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Manipulating Time Series Irreversibility Through Continuous Ordinal Patterns

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Abstract: Time irreversibility, i.e., the lack of invariance of a system under the operation of time reversal, has long attracted the attention of the statistical physics community, and has been shown to be a relevant marker of altered dynamics in many real-world problems. Here, I introduce and analyse the complementary problem of its manipulation. In other words, I ask whether, given a time series, it can be manipulated to achieve desired irreversibility while maintaining its original dynamics. I show how this problem can be tackled using Continuous Ordinal Patterns, a non-linear transformation of a time series based on the local structure created by neighbouring values. I further illustrate the relevance of this problem in the context of brain dynamics, determining that schizophrenic patients and control subjects are characterised by different "distances to irreversibility". Finally, I discuss some open questions, including the meaning of such manipulation from both theoretical and applied viewpoints.

Keywords: time series; time irreversibility; continuous ordinal patterns; schizophrenia; electroencephalography

1. Introduction

Time irreversibility, i.e., the invariance of a system or a time series under the operation of time reversal, has long attracted the attention of the statistical physics community [1]; more recently, efforts have been devoted to its numerical quantification in real-world and experimental time series [2]. Consider a system for which we can observe its dynamics; if a given property of the system is not the same in the normal (i.e., as observed forward in time) and time-reversed (i.e., observed backward in time) evolutions, we can then conclude that such a system breaks time symmetry and is irreversible. Real-world experience provides numerous examples of such time irreversibility: from ice cubes melting in a glass, to the mixing of initially separated gasses and liquids. Examples of time-reversible systems are also easy to find, e.g., in the movement of an ideal pendulum. The interest in this concept is motivated by the fact that time irreversibility reflects, at a macroscopic scale, important mechanisms driving the dynamics of a system at a microscopic level. To illustrate, these include non-linear dynamics, (linear or non-linear) non-Gaussian dynamics [3], the presence of dissipative forces (i.e., memory) [4], and the departure from equilibrium [5]. In other words, the observation of changes in the irreversibility of a system can indicate changes in the dynamics of the system itself, which may otherwise not be accessible to the researcher.

In the last two decades, many alternatives have been proposed to quantify irreversibility starting from real-world time series [2]. Note that this problem is not trivial, as, firstly, the definition of irreversibility is neutral with respect to the type of property that has to be evaluated; and, secondly, the extracted time series may represent only a part of the dynamics of the system—see Ref. [2] for a detailed discussion. A complementary problem that (to the best of our knowledge) has hitherto not been tackled is the manipulation of the irreversibility. In other words, given a time series representing the dynamics of a system,

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the aim is to construct a modified version of the same time series with higher (or lower) irreversibility. The underlying hypothesis is a simple one: by analysing the most efficient ways of manipulating irreversibility, one should be able to deduce which parts of the dynamics are driving it. Note that this idea, while being new in this context, is not new in general; for instance, a similar hypothesis buttresses tests on surrogate data, in which the objective is to test and rule out specific explanations of an observed result [6].

The problem of manipulating time irreversibility has trivial solutions. For instance, irreversibility is always reduced by noise [7], e.g., in observational cases, and can be increased by introducing memory in the signal [4]. Taking this to the extreme, a random permutation of any time series is reversible by construction, as any temporal structure is deleted; yet, such a result is trivial and of no use. Here, I thus add an additional restriction—i.e., I aim to construct time series of altered irreversibility that are maximally similar to the original one. This creates a trade-off between how much the time series have to be modified, compared to how much their irreversibility is manipulated.

While numerous ways of performing such manipulation can be envisioned, I here propose to do this by applying the recently proposed concept of Continuous Ordinal Patterns (COPs) [8]. These are a generalisation of the traditional ordinal pattern approach [9], in which not only the relative order of data points but also their magnitudes are taken into account, which have proven to be relevant in characterising time series, assessing irreversibility [8], and improving causality tests [10]. I propose a simple manipulation process in which, given a random COP, the time series is transformed according to it, and the result is discarded if it does not fulfill the required level of irreversibility. This process is repeated a large number of times in order to finally obtain the manipulated time series that has the highest linear correlation with the original one. COPs present several advantages in the context of the problem at hand. Firstly, as opposed to traditional ordinal patterns, COPs can be modified; hence, the result is not only time series-dependent but also pattern-dependent. Secondly, the application of a COP is akin to a non-linear transformation, and is thus expected to increase the time irreversibility; yet, as will be shown, it can also effectively be used to reduce it. Finally, COPs are very efficient from a computational point of view, allowing the fast analysis of large sets of time series.

After introducing the concept of COP (Section 2.1) and the numerical tests used to assess time irreversibility (Section 2.2), I show how this property can easily be manipulated, both towards higher (Section 3.2) and lower (Section 3.3) values, while retaining a high similarity with the original time series. This is illustrated using time series of known irreversibility, generated by well-known chaotic maps and dynamical systems. I further illustrate how this approach can be used to probe the dynamics of real-world systems. Specifically, I show in Section 4 how time series representing the brain activity of schizophrenic patients and matched control subjects initially have the same irreversibility, but that they yield significantly different results when manipulated. In other words, this pathology is modifying the normal brain dynamics by affecting its "distance to irreversibility". I conclude with a discussion of the importance of these results in Section 5, and further suggest future lines of research.

2. Methods

2.1. Continuous Ordinal Patterns

The manipulation of time series is here performed using the concept of Continuous Ordinal Patterns (COPs), a tool recently introduced to analyse time series [8] that is based on a variation on the celebrated ordinal pattern analysis proposed in 2002 by C. Bandt and B. Pompe [9].

Classical ordinal patterns are based on the idea of dividing the time series under analysis in short sub-windows, and on representing these through the permutation required to sort their values [11,12]. While being conceptually similar, COPs flip the analysis by calculating the distance of each sub-window to a given reference pattern, i.e., the COP. To illustrate, let us consider a time series X composed of n values $X = (x_1, x_2, \ldots, x_n)$. As in the case of ordinal pattern analysis, we are here going to consider sub-windows of it of size D, here fixed to 4. A COP is any set of values $\pi = (\pi_0, \pi_1, \ldots, \pi_{D-1})$ normalised in the range $[-1, 1]$. With this, we can then define a distance $\phi_\pi(t)$, assessing how well the COP represents the evolution of the data in the sub-window:

$$\phi_\pi(t) = \frac{1}{2D} \sum_{i=0}^{D-1} d_i = \frac{1}{2D} \sum_{i=0}^{D-1} |\pi_i - s_i^*|. \tag{1}$$

Note that s^* here represents a sub-window of the original data starting at time t, normalised in amplitude in the range $[-1, 1]$. $\phi_\pi(t) = 0$ implies that s^* and π are exactly equal, or, in other words, that the pattern π is a perfect representation of the dynamics within the sub-window s^*. Larger values of $\phi_\pi(t)$, on the other hand, imply that s^* and π are depicting different dynamics.

In summary, given a pattern π, the original time series can be transformed into a new one given by $\phi_\pi(t)$, representing the distance between the original time series and the COP. The transformation is non-linear in nature, and is hence expected to increase the irreversibility of the time series; yet, as will be shown below, this is not always the case.

2.2. Tests for Time Irreversibility

Given that irreversibility is defined as the presence of a property that is not preserved under a time-reversal operation, it is a vague concept; in other words, the definition does not specify which property must be quantified. Due to this, the scientific community has designed multiple independent tests to assess different aspects of time irreversibility, describing different characteristics in the data, and consequently yielding heterogeneous (and on occasion, contradictory) results. In order to account for such heterogeneity, I here consider four different tests that have been shown to perform well in a wide range of conditions, and briefly describe them below. The interested reader can refer to Ref. [2] for further details. In all cases, the software implementation corresponds to the library included in the previous review, which is freely available at (Irreversibility Tests Library at https://gitlab.com/MZanin/irreversibilitytestslibrary (accessed on 7 November 2024)).

- *BDS Statistics*. This test was originally proposed by Brock, Dechert, and Scheinkman, and was aimed at detecting the presence of low-dimensional chaos in real-world data—most prominently in economics and finance [13–15]. Only later, it was found that it also detects irreversibility, and has since been one of the fundamental tests in this context. Given a time series $x(t)$ composed of n observations, the BDS statistic is defined as

$$w_d(r, n) = \sqrt{n} \frac{C_m(r, n) - C_1(r, n)^m}{\sigma_m(r, n)}. \tag{2}$$

  Here, $C_m(r, n)$ is the sample correlation integral, i.e., a measure of the probability that two randomly selected points in the time series are within a certain distance, for an embedding dimension m and scaling parameter r. Additionally, $\sigma_m(r, n)$ is the estimated standard deviation of the statistic under the null hypothesis of independence. When values are independent and the time series is reversible, w_d is distributed according to a normal distribution of zero mean and unitary standard deviation. We here consider the standard value $m = 2$.
- *Visibility graphs*. This family of time series analysis tools are based on representing data as complex networks [16], i.e., mathematical objects composed of nodes (here, corresponding to individual data points), which are connected pairwise when the corresponding data values fulfil some geometrical rule [17]. Among this family, of relevance are directed Horizontal Visibility Graphs (dHVGs), in which connections between nodes (i.e., between pairs of time series values) are created if the line connecting both values is not obstructed by another intermediate point; or, in other words, if these values can "see" each other [18]. A time series is then accepted as irreversible if

the distributions of the number of links arriving at and departing from nodes (known respectively as the in- and out-degrees) are different in a statistically significant way, e.g., according to an Epps–Singleton test [19].

- *Permutation patterns tests*. These tests were independently proposed by several groups [20–22], and are based on representing the time series as sequences of permutation patterns, as previously described [9]. A time series is considered irreversible whenever the distributions of these patterns obtained in the original time series, and the backward version thereof, are different. I here use an embedding dimension of $D = 4$, and compare the frequency appearance of time-symmetric patterns using a binomial test [20].
- *Costa index*. This test was initially proposed to assess the irreversibility of time series representing heartbeat dynamics [23], but was later used in other contexts. It is based on the estimation of the number of times the time series increases or decreases, that is, $x(t) > x(t+1)$ vs. $x(t) < x(t+1)$. Note that, when looking at the dynamics backward in time, each instance of one type transforms into an instance of the other. Hence, irreversibility is defined as a statistically significant difference in the number of those instances, which can then be compared with that obtained for randomly shuffled time series.

3. Results

3.1. Manipulating Time Series' Irreversibility Through COP

I start this analysis by posing a basic question: supposing an initially time-reversible time series, does the application of a COP change its reversible nature? For this, I consider a time series $x(t)$, and the corresponding transformation $\phi_\pi(t)$ obtained by applying a random COP π to it. For the sake of simplicity, in what follows, I will assume that both time series are composed of n data points; note that in reality, $\phi_\pi(t)$ is shorter than $x(t)$ by construction, but that this can easily be solved by considering the first n values of the latter. We further assume that $x(t)$ is composed of independent random values, here drawn from a uniform distribution $\mathcal{U}(0,1)$, such that it is time-reversible by construction. We then calculate the difference in the irreversibility of $x(t)$ and $\phi_\pi(t)$, as measured by the p-value yielded by the four tests considered here when applied to both time series.

Figure 1 reports the probability distributions of the $\log_{10}$ of the p-value, for $x(t)$ (grey bars) and $\phi_\pi(t)$ (orange bars), and for the four irreversibility tests. It can be appreciated that the p-value is in general smaller, most drastically in the case of the BDS test. This is further confirmed by the insets, depicting scatter plots of the $\log_{10}$ of the p-value of the transformed time series, as a function of the original one; and by two-sample Kolmogorov–Smirnov tests between the pairs of distributions, see p-values inside each panel. In sum, it can be appreciated that the application of a random COP π generally results in an increase in the irreversibility of the time series, as initially hypothesised and in line with the non-linear nature of the transformation. This is nevertheless not always true: due to the stochastic nature of π, situations are observed in which the irreversibility is not changed, or is even reduced.

In order to obtain homogeneous results, I resort to the strategy proposed in refs. [8,10], involving the generation of a large set of random COPs π. For each one of them, the previous process is repeated—i.e., the time series is transformed into $\phi_\pi(t)$ and the irreversibility test is applied to it; finally, the transformation yielding the lowest p-value (i.e., the highest irreversibility) is retained. The left panel of Figure 2 reports the evolution of the median of the obtained p-values, as a function of the number of generated random patterns. It can be appreciated that 20 random patterns are enough to obtain time series that are irreversible in a statistically significant way (i.e., p-value < 0.01) in the majority of cases. It is also interesting to observe the wide difference between the BDS test on the one hand, and all other tests on the other; this is due to the fact that the former not only assesses irreversibility, but is rather designed to detect the presence of non-linearities in general. I finally evaluate the role of the time series length, obtaining that longer time series are associated with slightly smaller p-values; see the right panel of Figure 2.

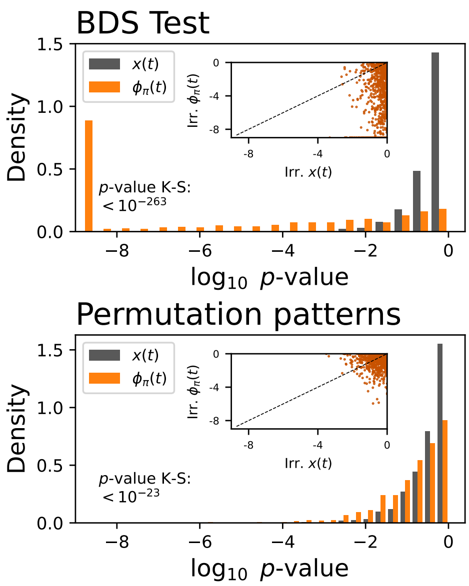

Figure 1. Comparison of the probability distributions of the $\log_{10}$ of the p-values, for, respectively, the original time series $x(t)$ (grey bars), and the transformation $\phi_\pi(t)$ (orange bars). Each panel reports the results for an irreversibility test, as described in Section 2.2. The insets further depict scatter plots of the $\log_{10}$ of the p-values of $\phi_\pi(t)$, as a function of those of $x(t)$; for reference, the dashed grey line represents the main diagonal. Finally, the text in each panel reports the p-value of a two-sample K-S test between the two probability distributions. Results correspond to time series of length 10^3.

The results presented in Figures 1 and 2 thus confirm that, as initially hypothesised, the application of COPs can increase the irreversibility of a time series—and make it more non-linear, as suggested by the different behaviour of the BDS test compared to the remainder ones. This increase is generally weak and further stochastic in nature; yet, stable results can be obtained by performing the same process multiple times.

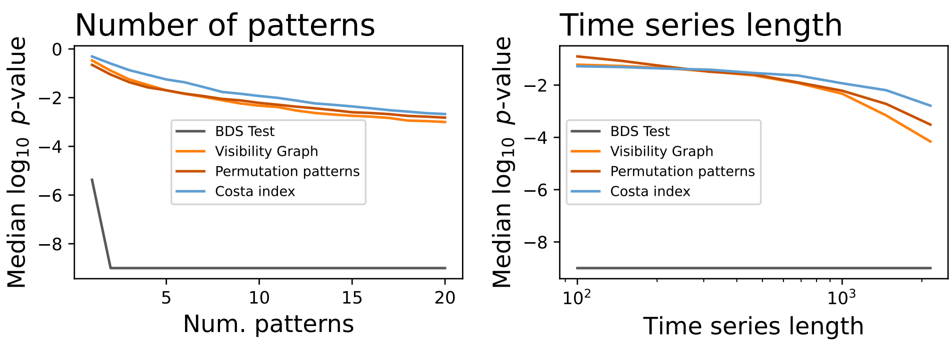

Figure 2. Using multiple random patterns. (**Left**) Evolution of the median $\log_{10}$ of the p-values obtained by each test (see legend for colour codes) on the transformed time series $\phi_\pi(t)$, as a function of the number of random patterns π considered, and for time series of length 10^3. (**Right**) Evolution of the same metric as a function of the time series length, when 10 random patterns π are used.

3.2. Increasing Irreversibility with Minimal Changes

Can the previous approach be used to increase the irreversibility of time series without substantially changing their dynamics? In order to tackle this question, I have modified the previous approach to include an assessment of the similarity between $\phi_\pi(t)$ and $x(t)$.

In detail, we firstly create a large set of random COPs (here 10^3); each one of them is applied to the original time series, and discarded whenever the p-value yielded by the irreversibility test is above a given threshold (here, 10^{-3}). We next calculate the linear correlations between $x(t)$ and the time series $\phi_\pi(t)$ created in the previous step; the pattern π for which such correlation is maximal in absolute value is finally selected as the best one. Note that this guarantees that the transformed time series $\phi_\pi(t)$ is both irreversible (with a p-value $< 10^{-3}$), and at the same time maximally similar to, $x(t)$. In order to include more flexibility in this process, we further create a new time series $y(t)$ by linearly combining the original and the transformed time series, i.e., $y(t) = (1-\alpha)x(t) + \alpha\phi_\pi(t)$, with $\alpha \in [0,1]$. By varying the mixing parameter α between 0 and 1, $y(t)$ morphs between the two time series; this can be used to balance the output between time series very similar to the original one (with $\alpha \to 0$), or that are maximally time-irreversible (with $\alpha \to 1$).

Figure 3 reports the evolution of the correlation coefficient ρ between $x(t)$ and $y(t)$, and of the median of the $\log_{10}$ of the p-value of the irreversibility test applied to $y(t)$, as a function of the mixing α. It can be appreciated that, as α increases, the resulting time series are more irreversible but also less similar to the original one—as expected by construction. Still, in all cases, it is possible to find a value of α for which the p-value is below 0.01 (i.e., the time series can be considered irreversible) while still displaying a $\rho > 0.8$.

The results of Figure 3 were obtained starting from time series $x(t)$ comprising random values drawn from a uniform distribution $\mathcal{U}(0,1)$. In order to check their generalisability, Figure 4 reports the two same metrics, for $\alpha = 1.0$, for the following:

- $\mathcal{U}$: The same uniform distribution $\mathcal{U}(0,1)$, included as a reference.
- $\mathcal{N}$: Random values drawn from a normal distribution $\mathcal{N}(0,1)$.
- *Arnold*: Time series corresponding to the x variable of an Arnold map, a chaotic and conservative (hence, reversible) map, defined as

$$x(t+1) = [x(t) + y(t)]mod(1), \tag{3}$$

$$y(t+1) = [x(t) + 2y(t)]mod(1). \tag{4}$$

- *O.-U.*: Ornstein–Uhlenbeck process, i.e., mean-reverting linear Gaussian process [3].
- *GBM*: Geometric Brownian motion described by the following Langevin equation [24]:

$$dx(t) = (1 - Z_t)x(t)[\delta dt + \sigma dW] + Z_t[x_0 - x(t)], \tag{5}$$

  where dW is the Wiener increment of the zero-mean $\langle dW \rangle = 0$ and correlation function $\langle dW_t dW_s \rangle = \delta(t-s)dt$, and δ the drift amplitude. By setting $\delta = 0$, the resulting time series is stationary and reversible [25].

As can be seen in Figure 4, all cases behave mostly similarly; the notable exception is the GBM process, yielding a very low correlation coefficient (always below 0.1, not reported for the sake of clarity). In other words, starting from time series that are time-reversible, the final result of the process can be different. Taking into account that the optimisation process is the same in all cases, this indicates that different time series are differently predisposed to become irreversible, or that the efficiency of the process changes depending on the nature of the analysed time series—in other words, that a "distance to irreversibility" can be defined. This will further be used in Section 4.

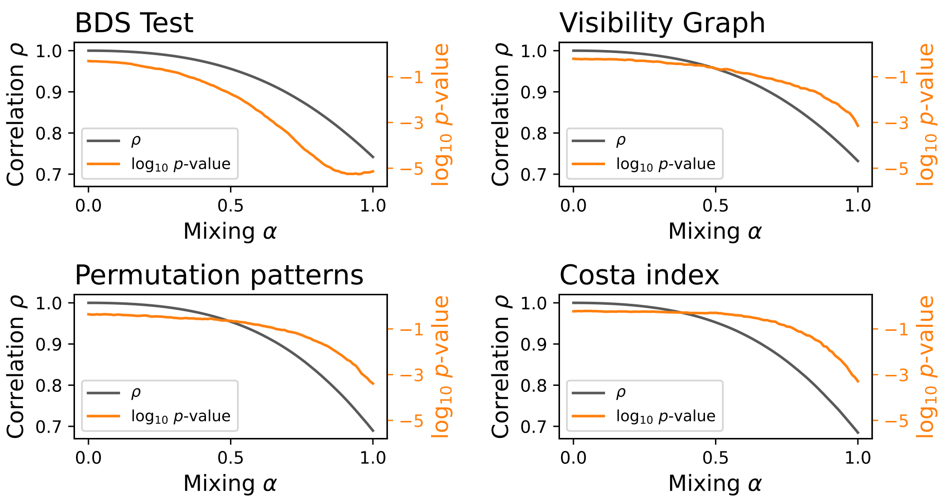

Figure 3. Increasing the irreversibility of random time series. Evolution of the linear correlation coefficient ρ between $x(t)$ and $y(t)$ (left Y axis, grey lines), and of the $\log_{10}$ of the p-values obtained by each irreversibility test on $y(t)$ (right Y axis, orange lines), as a function of the mixing parameter α—see main text for details.

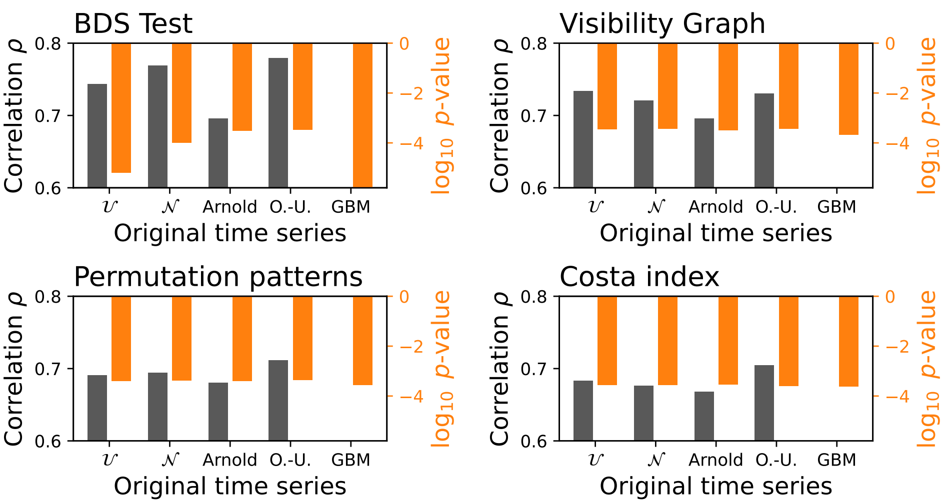

Figure 4. Linear correlation coefficient ρ (left Y axes, grey bars) between the original and manipulated time series, and $\log_{10}$ of the p-values (right Y axes, orange bars) obtained by each irreversibility test on latter time series, as a function of the type of time series.

3.3. Reducing Irreversibility

Up to now, I have focused on the problem of manipulating (initially reversible) time series to artificially increase their irreversibility, obtaining generally good results. The question that naturally follows is the following: can the process be reversed? That is, starting from irreversible time series, is it possible to manipulate them to obtain reversible ones? While this can trivially be obtained by adding some observational noise, I here want to see if the previous methodology can yield similar results.

The previous approach has been modified towards this aim. On the one hand, we start from irreversible time series generated with the chaotic Logistic map: $x(t+1) = ax(t)[1 - x(t)]$, with $a = 4.0$. Being a dissipative map, the resulting time series are irreversible by construction [26]. On the other hand, manipulated time series are retained whenever the p-value yielded by the irreversibility test is above a threshold of 0.6—i.e., to guarantee that they are time-reversible.

Figure 5 presents results similar, albeit slightly worse, than the previous case. Specifically, with values of α close to 1, it is possible to obtain reversible time series, with correlations between 0.7 and 0.8. The fact that lower correlations are obtained is not surprising, considering that the transformation induced by COPs is a non-linear one; this transformation thus naturally tends towards higher irreversibilities, and is here applied to obtain the opposite result. Additionally, and as previously mentioned, irreversibility can easily be reduced by introducing noise in the time series. I tested this idea by starting from time series generated with the Logistic map, then adding noise drawn from a uniform distribution until the irreversibility fell below what was obtained by the proposed transformation. Figure 6 then reports the average ρ, as obtained when introducing noise, as a function of the one obtained with the COP manipulation. Results are similar, except in the case of the Costa index: small quantities of noise are enough to confound this test and obtain large p-values, consequently obtaining time series very similar to the original ones. In sum, while the addition of an observational noise may be more efficient, the proposed optimisation procedure is still valid to reduce the irreversibility of time series.

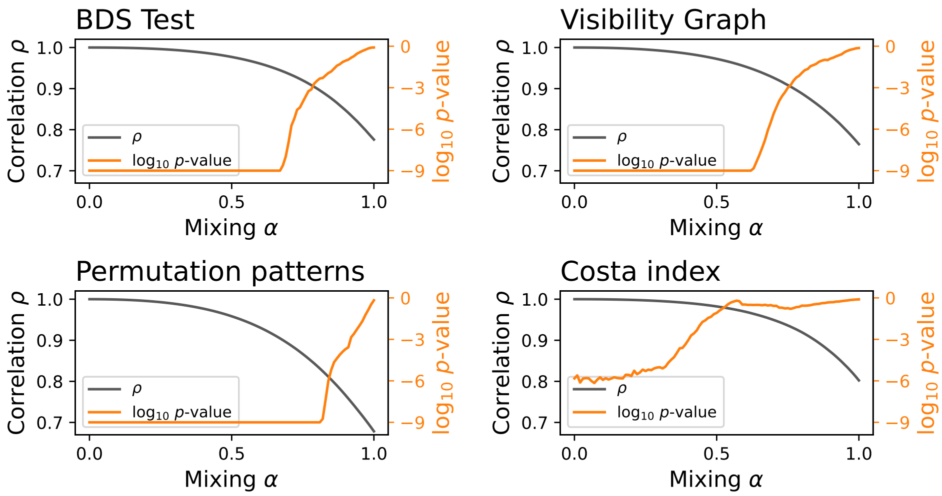

Figure 5. Reducing the irreversibility. Evolution of the linear correlation coefficient ρ between $x(t)$ and $y(t)$ (left Y axis, grey lines), and of the $\log_{10}$ of the p-values obtained by each irreversibility test on $y(t)$ (right Y axis, orange lines), as a function of the mixing parameter α—see main text for details.

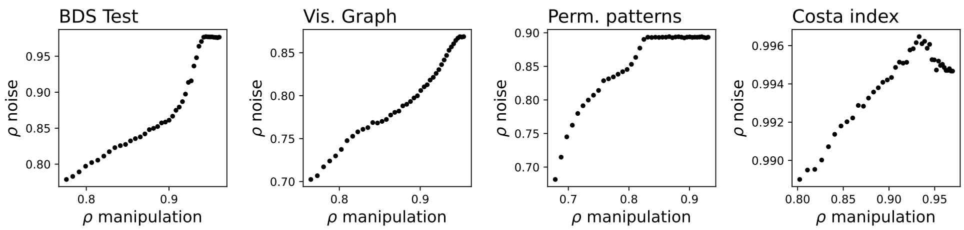

Figure 6. Scatter plots of the ρ obtained by adding random noise, as a function of the ρ obtained in the COP manipulation process. Each point corresponds to a different value of $\alpha > 0.6$, and noise was added to the original time series until the same level of reversibility in Figure 5 was achieved.

4. Manipulation of the Time Irreversibility of Brain Dynamics

One interesting conclusion that can be inferred from Figure 4 is that the manipulation process here proposed does not yield the same result depending on the characteristics of the original time series. For instance, for all four irreversibility tests, the resulting ρ is higher when the starting point is a time series created by the Ornstein–Uhlenbeck process, but substantially lower in the case of the Geometric Brownian Motion. Therefore, even if both types of time series are originally time-reversible, they have a different predisposition to be manipulated towards a higher irreversibility. In turn, this means that the result of the manipulation process can be used as a way of characterising the time series, beyond what a direct analysis of their irreversibility would yield.

In order to test this idea, I here analyse time series representing human brain activity. The computation performed by the human brain necessarily relies on non-linearities and memory; the underlying dynamics may thus be expected to be time irreversible. At the same time, we do not have direct access to such dynamics, but only to indirect measurements of it (e.g., through electroencephalography); and these are usually coarse-grained and noisy. As a result, irreversibility and its variation across conditions and pathologies can only reliably be measured in long time series [27–29].

I have considered the data set freely available at http://dx.doi.org/10.18150/repod.0107441 and described in Ref. [30], consisting of electroencephalographic (EEG) recordings corresponding to a condition of rest for 14 patients suffering from schizophrenia, and of the same number of matched control subjects. Patients comprise seven males, of 27.9 ± 3.3 years of age, and seven females, of 28.3 ± 4.1 years of age; all met the International Classification of Diseases ICD-10 criteria for paranoid schizophrenia (category F20.0). The same number of participants formed the healthy control group, with seven males of 26.8 ± 2.9 years of age, and seven females of 28.7 ± 3.4 years of age. The EEG data correspond to an eyes-closed resting state condition, and were recorded at 250Hz using the standard 10–20 EEG montage with 19 EEG channels: Fp1, Fp2, F7, F3, Fz, F4, F8, T3, C3, Cz, C4, T4, T5, P3, Pz, P4, T6, O1, and O2. The reference electrode was placed at FCz. The raw time series were considered in this study, with no additional artefact removal or filtering, beyond those already included in the original study [30]. Note that the choice of this condition was motivated by the availability of public data sets, and by the fact that strong changes in the irreversibility of EEG time series have previously been associated with this pathology [27]. At the same time, the analysis here presented could by applied to any other pathology/neurotype, including Alzheimer's and Parkinson's disease or autism spectrum disorder.

I start the analysis by illustrating the process on the first EEG electrode, i.e., Fp2. We calculate the irreversibility of its time series by randomly extracting 100 short segments (of 100 points each, i.e., 0.4 s) from all subjects, and obtaining p-values according to the Costa index test. As can be appreciated in the top left panel of Figure 7, most segments are time-reversible, with weak differences present between the two conditions. Specifically, a two-sample Kolmogorov–Smirnov test calculated between the two distributions yields a p-value of $9.77 \cdot 10^{-4}$. I then applied the manipulation procedure described in Section 3.2, aimed at increasing the irreversibility of these time series, and evaluated the resulting ρ between the manipulated and original segments. The results are represented in the top right panel of the same figure, and suggest the presence of a significant difference between both groups (p-value $< 10^{-195}$ according to the same test). In other words, even if both groups of segments have the same initial irreversibility, they present different distances to irreversibility, and this is smaller (i.e., manipulations of smaller amplitudes are needed to reach irreversibility, hence yielding a higher ρ) for schizophrenic patients. Such differences between control subjects and patients can be observed across all EEG channels, with p-values for the differences between ρs always being smaller than those yielded by the irreversibility test on the raw data—see, respectively, the orange and grey bars in the the middle panel of Figure 7.

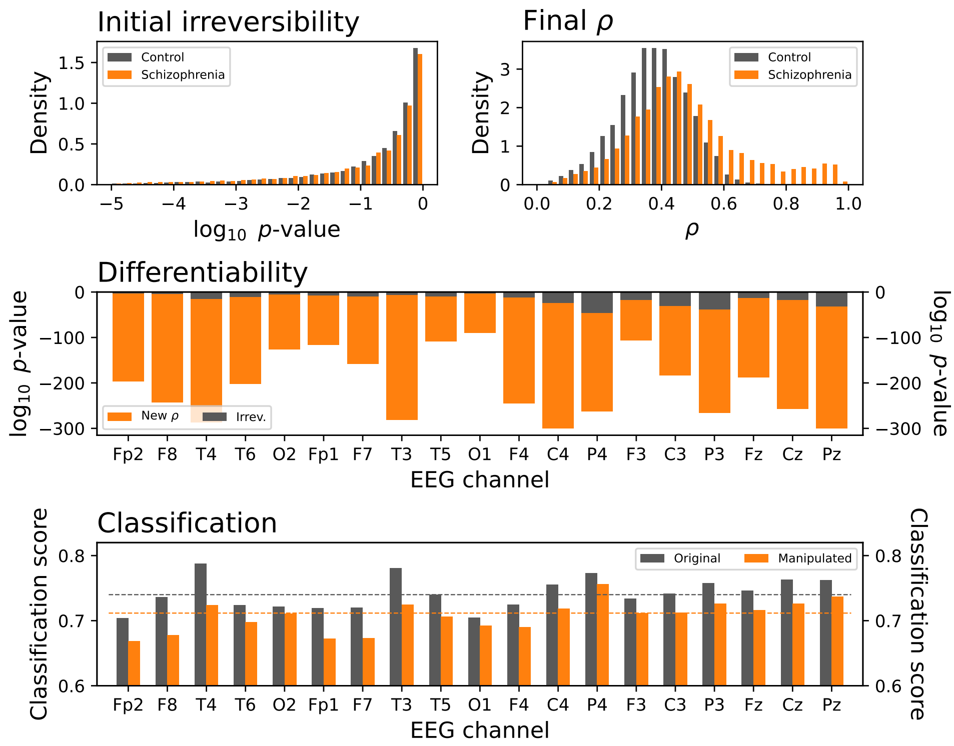

Figure 7. Manipulating irreversibility in the brain. (**Top left**) Probability distributions of the p-value obtained when applying the Costa index test, for control subjects (grey bars) and schizophrenic patients (orange bars), to time series corresponding to the Fp2 EEG electrode. (**Top right**) Probability distributions of ρ obtained after the manipulation of the same time series, for control subjects (grey bars) and schizophrenic patients (orange bars). (**Middle**) Orange bars: p-value yielded by a K-S test, comparing the distributions of ρ for control subjects and patients (i.e., depicted in the **top right** panel). Grey bars: p-value yielded by the same test when applied over the distributions of irreversibilities (i.e., what depicted in the top left panel). (**Bottom**) Score obtained by a ResNet model when trained to discriminate between control subjects and patients, using the original (grey bars) and manipulated (orange bars) time series.

If the manipulated time series displays a larger difference between the two groups in terms of time irreversibility, does this mean that they are more identifiable? In order to answer this question, I trained a Deep Learning (DL) model to discriminate between the two groups. The model is the Residual Networks (ResNet) model [31], i.e., a convolutional architecture that introduces the possibility of skipping connections between layers, allowing the network to learn residual functions and mitigate the vanishing gradient problem of very deep networks. This was chosen for its high efficiency in classifying sets of time series [32,33]. Key hyperparameters include the number of blocks (here set to 5), each comprising 5 layers of 32 filters of size 10, and the number of epochs, i.e., the number of times the whole data set is presented to the model during the training, here set to 200. The training was performed over half of the available segments chosen at random, with the validation on the remaining half—thus equivalent to two-fold cross-validation. Finally, the classification score was evaluated using the accuracy, i.e., the fraction of time series whose group was correctly identified by the model; in order to account for the stochasticity of the training process, the score here reported corresponds to the median over 100 independent realisations. The bottom panel of Figure 7 reports the average classification score for each EEG channel, obtained using the original (grey bars) and manipulated (orange bars) time series. Notably, the original time series yield larger classification scores across all channels—see also the median across them, represented by the dashed horizontal lines. In other words, even though the manipulation process allows for obtaining time series that are more different between the two conditions in terms of irreversibility, these are less different in general.

As a final test, Figure 8 reports the results when modifying time series corresponding to the Fp2 channel with the aim of reducing the time irreversibility; in other words, the two panels in Figure 8 correspond to the same panels in Figure 7 when applying the procedure described in Section 3.3. Notably, the results are very similar, with time series corresponding to schizophrenic patients also being easier to modify towards reduced irreversibility. As in the previous cases, this is confirmed by a two-sample Kolmogorov–Smirnov test, yielding p-values of 0.052 and $< 10^{-53}$ for the distributions in the left and right panels, respectively.

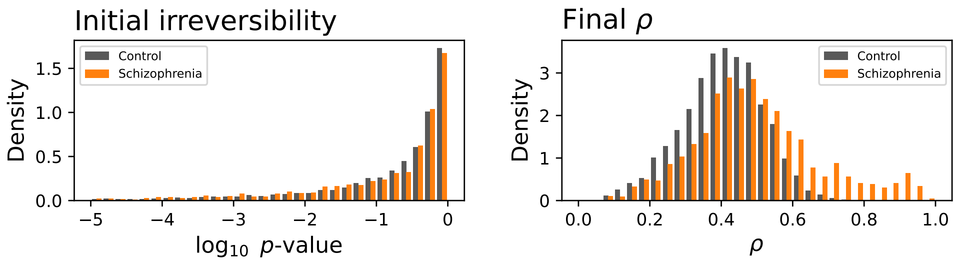

Figure 8. Reducing irreversibility in the brain. (**Left**) Probability distributions of the p-value obtained when applying the Costa index test, for control subjects (grey bars) and schizophrenic patients (orange bars). (**Right**) Probability distributions of ρ obtained after the time series manipulation aimed at reducing the time irreversibility, for control subjects (grey bars) and schizophrenic patients (orange bars). All results correspond to the Fp2 channel of the EEG data.

5. Discussion and Conclusions

While many tests have hitherto been proposed for the assessment of the time irreversibility of real-world time series [2], less attention has been devoted to the problem of manipulating this property. Here, I have analysed one possible approach for this, specifically using the concept of Continuous Ordinal Patterns (COP) [8]. In other words, given a time series, I here aimed at increasing (or decreasing) its irreversibility, while at the same time retaining its original dynamics (as measured by a linear correlation). The reader interested in reproducing the results will find a Python implementation of the algorithm available at https://gitlab.com/MZanin/irreversibilitytestslibrary. After proving that this is feasible in synthetic time series (Sections 3.2 and 3.3), I moved to the analysis of human brain data (Section 4).

As shown in Figures 4 and 7, the proposed manipulation process allows for describing time series through a new concept: the distance to irreversibility. Specifically, consider two time series that are both described as time-reversible by a given test; they may require modifications of different intensities to reach irreversibility, resulting in different values of ρ (i.e., the correlation between the original and manipulated versions of the time series). As demonstrated in the case of brain time series, this may be useful to discriminate between different conditions. These results lead to three interesting questions, which have hitherto never been tackled in the literature.

Firstly, what does this "distance from irreversibility" represent? While it may be tempting to interpret this as an amount of irreversibility, or the distance of the system to equilibrium, it is worth noting that this amount would manifest as differences in the initial p-values obtained by the tests. In other words, the time series analysed in Section 4 are very similar from an irreversibility point of view, and should therefore represent conditions in the brain with a similar distance to equilibrium. Two alternative explanations can be proposed. On the one hand, that the time series have actually different levels of irreversibility, which are nevertheless masked by the limited lengths of the considered segments. This is aligned with previous results indicating that long time series are required to detect irreversibility in EEG data [27]; yet, this seems at odd with the fact that time series of schizophrenic patients are more easier moved towards both higher and lower levels of irreversibility; see

Figures 7 and 8. On the other hand, this can be interpreted as the number of modifications that are required to start moving the dynamics towards a desired level of irreversibility, i.e., how much the dynamics has to be modified to increase (or decrease) the amount of entropy it produces. Nevertheless, note that the dynamics of the system is not changed, but rather the evolution of the observables used to describe it; hence, this distance tells us under which observational lens the system has to be observed to become time reversible or irreversible. In addition, these results have been obtained using a specific manipulation strategy, based on COPs, and have partially been compared with what can be obtained using observational noise; see Figure 6. Better insights on the meaning of manipulating irreversibility may emerge from considering alternative strategies, which will constitute an interesting open avenue for research.

Secondly, what does this "distance from irreversibility" represent in a neuroscience context? It is clearly related to different forms of workings of the brain, as shown by the important differences between control subjects and schizophrenic patients (Figure 7). Yet, what those differences are, and how they are related to the underlying pathology, are still open questions that ought to be tackled in the future, and that will have to be answered based on a better understanding of the physical meaning of the manipulation of irreversibility, as discussed above. Additionally, it will be important to understand how such differences are modulated by technical and methodological choices, e.g., the sampling frequency of the recorded time series, the application of filters to isolate specific frequency bands, or the length of the recorded time series—the latter related to the problem of the stationarity of brain dynamics, and hence, of its irreversibility [27]. In short, what has been proposed here may constitute a fertile ground for research, at the intersection between statistical physics and neurophysiology.

Finally, all results presented here focus on the irreversibility of individual time series, and of its manipulation; yet, an interesting open question is how these aspects interact when multiple time series are considered at the same time. This may be especially relevant in the case of electrophysiological brain signals, as different regions of the human brain are known to interact, both at rest and during cognitive tasks, creating network-like structures [34–36]. Extending the discussed approach to a multi-variate scenario is in principle feasible, provided one has access to a test to evaluate the irreversibility of multi-variate time series. To the best of our knowledge, such tests have yet to be proposed. Still, this topic may represent an interesting avenue of future research, especially in the context of networked and spatially extended real-world systems.

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Abbreviations

The following abbreviations are used in this manuscript:

BDS	Broock, Dechert, and Scheinkman
COPs	Continuous Ordinal Patterns
dHVG	Directed Horizontal Visibility Graph
DL	Deep Learning

EEG	Electroencephalography
GBM	Gaussian Brownian Motion
K-S	Kolmogorov–Smirnov
O.-U.	Ornstein–Uhlenbeck
ResNet	Residual Networks

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