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The Quenched g_A in Nuclei and Infrared Fixed Point in QCD

Mannque Rho [1,*] and Long-Qi Shao [2,3,4]

[1] Institut de Physique Théorique, Université Paris-Saclay, CNRS, CEA, 91191 Gif-sur-Yvette, France
[2] School of Fundamental Physics and Mathematical Sciences, Hangzhou Institute for Advanced Study, University of Chinese Academy of Sciences, Hangzhou 310024, China; shaolongqi22@mails.ucas.ac.cn
[3] Institute of Theoretical Physics, Chinese Academy of Sciences, Beijing 100190, China
[4] University of Chinese Academy of Sciences, Beijing 100049, China
* Correspondence: mannque.rho@ipht.fr

Abstract: The possible consequence of an infrared (IR) fixed point in QCD for $N_f = 2, 3$ in nuclear matter is discussed. It is shown in terms of d(ilaton)-χ effective field theory (dχEFT) incorporated in a generalized effective field theory implemented with hidden local symmetry and hidden scale symmetry that the superallowed Gamow–Teller transition in the doubly magic-shell nucleus ^{100}Sn recently measured at RIKEN indicates a large *anomaly-induced quenching* identified as a fundamental renormalization of g_A from the free-space value of 1.276 to ≈0.8. Combined with the quenching expected from strong nuclear correlations "*snc*", the effective coupling in nuclei $g_A^{\rm eff}$ would come to ∼1/2. If this result were reconfirmed, it would impact drastically not only nuclear structure and dense compact-star matter—where g_A figures in π-N coupling via the Goldberger-Treiman relation—but also in search for physics Beyond the Standard Model (BSM), e.g., $0\nu\beta\beta$ decay, where the fourth power of g_A figures.

Keywords: hidden symmetries; IR fixed-point; Landau-Migdal Fermi-liquid fixed-point; G*n*EFT; pseudo-conformality; anomaly-induced quenched g_A

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1. Introduction

Two recent measurements of the superallowed Gamow–Teller transition in the doubly magic closed-shell (DMCS) nucleus ^{100}Sn, one at GSI [1] and the other at RIKEN [2], give significantly different GT strengths. Put in the notation used in the two papers, the measured results are $\mathcal{B}_{\rm GT}^{\rm GSI} = 9.1^{+3.0}_{-2.6}$ and $\mathcal{B}_{\rm GT}^{\rm RIKEN} = 4.4^{+0.9}_{-0.7}$. In terms of the "extreme single-particle shell model (ESPM)" considered applicable for the DMCS nucleus ^{100}Sn, the effective g_A^* (throughout this paper, the superscripts "$*$" and "eff" both stand for density dependence, with the latter specifically for g_A in medium) in the ESPM gives the ranges

$$g_A^{*\rm GSI} \approx 0.9 - 1.0, \tag{1}$$
$$g_A^{*\rm RIKEN} \approx 0.6 - 0.7 \tag{2}$$

to be compared to the free-space value $g_A = 1.276$. In terms of the quenching factor denoted in the literature as q,

$$g_A^* = q g_A \tag{3}$$

$$q^{\rm GSI} \approx 0.71 - 0.78,\ q^{\rm RIKEN} \approx 0.47 - 0.54. \tag{4}$$

We prefer to quote the ranges here rather than the error bars since it is difficult to interpret them given the theoretical arguments injected in the quoted results, the reliability of which is hard to quantify in our approach. For the reason explained below, we will take $q^{\rm GSI} \approx 0.78$ giving $g_A^* \approx 1$ for GSI and $q^{\rm RIKEN} \approx 0.5$ giving $g_A^* \approx 0.6$ for RIKEN.

As argued [3,4] the "quenched $g_A^{\text{eff}} \approx 1$" with the quenching factor $q \equiv g_A^{\text{eff}}/g_A \approx 0.78$ observed in Gamow–Teller transitions in light nuclei [5–7] seems to be quite consistent with the GSI data, but not with the RIKEN result. Now, given that the RIKEN measurement [2], as claimed, must be improved with much smaller error bars, over the GSI one, this result taken seriously brings up in nuclear physics a strong tension in weak responses of nuclei to EW interactions between light and heavy nuclei. Additionally, in particle physics it has a drastic impact, so far unforeseen, of the quenched g_A in nuclear matter in the $0\nu\beta\beta$ process in heavy nuclei in search for new physics Beyond the Standard Model (BSM).

The objective of this paper is to expose for the first time the role of infrared fixed point and dilaton in QCD in nuclear interactions .

1.1. g_A^* and Where the Quenching Comes from

Let us briefly discuss the problem of the quenching of g_A in the nuclear medium.

The first thing to address is: Does the quenching involve a "fundamental" phenomenon when matching EFT to QCD or just a mundane nuclear effect appearing in finite density? It turns out, fortunately, that for superallowed Gamow–Teller transitions in doubly closed magic-shell (DCMS) nuclei one can reasonably—though not rigorously—answer this question. This is because there is a possibility of closely "mapping" what happens in DCMS nuclei in shell models to what can be treated in Landau–Fermi-liquid (LFL) theory [8,9] in the Fermi-liquid fixed-point (FLFP) approximation [10].

Shell-model analyses reviewed in [5–7] (and many other reviews that we will skip) rather persuasively suggested that for light nuclei, if the nuclear correlation is fully considered with proper wave function and effective operator, no *further* quenching seems to be needed in the sense defined in [11]. An interesting, very recent, development quotes the *sd* shell-model result in the mass range $A = 18$–39 in terms of the quenching factor for two effective shell-model interactions $q = 0.79 \pm 0.05$ and $q = 0.82 \pm 0.04$, giving $g_A^* \approx 1$ [7]. However in heavier nuclei, higher-order correlations involving large configuration space were difficult to put under control, making it difficult to reliably address the ab initio EFT calculation. There are two major issues raised in this conundrum, one in the nuclear many-body problem touching on the structure of dense (neutron-star) matter and the other hidden symmetries in QCD that become un-hidden in nuclear strong correlations. In addressing the problem involved, one cannot avoid an inherent fuzziness in what is "fundamental" and what is not in the observable quantities. In this article, we will try to give as precise a definition as feasible of what is meant to be fundamental. This issue arises because we are bound to work in the framework of an effective field theory (EFT).

The nuclear EFT we will be adopting is defined with the energy–momentum scale set by the chiral scale $\Lambda_\chi \sim 4\pi f_\pi \sim 1$ GeV at which the QCD degrees of freedom are to be integrated out. For the processes concerned in nuclear physics, the scale can in practice be brought down to the mass scale of the lowest-lying vector mesons (ρ, ω). We further implement the scalar dilaton σ (not to be confused with the σ in linear sigma model) as the pseudo-Goldstone boson for broken scale symmetry. Although the existence of an IR fixed point in QCD with the flavor number $N_f = 2, 3$ involved in nuclear processes remains still in controversy that began in 1970s, we will be arguing it should equally figure in dense nuclear interactions [12] as well as in particle physics involving scalar degrees of freedom in QCD [13,14]. One of the early roles of the dilaton in nuclear phenomena was recognized in the BR scaling [15]. It has figured importantly in our nuclear many-body approaches in the form of a generalized nuclear effective field theory that has been referred to as G*n*EFT by one of the authors for some time [12,16] involving both hidden local symmetry (HLS) [17] and hidden scale symmetry (HSS). By raising the relevant scale to capture the degrees freedom involved in those hidden symmetries, the G*n*EFT encompasses, at the mean-field order, the phenomena that are properly captured at higher chiral orders in perturbation in nuclear effective field theory, χ EFT, anchored on Weinberg's "Folk Theorem" on quantum effective field theory [18–20].

1.2. New Results

We start with a brief summary of the principal (new) results of this paper. From a recent, drastically new argument put forward by Zwicky [21–23] that there can exist an IR fixed point for two- or three-flavor QCD with the derivative of the β function with respect to the QCD coupling constant α_s (at the leading scale symmetry order) at the IR fixed point, β'_*, going to zero, resembling $\mathcal{N} = 1$ SUSY [24], we find from the most recent RIKEN experiment that the anomaly-induced quenching (AIQ) of g_A can be surprisingly big, making the value of g_A drop from the free-space value of 1.276 to $\sim$0.8. If the RIKEN result were reconfirmed by new measurements with accurately controlled theoretical inputs, then it would bring a totally new development not only in nuclear theory and nuclear astrophysics but also in searches for BSM. Since the axial coupling g_A is connected to the pion–nucleon coupling in nuclear medium via the Goldberger–Treiman relation, the strong AIQ would also basically revamp nuclear dynamics that is controlled by pion–nuclear interactions, that is, the nuclear physics anchored on chiral dynamics, which is the currently widely accepted paradigm in the nuclear physics community.

On the other hand, if the RIKEN data were shown to be unreliable or defective, then one could arrive at an understanding of why g_A is quenched to $g_A^* = 1$ in terms of strongly correlated nuclear dynamics with local and scale symmetries of QCD hidden in the nuclear matter. It would also clarify the role of the IR fixed point in nuclear physics. This suggests that settling the issue of the superallowed GT transition in ^{100}Sn is urgently needed.

2. The G*n*EFT

Here is a brief outline of the EFT framework we are working in. The implementation to chiral Lagrangian of hidden local symmetry fields is well established [17]. What we need to address then is how to implement hidden scale symmetry. The dilaton, the Nambu–Goldstone boson of spontaneously broken scale symmetry, has appeared in various different contexts in the literature with different definitions including going beyond the SM. We are interested in scale symmetry in QCD with the flavor numbers $N_f \leq 3$ relevant to nuclear dynamics. The scheme we adopt is the "genuine dilaton (GD)" proposed by Crewther [13,14], the characteristic feature of which is shared with the "conformal dilaton" phase in QCD (CD-QCD) developed more recently by Zwicky [21,23]. There may be other approaches but we find them best applicable to our problem. There are some differences in details between the two, but they do not affect in our scheme that exploits the notion of emergence of the scale symmetry. Briefly stated, the phase CD-QCD is different from the conformal window [25–28] in the N_c—N_f phase diagrams in that the quark condensate $\langle\bar{q}q\rangle$ exists in the CD-QCD phase, thus it can cause spontaneous breaking of scale symmetry in the chiral limit and give a large decay constant to the dilaton. This gives mass to other hadrons through the Goldberger–Treiman type relation. The scale symmetry that figures in GD (and also in CD-QCD) is hidden in the sense that it *emerges only* in the deep IR region. The process we are concerned with, as our work indicates, takes place not far from the IR fixed point, so the strong controversy in the field does not appear to seriously affect our reasoning that figures in nuclear dynamics.

The model we use, G*n*EFT, is a coarse-grained macroscopic approach formulated to address high-density phenomena taking place in massive compact-star matter at a density of $\sim$(5–10)n_0 (where n_0 is the nuclear equilibrium density $\sim$0.16 fm^{-3}). It has fared surprisingly well with no serious tension with the recent gravity-wave astrophysical data. It turns out that the formulation briefly explained below, anchored on Landau–Fermi-liquid structure of strongly correlated fermions (since the pion field is extremely important in nuclear dynamics, one should properly call it Landau–Migdal–Fermi-liquid, but although we omit Migdal, it should be understood as such), applies from $\sim n_0$ to the core density of neutron stars. How this is accomplished is described in detail invoking "pseudo-conformality (PC)" in [12,29]. We skip the details here but mention that the formulation involving "PC" applies very well to the present problem we are concerned with, namely heavy nuclei.

The chiral-scale Lagrangian from which we start, constructed by Crewther and Tunstall [14] including the leading anomalous dimension terms via the Callan–Symanzik renormalization group equation, is of the form (this formula essentially applies to the CD-QCD scheme [21] with, however, a crucially significant impact on the anomalous terms with $d \neq 4$ in the g_A problem that we will come to in Section 3.2):

$$\mathcal{L}_{\mathrm{d}\chi\mathrm{EFT}} =: \mathcal{L}_{\mathrm{inv}}^{d=4} + \mathcal{L}_{\mathrm{anom}}^{d>4} + \mathcal{L}_{\mathrm{mass}}^{d<4}, \tag{5}$$

where d is the scaling dimension. For $\mathcal{L}_{\mathrm{anom}}^{d>4}$, $d = 4 + \gamma_{G^2}(\alpha_s)$ with α_s the QCD coupling constant and $\gamma_{G^2}(\alpha_s)$ the anomalous dimension of the gluonic operator $G_{\mu\nu}G^{\mu\nu}$. For $\mathcal{L}_{\mathrm{mass}}^{d<4}$, $d = 3 - \gamma_m(\alpha_s)$ where $\gamma_m(\alpha_s)$ is the anomalous dimension of the bilinear quark operator $\bar{q}q$. It is assumed that we are near the IR fixed point in the low-energy and density regime of QCD, $\alpha_s \lesssim \alpha_{\mathrm{IR}}$ with α_{IR} the QCD coupling constant at the IR fixed point, so the anomalous dimension has the expansion with respect to $\delta_{\alpha s} \equiv O(\alpha_s - \alpha_{\mathrm{IR}})$.

$$\gamma_{G^2}(\alpha_s) \equiv \beta'(\alpha_s) - \beta(\alpha_s)/\alpha_s = \beta'_* + O(\delta_{\alpha s}) \tag{6}$$

with the anomalous dimension β'_*, the derivative of β function at the IR fixed point. Ignoring higher order terms in $\delta_{\alpha s}$, we have

$$\begin{aligned}
\mathcal{L}_{\mathrm{inv}}^{d=4} &= \{c_1\mathcal{K} + c_2\mathcal{K}_\sigma + c_3(\chi/f_\sigma)^2\}(\chi/f_\sigma)^2, \\
\mathcal{L}_{\mathrm{anom}}^{d>4} &= \{(1-c_1)\mathcal{K} + (1-c_2)\mathcal{K}_\sigma + c_4(\chi/f_\sigma)^2\}(\chi/f_\sigma)^{2+\beta'_*}, \\
\mathcal{L}_{\mathrm{mass}}^{d<4} &= \mathrm{Tr}(MU^\dagger + UM^\dagger)(\chi/f_\sigma)^{3-\gamma_m}.
\end{aligned} \tag{7}$$

Here,

$$\mathcal{K} = \frac{1}{4} f_\pi^2 \mathrm{Tr}(\partial_\mu U \partial^\mu U^\dagger) \quad \text{and} \quad \mathcal{K}_\sigma = \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma \tag{8}$$

and $U = exp(i\boldsymbol{\pi}/f_\pi)$ is the nonlinear realization of pion field, $\chi/f_\sigma = exp(\sigma/f_\sigma)$ is the conformal compensator field for the dilaton σ, c_i for $i = 1,2,3,4$ are parameters and M is quark mass matrix. Two counting schemes are involved: the scale and chiral power counting in the Lagrangian. Near the IR fixed point, $c_{1,2} = 1 + O(M), c_{3,4} = O(M)$ where $O(M) \sim O(m_\pi^2) \sim O(p^2)$ in the chiral power counting. When approaching the IR fixed point, $M \to 0, c_{1,2} \to 1, c_{3,4} \to 0$ such that only the $d = 4$ term survives.

In GnEFT [16], the many-body problem in nuclear matter is handled by a renormalization-group (RG) approach to interacting fermions on the Fermi sphere [10]. It has been shown that implemented with the hidden local symmetric and hidden scale-invariant fields, the mean field approximation of the (hidden local and scale symmetric) chiral Lagrangian with its parameters BR-scaling becomes equal to what corresponds to the Landau–Fermi-liquid fixed point theory of many-nucleon systems [8,12,29].

Now applying the same Callan–Symanzik RG manipulation to the nucleon axial-current $J_{5\mu}$ coupling to the weak external field $\mathcal{W}^\mu$, one obtains the weak Lagrangian that we need. To the leading chiral-order and leading-order anomalous dimension, it is of the form

$$\mathcal{L}_{weak} = J_{5\mu}^a \mathcal{W}^{a_\mu} \tag{9}$$

with

$$J_{5\mu}^{\pm} = Q_{ssb}(\chi) g_A \bar{\psi} \tau^{\pm} \gamma_\mu \gamma_5 \psi \tag{10}$$

where

$$Q_{ssb}(\chi) = c_A + (1 - c_A)(\frac{\chi}{f_\sigma})^{\beta'_*} \tag{11}$$

stands for the anomaly-induced effect inherited from QCD. Here, c_A is an undetermined parameter as c_i with $i = 1, 2, 3, 4$. Apart from Q_{ssb} in (10), $\mathcal{L}_{weak}$ is scale-invariant. The deviation from possible scale-invariance resides in (11). In GnEFT with the "vacuum" given by the medium with the baryon density n, we expand

$$\chi = \langle\chi\rangle^* + \chi' \tag{12}$$

with χ' representing the fluctuation dilaton field, the $*$ the density dependenc and $f_\sigma = \langle\chi\rangle_{n=0}$ in the medium-free space. If we ignore the fluctuating dilaton-field contributions that enter at higher loop orders, we have $Q_{ssb}(\langle\chi\rangle_{n=0}) = 1$, so there is no scale symmetry breaking effect at the leading order on g_A in the matter-free vacuum. One can therefore say that the anomaly-induced effect, hidden in the matter-free vacuum, can be revealed primarily by the presence of baryonic matter.

Going into finite density,

$$\langle\chi\rangle^*/f_\sigma = f_\sigma^*/f_\sigma = \Phi(n) \tag{13}$$

can be considered to be an "order parameter" that characterizes the vacuum structure modified by medium, the uppercase $*$ standing for in-medium quantity. The quantity $\Phi(n)$ is referred to in the literature as the "BR scaling factor." It governs how parameters in the dχEFT scale with density.

In the order we are considering, the axial current we are dealing with is given by

$$J_{5\mu}^{\pm} = q_{ssb} g_A \bar{\psi}\tau^{\pm}\gamma_\mu\gamma_5\psi \tag{14}$$

with

$$q_{ssb} = c_A + (1 - c_A)\Phi^{\beta'_*}. \tag{15}$$

We replaced the $\Phi(n)$ with Φ. q_{ssb} multiplies the coupling constant g_A in nuclear axial processes, hence represents a "fundamental renormalization" of the coupling constant $g_A \to g_A^{inherit} \equiv q_{ssb} g_A$ inherited from QCD which becomes manifested importantly in the medium, to be distinguished from what is given in standard many-body nuclear correlations. It is worth noting that *the effect of β'_*, which is hidden in the vacuum, is "exposed" in finite density in the axial current* for $\Phi \neq 1$. The physical transition matrix element of the current (14) in nuclei will then be the *full* nuclear matrix element $M_{\rm nucl}$ of the operator $j_5^{\rm nucl} = \bar{\psi}\tau^{\pm}\gamma_\mu\gamma_5\psi$ multiplied by the axial constant $g_A^{inherit}$. These two quantities are not entirely separate from each other because the BR scaling Φ must figure in both $M_{\rm nucl}$ and the effective axial constant. Now the question raised is how it figures in $g_A^{\rm eff}$ and $M_{\rm nucl}$, such as does it depend on density interplaying in both etc.? It turns out, fortunately, that there is very little interplay between the two.

There are two quantities to be considered. The first is the full nuclear correlations that we refer to as "mundane nuclear effect" (MNE in short) and the second, the main issue in this paper, is the "fundamental quenching factor" $q_{ssb} - 1 \neq 0$ referred to as "AIQ" for anomaly-induced quenching. Now, in order to minimize the possible overlap between MNE and AIQ we focus on the superallowed Gamow–Teller transition involving zero momentum transfer $q = 0$ and zero energy transfer $\omega = 0$. The strategy adopted [4] is to map the "Extreme Single Particle (shell) Model" (ESPM for short) in doubly magic closed shell nuclei to "Fermi-liquid fixed-point" (FLFP) approximation [10] at $(q, \omega) \to (0, 0)$ with $q/\omega \to 0$ on the Fermi surface. The FLFP approximation is best applicable for a quasi-particle making the superallowed transition *on the Fermi surface* with loop contributions suppressed in nuclear matter. The resulting *full* GT matrix element is then given by

$$M^{\rm L} = \frac{1}{2} q^L_{snc} g_A \langle\sigma\tau\rangle \tag{16}$$

where $\langle \sigma\tau \rangle$ is the single quasi-nucleon matrix element of the GT operator. The superscript L represents, from here on, the Landau–Fermi-liquid fixed-point quantity. The factor q^L_{snc} given in (22) below, with the subscript *snc* representing strong nuclear correlation, is the factor that captures the *total* matrix element in the EFT adopted. The product

$$g^L_A = q^L_{snc} g_A \tag{17}$$

is then the LFL fixed-point prediction for the single-quasi-particle coupling constant that captures the complete nuclear correlation. It is related to the $g^{\rm eff}_A$ mentioned in the Introduction.

2.1. q^L_{snc} as Landau–Fermi-Liquid Fixed-Point Quantity

To proceed, we review first how the FLFP quantity q_{snc} is derived. This was derived a long time ago [8]. It comes from the combination of chiral symmetry with the hidden symmetries for highly correlated fermions on Fermi sphere treated in renormalization-group approach [10]. The basic idea is to arrive at Landau– Fermi-liquid theory at the Fermi-liquid fixed point for the process of a quasi-proton on the Fermi surface, making the $q/\omega \to 0$ β^+ transition to a quasi-neutron corresponding to the ^{100}Sn superallowed GT transition arriving at Equation (16). This result obtained in [8] has an extremely simple form

$$q^L_{snc} = (1 - \frac{1}{3}\Phi\tilde{F}^\pi_1)^{-2} \tag{18}$$

where $\Phi = f^*_\chi/f_\chi$ and $\tilde{F}^\pi_1 = (m_N/m^L_N)F^\pi_1$ with the Landau mass m^L_N is the Landau interaction parameter for the pion exchange. This simple structure encodes low-energy theorems, Ward identities, etc., in the large N_c limits that are satisfied in the skyrmion model for baryons. Now, in the GD (and most likely also in Zwicky's CD-QCD) scheme, Φ satisfies

$$\Phi = f^*_\sigma/f_\sigma \approx f^*_\chi/f_\chi \simeq f^*_\pi/f_\pi \tag{19}$$

so Φ is available from deeply bound pionic nuclei up to near n_0 [30], hence it is a known quantity. (It should be mentioned that for large number of flavors, say, $N_f \gtrsim 10$ as in the conformal window scenario for dilatonic Higgs model, the ratio $f_\chi/f_\pi << 1$ [31], whereas in GD and CD-QCD $f_\chi \approx f_\pi$. The dilatons could have different properties between the two regimes.) It turns out that q_{snc} as given in (18) is highly insensitive to density near n_0. It comes out to be

$$q^L_{snc} \approx 0.78 \ \text{ for } n = n_0. \tag{20}$$

This leads to

$$g^L_A \approx 1. \tag{21}$$

It is predicted that this result holds for densities $n \approx (0.8 - 1.1)n_0$, hence in light nuclei as well as in heavy nuclei.

This prediction in the Landau–Fermi-liquid fixed point for the axial current is in the same class as the nuclear response in the EM currents. Here, the situation is even more straightforward. An illustrative case is the EM orbital current in the mean-field treatment of GnEFT, which reproduces precisely Migdal's finite Fermi-liquid formula [32] $\vec{J} = \frac{\vec{k}}{m_N}\left(\frac{1+\tau_3}{2} + \delta g_l\right)$ with $\delta g_l = \frac{1}{6}(\tilde{F}'_1 - \tilde{F}_1)\tau_3$ where $\tilde{F}_1$ and $\tilde{F}'_1$ are Landau–Migdal interaction parameters expressed in terms of the parameters of the Lagrangian involved in GnEFT. There are two remarkable results in this formula. First, the orbital current is given in terms of the vacuum nucleon mass—instead of the Landau mass m_L—satisfying the Kohn theorem [33]. The other is that the prediction for the nuclear anomalous gyromagnetic ratio [8]—with the soft-pion theorems playing the crucial role—$\delta g^p_l(n_0) \simeq 0.21$ agrees with what is measured in the Pb region, $\delta g_l^{\text{proton}} = 0.23 \pm 0.03$ [34]. This quantity fails to be explained by standard χEFT which gives $\sim$0.07, far short of the experimental value.

2.2. q_{snc} in ESPM

We now turn to the ESPM. For this we take the superallowed GT transition in ^{100}Sn nucleus, which has the proton and neutron shells completely filled at 50/50 magic shells. The transition involved in the ESPM is the pure superallowed transition of a proton (denoted π) $\pi g_{9/2}$ in the completely filled orbital to a neutron (ν) in the empty spin-orbit partner $\nu g_{7/2}$ orbital of ^{100}In. This offers the simplest possible structure of the daughter state that is of a pure $\nu g_{7/2}$ particle-$\pi g_{9/2}$ hole state to which the ESPM can be applied. Now, if one assumes that the final (daughter) state reached in the GT process has ignorable mixing with other particle-hole states, which is more or less the case [35], then the mapping from the FLFP approximation in the Fermi-liquid to the ESPM would be as exact as feasible. In reality, one cannot expect a total non-mixing even in this double magic-shell configurations. The same is the case in the Landau–Fermi-liquid fixed-point approximation based on $\bar{N}$ going to ∞. The question then remains how exactly the transition from the ^{100}Sn ground state to the pure ($\nu g_{7/2}$) particle-($\pi g_{9/2}$) hole state in ^{100}In can be extracted *from experiments*. This would require as accurate an account as possible of the theoretical mixing in the daughter state.

3. Observation and Prediction

3.1. Experiments

Here, we look at what Nature says in the ^{100}Sn transition. Equation (15) for q_{ssb} involves two unknowns, c_A and β'_*. Neither is available by lattice QCD or experiments. Therefore, we cannot make an unambiguous (theoretical) prediction with the formula (15).

To proceed, we need to resort to experimental decay rates to extract the nuclear matrix element M_{nucl} from experiments so as to obtain q_{snc}. For this, an ab initio no-core shell-model calculation that takes into as full an account of nuclear correlations as feasible could provide the necessary information. Up to date, however, one crucially important theoretical ingredient has been missing for that feat. For accurately calculating superallowed GT transitions in heavy nuclei, one must take into account the nuclear tensor force with its strength decreasing with increasing density. This decrease is caused by the tendency of cancellation between the tensor forces given by the exchange of the pion and the isovector meson ρ subject to the BR scaling. This mechanism makes the Gamow–Teller response strength change from low density to higher density. This feature is associated with Migdal's g'_0 interaction associated with what's called the "Ericson–Ericson–Lorentz–Lorenz (E^2-L^2) effect" in condensed matter physics applied to the Δ-hole effect in pion–nuclear processes. However, up to date no such calculations have been performed. Since there seems to be prevalent disagreements from aficionados of ab initio chiral EFT many-body approaches to nuclear interactions, it might be worth making a comment on what is mentioned above that was made elsewhere. There is what is heralded as "first-principles" resolution of the quenched g_A problem in the ^{100}Sn GT transition [36]. As pointed out in [3], the authors of [36] fit the GSI data, not the more "improved" RIKEN, which is not even cited therein. As for the strategy of "ab initio first principles approach" adopted in [36], it has two defects shared by all other "first-principles" calculations so far published: First, the nuclear tensor forces in G*n*EFT controlled by the BR scaling as a function of density are not properly taken into account. Second, the many-body currents in the Gamow–Teller channel are not "protected by chiral symmetry" and hence cannot be controlled at a low order of chiral expansion; this in stark contrast to the axial-charge transition mentioned in Sect. III C. The first point is the main point of G.E. Brown, reproduced in [37].

The ESPM-Landau FLFP mapping applicable for the doubly magic closed-shell structure can allow one to bypass this stumbling block. The mapping is given by the Landau fixed-point pionic interaction $\tilde{F}_1^\pi$ multiplied by the in-medium scaling of the dilaton decay constant Φ

$$q_{snc}^L = (1 - \frac{1}{3}\Phi\tilde{F}_1^\pi)^{-2} \simeq 1/g_A \simeq 0.78. \quad (22)$$

As remarked, it is applicable to heavy nuclei as well as to nuclear matter. It was first obtained numerically in [3,8]. As in standard chiral EFT, one would have to make corrections to the Landau's FLFP approximation result (22). How to perform such corrections in this Fermi-liquid approach has been recently formulated in condensed matter physics in terms of the nonlinear bosonization approach with the coadjoint orbit method [38]. Our problem is much more involved than in condensed matter systems because there are the pion and the hidden symmetry bosons (both HLS and HSS) coupled to Fermi surface fluctuations. The first attempt to include such corrections indicate, however, the correction to the FLFP approximation to (22) is quite small, coming at $O(10^{-4})$ in a wide range of matter densities [39]. Further effort needs to be made but this is more or less what is found in light nuclei for g_A^{eff}, which indicates that with no AIQ effect there is little dependence on density [5–7]. As noted below, this also hints at the approach to the dilaton-limit fixed point [40] at some high density at which g_A must go to 1.

Let us now look at the experiments available in ^{100}Sn where the mapping between the shell model and Landau–Fermi-liquid model could be made. As mentioned at the beginning of this paper, there are two experiments, one from GSI [1] and another from RIKEN [2].

- **GSI**: In this experiment, the transition zeroing-in on ∼95% of the final daughter state of the pure $(\nu g_{7/2})$particle-$(\pi g_{9/2})$hole configuration has been reported. The resulting q_{snc}^{GSI} comes out close to q_{snc}^L [4], leading to

$$q_{ssb}^{\text{GSI}} \approx (0.9 - 1.0). \quad (23)$$

  A detailed account of the mixing in the final state involving theoretical inputs seems to give a somewhat smaller q_{ssb} but it is not clear how reliable the mixing can be estimated. We choose not to rely on this analysis. Modulo the ∼5% uncertainty, however, it seems safe to conclude q_{ssb}^{GSI} given by (23) indicates there is no appreciable AIQ. This gives then what one might call "pure quasi-nucleon constant" $g_A^{pqn} = q_{snc} \times g_A = 0.78 \times 1.276 = 1$. This is equal to g_A^L (17) predicted by FLFP. As noted, this is consistent with the DLFP result [40] but at a much higher density in G*n*EFT where an intricate interplay between the attraction due to the dilaton exchange and the ω repulsion plays a crucial role [12,29,41].
- **RIKEN**: The more recent measurement at RIKEN for the same transition, improved over the GSI result with smaller error bars, comes out to be drastically different and points to a major AIQ effect. As it stands, this experimental result is the most clear-cut, if not in error, indication for the possible evidence for the AIQ. It is difficult to assess the accuracy with which the nuclear correlations between the neighboring states near the pure $(\nu g_{7/2})$particle-$(\pi g_{9/2})$hole configuration are taken into account in arriving at the experimental result of q_{snc}. It would require highly accurate theoretical inputs to q_{snc}, which we are unable to assess whether the RIKEN analysis provides. Assuming the q_{snc}^L given by the mapping for q_{snc}, the RIKEN result implies a significant effect that deviates from $q_{ssb}^L = 1$,

$$q_{ssb}^{\text{RIKEN}} \approx (0.6 - 0.7). \quad (24)$$

  This result is saying that the "fundamental" g_A inherited from QCD is ∼0.8–0.9, which is considerably renormalized from 1.276. Now, given that this is an intrinsic QCD effect, it should apply to *ALL* weak processes at all kinematics in the nuclear medium, not just superallowed Gamow–Teller transitions at $(q,\omega) \to (0,0)$. Even more significantly, in the process $0\nu\beta\beta$ transitions in nuclei where the momentum transfer can be of order ∼100 MeV, the axial coupling constant appears at fourth power in the cross-section.

So the big question is: Which one is correct?

3.2. Prediction by CD-QCD

Up to this point, there was no answer. What is new is that one may be able to answer the question with the coming experimental facilities and the new theoretical development. The recent development on the IR fixed point in the CD-QCD scheme anchored on soft dilaton theorems in dχEFT of Zwicky [21] predicts, analogously to the $\mathcal{N} = 1$ SUSY case [24], the anomalous dimension

$$\beta'_* = 0. \tag{25}$$

This prediction eliminates the gluonic trace anomaly term with $d > 4$ in Equation (7), attributing the QCD trace anomaly to the quark mass term. This would come about if the gluonic trace anomaly in some sense were integrated out, with the QCD gauge coupling running logarithmically near the IR fixed point [21]. Given that the would-be dilaton, if identified with the $f_0(500)$, has a mass $\gg m_\pi$, it could be that the correction to the β'_* in the exponent of (χ/σ) may not be negligible.

What this means is that Q_{ssb} in (10) in the leading chiral and scale order becomes

$$Q_{ssb}(\chi) = c_A + (1 - c_A)(\frac{\chi}{f_\sigma})^{\beta'_*} \to q_{ssb} = 1. \tag{26}$$

Hence, the quenching of g_A would be entirely due to the mundane nuclear effect. But this prediction is clearly at odds with the RIKEN result (24). This would apply not just to the axial current but also to other channels in the GnEFT Lagrangian. Here, the higher scale-order term $O(\alpha_s - \alpha_{\rm IR})$ is taken to be negligible. But it is not obvious that the system lies very near the IR fixed point, so that $O(\alpha_s - \alpha_{\rm IR})$ may not be ignorable. In fact the would-be free-space dilaton mass, if identified with $f_0(500)$ in the CD-QCD scheme, must be higher than the pion mass, so the correction to β'_* in the exponent to Φ may not be ignorable unless the system is driven close to the IR fixed point by external disturbance, say, baryon density. We cannot give a convincing simple argument at the moment, but the fact that the phenomenology in GnEFT with the topology change as hadron–quark crossover gives the precocious pseudo-conformal sound velocity $v_s^2/c^2 \approx 1/3$ in compact stars at a density as low as $n \gtrsim 3n_0$ is consistent with $\beta'_* \approx 0$.

3.3. Evidence For and Against $q_{ssb}^{\rm RIKEN} \approx (0.6 - 0.7)$

The dilaton-limit fixed point in GnEFT is expected to appear at a much higher density than normal n_0, encompassing what is expected in massive compact stars. How the effect relevant to normal nuclear matter persists from n_0, where the g_A problem lies to $\sim(5-7)n_0$ probed in compact stars cannot be given a simple description. But as defined, the AIQ should apply to the axial coupling constant g_A figuring in *ALL* processes involving the nucleon axial current independently of the kinematics. Furthermore, this applies to ALL processes where the pion–nucleon coupling figures as expected from the Goldberger–Treiman relation between g_A and $g_{\pi NN}$. One may then ask how the currently successful χEFT in nuclear physics that is anchored on soft pions—which includes nuclear astrophysical observations—escapes "torpedos" of the fundamentally quenched g_A?

Among numerous processes in nuclear dynamics, there is one case that seems to be strongly against $q_{ssb}^{\rm RIKEN}$. It is the first-forbidden beta decay in nuclei dominated by the axial-charge operator $J_{05}^{\pm}$. For this process, what was dubbed as "chiral filtering mechanism" was postulated in formulating the meson-exchange axial currents in χEFT [42]. There it was shown that the nuclear axial current had drastically different power expansion for the time-component of the axial current from the space component. The former was seen dominated by the soft-pion exchange while the latter was suppressed, in particular for the Gamow–Teller transitions. Indeed, this soft-pion mechanism led to an enhanced axial-charge transition where $q_{ssb} = 1$ [43] was well confirmed by an experiment in Pb nuclei [44]. This result has been further supported with $A = 12$ nuclei [45], indicating relatively weak dependence on density. Note that the AIQ factor for the axial current controlled by soft-dilaton effects [21] should apply exactly as in the superallowed GT transition and the axial charge operator that enters in the first-forbidden process is primarily controlled by soft-pion effects. The precise agreement between theory and experiment would surely be destroyed by the AIQ of the size given by the RHIC. This interplay of soft-dilaton and soft-pion involved in providing nuclear model independence could be given a "first-principle" test.

There are, however, certain axial processes being discussed in the literature that may be indicative of an appreciable quenching of q_{ssb}. As stressed by Suhonen (private communication from J. Suhonen), there is a suggestion that measuring β-decay spectral shape instead of superallowed GT transitions would allow the AIQ to be better extracted. Indeed, there seems to be a signal [46] for the $(1 - q_{ssb}) \sim (0.3 - 0.4)$ comparable to what is indicated by the RIKEN data. Here, the nucleon wavefunction involved probes different kinematics from the superallowed GT transition, so the LFL fixed-point-to-ESPM mapping where the nuclear tensor forces play a key role for determining q^L_{snc} cannot be applied.

In fact, the spectral shape involves nuclear matrix elements with the structure of the current operator (i.e., many-body meson-exchange currents) and the BR-scaled tensor forces are appreciably different from what enters in the superallowed GT transitions, so sorting out the nuclear effect to handle the intricacy between "fundamental" and "mundane" nuclear effects would seem to require a drastically different approach.

4. Conclusions

In this paper, we provided what we consider to be a solution to the long-standing mystery of the quenched g_A in nuclear GT transitions. What turns out to play a key role here is the presence of an IR fixed-point in QCD with two or three flavors, so far unobserved, at which hidden scale symmetry with GD [13] or CD-QCD [21] intervenes. Combined with hidden local symmetry [17,47], implemented with a hadron–quark continuity in terms of a topology change, this GD/CD-QCD scheme has so far met with *no tension* with modern developments with compact-star observables.

A most striking observation in the scheme with the IR fixed point is that the anomalous dimension at the fixed point, β'_*, comes out to be zero [21]. As shown in this paper, if higher-order scale-chiral contributions cannot be ignored as assumed so far, this would imply that there can be an important renormalization of the axial-current coupling constant from the free-space value 1.276 to $\sim 1/2$ in nuclear matter leading to a dramatic quenching of g_A in nuclear medium—which has not been seen so far in nuclear processes where pion-nuclear coupling is involved. This would also lead to a $\sim (1/2)^4$ quenching in $2\nu\beta\beta$ and $0\nu\beta\beta$ processes, which is highly relevant for the ongoing BSM search. The culprit for this possibility of a "humongous" effect is the recent RIKEN experiment in the superallowed Gamow–Teller transition in the doubly closed magic-shell nucleus ^{100}Sn. An unambiguous reconfirmation or invalidation of this result is called for. If it turned out to be either definitively reconfirmed or invalidated, then it would either support or rule out the possible IR fixed point with GD/QCD-CD, with this coming from nuclear physics.

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