; García Coni, A.; Lopez-Ramon, M.F.; Navarro-Pardo, E. Development of Perceptual Inhibition in Adolescents—A Critical Period? *Symmetry* **2021**, *13*, 457. [CrossRef]
12. Castro-Palacio, J.C.; Fernández-De-Córdoba, P.; Isidro, J.M.; Sahu, S.; Navarro-Pardo, E. Human Reaction Times: Linking Individual and Collective Behaviour through Physics Modeling. *Symmetry* **2021**, *13*, 451. [CrossRef]
13. Wu, S.-F.; Chang, W.-T. The Evaluation on the Process Capability Index C_L for Exponentiated Frech'et Lifetime Product under Progressive Type I Interval Censoring. *Symmetry* **2021**, *13*, 1032. [CrossRef]

14. Lin, H.; Liu, L.; Zhang, Z. Hedging and Evaluating Tail Risks via Two Novel Options Based on Type II Extreme Value Distribution. *Symmetry* **2021**, *13*, 1630. [CrossRef]
15. Hall, P. Asymptotic Theory of Triple Sampling for Sequential Estimation of a Mean. *Ann. Stat.* **1981**, *9*, 1229. [CrossRef]
16. Velazquez, L.; Castro-Palacio, J.C. Improving the efficiency of Monte Carlo simulations of systems that undergo temperature-driven phase transitions. *Phys. Rev. E* **2013**, *88*, 013311. [CrossRef]
17. Velazquez, L.; Castro-Palacio, J.C. Extended canonical Monte Carlo methods: Improving accuracy of microcanonical calculations using a reweighting technique. *Phys. Rev. E* **2015**, *91*, 033308. [CrossRef]
18. Fernández de Córdoba, P.; Nieves, J.; Oset, E.; Vicente-Vacas, M.J. Coherent pion production in the (He-3,T) reaction in nuclei. *Phys. Lett. B* **1993**, *319*, 416–420. [CrossRef]
19. Castro-Palacio, J.C.; Isidro, J.M.; Navarro-Pardo, E.; Velázquez-Abad, L.; Fernández-De-Córdoba, P. Monte Carlo Simulation of a Modified Chi Distribution with Unequal Variances in the Generating Gaussians. A Discrete Methodology to Study Collective Response Times. *Mathematics* **2021**, *9*, 77. [CrossRef]
20. Ortigosa, N.; Orellana-Panchame, M.; Castro-Palacio, J.C.; Fernández de Córdoba, P.; Isidro, J.M. Monte Carlo Simulation of a Modified Chi Distribution Considering Asymmetry in the Generating Functions: Application to the Study of Health-Related Variables. *Symmetry* **2021**, *13*, 924. [CrossRef]
21. Allouba, H. L-Kuramoto–Sivashinsky SPDEs in one-to-three dimensions: L-KS kernel, sharp Hölder regularity, and Swift–Hohenberg law equivalence. *J. Differ. Equ.* **2015**, *259*, 6851–6884. [CrossRef]
22. Allouba, H. A brownian-time excursion into fourth-order PDEs, linearized kuramoto–sivashinsky, and BTP-SPDEs on $R_+ \times R^d$. *Stoch. Dyn.* **2006**, *6*, 521–534. [CrossRef]
23. Allouba, H. A linearized Kuramoto–Sivashinsky PDE via an imaginary-Brownian-time-Brownian-angle process. *C. R. Math. Acad. Sci. Paris* **2003**, *336*, 309–314. [CrossRef]
24. Allouba, H.; Xiao, Y. L-Kuramoto–Sivashinsky SPDEs vs. time-fractional SPIDEs: Exact continuity and gradient moduli, 1/2-derivative criticality, and laws. *J. Differ. Equ.* **2017**, *263*, 1552–1610. [CrossRef]
25. Moret-Tatay, C.; Leth-Steensen, C.; Irigaray, T.Q.; Argimon, I.I.L.; Gamermann, D.; Abad-Tortosa, D.; Oliveira, C.; Sáiz-Mauleón, B.; Vázquez-Martínez, A.; Navarro-Pardo, E.; et al. The Effect of Corrective Feedback on Performance in Basic Cognitive Tasks: An Analysis of RT Components. *Psychol. Belg.* **2016**, *56*, 370–381. [CrossRef] [PubMed]
26. Moret-Tatay, C.; Moreno-Cid, A.; Argimon, I.I.L.; Quarti Irigaray, T.; Szczerbinski, M.; Murphy, M.; Vázquez-Martínez, A.; Vázquez-Molina, J.; Sáiz-Mauleón, B.; Navarro-Pardo, E.; et al. The effects of age and emotional valence on recognition memory: An ex-Gaussian components analysis. *Scand. J. Psychol.* **2014**, *55*, 420–426. [CrossRef]
27. Moret-Tatay, C.; Gamermann, D.; Navarro-Pardo, E.; Fernández de Córdoba, P. ExGUtils: A python package for statistical analysis with the ex-gaussian probability density. *Front. Psychol.* **2018**, *9*, 612. [CrossRef]
28. Castro-Palacio, J.C.; Fernández-De-Córdoba, P.; Isidro, J.M.; Navarro-Pardo, E.; Selvas Aguilar, R. Percentile Study of χ Distribution. Application to Response Time Data. *Mathematics* **2020**, *8*, 514. [CrossRef]
29. Iglesias-Martínez, M.E.; Hernaiz-Guijarro, M.; Castro-Palacio, J.C.; Fernández-de-Córdoba, P.; Isidro, J.M.; Navarro-Pardo, E. Machinery Failure Approach and Spectral Analysis to Study the Reaction Time Dynamics over Consecutive Visual Stimuli: An Entropy-Based Model. *Mathematics* **2020**, *8*, 1979. [CrossRef]