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Stability Analysis of a Mathematical Model for Adolescent Idiopathic Scoliosis from the Perspective of Physical and Health Integration

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Abstract: In this paper, we take physical and health integration as the entry point. Firstly, based on the transformation mechanism of adolescent idiopathic scoliosis we construct a time delay differential model. Moreover, using the theory of characteristic equation we discuss the stability of a positive equilibrium under the delays of $\tau = 0$ and $\tau \neq 0$. Furthermore, through numerical simulation, it has been verified the delay, τ, exceeds a critical value, the positive equilibrium loses its stability and Hopf bifurcation occurs. Lastly, we determine that sports have a positive effect on adolescent idiopathic scoliosis, directly reducing the number of people with adolescent idiopathic scoliosis.

Keywords: adolescent idiopathic scoliosis; physical and health integration; time delay; stability; Hopf bifurcation

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1. Introduction

In the report of the 20th National Congress, President Xi Jinping pointed out the need to "Extensively carry out national fitness activities, strengthen youth sport work, promote the development of mass sport, and accelerate the construction of a sporting country". Adolescent health has drawn top-level national attention. In 2019, an excellent rate for high-quality health standards among students aged 6–22 in China of 23.8% was achieved, and the areas with a highly excellent rate were the economically developed eastern and coastal areas [1]. The rate of abnormal spinal curvatures among middle school students is 2.8%, and this is mainly due to abnormal postural curvature [2]. In 2022, China Central Television News Channel's Chaowen Tianxia program reported that adolescent idiopathic scoliosis is the third-largest disease among adolescents after obesity and myopia. The number of primary and secondary school students with scoliosis in China is expected to exceed 5 million, with an annual increase of 300,000 [3]. Therefore, it is urgent to enhance the physique of adolescents and improve their physical health. How to rehabilitate adolescent idiopathic spinal curvatures has become a new topic for scholars.

Adolescent idiopathic scoliosis is a form of structural scoliosis of the spine, and accounts for over 80% of all scoliosis populations [4]. Adolescent idiopathic scoliosis is a common orthopedic disease, and it affects the alignment, growth and function of the spine, and the physical and mental health of adolescents especially during the growth and development period. If there is not a timely intervention made for adolescent idiopathic scoliosis, with the increase in lateral deformity, it will be accompanied by abnormal posture, lower back pain, decreased cardiovascular function, decreased respiratory function, decreased athletic ability and other functional problems, and then create conditions such as depression, anxiety and other psychological problems [5]. So far, the specific etiology of adolescent idiopathic scoliosis is unclear, and scholars from various academic circles, such as genetics, hormones, growth abnormalities, biomechanics, and neuromuscular theories [6], have conducted extensive research on it and formed many theories but it is recognized somewhat that long durations of abnormal posture in adolescents are a cause. Moreover, Chen [7] believes that high-intensity studying and a lack of sleep are the causes of rising incidence and increased severity of scoliosis among children and adolescents in China, while a lack of participation in sports is the direct cause of an increasing number of scoliosis cases.

The International Society for Scoliosis Research used Cobb's method to measure the angle of the lateral curvature of the spine in X-ray films. X-ray films are taken with the patient in an upright position. Cobb angles exceeding 10° define scoliosis. For the angle of mild adolescent idiopathic scoliosis, there is no exact value. Generally speaking, when the Cobb angle is between 10° and 25°, it indicates mild adolescent idiopathic scoliosis; angles of 25° to 45° indicate to moderate adolescent idiopathic scoliosis; an angle greater than 45° indicates severe adolescent idiopathic scoliosis. In medicine, individuals without scoliosis and those whose Cobb angles are less than 10° are defined as healthy individuals, while moderate adolescent idiopathic scoliosis usually requires the use of corrective braces for treatment, and severe adolescent idiopathic scoliosis requires surgery.

Most of the literature on adolescent idiopathic scoliosis is from the perspective of pathogenesis and biomechanics [8,9]. The theory of differential equation plays a very important role in not only practical applications, but also theoretical research such as that of Slyn'ko and Tunç [10] and Tunç and Tunç [11]; it is commonly used to study infectious diseases such as COVID-19 and HIV [12–14], as well as skin cancer, leukemia and other non-communicable diseases [15,16]. However, there are few researchers who have studied adolescent idiopathic scoliosis by constructing differential models and using stability theory. Physical and health integration is the in-depth combination of two major systems for sports and health, and is the interaction and sharing of resources such as technology, talents, and facilities in the two systems. Compared to physical and medical integration, it has the characteristics of extensive coverage, a diversity of actions and effective results. It is one of the important ways to realize the healthy China strategy. Furthermore, starting from physical and health integration, we investigate the trend of adolescent idiopathic scoliosis and the influence of sports on adolescent idiopathic scoliosis from a mathematical perspective through theoretical analysis and numerical simulation, and provide a new research method for studying adolescent idiopathic scoliosis. In this paper, we refer individuals without scoliosis and those with a Cobb angle less than of 10 degrees as susceptible individuals, those with a Cobb angle between 10 degrees and 45 degrees as affected individuals, and those who require surgical treatment as surgical individuals. Therefore, adolescents are classified into three groups: susceptible individuals, infected individuals and surgical individuals.

This paper is organized as follows: in Section 2, we construct the model of adolescent idiopathic scoliosis; in Section 3, we investigate stability including stability switches of the system (2); in Section 4, applications with numerical simulations are described to illustrate our results; in Section 5, we discuss the effect of sports on adolescent idiopathic scoliosis; in Section 6, we conclude the paper.

2. Formulation of Mathematical Model

Adolescent idiopathic scoliosis, a disease with a persistent deviation of the spine from the midline of the body, making the spine curved or "S"-shaped to the side, is not an infectious disease. Let susceptible individuals be denoted by $h(t)$ and affected individuals be denoted by $c(t)$. For adolescents, from childhood to adolescence, $h(t)$ increases logistically. Under acquired or congenital influence, some adolescents have scoliosis, and the transfer rate from $h(t)$ to $c(t)$ is μ. Adolescent idiopathic scoliosis is a reversible disease; most $c(t)$ can recover with the help of a brace and sports. The recovery rate with a brace is β', and that with sports is α. However, a small number of patients cannot use a brace effectively and do not participate in appropriate sports, so they require surgery after the aggravation of the condition. It takes about 3 months to fully recover after surgery. After recovery, teenagers are aware of the importance of daily sitting posture and sports, both psychologically and physically, which will make them change bad habits and strengthen sports performance, and make them no longer suffer from scoliosis in the adolescent stage. Therefore, the surgical individual is not considered in the model's construction. This means that the conversion process between susceptible individuals and affected individuals is represented by the flowchart provided in Figure 1.

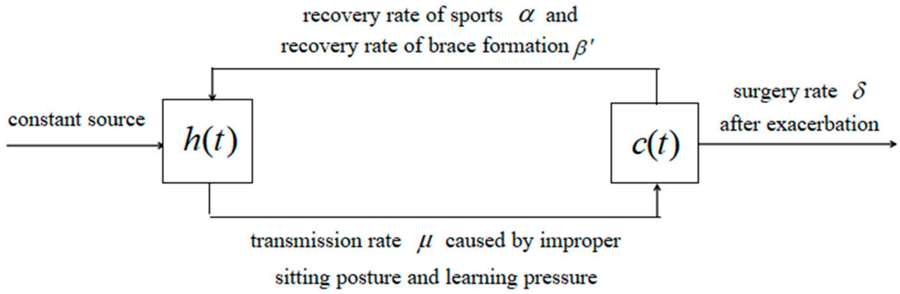

Figure 1. Transformation process between susceptible individuals $h(t)$ and affected individuals $c(t)$.

Furthermore, time delay refers to the phenomenon of delay, and reflects the dependence of the system on both the current and the past state. In the models for many biological processes, time delay is usually introduced to more accurately describe the change in objective things [17–20]. When we construct a model of adolescent idiopathic scoliosis, in order to make our research more practical, we need to consider the impact of time delay. During the onset of adolescent idiopathic scoliosis, the transition period from individuals being susceptible to affected is above 3–6 months, and the recovery period for individuals from being affected to susceptible is above 3–6 months under the combined action of sports and brace use (except for those who need surgical treatment, from affected individuals to surgical individuals). Therefore, we use a time delay, τ, to represent the transition period and the recovery period in this paper.

Based on the above analysis and the transformation mechanism of adolescent idiopathic scoliosis, we obtain the following system,

$$\begin{cases} h'(t) = rh(t)\left(1 - \frac{h(t)}{K}\right) + (\alpha + \beta')c(t-\tau) - \mu h(t-\tau), \\ c'(t) = \mu h(t-\tau) - (\alpha + \beta')c(t-\tau) - \delta c(t), \end{cases} \tag{1}$$

where μ is the transmission rate, α is the recovery rate from sports, β' is the recovery rate from brace use, and δ is the surgery rate. $h(t)$ is assumed to grow logistically with the growth rate, r, and carrying capacity, K. τ is the time delay, and indicates the transmission period and recovery period. All parameters in μ, α, β', δ, r, K are positive.

The initial conditions for the delayed system above take the form of

$$h_0(\theta) = \phi_1(\theta) \geq 0, \;\; c_0(\theta) = \phi_2(\theta) \geq 0, \;\; \theta \in [-\tau, 0], \; h(0) > 0, c(0) > 0,$$

where $\phi = (\phi_1, \phi_2) \in C([-\tau, 0], R^2_+)$, $R^2_+ = \{(h, c) : h \geq 0, c \geq 0\}$, $\|\phi\| = \max\{|\phi(\theta)| : \theta \in [-\tau, 0]\}$, and $|\phi|$ are any norm in R^2_+. As usual, we use the conventional notation $h_t(\theta) = h(t + \theta)$ for $\theta \in [-\tau, 0]$. In this paper, for the biological reasons, we only consider that and $h(\theta) > 0, c(\theta) > 0$ are all continuously differentiable in $-\tau \leq \theta \leq 0$ and $h(0) > 0, c(0) > 0$.

System (1) also can be rewritten as

$$\begin{cases} h'(t) = rh(t)\left(1 - \frac{h(t)}{K}\right) + \beta c(t-\tau) - \mu h(t-\tau), \\ c'(t) = \mu h(t-\tau) - \beta c(t-\tau) - \delta c(t), \end{cases} \tag{2}$$

where $\beta = \alpha + \beta'$ is the recovery rate aided by the combined action of sports and brace use.

The steady states occur by setting the left-hand side of system (2) to zero, i.e.,

$$\begin{cases} rh(t)\left(1-\frac{h(t)}{K}\right)+\beta c(t)-\mu h(t)=0, \\ \mu h(t)-(\beta+\delta)c(t)=0. \end{cases}$$

so we can obtain the following conclusion.

Lemma 1. *We assume that the functions $c(t)$ and $h(t)$ are continuously differentiable. When the condition*

$$\frac{\mu\delta}{r(\beta+\delta)}<1 \qquad (H_0)$$

holds, system (2) has a unique positive equilibrium, $E^*(h^*, c^*)$, where

$$h^*=K\left(1-\frac{\mu\delta}{r(\beta+\delta)}\right),\ c^*=\frac{\mu h^*}{\beta+\delta}.$$

Obviously, we can obtain $h^*<K$ from the expression h^*.

3. Stability Analysis

In this section, we mainly provide the stability of the positive equilibrium and the stability switch of the system (2). Firstly, the characteristic equation of the linearized system (2) is given, and then the stability of the positive equilibrium and stability switch of the system (2) are determined via the sign of the characteristic root.

Let $h(t)=h^*+H(t)$ and $c(t)=c^*+C(t)$. Then, the linearized system (2) is

$$\begin{cases} H'(t)=h^*\left[\left(r-\frac{2rh^*}{K}\right)H(t)-\mu H(t-\tau)+\beta C(t-\tau)\right], \\ C'(t)=c^*[\mu H(t-\tau)-\beta C(t-\tau)-\delta C(t)]. \end{cases} \tag{3}$$

Then, the corresponding Jacobian matrix of system (3) is

$$\begin{pmatrix} r-\frac{2rh^*}{K}-\mu e^{-\lambda\tau} & \beta e^{-\lambda\tau} \\ \mu e^{-\lambda\tau} & -\beta e^{-\lambda\tau}-\delta \end{pmatrix},$$

so the characteristic equation of system (3) is

$$\begin{vmatrix} \lambda-r+\frac{2rh^*}{K}+\mu e^{-\lambda\tau} & -\beta e^{-\lambda\tau} \\ -\mu e^{-\lambda\tau} & \lambda+\beta e^{-\lambda\tau}+\delta \end{vmatrix}=0,$$

that is,

$$\lambda^2+(\beta+\mu)\lambda e^{-\lambda\tau}+\left[\mu\delta+\beta\left(\frac{2rh^*}{K}-r\right)\right]e^{-\lambda\tau}+\left(\delta+\frac{2rh^*}{K}-r\right)\lambda+\delta\left(\frac{2rh^*}{K}-r\right)=0. \tag{4}$$

Let $a_1=\beta+\mu>0$, $a_2=\mu\delta+\beta\left(\frac{2rh^*}{K}-r\right)$, $a_3=\delta+\frac{2rh^*}{K}-r$, $a_4=\delta\left(\frac{2rh^*}{K}-r\right)$. Then, Equation (4) becomes

$$\lambda^2+a_1\lambda e^{-\lambda\tau}+a_2e^{-\lambda\tau}+a_3\lambda+a_4=0. \tag{5}$$

When $\tau=0$, Equation (4) is

$$\lambda^2+(a_1+a_3)\lambda+a_2+a_4=0. \tag{6}$$

Therefore, if

$$a_1+a_3>0,\ a_2+a_4>0 \tag{7}$$

holds, all roots of Equation (6) have negative real parts.

Note 1. *If $h^* > \frac{K}{2}$, then $a_2 > 0$, $a_3 > 0$, $a_4 > 0$, and condition (7) holds. Meanwhile, the condition $h^* > \frac{K}{2}$ is equivalent to*

$$\frac{\mu\delta}{r(\beta+\delta)} < \frac{1}{2}. \qquad (H_1)$$

Obviously, when (H_1) holds, (H_0) holds. That is, there is a positive equilibrium E^*.

Based on the above analysis and Routh Hurwitz criterion [21], we have the following conclusion.

Theorem 1. *If conditions (H_0) and (7) hold, then the positive equilibrium, E^* , of system (2) is asymptotically stable for $\tau = 0$.*

Note 2. *At the same time, when (H_1) holds, then (H_0) and (7) both hold.*

Next, we want to confirm whether or not the real part of the root, λ, for the characteristic equation, Equation (5), will increase to zero and eventually become positive as τ changes. When $\tau \neq 0$, let $\lambda = i\omega(\omega \neq 0$, since $a_2 + a_4 > 0)$; through Equation (5) we can obtain

$$\begin{cases} a_1\omega\sin\omega\tau + a_2\cos\omega\tau = \omega^2 - a_4, \\ a_1\omega\cos\omega\tau - a_2\sin\omega\tau = -a_3\omega. \end{cases} \tag{8}$$

Thus, we have $(a_1\omega)^2 + a_2^2 = (\omega^2 - a_4)^2 + (a_3\omega)^2$, that is

$$\omega^4 + (a_3^2 - a_1^2 - 2a_4)\omega^2 + a_4^2 - a_2^2 = 0, \tag{9}$$

and the root of Equation (9) is

$$\omega_\pm^2 = \frac{(a_1^2 - a_3^2 + 2a_4) \pm \sqrt{(a_1^2 - a_3^2 + 2a_4)^2 - 4(a_4^2 - a_2^2)}}{2}. \tag{10}$$

Therefore, when the condition

$$a_1^2 - a_3^2 + 2a_4 < 0 \text{ and } a_4^2 - a_2^2 > 0 \text{ or } (a_1^2 - a_3^2 + 2a_4)^2 < 4(a_4^2 - a_2^2) \tag{11}$$

holds, $\omega_\pm^2$ values are all not positive, that is, (9) has no positive root, which means (5) has no pure imaginary root. Since Equation (6) has negative real parts, according to Rouche's theorem, the characteristic equation, Equation (5), also has negative real parts for the roots. We can draw the following conclusion.

Theorem 2. *If conditions (7) and (11) hold, all roots of the characteristic equation, Equation (5), have negative real parts for the arbitrary $\tau > 0$.*

Otherwise, when

$$a_4^2 - a_2^2 < 0 \text{ or } \begin{cases} a_1^2 - a_3^2 + 2a_4 > 0 \text{ and} \\ (a_1^2 - a_3^2 + 2a_4)^2 = 4(a_4^2 - a_2^2) \end{cases} \tag{12}$$

holds, Equation (9) has a positive root, ω_+^2. If

$$a_1^2 - a_3^2 + 2a_4 > 0, a_4^2 - a_2^2 > 0 \text{ and } (a_1^2 - a_3^2 + 2a_4)^2 > 4(a_4^2 - a_2^2) \tag{13}$$

holds, Equation (9) has two positive roots $\omega_\pm^2$. If the conditions (12) and (13) hold, Equation (5) has pure imaginary root when τ reaches a certain value. The critical value $\tau_j^\pm$ of τ determined by (8) is

$$\tau_j^\pm = \frac{1}{\omega_\pm}\arccos\left\{\frac{(a_2 - a_1a_3)\omega_\pm^2 - a_2a_4}{a_1^2\omega_\pm^2 + a_2^2}\right\} + \frac{2j\pi}{\omega_\pm}, j = 0, 1, 2, \ldots. \tag{14}$$

From the above analysis, we can draw the following conclusion.

Lemma 2. *(1) When (7) and (12) hold and* $\tau = \tau_j^+$ *, (5) has a pair of pure roots,* $\pm i\omega_+$*. (2) When (7) and (13) hold and* $\tau = \tau_j^+$ $(\tau = \tau_j^-)$*, (5) has a pair of pure roots,* $\pm i\omega_+$ $(\pm i\omega_-)$*.*

When $\tau > \tau_j^+$ and $\tau < \tau_j^+$, we would expect that the real parts of certain characteristic roots of (5) become positive. For convenience, we denote $\lambda_j^\pm = \alpha_j^\pm(\tau) + i\omega_j^\pm(\tau)$, $j = 0, 1, 2, \dots$; meanwhile, the roots of (5) satisfy $\alpha_j^\pm(\tau_j^\pm) = 0$ and $\omega_j^\pm(\tau_j^\pm) = \omega_\pm$. We can verify that the transversality conditions $\frac{d}{d\tau}\mathrm{Re}\lambda_j^+(\tau_j^+) > 0$ and $\frac{d}{d\tau}\mathrm{Re}\lambda_j^-(\tau_j^-) < 0$ hold. It follows that $\tau_j^\pm$ represents bifurcation values. Therefore, we have the following theorem on the distribution of eigenvalues.

Theorem 3. *Let* $\tau_j^\pm$ *be as defined in (14).*

(1) *If (7) and (12) hold, all roots of Equation (5) have negative real parts for* $\tau \geq 0$.
(2) *If (7) and (12) hold, all roots of Equation (5) have negative real parts for* $\tau \in [0, \tau_0^+)$ *; Equation (5) has a pair of pure roots,* $\pm i\omega_+$ *, for* $\tau = \tau_0^+$ *; Equation (5) has at least one root with a positive real part for* $\tau > \tau_0^+$.
(3) *If (7) and (13) hold, then there exists a positive integer, k , such that there are k switches from stability to unstability to stability. That is, when*

$$\tau \in [0, \tau_0^+],\ (\tau_0^-, \tau_1^+), \dots,\ (\tau_{k-1}^-, \tau_k^+),$$

all roots of Equation (5) have negative real parts; when

$$\tau \in [\tau_0^+, \tau_0^-),\ [\tau_1^+, \tau_1^-), \dots,\ [\tau_{k-1}^+, \tau_{k-1}^-) \text{ and } \tau > \tau_k^+,$$

Equation (5) has at least one root with a positive real part.

Note 3. *The case (3) of Theorem 3 shows that when the time delay,* τ *, exceeds the critical value,* $\tau_j^+, j = 0, 1, 2, \dots, k-1$ *, the positive equilibrium,* E^* *, of system (2) loses stability, and system (2) exists as a Hopf bifurcation [22].*

4. Numerical Simulation

In this part, we use numerical simulation to show the interesting dynamical behavior of system (2). From the following examples, it can be seen that with the change in the time delay, τ, the stability of the positive equilibrium of system (2) will also change, and there is a Hopf bifurcation, which is also a numerical verification of the conclusion in the second part.

Application 1. *Let* $r = 0.02$, $K = 10^8$, $\beta = 0.7$, $\delta = 0.3$, $\mu = 0.02$ *; then, system (2) becomes*

$$\begin{cases} h'(t) = 0.02h(t)\left(1 - 10^{-8}h(t)\right) + 0.7c(t) - 0.02h(t), \\ c'(t) = 0.02h(t) - 0.7c(t) - 0.3c(t). \end{cases} \tag{15}$$

Via simple numerical computation, the positive equilibrium of system (15) is $E^* = (7 \times 10^7, 1.4 \times 10^6)$ *, and* $a_1 = 0.7200, a_2 = 0.0116, a_3 = 0.3080, a_4 = 0.0024, \frac{\mu\delta}{r(\beta+\delta)} = 0.3 < \frac{1}{2}$. *Therefore, the condition* (H_1) *holds; that is,* (H_0) *and (7) are satisfied, and then Theorem 1 holds.*

From Figure 2, it can be seen that $(h(t), c(t))$ tends towards $(h^*, c^*) = (7 \times 10^7, 1.4 \times 10^6)$ as $t \to +\infty$ for $\tau = 0$. That is, the positive equilibrium, E^*, of system (2) is asymptotically stable for $\tau = 0$. Meanwhile $a_1^2 - a_3^2 + 2a_4 = 0.4283, a_4^2 - a_2^2 = -1.288 \times 10^{-4}$, and $(a_1^2 - a_3^2 + 2a_4)^2 - 4(a_4^2 - a_2^2) = 0.1840$, so condition (12) holds. Via (10) and (14), we obtain $\tau_0^+ = 3.0366$.

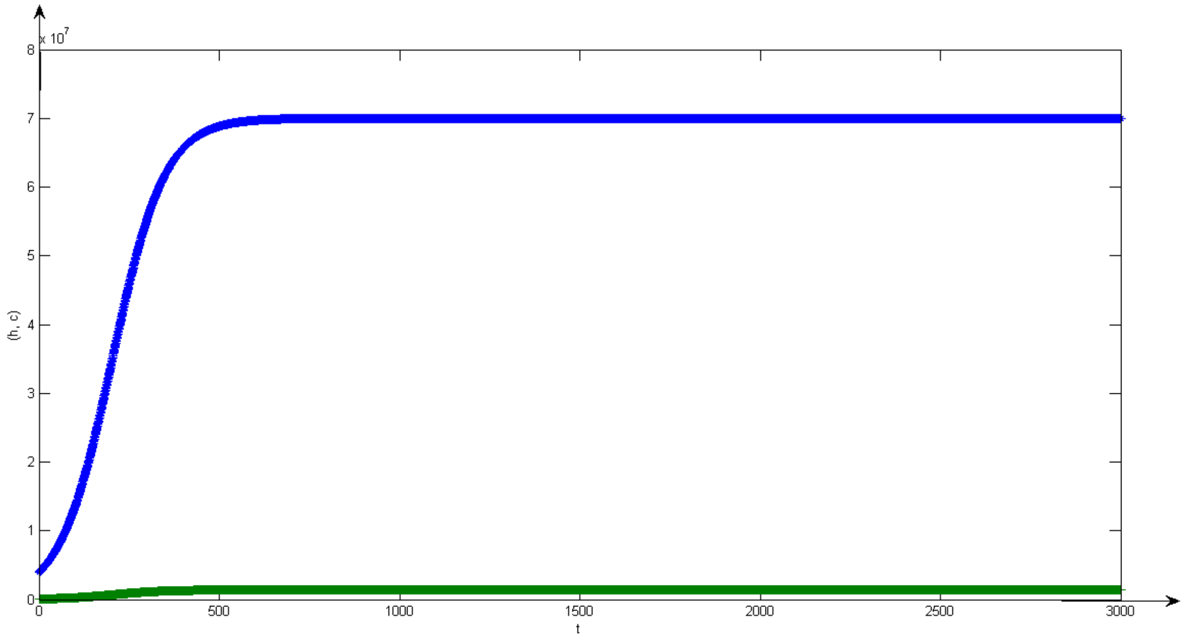

Figure 2. The curves of system (15), where the blue and green curves represent $h(t)$ and $c(t)$, respectively.

Application 2. *Based on Application 1, when* $\tau = 3$ *, the corresponding system is*

$$\begin{cases} h'(t) = 0.02h(t)\big(1 - 10^{-8}h(t)\big) + 0.7c(t-3) - 0.02h(t-3), \\ c'(t) = 0.02h(t-3) - 0.7c(t-3) - 0.3c(t). \end{cases} \tag{16}$$

where $\tau = 3 < \tau_0^+$ *, the curves of* t *and* $h(t)$ *,* t *and* $c(t)$ *are shown in* Figure 3.

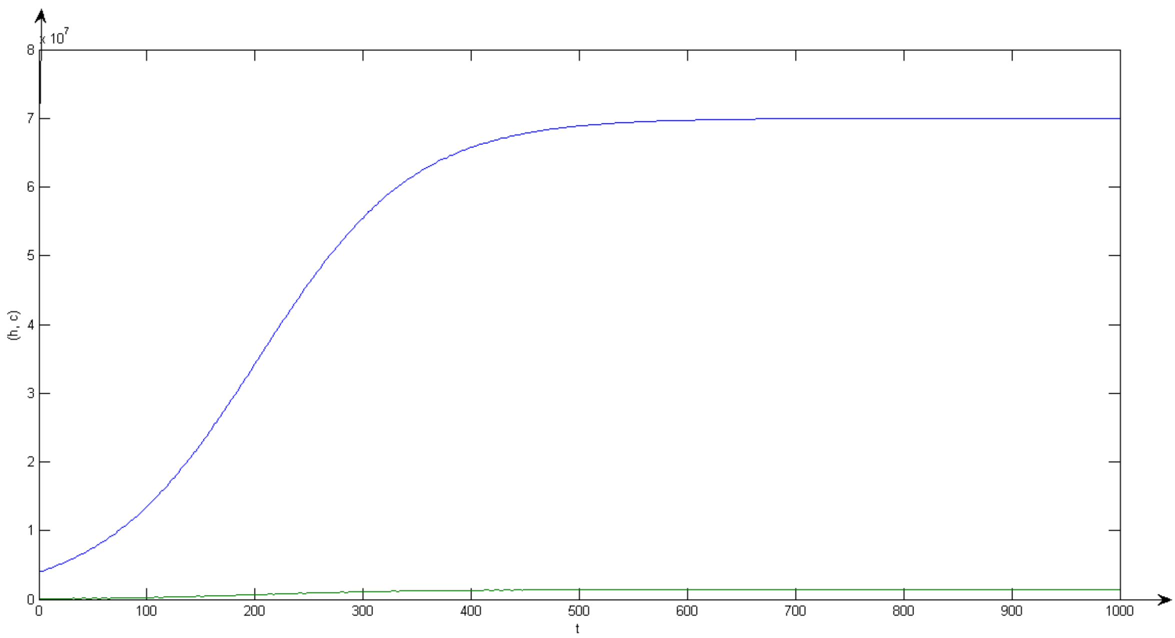

Figure 3. The curves of t and $h(t)$, and t and $c(t)$ in system (16) for $\tau = 3$, where the blue and green curves respresent $h(t)$ and $c(t)$, respectively.

From Figure 3, it can be seen that $(h(t), c(t))$ still tends towards $(h^*, c^*) = (7 \times 10^7, 1.4 \times 10^6)$ as $t \to +\infty$ for $\tau = 3$, indicating that the positive equilibrium, E^*, of system (2) is still asymptotically stable.

Application 3. *Based on Application 1, when* $\tau = 3.1$ *, the corresponding system is*

$$\begin{cases} h'(t) = 0.02h(t)\left(1 - 10^{-8}h(t)\right) + 0.7c(t-3.1) - 0.02h(t-3.1), \\ c'(t) = 0.02h(t-3.1) - 0.7c(t-3.1) - 0.3c(t). \end{cases} \quad (17)$$

When $\tau = 3.1 > \tau_0^+$ *the curves of* t *and* $h(t)$ *, and* t *and* $c(t)$ *are shown in* Figure 4.

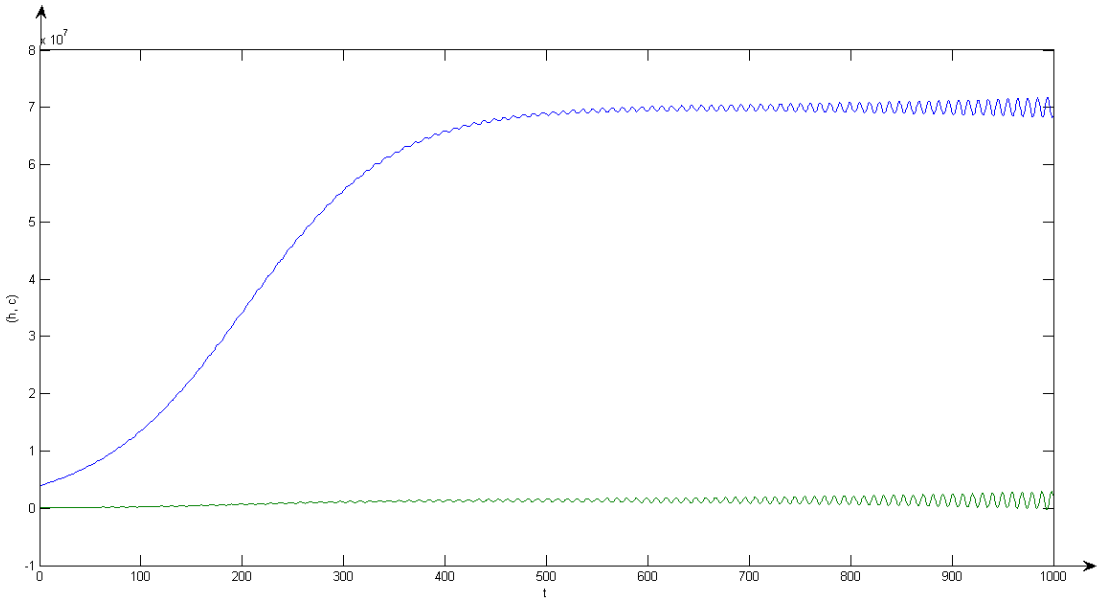

Figure 4. The curves of t and $h(t)$, and t and $c(t)$ in system (17) with $\tau = 3.1$ and $t = 1000$, where the blue and green curves respresent $h(t)$ and $c(t)$, respectively.

Moreover, the orbits of $h(t)$ and $c(t)$ under the same conditions are in Figure 5. Further, we extend t to 2000 and obtain the orbits of $h(t)$ and $c(t)$, as shown in in Figure 6.

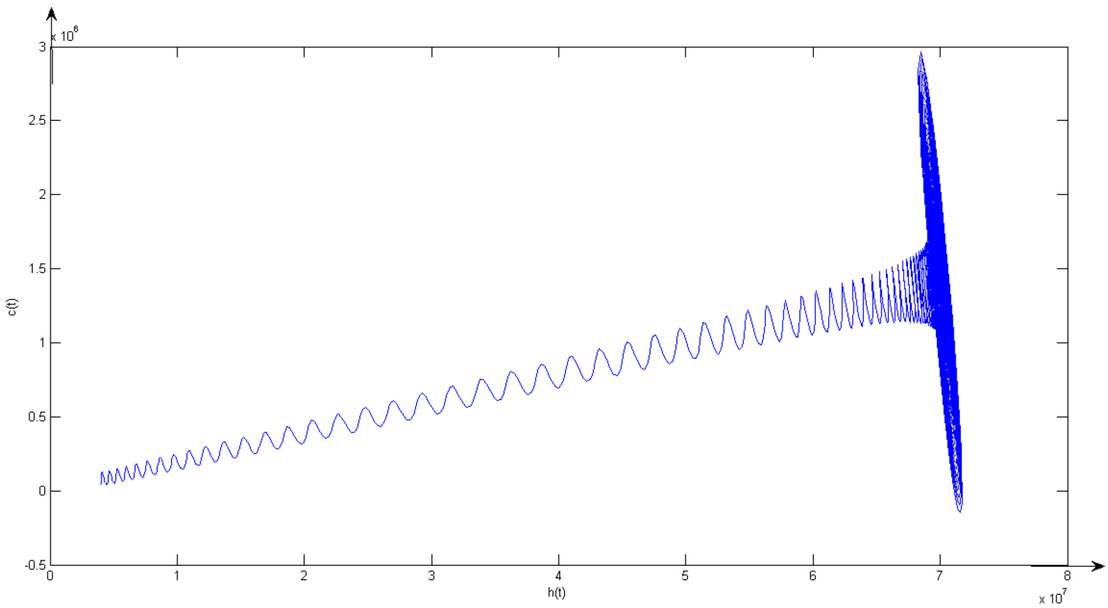

Figure 5. Orbits of $h(t)$ and $c(t)$ for system (17) with $\tau = 3.1$ and $t = 1000$.

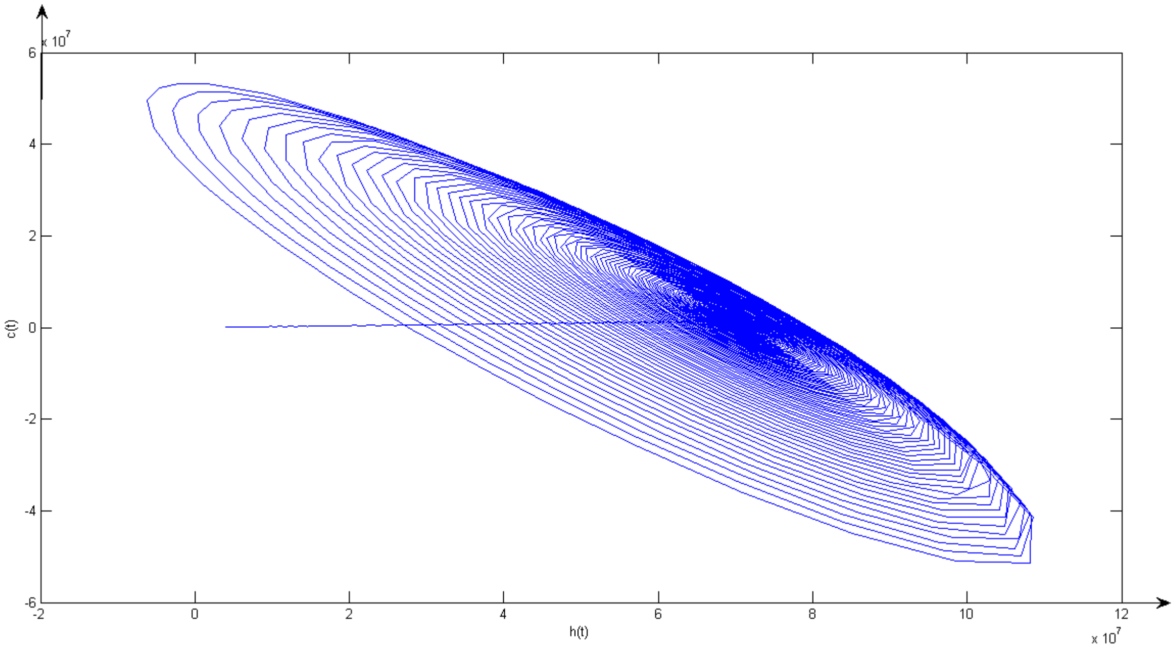

Figure 6. Orbits of $h(t)$ and $c(t)$ for system (17) with $\tau = 3.1$ and $t = 2000$.

From Applications 1–3, it can be seen that the positive equilibrium, E^*, is stable for $\tau \in [0, \tau_0^+]$, and the positive equilibrium, E^*, is unstable for $\tau > \tau_0^+$, which verifies the conclusion of Theorem 3(3). Therefore, τ_0^+ is a critical value, and Hopf bifurcation occurs at $\tau = \tau_0^+$.

5. Discussion

The bifurcation problem is an important problem in the study of dynamical systems and nonlinear differential equations, and its research objects are structurally unstable systems. The bifurcation phenomenon refers to the sudden changes in certain properties of the system (such as the equilibrium state, periodic phenomena, stability, etc.) when parameters change and exceed certain critical values in a parameter-dependent system. In this paper, time delay, τ, is not only the transition period from individuals being susceptible to affected but also the recovery period from individuals being affected to susceptible under the combined action of sports and brace use. In Applications 1–3, when $\tau \in [0, \tau_0^+]$ the positive equilibrium, E^*, is stable, which means that the number of susceptible individuals $h(t) \to 7 \times 10^7$ as $t \to +\infty$, and the number of affected individuals $c(t) \to 1.4 \times 10^6$ as $t \to +\infty$. When $\tau > \tau_0^+$, the positive equilibrium, E^*, is unstable, which means that $h(t)$ and $c(t)$ do not tend toward $(7 \times 10^7, 1.4 \times 10^6)$ for a large $t > 0$. The transmission period and recovery period are 3–6 months, which is basically greater than 3.0366, and E^* is unstable, which shows sports have a positive effect on adolescent idiopathic scoliosis. Therefore, with the help of sports and brace use, the number of affected individuals is not fixed at a large value, but rather far away from this large fixed value.

In system (1), the recovery rate is denoted by $\beta = \alpha + \beta'$, where α is the recovery rate of movement, β' is the recovery rate from brace use; since the recovery rate from brace use is relatively fixed, we mainly refer to the recovery rate from sports when we discuss recovery rate in this article. Next, in order to observe the impact of β on stability, the parameters r, K, β, δ, μ in Applications 1–3 remain unchanged, but the value of β is changed.

We consider the case of $\beta < 0.7$. Assuming $\beta = 0.6$, we determine that $\tau_0^+ = 3.7916$, so $E^*(6.6667 \times 10^7, 1.4815 \times 10^6)$ is stable for $\tau \in [0, \tau_0^+]$, and $E^*(6.6667 \times 10^7, 1.4815 \times 10^6)$ is unstable for $\tau > \tau_0^+$; with $\beta = 0.5$, $\tau_0^+ = 5.0772$, $E^*(6.2500 \times 10^7, 1.5625 \times 10^6)$ is stable for $\tau \in [0, 5.0772]$, and $E^*(6.2500 \times 10^7, 1.5625 \times 10^6)$ is unstable for $\tau > 5.0772$. It is easy to see that as the parameter β decreases and τ_0^+ becomes larger, the chance of $\tau > \tau_0^+$ decreases.

However, when $\beta > 0.7$, let $\beta = 0.8$ and $\tau_0^+ = 2.5364$; $E^*(7.2727 \times 10^7, 1.3223 \times 10^6)$ is stable for $\tau \in [0, 2.5364]$, and $E^*(7.2727 \times 10^7, 1.3223 \times 10^6)$ is unstable for $\tau > 2.5364$; let $\beta = 0.85$ and $\tau_0^+ = 2.3442$; $E^*(7.3913 \times 10^7, 1.2854 \times 10^6)$ is stable for $\tau \in [0, 2.3442]$, and $E^*(7.3913 \times 10^7, 1.2854 \times 10^6)$ is unstable for $\tau > 2.3442$. It is easy to see that as the parameter β increases and τ_0^+ becomes smaller, the chance of $\tau > \tau_0^+$ decreases. That is, participating in an adolescent idiopathic scoliosis intervention process through sports can improve the overall condition of adolescent idiopathic scoliosis.

Moreover, it also can be found that the positive equilibrium $E^*(h^*, c^*)$ of system (2) changes with the continuous increase in parameter β during the change process of parameter β above; the number of susceptible individuals, $h(t)$, continuously increases from $6.2500 \times 10^7 (\beta = 0.5)$ to $7.3913 \times 10^7 (\beta = 0.85)$; the number of infected individuals $c(t)$ continuously decreases from $1.5625 \times 10^6 (\beta = 0.5)$ to $1.2854 \times 10^6 (\beta = 0.85)$. This shows that participation in sports directly affects the prevalence of adolescent idiopathic scoliosis. However, participation in sports here requires scientific sports prescriptions as a medium, such as corrective exercises, asymmetrical crawling and breathing training and other methods. In the specific intervention implementation process, it is also necessary to combine scientific training with the patient's bending degree, the comfort of the support, and the patient's physical fitness.

6. Conclusions

In this paper, we have investigated the stability of an adolescent idiopathic scoliosis system from the perspective of physical and health integration. We started with the generation mechanism of adolescent idiopathic scoliosis, and obtained a differential system for the transformation of adolescent idiopathic scoliosis. Afterwards, we qualitatively analyzed the stability of the positive equilibrium and the stability switches of the system with respect to the delay parameter. Moreover, it was also interesting to investigate the stability switches and Hopf bifurcation with respect to the delay parameter via numerical simulations. Lastly, we discussed the effect of sports on adolescent idiopathic scoliosis.

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