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Preassigned-Time Bipartite Flocking Consensus Problem in Multi-Agent Systems

Xiejun Cheng [1,†], Jiashang Yu [2,*], Xiurong Chen [3,*,†], Jiaju Yu [1,3,4] and Bing Cheng [1,3]

1 School of Business, Jiangsu Second Normal University, Nanjing 211200, China; chxj@jssnu.edu.cn (X.C.); jiaju_yu@qau.edu.cn (J.Y.); chengbing1218@qau.edu.cn (B.C.)
2 School of Mathematics and Statistics, Heze University, Heze 274700, China
3 School of Science and Information, Qingdao Agricultural University, Qingdao 266109, China
4 School of Mathematics and Finance, Chuzhou University, Chuzhou 239000, China
* Correspondence: yujiashang@hezeu.edu.cn (J.Y.); xiurongchen@qau.edu.cn (X.C.); Tel.: +86-0532-58957444 (J.Y.)
† These authors contributed equally to this work.

Abstract: This article is concerned with the bipartite flocking problem in multi-agent systems. Our contributions can be summarized as follows. Firstly, a class of preassigned-time consensus protocols is proposed to solve the issue of multi-agent systems. Secondly, with the aid of the symmetric properties of the graph theory and the Lyapunov stability theorem, we prove that agents can be divided into two disjointed clusters in a finite time, and they move to opposite directions at the same magnitude and speed. The protocol is novel among existing fixed/finite-time protocols in that the associated settling time is a preassigned constant and a parameter of the protocol. Moreover, it is proven that the diameters of the clusters are bounded and independent of other the protocol parameters. These results are demonstrated through both theoretical analysis and simulation examples.

Keywords: bipartite-flocking consensus; preassigned time; multi-agent systems; structural balanced graph

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1. Introduction

It is well known that many interesting phenomena can be modeled as multi-agent systems because agents in the systems can both collaborate and compete with each other; many prototypical results can be found in the existing literature. For example, in [1], Fan et al. investigated the bipartite flocking issue in double-integrator dynamic systems in virtue of LaSalle's invariance principle, Barbalat's Lemma, and graph theory. The cluster flocking consensus in the Cucker–Smale model was dealt with in [2–5].

It is worth noticing that the convergence time of the protocols in the above works are usually infinite. For practical applications, the convergence time (settling time) should be finite to realize the precise consensus of multi-agent systems. To achieve finite convergence, many finite-time or fixed-time bipartite flocking protocols were proposed in literature, such as some fixed-time bipartite protocols that were provided in [6,7]. However, for finite-time protocols and fixed-time protocols, the corresponding settling times cannot be arbitrarily preassigned and also depend on the protocol parameters or initial conditions. In addition, it can be seen in references [6,7] that there exist at least two problems with the settling time T^* that are worth discussing.

Problem 1: T^* is dependent on not only on the parameters of the protocol but also the topological information. The relationship between the settling time and the parameters is complex.

Problem 2: To adjust T^*, we can enlarge the strength parameters or adjust the power parameters; however, the magnitude of T^* always varies greatly in response to even small adjustments of the power parameter. That is to say, adjustment of the settling time is too sensitive.

To the best of our knowledge, the above two problems have not been solved at present in the existing literature. In real applications, a controlled system is usually required to achieve a bipartite flocking consensus in a preset period according to the actual requirements, and at the same time, the period is required to be independent of the initial conditions and the other design parameters. That is to say, a preassigned-time consensus should be realized under the designed protocol.

However, there are few existing results on preassigned-time bipartite flocking in multi-agent systems in spite of the fact that there have been many results on finite-time bipartite flocking [8–14] and asymptomatic flocking problems [15–17]. Different from asymptotic bipartite flocking (finite- or fixed-time flocking), preassigned-time bipartite flocking requires agents to reach some state of interest within a preassigned time according to the requirements of the control design. Hence, it is significant, necessary, and valuable to discuss preassigned-time flocking consensus problems in multi-agent systems.

Motivated by the above observations and reference [18], the aim of this work is to investigate the bipartite flocking consensus problem in multi-agent systems, and the aim of the manuscript is to design a new class of consensus protocol that can solve the consensus problem in multi-agent systems. It is notable that the settling time corresponding to the protocol is a tunable parameter and the diameter of the cluster is estimated. We devote this work to the investigation of the preassigned-time bipartite flocking consensus problem.

Our contributions are summarized up as follows. Firstly, a class of preassigned-time bipartite flocking protocols is proposed. It is worth noting that—different from the existing fixed-time consensus protocols in [19–21], which can solve the preassigned-time consensus problem in multi-agent systems in virtue of the protocol having a time-varying gain value—a constant gain is introduced to the protocol in this work. In particular, the constant is independent of the initial conditions, the other parameters, and the topological information. Certainly, the techniques in [6,7,12–14,22] all fail to solve the preassigned-time bipartite flocking problem. Secondly, with the aid of structural balance graph theory and Lyapunov stability theorem, the feasibility of the protocol is proven by rigorous proof. Thirdly, some sufficient conditions are given to guarantee realization of the preassigned-time biparatite flocking consensus. To the best of our knowledge, this the first time that a constant gain has been discussed in the context of preassigned-time bipartite flocking consensus issues. Thus, the results obtained in this article are novel at present.

The rest of the article is organized as follows. Section 2 introduces some preliminaries and the problem formulation. The preassigned-time bipartite flocking results are presented in Section 3. In Section 4, one simulation example is provided to verify the theoretical results. Section 5 concludes the article with some conclusions.

2. Preliminaries and Problem Formulation

2.1. Preliminaries

Signed graphs $G(A) = G(V, E, A)$ are necessary tools for describe the communication topology of multi-agent systems, where $V = \{v_i | \, i \in \Pi_N\}$ is the vertex set, $E = \{(v_i, v_j)|, v_i, v_j \in V\}$ denotes the edge set, and $A = [a_{ij}] \in R^{N\times N}$ is the weighted adjacency matrix, which satisfies $a_{ij} \neq 0$ if $(v_i, v_j) \in E$ and $a_{ij} = 0$ otherwise. Moreover, self-loops are not considered in the paper; therefore, $a_{ii} = 0$ for any $i \in \Pi_N$. The corresponding Laplacian matrix $L = [l_{ij}] \in R^{N\times N}$ for a signed graph $G(A)$ satisfies $l_{ij} = -a_{ij}$ if $i \neq j$, and $l_{ii} = \sum_{j=1}^{N} |a_{ij}|$, $i, j \in \Pi_N$. When the relationship between agent i and agent j is cooperative, $a_{ij} \geq 0$; otherwise, a competitive relationship is present, in which case $a_{ij} \leq 0$.

Definition 1 ([23])**.** *For an undirected signed graph $G(A)$, if there is a bipartition for the vertex set V such that $V = V_1 \cup V_2$, $V_1 \cap V_2 = \emptyset$, $a_{ij} \geq 0$ for any v_i, $v_j \in V_r$, $a_{ij} \leq 0$ for $v_i \in V_r$, $v_j \in V_{3-r}$, and $r = 1, 2$, then signed graph the $G(A)$ is said to be structurally balanced; otherwise, it is called structurally unbalanced.*

Lemma 1 ([23])**.** *For a connected signed graph $G(A)$, it is said to be a structurally balanced graph if and only if there is a matrix $S = diag\{\sigma_1, \cdots, \sigma_N\}$ where $\sigma_i = 1$ or $\sigma_i = -1$ such that $SAS \geq 0$ and $SS^T = I_N$. In addition, the second smallest eigenvalue of matrix L is positive.*

Lemma 2 ([24])**.** *If numbers $0 \leq \xi_i \in R$, $i \in \Pi_n$, then the following inequality holds*

$$\sum_{i=1}^{n} \xi_i^p \geq min_{p\geq 0}\left\{1, n^{1-p}\right\}\left(\sum_{i=1}^{n} \xi\right)^p$$

Lemma 3 ([25])**.** *If a positive definite, unbounded, and radial function $V(x) : R^n \to R_+$ exists satisfying the inequality*

$$\dot{V}(x) \leq -\frac{T}{T_p}(\alpha V^p(x) + \beta V^q(x)), \tag{1}$$

where the parameters are $0 \leq p < 1 < q$, $m_p = \frac{1-p}{q-p}$, $m_p = \frac{q-1}{q-p}$, and $T = \frac{1}{\alpha(q-p)}(\frac{\alpha}{\beta})^{m_p} B(m_p, m_q)$; $B(\cdot,\cdot)$ is the beta function; and $T_p > 0$ is a constant, then $V(x)$ reaches zero in a finite time, and the associated settling time function satisfies $T(x_0) \leq T_p$.

2.2. Problem Statement

Consider a multi-agent system composed of N agents, and the associated communication topology is a structurally balanced graph $G(A)$. The dynamics of i-th agent satisfy the following equations [22]:

$$\begin{aligned} &\dot{x}_i(t) = v_i(t),\ \dot{v}_i(t) = u_i(t), \\ &x_0 = x_i(0),\ v_0 = v_i(0),\ i \in \Pi_N. \end{aligned} \tag{2}$$

where $x_i \in R^d$ and $v_i \in R^d$ are the position of i-th agent and the velocity of i-th agent, respectively. $u_i(t)$ denotes the input, and it is also said to be the consensus protocol that needs to be designed. For the specific physical meaning of system (2), readers can refer to [22] (it is omitted here). In addition, to simplify the analysis, we choose a value of $d = 1$ in this article. When $d > 1$, it can be dealt with using the Kronecker product as well. Here, it is assumed that each agent is fitted with position and velocity onboard sensors and that they work instantaneously. In addition, the communication topologies for position and velocity are not the same; in fact, unmanned aerial vehicles can be described by system (2).

Before proceeding, the preassigned-time bipartite flocking consensus is defined as follows.

Definition 2 ([22])**.** *For any given initial conditions, the preassigned-time bipartite flocking consensus of system (2) is realized if the following conditions hold:*

(1) For any $i, j \in V$, we have that $\lim_{t\to T_p} v_i(t) = \kappa_{ij} \lim_{t\to T_p} v_j(t)$ and $v_i(t) = \kappa_{ij} v_j(t)$ for $t \geq T_p$, where $\kappa_{ij} = 1$ if $i, j \in V_r$, $r \in \Pi_2$; otherwise $\kappa_{ij} = -1$, and V_1 and V_2 are a bipartition of V in Definition 1. Moreover, the settling time T_p is a constant which can be preassigned according to practical requirement.

(2) We can find a positive constant $C > 0$ (the diameter of the cluster) such that $\sup_{t\geq 0} \|x_i(t) - x_j(t)\| \leq C$, for any $i, j \in V_r$, $r \in \Pi_2$, where $\|\cdot\|$, $\|\cdot\|$ stands for 2-norm.

Remark 1. *Here, the parameter T_p is independent of a system's initial states and the other protocol parameters, and it may be designed according to a designers' requirement. For the agents in the same cluster V_r, Condition (1) of Definition 2 implies that agents move at the same speed within a preassigned time. For agents in different clusters, they are to move at the same speed in magnitude but in opposite directions. Condition (2) in Definition 2 shows that the diameter of clusters must be bounded by some constant. Thus, only the two conditions holding simultaneously can guarantee that a preassigned-time bipartite flocking consensus is achieved.*

3. Preassigned-Time Bipartite Flocking Consensus

This article aims to design a class of protocols that can solve the bipartite flocking consensus problem in system system (2). For this purpose, we design the following consensus protocol $u_i(t)$ for the i-th agent

$$u_i(t) = \frac{T}{T_p} \sum_{k=1}^{2} l_k \left(\sum_{j=1}^{N} a_{ij} \big(v_j(t) - sign(a_{ij}) v_i(t) \big) \right)^{\lambda_k} \tag{3}$$

where $i \in \Pi_N$, $T_p > 0$ is constant, power parameters $0 < \lambda_1 < 1 < \lambda_2$, gain coefficients $l_k > 0, k = 1, 2$, T is to be given later.

Now, we are in position to state our main results.

Theorem 1. *If graph $G(A)$ is a connected undirected signed graph and satisfies structurally balanced conditions, then the preassigned-time bipartite flocking consensus of multi-agent systems system(2) can be realized under protocol (3). Moreover, the corresponding settling time is upper bounded by T_p.*

Proof. The proof of theorem will be finished through two steps.
Step 1: Due to the fact that the signed graph $G(A)$ satisfies structurally balanced conditions, it follows from Lemma 1 that we could find a diagonal matrix $S = diag\{\sigma_1, \cdots, \sigma_N\}$ such that $SAS \geq 0$. Take the linear transform $z(t) = Sv(t)$, where $z(t) = [z_1(t), \cdots, z_N(t)]^T$, $v(t) = [v_1(t), \cdots, v_N(t)]^T$; then, we have $z(0) = Sv(0)$, $sign(a_{ij}) = \sigma_i \sigma_j$. So, we can obtain $\dot{z}_i(t) = \sigma_i \dot{v}_i(t)$, $\dot{x}_i(t) = \sigma_i \dot{z}_i(t)$. Further, we have

$$\begin{aligned} ll\dot{z}_i(t) \quad &= \sigma_i \tfrac{T}{T_p} \textstyle\sum_{k=1}^{2} l_k \left(\sum_{j=1}^{N} a_{ij} \big(v_j(t) - sign(a_{ij}) v_i(t) \big) \right)^{\lambda_k} \\ &= \tfrac{T}{T_p} \textstyle\sum_{k=1}^{2} l_k \left(\sum_{j=1}^{N} |a_{ij}| \big(z_j(t) - z_i(t) \big) \right)^{\lambda_k}. \end{aligned} \tag{4}$$

The candidate Lyapunov function is taken as

$$V(t) = \frac{1}{2} z^T(t) \hat{L} z(t) = \frac{1}{4} \sum_{j=1}^{N} |a_{ij}| \big(z_i(t) - z_j(t) \big)^2. \tag{5}$$

where $\hat{L} = SLS$. The derivative of $V(t)$ along system (4) can be obtained as follows.

$$\begin{aligned} \frac{dV(t)}{dt} =& \frac{T}{T_p} l_1 \sum_{i=1}^{N} \left(\sum_{j=1}^{N} |a_{ij}| (z_i - z_j) \right)^{1+\lambda_1} \\ &+ \frac{T}{T_p} l_2 \sum_{i=1}^{N} \left(\sum_{j=1}^{N} |a_{ij}| (z_i - z_j) \right)^{1+\lambda_2} \\ =& \frac{T}{T_p} l_1 \sum_{j=1}^{N} \left[\sum_{i=1}^{N} |a_{ij}| (z_i - z_j)^2 \right]^{p} \\ &+ \frac{T}{T_p} l_2 \sum_{j=1}^{N} \left[\sum_{i=1}^{N} |a_{ij}| (z_i - z_j)^2 \right]^{q}. \end{aligned} \tag{6}$$

Because $0 < \lambda_1 < 1 < \lambda_2$, we have $\frac{1}{2} < p = 1 + \lambda_1 < 1 < q = 1 + \lambda_2$. According to Lemma 2, we have

$$\sum_{j=1}^{N} \left[\sum_{i=1}^{N} |a_{ij}| (z_i - z_j)^2 \right]^{p} \geq \left(\sum_{i=1}^{N} \sum_{j=1}^{N} |a_{ij}| (z_i - z_j)^2 \right)^{p}. \tag{7}$$

$$\sum_{j=1}^{N}\left[\sum_{i=1}^{N}|a_{ij}|(z_i-z_j)^2\right]^q \geq N^{1-q}\left(\sum_{i=1}^{N}\sum_{j=1}^{N}|a_{ij}|(z_i-z_j)^2\right)^q. \tag{8}$$

Based on inequalities (6)–(8)we can obtain

$$\begin{aligned}\frac{dV(t)}{dt} \leq &-\frac{T}{T_p}l_1\left(\sum_{i=1}^{N}\sum_{j=1}^{N}|a_{ij}|(z_i-z_j)^2\right)^p \\ &-\frac{T}{T_p}l_2N^{1-q}\left(\sum_{i=1}^{N}\sum_{j=1}^{N}|a_{ij}|(z_i-z_j)^2\right)^q.\end{aligned} \tag{9}$$

Note that matrix $\hat{L} = SLS$ is positive and semi-definite. Then, there must be a unique matrix Q, which is a semi-positive definite matrix such that $\hat{L} = QQ^T$. Thus, according to Theorem 4.1 in [8], we can obtain the following inequality:

$$\begin{aligned}\sum_{i=1}^{N}\sum_{j=1}^{N}|a_{ij}|(z_i-z_j)^2 &= (\hat{L}z(t))^T(\hat{L}z(t)) \\ &= 2z^TQQ^TQQ^Tz(t) \geq 2\lambda_2(\hat{L})V(t).\end{aligned}$$

Combining inequality (9) and the above inequality, for $V(t) \neq 0$, we have

$$\frac{dV(t)}{dt} \leq -\frac{T}{T_p}\left[\alpha(V(t))^p + \beta(V(t))^q\right]. \tag{10}$$

where $p = \frac{1+\lambda_1}{2}$, $q = \frac{1+\lambda_2}{2}$, $\alpha = l_1(2\lambda_2(L))^p$, $\beta = N^{1-q}l_2(2\lambda_2(L))^q$, $\lambda_2(L)$ [26] denotes the second smallest eigenvalue of matrix L, T can be obtained using Lemma 3. Invoking Lemma 3 and the comparison principle, we can conclude that $V(t)$ converges to zero in a preassigned time T_p, that is to say,

$$V(t) \equiv 0 \; for \; t \geq T_p. \tag{11}$$

That is to say, $\sigma_i v_i = \sigma_j v_j$ for $t \geq T_p$.
Therefore, the following result can be obtained

$$\lim_{t\to T_p} v_i(t) = \sigma_i\sigma_j \lim_{t\to T_p} v_j(t) \; and \; v_i(t) = \sigma_i\sigma_j v_j(t) \tag{12}$$

for $t \geq T_p$.
Because $z_i(t) = \sigma_i v_i(t)$, $\sigma_i \in \{-1,1\}$, we obtain $\lim_{t\to T_p} v_i(t) = \kappa_{ij}\lim_{t\to T_p} v_j(t)$ and $v_i(t) = \kappa_{ij}v_j(t)$ for $t \geq T_p$, where $\kappa_{ij} = 1$ when $i,j \in V_r$, $r \in \Pi_2$, $\kappa_{ij} = -1$ otherwise.
The above equalities imply that condition (1) in Definition 2 holds. Note that here T_p is the preassigned time, which is independent of other protocol parameters, initial conditions, and topology information. Therefore, T_p can be set according to designers' requirements.
Step 2: For any $i,j \in V_r$, $r = 1,2$, where V_r is same as that in Definition 1. Then, the following candidate Lyapunov functions are considered.

$$\bar{V}_r = \sum_{i,j\in V_r}\|v_i(t) - v_j(t)\|^2 \tag{13}$$

and

$$\bar{X}_r = \sum_{i,j\in V_r}\|x_i(t) - x_j(t)\|^2. \tag{14}$$

Then, Equation (11) implies that $\bar{V}_r = 0$ for $t \geq T_p$, and combination with Equation (10), it also means that $\bar{V}_r$ is bounded when time $t \geq 0$. In other words, one can find a constant

$K > 0$ that satisfies $\bar{V}_r \le K$. For more details, one can refer to references [7,22]; for the sake of brevity, we omit them here. Therefore, for any $i, j \in V_r$. Invoking the triangle inequality and Cauchy–Schwartz inequality, we obtain:

$$\begin{aligned}\dot{\bar{X}}_r &= 2\sum_{i,j\in V_r}\langle x_i(t)-x_j(t), v_i(t)-v_j(t)\rangle \\ &\le 2\sum_{i,j\in V_r}\|x_i(t)-x_j(t)\|\|v_i(t)-v_j(t)\| \\ &\le 2\bar{X}_r^{\frac{1}{2}}\bar{V}_r^{\frac{1}{2}}.\end{aligned} \tag{15}$$

Integrating differential inequality inequality (15) in the interval $[0, t]$ yields

$$\bar{X}_r^{\frac{1}{2}} \le \bar{X}_r(0)^{\frac{1}{2}} + K^{\frac{1}{2}}T_p$$

Combining (14), we can obtain

$$\max_{i\neq j\in V_r}\|x_i(t)-x_j(t)\| \le \bar{X}_r \le \left(\bar{X}_r(0)^{\frac{1}{2}} + K^{\frac{1}{2}}T_p\right)^2 = C_r. \tag{16}$$

Further, we have

$$\max_{i\neq j\in V_r}\|x_i(t)-x_j(t)\| \le \max_{r\in\Pi_2}\{C_r\} = C.$$

This implies that the Condition (2) in Definition 2 holds as well. This completes the proof. □

Remark 2. *Step 1 shows that the velocities of agents can realize a bipartite consensus in a preassigned time.*

Remark 3. *Step 2 shows that positions of agents are bounded in preassigned time.*

Remark 4. *For Theorem 1, only undirected connected topology graph is employed to discuss the bipartite flocking consensus problem, and the associated settling time is a preassigned constant which is independent of any other elements such as other protocol parameters, network connectivity, and so on. Moreover, it follows from (16) that the diameter of each cluster is influenced by the initial states $X^{\frac{1}{2}}(0)$, upper bound $K > 0$, and preassigned time T_p. This is different from the fixed-time bipartite flocking protocol in [6,7,22], which cannot ensure that the diameter of each cluster is independent of network connectivity $\lambda_2(L)$ at all. Consensus protocol (3) can ensure the diameter of each cluster is independent of $\lambda_2(L)$ and the parameters of the protocol. Therefore, bipartite flocking consensus is realized in finite time T_p, which can be set according to the designers requirement. These results are novel in comparison to the existing results [6,7,22].*

4. Simulations

In this section, one simulation example is performed to illustrate the associated theoretical results in Section 3. To compare to the results in [22], the numerical example in [22] is adopted here. The example is a multi-agent system composed of six agents, and the associated weighted adjacency matrix is A, which is defined as

$$A = \begin{bmatrix} 0 & 3 & 0 & -5 & -1 & 0 \\ 3 & 0 & 2 & 0 & -4 & 1 \\ 0 & 2 & 0 & 0 & 0 & 0 \\ -5 & 0 & 0 & 0 & 0 & 0 \\ -1 & -4 & 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 & 3 & 0 \end{bmatrix},$$

The structure of balanced graph can be described by the weighted adjacency matrix A; thus, the structure of graph is Figure 1. The protocol parameters are chosen as $l_k = 1$,

$k \in \Pi_2$, $\lambda_1 = 0.3$, $\lambda_2 = 1.9$. Preassigned times are chosen as $T_p = 2$ and $T_p = 3$. The initial state is chosen as $x_0 = [4, 2, 0, -2, -4, -6]^T$, $v_0 = [1, 2, 3, 4, 5, -6]^T$, which are same as parameters as in [6,7,22]. The associated numerical results are shown in Figures 2 and 3, and the bipartite flocking consensus is realized in preassigned time T_p. The diameters corresponding to $T_p = 3$ and $T_p = 2$ are about 2.1 and 2.7, respectively. The initial state is chosen as $x_0 = [10, 12, 8, -8, -14, -16]$, $T_p = 3$, and the diameter is about 6.15. The control signal is shown in Figure 4, which illustrates that the control protocol is distributed as well. This simulation supports our theoretical results well, that is to say, the figure verified effectiveness of the protocol (3). Figure 5 corresponds to unstructured balanced graph. It is obvious that a bipartite flocking consensus is not realized.

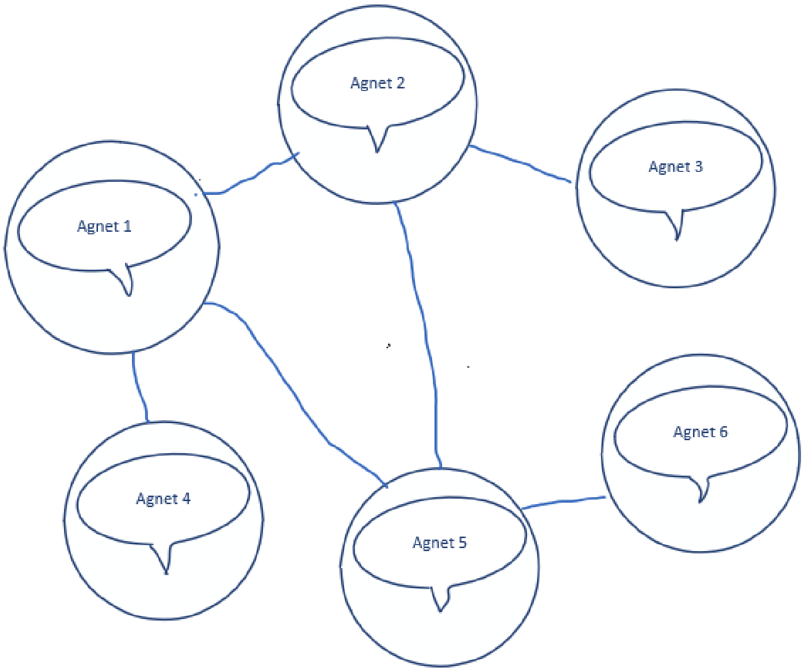

Figure 1. The communication topology of multi-agent systems (2).

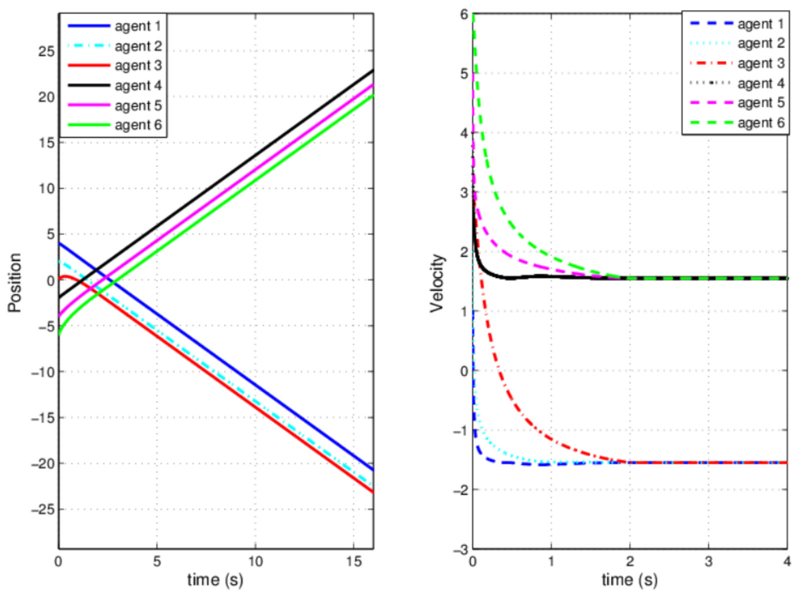

Figure 2. The positions and velocities of agents with $T_p = 2$.

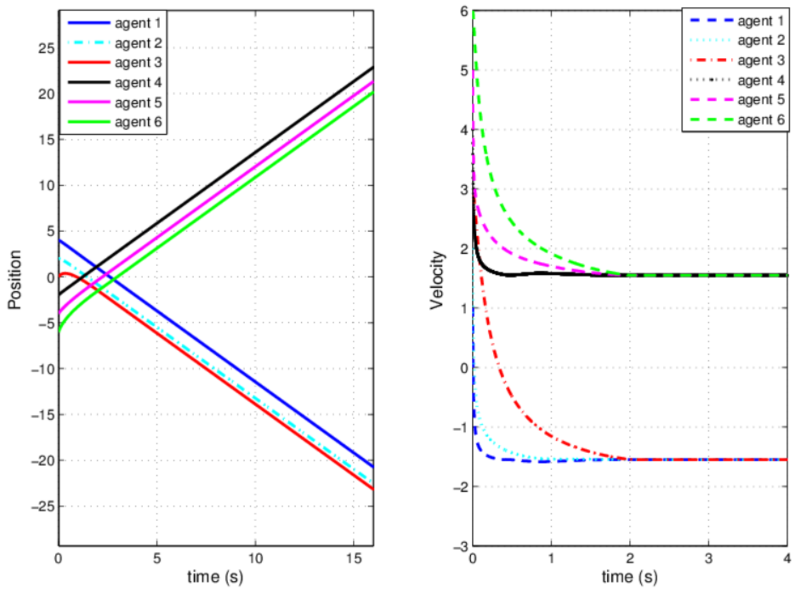

Figure 3. The positions and velocities of agents with $T_p = 3$.

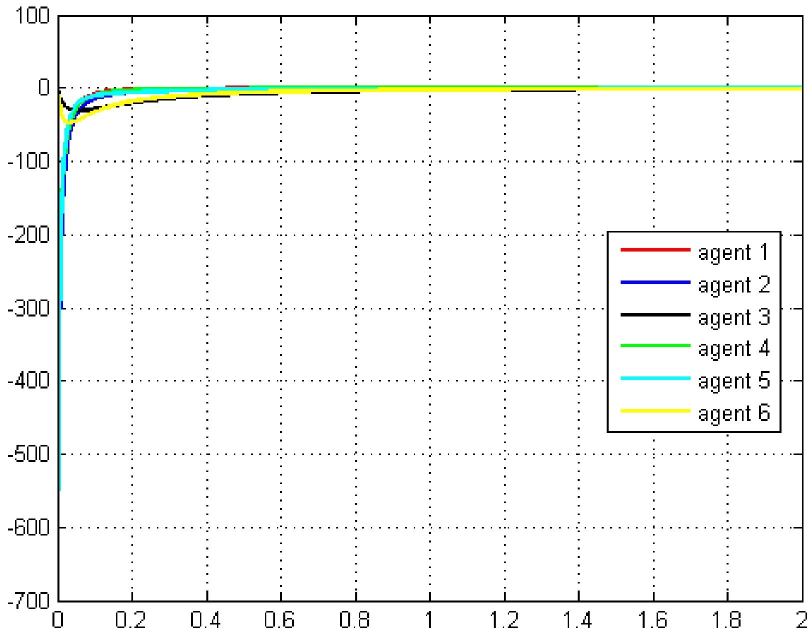

Figure 4. The control signal u_i of multi agent systems (2) with $T_p = 2$.

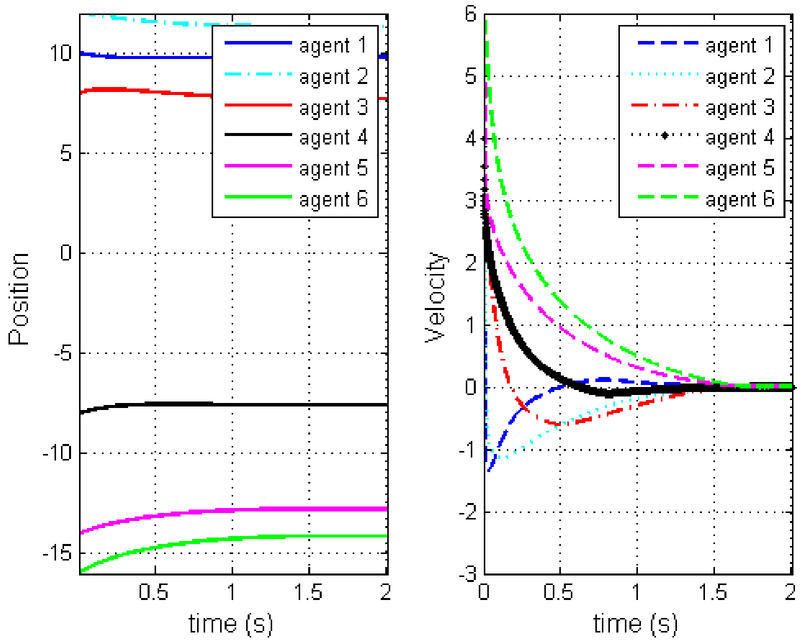

Figure 5. The positions and velocities of agents under unstructured balanced graph.

5. Conclusions

In this paper, the bipartite flocking consensus issue is investigated. A new class of preassigned-time consensus protocols is proposed to realize consensus in multi-agent systems. With the aid of structurally balanced graph theory and the Lyapunov stability theorem, the consensus analysis is completed. It is proven that the settling time is a prior constant which can be set according to requirements. In addition, theoretical analysis shows that the diameter of the clusters is bounded. However, there remains work that is also worth researching, such as systems under switching topology, event-triggered mechanism protocols, and consensus of multi-agent systems; our future work will investigate these issues.

6. Notations

R^n and R denote the n-dimensional real number space and the real number set, respectively. $R^{n\times m}$ denotes the $n \times m$ dimensional matrix space. $\Pi_N = \{1, \cdots, N\}$ is an index set. N is a positive integrator number. I_N denotes an $N \times N$ dimensional identity matrix, and $A \geq 0$ $(A > 0)$ implies that matrix A is a positive semi-definite matrix (positive definite matrix).

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References

1. Ha, S.Y.; Ko, D.; Zhang, Y. Critical coupling strength of the Cucker-Smale model for flocking. *Math Models Methods Appl. Sci.* **2017**, *27*, 1051–1087. [CrossRef]
2. Cho, J.; Ha, S.-Y.; Huang, F.; Jin, C.; Ko, D. Emergence of bi-cluster flocking for agent-based models with unit speed constraint. *Anal. Appl.* **2016**, *14*, 39–73. [CrossRef]
3. Cho, J.; Ha, S.-Y.; Huang, F. Emergence of bi-cluster flocking for the Cucker-Smale model. *Math. Model. Methods Appl. Sci.* **2016**, *26*, 1191–1218. [CrossRef]
4. Ru, L.; Xue, X. Multi-cluster flocking behavior of the Hierarchical Cucker-Smale Mode. *J. Frankl. Inst.* **2017**, *354*, 2371–2392. [CrossRef]
5. Ha, S.; Ko, D.; Zhang, Y.; Zhang, X. Emergent dynamics in the interactions of Cucker-Smale ensembles. *Kinet. Relat. Model.* **2017**, *10*, 689–723. [CrossRef]
6. Liu, H.; Wang, X.; Huang, Y.; Liu, Y. A new class of fixed-time bipartite flocking protocols for multi-agent systems. *Appl. Math. Model.* **2020**, *84*, 501–521. [CrossRef]
7. Yu, J.; Yu, S. An improved fixed-time bipartite flocking protocol for nonlinear multi-agent systems. *Int. J. Control* **2020**, *95*, 900–905. [CrossRef]
8. Deng, Q.; Wu, J.; Han, T.; Yang, Q.; Cai, X. Fixed-time bipartite consensus of multi-agent systems with disturbances. *Phys. A* **2019**, *516*, 37–49. [CrossRef]
9. Liu, X.; Cao, J.; Xie, C. Finite-time and fixed-time bipartite consensus of multi-agent systems under a unified discontinuous control protocol. *J. Frankl. Inst.* **2019**, *356*, 734–751. [CrossRef]
10. Chen, G.; Lewis, F.; Xie, L. Finite-time distributed consensus via binary control protocols. *Automatica* **2011**, *47*, 1962–1968. [CrossRef]
11. Ning, B.; Han, Q. Prescribed finite-time consensus tracking for multi-agent systems with nonholonomic chained-form dynamics. *IEEE Trans. Autom. Control.* **2019**, *64*, 1686–1693. [CrossRef]
12. Ning, B.; Han, Q.; Zuo, Z.; Jin, J.; Zheng, J. Collective behaviors of mobile robots beyond the nearest neighbor rules with switching topology. *IEEE. Trans. Cybern.* **2018**, *48*, 1577-1590. [CrossRef]
13. Zhan, X.; Wu, J.; Jiang, T.; Jiang, X. Optimal performance of networked control systems under the packet dropouts and channel noise. *ISA Trans.* **2015**, *58*, 214–221. [CrossRef]
14. Zhan, X.; Cheng, L.; Wi, J.; Yang, Q.; Han, T. Optimal modified performance of MIMO networked control systems with multi-parameter constraints. *ISA Trans.* **2019**, *84*, 111–117. [CrossRef]
15. Liu, H.; Wang, X.; Liu, Y.; Li, X. On non-collision flocking and line-shaped spatial configuration for a modified singular Cucker-Smale model. *Commun. Nonlinear Sci. Numer. Simulat.* **2019**, *75*, 280–301. [CrossRef]
16. Wang, X.; Wang, L.; Wu, J. Impacts of time delay on flocking dynamics of a two-agent flock model. *Commun. Nonlinear Sci. Numer. Simulat.* **2019**, *70*, 80–88. [CrossRef]
17. Ahn, S.M.; Choi, H.; Ha, S.Y.; Lee, H. On collision-avoiding initial configurations to Cucker-Smale type flocking models. *Commun. Math. Sci.* **2012**, *10*, 625–643. [CrossRef]
18. Hu, C.; Jiang, H. Special Functions-Based Fixed-Time Estimation and Stabilization for Dynamic Systems. *IEEE Trans. Syst. Man, Cybern. Syst.* **2021**, *52*, 3251—3262. [CrossRef]
19. Liu, D.; Liu, Z.; Chen, C.P. Prescribed-time containment control with prescribed performance for uncertain nonlinear multi-agent systems. *J. Frankl. Inst.* **2020**, *358*, 1782–1811. [CrossRef]
20. Wang, Y.; Song, Y.; Hill, D.J. Prescribed-time consensus and containment control of networked multiagent systems. *IEEE Trans. Cybern.* **2018**, *49*, 1138–1147. [CrossRef]
21. Chen, X.; Yu, H.; Hao, F. Prescribed-time event-triggered bipartite consensus of multiagent systems. *IEEE Trans. Cybern.* **2020**, *52*, 2589–2598. [CrossRef] [PubMed]
22. Xiao, Q.; Liu, H.; Wang, X.; Huang, Y. A note on the fixed-time bipartite flocking for nonlinear multi-agent systems. *Appl. Math. Lett.* **2020**, *99*, 105973. [CrossRef]
23. Altafini, C. Consensus problems on networks with antagonistic interactions. *IEEE. Trans. Automat. Control.* **2013**, *58*, 935–946. [CrossRef]
24. Hardy, G.; Littlewood, J. *Inequalities*; Cambridge University Press: Cambridge, UK, 1952.
25. Aldana-López, R.; Gómez-Gutiérrez, D.; Jiménez-Rodrguez, E.; Sánchez-Torres, J.D.; Defoort, M. Enhancing the settling time estimation of a class of fixed-time stable systems. *Int. J. Robust. Nonlin.* **2019**, *29*, 4135–4148. [CrossRef]
26. Jäntschi, L. The eigenproblem translated for alignment of molecules. *Symmetry* **2019**, *11*, 1027. [CrossRef]