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Editorial

Symmetry in Ordinary and Partial Differential Equations and Applications

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This Special Issue of the journal *Symmetry* is dedicated to recent progress in the field of nonlinear differential problems. It therefore collects papers on existence and non-existence criteria, multiplicity, and regularity results of solutions, as well as applications to real-life problems. Special emphasis is put on capacity and variational methods and on regularization and approximation procedures. These are attractive topics that have been developed on consolidated theories for continuous and discrete systems, especially in recent decades. More precisely, the main problems are investigated by imposing different boundary conditions and appropriate initial conditions on the data. A number of contributions in this Special Issue focus on nonlinear operators as tools to describe the dynamic aspects of systems, where symmetries and asymmetries play key roles. The volume gathers eighteen research works authored by leading experts in the field, as well as by young and promising researchers. In this editorial, we briefly summarize these contributions to highlight the salient points of this volume.

Tudorache and Luca [1] establish the existence of positive solutions to a Riemann–Liouville fractional-type differential equation with sequential derivatives, a positive parameter, and a sign-changing singular nonlinearity. The strategy is based on the Guo–Krasnosel'skii fixed-point theorem.

Kanguzhin and Auzerkhan [2] develop a one-dimensional mathematical model of the vibrations of structures consisting of elastic and weakly curved rods. Hence, they use differential equations to model the longitudinal and transverse vibrations of these elastic rods. Suitable conjugation conditions ensure the all-around decidability and symmetry of the problems on a star graph.

Sunday et al. [3] deal with step-size methods for integrating stiff differential systems. Hence, they present a method that can vary the step size within a defined integration interval using a Lagrange interpolation polynomial as a basis function via its integration at selected limits.

Folly-Gbetoul [4] performs a Lie analysis of $(2k+2)$th-order difference equations and obtains $k+1$ non-trivial symmetries. Using these symmetries, the author establishes suitable exact solutions and provides the conditions needed to obtain the convergence of the solutions.

Zuo et al. [5] pose a boundary value problem for biharmonic operators on the Heisenberg group. Hence, they establish the existence of weak solutions to the main problem. The strategy is based on a version of the mountain pass theorem and classical variational tools.

Elsayed et al. [6] develop a qualitative analysis of certain properties of higher-order nonlinear difference equations. Hence, they discuss local and global stability, as well as the boundedness of solutions.

Tudorache and Luca [7] continue the study of positive solutions to fractional-type differential equations (see [1]). This time, they deal with sequential derivatives, a positive parameter, and a non-negative singular nonlinearity, supplemented with integral and multipoint boundary conditions. The strategy is based on the Krein–Rutman theorem, together with suitable fixed-point index theorems.

Kazakov [8] studies a class of nonlinear second-order evolutionary parabolic partial differential equations. Hence, the author establishes the existence of piecewise analytical solutions and obtains exact solutions. The strategy is based on the Cauchy–Kovalevskaya theorem (for the majorant method) and on the Clarkson–Kruskal direct method.

Piotrowska and Sajewski [9] study the problem of describing voltage dynamics in electric circuits, involving appropriate fractional-order elements in the analysis. The authors design a linear system of state equations by using the two-parameter operator conformable derivative in the Caputo sense.

Léandre [10] studies a new approximation, with respect of the parametrix method, of the solution to a parabolic equation whose generator is of a large order and of the Hoermander form. Hence, the author generalizes the approximation of Stratonovitch diffusion processes to a higher-order generator.

Ku et al. [11] investigate some numerical methods for solving partial differential equations. Hence, they design a collocation scheme involving multiquadric radial basis functions, including multiquadric and inverse multiquadric functions, without the shape parameter. The authors separate the center points from the interior points; hence, the center points are regarded as fictitious sources collocated outside the domain.

Jleli et al. [12] study the large time behavior of the inhomogeneous damped wave equation with nonlinear memory in the right-hand side. Under sufficient conditions on the nonhomogeneous term, they establish nonexistence results by using the test function method.

Jleli et al. [13] investigate the nonexistence of global solutions to some classes of fractional in time nonlinear Schrödinger equations under suitable initial data and involving the combined effects of absorption and dispersion terms. Hence, they establish nonexistence results by using specific test and cut-off functions.

Cherniha [14] present a discussion of a Lie symmetry analysis of a certain Burgers–Fitzhugh–Nagumo-type equation. Hence, the author analyzes the problem of finding exact solutions for the main equation.

Yusuf et al. [15] consider the problem of describing quantum systems interacting with the environment. Hence, they obtain the existence of fixed points to certain quantum operations by using an appropriate order-preserving map. The strategy is based on fixed-point arguments for contractive maps in generalized metric spaces.

Alqahtani et al. [16] consider the large time behavior of solutions to certain exterior problems with a Schrödinger operator in the principal part. Hence, they establish nonexistence results in the case of a nonhomogeneous Neumann boundary condition. The strategy is based on the test function method.

Hussain et al. [17] study a spatially two-dimensional Burgers–Huxley equation that depicts the interaction between convective phenomena, nerve proliferation in neurophysics, as well as motion in liquid crystals. Using the Lie symmetry method, the authors establish the group invariant solutions for the main equation. Hence, they calculate some power series solutions, adopting the power series method.

Jabeen et al. [18] obtain sufficient conditions for the existence of mild solutions to certain impulsive evolution differential equations with causal operators in separable Banach spaces. The strategy is based on strongly continuous semigroups theory, together with a concept of measure of noncompactness and the Schauder fixed-point theorem.

Conflicts of Interest: The author declares no conflict of interest.

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