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On Prešić-Type Mappings: Survey

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Abstract: This paper is dedicated to the memory of the esteemed Serbian mathematician Slaviša B. Prešić (1933–2008). The primary aim of this survey paper is to compile articles on Prešić-type mappings published since 1965. Additionally, it introduces a novel class of symmetric contractions known as Prešić–Menger and Prešić–Ćirić–Menger contractions, thereby enriching the literature on Prešić-type mappings. The paper endeavors to furnish young researchers with a comprehensive resource in functional and nonlinear analysis. The relevance of Prešić's method, which generalizes Banach's theorem from 1922, remains significant in metric fixed point theory, as evidenced by recent publications. The overview article addresses the growing importance of Prešić's approach, coupled with new ideas, reflecting the ongoing advancements in the field. Additionally, the paper establishes the existence and uniqueness of fixed points in Menger spaces, contributing to the filling of gaps in the existing literature on Prešić's works while providing valuable insights into this specialized domain.

Keywords: metric space; Prešić-type mapping; Kannan–Prešić-type mapping; G-Prešić operator; Prešić–Picard sequence; weakly Prešić–Picard sequence; Prešić–Menger-type mapping; Prešić–Ćirić–Menger-type mapping

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1. Introduction and Preliminaries

Let X be a nonempty set and $u \in X$. Then, u is called a fixed point of a mapping $T : X \to X$ if $Tu = u$. The existence of fixed points of self-mappings has been considered by several authors in different spaces. Most of the results on fixed points are the generalizations of the famous Banach contraction principle, which ensures the existence and uniqueness of the fixed point of self-mappings defined on complete metric spaces. It states that: if $T : X \to X$ is a Banach contraction on a complete metric space (X, d), that is, T satisfying the condition:

$$d(Tx, Ty) \leq \lambda \cdot d(x, y), \tag{1}$$

for all $x, y \in X$, where $\lambda \in [0, 1)$, then T has a unique fixed point (say) $u \in X$. Moreover, if $x \in X$ is an arbitrary point, then the sequence $x_n = T^n x$ is convergent and $\lim_{n \to +\infty} x_n = u$.

After the appearance of the famous result of Banach in 1922 [1] about the unique fixed point of a contractive mapping defined on the complete metric space, many researchers successfully generalized that result. For the last 100 years or more, many mathematicians have continued to work on the generalization of Banach's result. Details can be found in the recent monograph by Lj.Ćirić [2], as well as the extensive work of B.E. Rhoades [3]. See also [4]. All these generalizations went in two directions.

The first one is the change of some of the three axioms of the metric space. This is how various classes of general metric spaces arose, such as partial metric spaces, metric-like spaces, b-metric spaces, b-metric-like spaces, partial b-metric spaces, G-metric spaces, G_b-metric spaces, S-metric spaces, S_b-metric spaces and others.

The second one is the generalization of the right-hand side $\lambda \cdot d(x,y)$ in (1), in the sense that one of the following expressions is taken instead of $\lambda \cdot d(x,y)$:

$$a \cdot d(x,Tx) + b \cdot d(y,Ty),\ a \geq 0,\ b \geq 0,\ a+b < 1 \text{ (Kannan)}$$

$$a \cdot d(x,Ty) + b \cdot d(Tx,y),\ a \geq 0,\ b \geq 0, a+b < 1 \text{ (Chatterjea)}$$

$$a \cdot d(x,y) + b \cdot d(x,Tx) + c \cdot d(y,Ty), a \geq 0, b \geq 0, c \geq 0, a+b+c < 1 \text{ (Reich)}$$

$$a \cdot d(x,y) + b \cdot d(x,Tx) + c \cdot d(y,Ty) + \delta \cdot d(x,Ty) + e \cdot d(Tx,y), a \geq 0, b \geq 0, c \geq 0,$$
$$\delta \geq 0, e \geq 0, a+b+c+\delta+e < 1 \text{ (Hardy–Rogers)}$$

$$\lambda \max\left\{d(x,y), \frac{d(x,Tx)+d(y,Ty)}{2}, \frac{d(x,Ty)+d(Tx,y)}{2}\right\} \text{ (Ćirić's generalized contraction I)}$$

$$\lambda \max\left\{d(x,y), d(x,Tx), d(y,Ty), \frac{d(x,Ty)+d(Tx,y)}{2}\right\} \text{(Ćirić's generalized contraction II)}$$

$$\lambda \max\{d(x,y), d(x,Tx), d(y,Ty), d(x,Ty), d(Tx,y)\} \text{ (Ćirić's quasicontraction)}$$

or

$$\varphi(d(x,y)) \text{ where, } \varphi : [0,+\infty) \to [0,+\infty) \text{ (Boyd–Wong i.e., . weakly contraction).}$$

Now, we list the next two important and well-known contractive conditions, which are significantly different from the previous ones in that they have both sides left and right and in which the left is different from $d(Tx,Ty)$. The first is called Meir–Keeler and it reads:

For all $\varepsilon > 0$, there exists $\delta > 0$ such that for every x, y in X, the next implication holds

$$\varepsilon \leq d(x,y) < \varepsilon + \delta \text{ implies } d(Tx,Ty) < \varepsilon$$

(Meir–Keeler contraction).

The second refers to the highly famous contractive condition proposed by D. Wardowski in 2012, which extends the renowned Banach result. It reads as follows:

There exists $\tau > 0$ so that whenever $d(Tx,Ty) > 0$, the inequality $\tau + F(d(Tx,Ty)) \leq F(d(x,y))$ holds, where F is a function that maps $(0,+\infty)$ to $\mathbb{R}$ and satisfies the following three conditions:

(F1) F is strictly increasing, i.e., for all $\alpha, \beta \in \mathbb{R}_+$ such that $\alpha < \beta$, $F(\alpha) < F(\beta)$;

(F2) For each sequence $\{\alpha_n\}_{n\in\mathbb{N}}$ of positive numbers, $\lim_{n\to+\infty} \alpha_n = 0$ if and only if $\lim_{n\to+\infty} F(\alpha_n) = -\infty$;

(F3) There exists $k \in (0,1)$ such that $\lim_{\alpha\to 0^+} \alpha^k F(\alpha) = 0$.

In 1965, S. Prešić [5] extended the Banach principle for the mappings defined from product X^k (where k is a positive integer) into the space X and proved the following theorem.

Theorem 1. *Let (X,d) be a complete metric space, k a positive integer and $T : X^k \to X$ a mapping satisfying the following contractive type condition:*

$$d(T(x_1,x_2,\ldots,x_k), T(x_2,x_3,\ldots,x_k,x_{k+1})) \leq \sum_{i=1}^{k} q_i d(x_i,x_{i+1}), \tag{2}$$

for every $x_1, x_2, \ldots, x_k, x_{k+1} \in X$, where $q_1, q_2, \ldots, q_k$ are nonnegative constants such that $q_1 + q_2 + \cdots + q_k < 1$.

Then there exists a unique point $u \in X$ such that $T(u, u, \ldots, u) = u$. Moreover, if $x_1, x_2, \ldots, x_k$ are arbitrary points in X and for $n \in \mathbb{N}$, $x_{n+k} = T(x_n, x_{n+1}, \ldots, x_{n+k-1})$, then the sequence $\{x_n\}$ is convergent and

$$\lim_{n\to+\infty} x_n = T\left(\lim_{n\to+\infty} x_n, \ldots, \lim_{n\to+\infty} x_n\right).$$

A mapping satisfying (2) is referred to as a Prešić-type contraction. These types of contractions have found widespread applications across various mathematical domains. One notable application lies in the convergence of sequences, where the principles of Prešić-type contractions have been instrumental in understanding the behavior of convergent sequences [5,6]. Additionally, they have been employed in solving nonlinear difference equations, offering valuable insights into the dynamics of such equations [7,8]. Furthermore, Prešić-type contractions have proven effective in addressing nonlinear inclusion problems, providing techniques for determining solutions in complex nonlinear systems [9]. Moreover, they have been instrumental in addressing convergence issues related to nonlinear matrix difference equations, offering methods to analyze the behavior and stability of such equations under various conditions [10]. The significance and versatility of Prešić-type contractions in modern mathematics are underscored by their diverse applications. These contractions transcend theoretical frameworks, offering valuable insights and tools for understanding and solving problems across various mathematical disciplines. Their utility is exemplified through numerous practical applications, supported by numerical examples such as those detailed in References [11–13]. From optimization problems to dynamical systems, the effectiveness of Prešić-type contractions illuminates their relevance in addressing real-world challenges and advancing with the new trend in mathematical theory [14,15].

The first work [16], which extended the scope of previous studies, dates back to 2007. This work represents the initial positive outcome subsequent to the publications of Prešić's works [5,6] in 1965 (also see [17]).

Theorem 2. *Let (X, d) be a complete metric space, k a positive integer and $T : X^k \to X$ a mapping satisfying the following contractive type condition*

$$d(T(x_1, x_2, \ldots, x_k), T(x_2, \ldots, x_k, x_{k+1})) \leq \lambda \max\{d(x_i, x_{i+1}) : 1 \leq i \leq k\}, \tag{3}$$

where $\lambda \in (0, 1)$ is a constant and $x_1, \ldots, x_{k+1}$ are arbitrary elements in X. Then, there exists a point x in X such that $T(x, \ldots, x) = x$. Moreover, if $x_1, x_2, x_3, \ldots, x_k$ are arbitrary points in X and for $n \in \mathbb{N}$,

$$x_{n+k} = T(x_n, x_{n+1}, \ldots, x_{n+k-1}),$$

then the sequence $\{x_n\}_{n=1}^{+\infty}$ is convergent and

$$\lim_{n\to+\infty} x_n = T\left(\lim_{n\to+\infty} x_n, \lim_{n\to+\infty} x_n, \ldots, \lim_{n\to+\infty} x_n\right).$$

If, in addition, we suppose that on diagonal $\triangle \subset X^k$,

$$d(T(u, \ldots, u), T(v, \ldots, v)) < d(u, v)$$

holds for all $u, v \in X$, with $u \neq v$, then x is the unique point in X with $T(x, x, \ldots, x) = x$.

Further, new generalizations of Prešić's result went in the direction of replacing the right-hand side in (2) with a more general expression similar to the one in paper [16]. For this purpose, using already well-known contractive conditions such as Kannan, Chatterjea, Reich, Hardy–Rogers, Ćirić, Boyd–Wong, Rus, Matkowski and others, the new contractive conditions and corresponding new results were obtained: Prešić–Kannan, Prešić–Chatterjea, Prešić–Reich, Prešić–Hardy–Rogers, Prešić–Ćirić, Prešić–Boyd–Wong and others. Taking

$k = 1$, in these results, we obtained the old well-known results such as Kannan, Chatterjea, Reich, Hardy–Rogers, Ćirić, Boyd–Wong and others.

Let k be a positive integer. If T is a mapping from X^k to X and if $(x_1, x_2, \ldots, x_k)$ is a given arbitrary point in X^k, then $x_{n+k} = T(x_n, x_{n+1}, \ldots, x_{n+k-1})$ defines the so-called Prešić–Picard sequence in X. Taking $k = 1$, we obtain the standard Picard sequence in X.

Important Notice

It is evident that not only in the definition of the Prešić contraction but also in all the results presented in the previously published papers [11–16,18–59], one can set $k = 2$. Thus, Prešić's contractive condition can be expressed as follows: There are nonnegative numbers a and b with $a + b < 1$ such that for every three points x, y, z from X: $d(T(x, y), T(y, z)) \leq a \cdot d(x, y) + b \cdot d(y, z)$. Note also that in this case, the Prešić–Picard sequence has the form $x_{n+2} = T(x_n, x_{n+1})$. Essentially, by assuming that $k = 2$, no less general results are obtained compared to those in the above-mentioned works. The only thing that is avoided is the need for technically more intricate formulations.

Let us also recall that when $k = 2$, the expression $d(T(x, y), T(y, z))$ corresponds to the left side of the original Prešić condition (2), namely $d(T(x_1, x_2, \ldots, x_k), T(x_2, x_3, \ldots, x_k, x_{k+1}))$. In both instances, there exists a specific order: $(x, y), (y, z)$, or $(x_1, x_2, \ldots, x_k), (x_2, x_3, \ldots, x_k, x_{k+1})$, which must be taken into consideration when establishing the results.

It is not challenging to streamline all the previously established results from Prešić's mappings into a new, more suitable form, leveraging the fact that $k = 2$. This approach proves to be easier and simpler to articulate. Now, let us elucidate the procedure for locating a point x within X such that for a given mapping T from X^2 to X, the condition $T(x, x) = x$ is satisfied. Initially, a Prešić–Picard sequence is defined for two given points x_0 and x_1 from X, where $x_{n+2} = T(x_n, x_{n+1})$ for $n = 0, 1, 2, \ldots$. If $x_0 = x_1 = x_2$, then evidently $x_0 = T(x_0, x_0)$ serves as a fixed mapping point of T in the Prešić sense. However, if $x_0 \neq x_1$, the sequence $x_{n+2} = T(x_n, x_{n+1})$ is explored, assuming that x_n is distinct from x_{n+1} for each n.

2. Application of Rules $x, y; y, z$

In this section, we aim to reframe well-known published results employing a Prešić-type contraction. By setting $k = 2$, we examine the mapping T from X^2 to X, where (X, d) represents the given metric space. It is established that X^2 can be equipped with a metric induced by the metric d on a nonempty set X. Among these metrics, one is expressed by the formula $D((x, y), (u, v)) = d(x, u) + d(y, v)$, while the other is defined as $D_1((x, y), (u, v)) = \max\{d(x, u), d(y, v)\}$. Now, Prešić's theorem from [5,6] in the new $x, y; y, z$ environment can be stated as follows.

Theorem 3. *Let (X, d) be a complete metric space and $T : X^2 \to X$ a mapping satisfying the following contractive type condition*

$$d(T(x, y), T(y, z)) \leq q_1 d(x, y) + q_2 d(y, z),$$

for every x, y, z in X, where q_1, q_2 are nonnegative constants such that $q_1 + q_2 < 1$. Then, there exists a unique point u in X such that $T(u, u) = u$. Moreover, if x_1, x_2, x_3 are arbitrary points in X and for $n \in \mathbb{N}$,

$$x_{n+2} = T(x_n, x_{n+1}),$$

then there, the sequence $\{x_n\}_{n=1}^{+\infty}$ is convergent and

$$\lim_{n\to+\infty} x_n = T\left(\lim_{n\to+\infty} x_n, \lim_{n\to+\infty} x_n\right).$$

Similarly, we can obtain the Ćirić–Prešić result from [16] in a new $x, y; y, z$ environment.

3. Prešić-Type Mappings in Menger Spaces

In this section, we introduce a novel class of contractions known as Prešić–Menger and Prešić–Ćirić–Menger contractions, representing probabilistic versions of their conventional counterparts. Within this framework, we establish the existence and uniqueness of fixed points in Menger spaces [60]. The results presented here address a gap in the existing literature on Prešić's works, providing valuable insights into this specialized domain.

Let us begin by revisiting some fundamental notations, definitions and topological properties of Menger spaces. For further elucidation, readers are encouraged to consult [61].

Definition 1. *A map $\xi : [0,+\infty) \to [0,1]$ is called a distance distribution function if the following conditions are verified:*

1. *ξ is left continuous on $[0,+\infty)$;*
2. *ξ is nondecreasing;*
3. *$\xi(0) = 0$ and $\xi(+\infty) = 1$.*

We denote by Δ^+ the class of all distance distribution functions. The subset $D^+ \subset \Delta^+$ is the set $D^+ = \left\{\xi \in \Delta^+ : \lim_{x\to+\infty} \xi(x) = 1\right\}$.

A specific element of D^+ is the Heavyside function ϵ_0 defined as:

$$\epsilon_0(x) = \begin{cases} 0 & \text{if} \quad x = 0, \\ 1 & \text{if} \quad x > 0. \end{cases}$$

Definition 2. *A triangular norm (briefly t-norm) is a mapping $\urcorner : [0,1] \times [0,1] \to [0,1]$ such that for all $x, y, z \in [0,1]$, the following conditions are satisfied:*

1. *$\urcorner(x,y) = \urcorner(y,x)$;*
2. *$\urcorner(x,\urcorner(y,z)) = \urcorner(\urcorner(x,y),z)$;*
3. *$\urcorner(x,y) < \urcorner(x,z) \quad for \quad y < z$;*
4. *$\urcorner(x,1) = \urcorner(1,x) = x$.*

The most basic t-norms are: $\urcorner_M(x,y) = min(x,y)$, $\urcorner_P(x,y) = x \cdot y$ and $\urcorner_L(x,y) = max(x+y-1,0)$.

Definition 3. *If $\urcorner$ is a t-norm and $(x_n)_{n\in\mathbb{N}}$ is a sequence of numbers in $[0,1]$, $\urcorner_{i=1}^n x_i$ is defined recurrently by $\urcorner_{i=1}^1 x_i = x_1$ and $\urcorner_{i=1}^n x_i = \urcorner(\urcorner_{i=1}^{n-1} x_i, x_n)$, for all $n \geq 2$.*

$\urcorner$ can also be extended to countable infinitary operation by defining $\urcorner_{i=1}^{+\infty} x_i$ for any sequence $(x_i)_{i\in\mathbb{N}}$ as $\lim_{n\to+\infty} \urcorner_{i=1}^n x_i$.

Definition 4 ([62])**.** *We say that a t-norm $\urcorner$ is of H-type if the family $(\urcorner^n(x))_{n\in\mathbb{N}}$ is equicontinuous at the point $x = 1$, that is*

$$for\ all\ \epsilon \in (0,1),\ there\ exists\ \lambda \in (0,1):\ t > 1-\lambda\ implies\ \urcorner^n(t) > 1-\epsilon\ \ for\ all\ n \geq 1.$$

Definition 5. *The triple $(X, F, \urcorner)$ where X is a nonempty set, F is a function from $X \times X$ into Δ^+ and $\urcorner$ is a t-norm is called a Menger space if the following conditions are satisfied for all $p, q, r \in X$ and $x, y > 0$:*

- *(i)* $F_{p,p} = \epsilon_0$;
- *(ii)* $F_{p,q} \neq \epsilon_0 \ \ if \ p \neq q$;
- *(iii)* $F_{p,q} = F_{q,p}$;
- *(iv)* $F_{p,q}(x+y) \geq \urcorner(F_{p,r}(x), F_{r,q}(y))$.

$(X, F, \urcorner)$ is a Hausdorff topological space in the topology induced by the family of (ϵ, λ)-neighborhoods:

$$\mathcal{N} = \{N_p(\epsilon, \lambda) : \ p \in X, \ \epsilon > 0 \ \text{ and } \ \lambda > 0\},$$

where

$$\mathcal{N}_p(\epsilon, \lambda) = \{q \in X : \ F_{p,q}(\epsilon) > 1 - \lambda\}.$$

Definition 6. *Let $(X, F, \urcorner)$ be a Menger space. A sequence $\{x_n\}$ in X is said to be:*

1. *Convergent to $x \in X$ if for any given $\epsilon > 0$ and $\lambda > 0$ there exists a positive integer $N(\epsilon, \lambda)$ such that $F_{x_n,x}(\lambda) > 1 - \epsilon$ whenever $n \geq N$.*
2. *A Cauchy sequence if for any $\epsilon > 0$ and $\lambda > 0$ there exists a positive integer $N(\epsilon, \lambda)$ such that $F_{x_n,x_m}(\lambda) > 1 - \epsilon$ whenever $n, m \geq N$.*

A Menger space $(X, F, \urcorner)$ is said to be complete if each Cauchy sequence in X is convergent to some point in X.

Now, we will consider the gauge functions from the class Γ^k of all mapping $\varphi : [0,1]^k \to [0,1]$ that verified the following requirement:

1. φ is a continuous and an increasing function.
2. $\varphi(t, \ldots, t) \geq t$ for all $t \in [0,1]$.

Before stating the first result, we introduce the definition of the Prešić contraction in the sense of Menger spaces.

Definition 7. *Let $(X, F, \urcorner)$ be a Menger space, k a positive integer and $\varphi \in \Gamma^k$. A mapping $f : X^k \to X$ is called a Prešić–Menger contraction if*

$$F_{f(x_1,x_2,\ldots,x_k),f(x_2,x_3,\ldots,x_{k+1})}(qt) \geq \varphi\big(F_{x_1,x_2}(t), F_{x_2,x_3}(t), \ldots, F_{x_k,x_{k+1}}(t)\big), \tag{4}$$

where $x_1, x_2, \ldots, x_k \in X$, $0 < q < 1$ and $t > 0$.

We now present our initial result as follows:

Theorem 4. *Let $(X, F, \urcorner)$ be a complete Menger space, k a positive integer and $f : X^k \to X$ a probabilistic Prešić contraction. Then, there exists a unique point x in X such that $x = f(x, x, \ldots, x)$. Moreover, if $x_1, x_2, \ldots, x_k$ are arbitrary points in X and for $n \in \mathbb{N}$, $x_{n+k} = f(x_n, x_{n+1}, \ldots, x_{n+k-1})$, then the sequence $\{x_n\}$ is convergent and*

$$\lim_{n \to +\infty} x_n = f\left(\lim_{n \to +\infty} x_n, \ldots, \lim_{n \to +\infty} x_n\right).$$

Proof. Suppose $x_1, x_2, \ldots, x_k$ are arbitrary points in X, we define a sequence as the following

$$x_{n+k} = f(x_n, x_{n+1}, \ldots, x_{n+k-1}),$$

and we put $\alpha_n = F_{x_n,x_{n+1}}(qt)$. We will show by induction that

$$\alpha_n \geq \left(\frac{K - \theta^n}{k + \theta^n}\right)^2, \tag{5}$$

where $\theta = \frac{1}{q}$ and $K = min\left\{\frac{\theta(1+\sqrt{\alpha_1})}{1-\sqrt{\alpha_1}}, \frac{\theta^2(1+\sqrt{\alpha_2})}{1-\sqrt{\alpha_2}}, \ldots, \frac{\theta^k(1+\sqrt{\alpha_k})}{1-\sqrt{\alpha_k}}\right\}$.

Clearly, from the definition of K, we see that (5) is true for $n = 1$. Then, for $n+k$ we have

$$\begin{aligned}
\alpha_{n+k} &= F_{x_{n+k},x_{n+k+1}}(qt) \\
&\geq \varphi\big(F_{x_n,x_{n+1}}(t), F_{x_{n+1},x_{n+2}}(t), \ldots, F_{x_{n+k},x_{n+k+1}}(t)\big) \\
&= \varphi(\alpha_n, \alpha_{n+1}, \ldots, \alpha_{n+k-1}) \\
&\geq \varphi\left(\left(\frac{K-\theta^n}{K+\theta^n}\right)^2, \left(\frac{K-\theta^{n+1}}{K+\theta^{n+1}}\right)^2, \ldots, \left(\frac{K-\theta^{n+k-1}}{K+\theta^{n+k-1}}\right)^2\right) \\
&\geq \varphi\left(\left(\frac{K-\theta^{n+k-1}}{K+\theta^{n+k-1}}\right)^2, \left(\frac{K-\theta^{n+k-1}}{K+\theta^{n+k-1}}\right)^2, \ldots, \left(\frac{K-\theta^{n+k-1}}{K+\theta^{n+k-1}}\right)^2\right) \\
&\geq \left(\frac{K-\theta^{n+k-1}}{K+\theta^{n+k-1}}\right)^2 \\
&\geq \left(\frac{K-\theta^{n+k}}{K+\theta^{n+k}}\right)^2.
\end{aligned}$$

Thus, inductive proof of (5) is complete.

Now, for $p \in \mathbb{N}$ and $t \in [0, +\infty)$, we have

$$\begin{aligned}
F_{x_n,x_{n+p}}(t) &\geq \urcorner(F_{x_n,x_{n+1}}(\frac{t}{2}), F_{x_{n+1},x_{n+p}}(\frac{t}{2})) \\
&\geq \urcorner^p\left(F_{x_n,x_{n+1}}(\frac{t}{2}), F_{x_{n+1},x_{n+2}}(\frac{t}{2^2}), \ldots, F_{x_{n+p-1},x_{n+p}}(\frac{t}{2^p})\right) \\
&\geq \urcorner^p\left((\frac{K-2^n}{K+2^n})^2, (\frac{K-2^{2n}}{K+2^{2n}})^2, \ldots, (\frac{K-2^{np}}{K+2^{np}})^2)\right) \\
&\geq \urcorner_{i=1}^{+\infty}((\frac{K-2^{ni}}{K+2^{ni}})^2) \\
&\geq 1-\epsilon.
\end{aligned}$$

Hence, $\{x_n\}$ is a Cauchy sequence in X and since X is complete, there is z in X such that $x_n \to z$ as $n \to +\infty$.

Now, we prove that $f(z, z, \ldots, z) = z$. In fact, we have

$$\begin{aligned}
F_{f(z,z,\ldots,z),z}(t) &= \lim_{n\to+\infty} F_{f(z,z,\ldots,z),x_{n+k}}(t) \\
&= \lim_{n\to+\infty} F_{f(z,z,\ldots,z),f(x_n,x_{n+1},\ldots,x_{n+k-1})}(t) \\
&\geq \lim_{n\to+\infty} \urcorner^{k-1}\Bigg(F_{f(z,z,\ldots,z),f(z,z,\ldots,z,x_n)}\left(\frac{t}{2}\right), \\
&\qquad F_{f(z,z,\ldots,z,x_n),f(z,z,\ldots,z,x_n,x_{n+1})}\left(\frac{t}{2^2}\right), \ldots, \\
&\qquad F_{f(z,x_n,\ldots,x_{n+k-2}),f(x_n,x_{n+1},\ldots,x_{n+k-1})}\left(\frac{t}{2^{k-1}}\right)\Bigg) \\
&\geq \lim_{n\to+\infty} \urcorner^{k-1}(\varphi\{F_{z,z}(t), F_{z,z}(t), \ldots, F_{z,x_n}(t)\}, \\
&\qquad \varphi\{F_{z,z}(t), \ldots, F_{z,x_n}(t), F_{x_n,x_{n+1}}(t)\}, \ldots, \\
&\qquad \varphi\{F_{z,x_n}(t), F_{x_n,x_{n+1}}(t), \ldots, F_{x_{n+k-2},x_{n+k-1}}(t)\}) \\
&\to 1 \quad \text{as } n \to +\infty.
\end{aligned}$$

Thus, $F_{f(z,z,\ldots,z),z}(t) = 1$, which means that z is a fixed point of f, which is proof also for

$$\lim_{n\to+\infty} x_n = f\left(\lim_{n\to+\infty} x_n, \ldots, \lim_{n\to+\infty} x_n\right).$$

Finally, to show the uniqueness, we suppose that there exists $z' \in X$ such that $z' = f(z', z', \ldots, z')$. Then, from (12) we obtain

$$\begin{aligned} F_{z,z'}(qt) &= F_{f(z,z,\ldots,z),f(z',z',\ldots,z')}(t) \\ &\geq \varphi(F_{z,z'}(t), F_{z,z'}(t), \ldots, F_{z,z'}(t)) \\ &\geq F_{z,z'}(t), \end{aligned}$$

which implies that $z = z'$. Thus, z is the unique point of f in X. □

Let us define the Prešić–Ćirić–Menger contraction in the framework of probabilistic metric spaces.

Definition 8. *Let $(X, F, \daleth)$ be a Menger space and k a positive integer. A mapping $f : X^k \to X$ is called a probabilistic Prešić–Ćirić–Menger contraction if*

$$F_{f(x_1,x_2,\ldots,x_k),f(x_2,x_3,\ldots,x_{k+1})}(qt) \geq \min_{1\leq i\leq k}\{F_{x_i,x_{i+1}}(t)\} \tag{6}$$

where $x_1, x_2, \ldots, x_k \in X$, $0 < q < 1$ and $t > 0$.

Expanding upon Theorem 4, we present the following theorem as a broader generalization.

Theorem 5. *Let $(X, F, \daleth)$ be a complete Menger space under a t-norm of H-type, k a positive integer and $f : X^k \to X$ a Prešić–Ćirić–Menger contraction. Then, there exists a fixed point z in X such that $z = f(z, \ldots, z)$. Moreover, if $x_1, x_2, \ldots, x_k$ are arbitrary points in X and for $n \in \mathbb{N}$, $x_{n+k} = f(x_n, x_{n+1}, \ldots, x_{n+k-1})$, then the sequence $\{x_n\}$ is convergent and*

$$\lim_{n\to+\infty} x_n = f\left(\lim_{n\to+\infty} x_n, \ldots, \lim_{n\to+\infty} x_n\right).$$

If, in addition, we suppose that on the diagonal of X^k we have for any $z, z' \in X$ such that $z \neq z'$ and $t > 0$,

$$F_{f(z,\ldots,z),f(z',\ldots,z')}(t) > F_{z,z'}(t). \tag{7}$$

then z is unique.

Proof. Suppose $x_1, x_2, \ldots, x_k$ are arbitrary points in X. We define a sequence as the following

$$x_{n+k} = f(x_n, x_{n+1}, \ldots, x_{n+k-1}) \quad for\ all\ n \in \mathbb{N},$$

and consider

$$\psi(t) = \min_{1\leq i\leq k}\left\{\theta^i F_{x_i,x_{i+1}}(t)\right\} \ for\ all\ t > 0,$$

where $\theta = q^{-k}$. We will show by induction that

$$F_{x_n,x_{n+1}}(t) \geq \frac{1}{\theta^n}\psi(t) \ \ for\ all\ n \in \mathbb{N}\ and\ t > 0. \tag{8}$$

By the definition of $\psi(t)$, it is obvious that (8) is true for $n = 1, 2, \ldots, k$. Let the following k inequalities hold, for $t > 0$,

$$F_{x_n,x_{n+1}}(t) \geq \frac{1}{\theta^n}\psi(t),\ F_{x_{n+1},x_{n+2}}(t) \geq \frac{1}{\theta^{n+1}}\psi(t), \ldots, F_{x_{n+k-1},x_{n+k}}(t) \geq \frac{1}{\theta^{n+k-1}}\psi(t).$$

Then, from $\theta > 1$ and the contractivity condition, we obtain,

$$\begin{aligned} F_{x_{n+k},x_{n+k+1}}(qt) &\geq \min_{1\leq i\leq k}\{F_{x_{n+i},x_{n+i+1}}\} \\ &\geq \min_{1\leq i\leq k}\left\{\frac{1}{\theta^{n+i}}\psi(t)\right\} \\ &\geq \frac{1}{\theta^{n+k}}\psi(t). \end{aligned}$$

Therefore, (8) is true for all $n \in \mathbb{N}$. Now, we show that $\{x_n\}$ is a Cauchy sequence. Let $\epsilon \in (0,1)$ and $t > 0$. For $n, m \in \mathbb{N}$ with $m > n$. By using (8), we obtain

$$\begin{aligned} F_{x_n,x_m}(qt) &\geq \daleth\left(F_{x_n,x_{n+1}}(\frac{t}{2}), F_{x_{n+1},x_m}(\frac{t}{2})\right) \\ &\geq \daleth^{m-n}\left(F_{x_n,x_{n+1}}(\frac{t}{2}), F_{x_{n+1},x_{n+2}}(\frac{t}{2^2}), \ldots, F_{x_{m-2},x_{m-1}}(\frac{t}{2^{m-n-2}}), F_{x_{m-1},x_m}(\frac{t}{2^{m-n-1}})\right) \\ &\geq \daleth^{m-n}\left(\frac{1}{\theta^n}\psi(\frac{t}{2}), \frac{1}{\theta^{n+1}}\psi(\frac{t}{2^2}), \ldots, \frac{1}{\theta^{m-2}}\psi(\frac{t}{2^{m-n-2}}), \frac{1}{\theta^{m-1}}\psi(\frac{t}{2^{m-n-1}})\right) \\ &\geq \daleth^{m-n}\left(\frac{1}{\theta^n}\psi(\frac{t}{2}), \frac{1}{\theta^n}\psi(\frac{t}{2^2}), \ldots, \frac{1}{\theta^n}\psi(\frac{t}{2^{m-n-2}}), \frac{1}{\theta^n}\psi(\frac{t}{2^{m-n-1}})\right). \end{aligned}$$

Next, by taking $\vartheta = \inf_{t>0}\psi(t)$, we obtain

$$F_{x_n,x_m}(qt) \geq \daleth^{m-n}\left(\frac{1}{\theta^n}\vartheta\right). \tag{9}$$

Now, let $\epsilon > 0$ be given. Since $\daleth$ is a t-norm of H-type, there exists $\lambda \in (0,1)$ such that $\daleth^n(t) > 1-\epsilon$ for all $n \in \mathbb{N}$ when $t > 1-\lambda$. Then, by choosing $\frac{1}{\theta^{n_0}}\vartheta > 1-\lambda$ for all $n > n_0$ we obtain

$$F_{x_n,x_m}(qt) > 1-\epsilon \ \ \textit{for all } n, m > n_0 \textit{ and } t > 0.$$

Hence, $\{x_n\}$ is a Cauchy sequence. Since X is complete, there is some $x \in X$ such that $x_n \to z$ as $n \to +\infty$.

We will show that z is a fixed point of f. In fact, for any $n \in \mathbb{N}$ and $t > 0$, we have

$$\begin{aligned} F_{x_{n+k},f(z,\ldots,z)}(t) &= F_{f(x_n,\ldots,x_{n+k-1}),f(z,\ldots,z)}(t) \\ &\geq \daleth^k\Bigg(F_{f(x_n,\ldots,x_{n+k-1}),f(x_{n+1},\ldots,x_{n+k-1},z)}\left(\frac{t}{2}\right), \\ &\quad F_{f(x_n,\ldots,x_{n+k-1},z),f(x_{n+1},\ldots,x_{n+k-1},z,z)}\left(\frac{t}{2^2}\right), \\ &\quad \ldots, \\ &\quad F_{f(x_{n+k-1},z,\ldots,z),f(z,\ldots,z)}\left(\frac{t}{2^{k-1}}\right)\Bigg). \end{aligned} \tag{10}$$

Using (6), we have for all $t > 0$

$$F_{f(x_n,\ldots,x_{n+k-1}),f(x_{n+1},\ldots,x_{n+k-1},z)}(\frac{t}{2}) \geq min\left\{\min_{1\leq i\leq k}\{F_{x_{n+i-1},x_{n+i}}(t)\}, F_{x_{n+k-1},z}(t)\right\}. \tag{11}$$

Letting $n \to +\infty$ in (11), we obtain

$$\lim_{n\to+\infty} F_{f(x_n,\ldots,x_{n+k-1}),f(x_{n+1},\ldots,x_{n+k-1},z)}(t) = 1$$

Similarly, for the other components in (10), we achieve

$$\lim_{n\to+\infty} F_{x_{n+k},f(z,\ldots,z)}(t) = 1.$$

Thus, z is a fixed point of f.

Finally, to prove uniqueness, we suppose that $z' \in X$ exists such that $z' = f(z',\ldots,z')$ with $z \neq z'$. Then, from the diagonal condition (7), we have for all $t > 0$

$$F_{z,z'}(t) = F_{f(z,\ldots,z),f(z',\ldots,z')}(t) > F(z,z'),$$

which is a contradiction. Hence, z is unique. □

Application of Rules $(x,y);(y,z)$ in Menger Spaces

The Prešić–Menger theorem in the $(x,y);(y,z)$ context is stated as follows:

Theorem 6. *Let $(X,F,\urcorner)$ be a complete Menger space, $\varphi \in \Gamma^2$ and $f : X\times X \to X$ a mapping satisfying the following contractive-type condition*

$$F_{f(x,y),f(y,z)}(q(t_1+t_2)) \geq \varphi\big(F_{x,y}(t_1),F_{y,z}(t_2)\big), \tag{12}$$

where $x,y,z \in X$, $0 < q < 1$ and $t_1,t_2 > 0$. Then, there exists a unique point x in X such that $u = f(u,u)$. Moreover, if x_1,x_2,x_3 are arbitrary points in X and for $n \in \mathbb{N}$, $x_{n+2} = f(x_n,x_{n+1})$, then the sequence $\{x_n\}$ is convergent and

$$\lim_{n\to+\infty} x_n = f\left(\lim_{n\to+\infty} x_n, \lim_{n\to+\infty} x_n\right).$$

Similarly, we can obtain the Ćirić–Prešić result in a different setting, where the environment is defined by the relationships between x, y, and z as $(x,y);(y,z)$.

Theorem 7. *Let $(X,F,\urcorner)$ be a complete Menger space under a t-norm of H-type and $f : X\times X \to X$ is mapping satisfying the following contractive-type condition*

$$F_{f(x,y),f(y,z)}(q(t_1+t_2)) \geq \min\{F_{x,y}(t_1),F_{y,z}(t_2)\}, \tag{13}$$

where $x,y,z \in X$, $0 < q < 1$ and $t_1,t_2 > 0$. Then, there exists a fixed point u in X such that $u = f(u,u)$. Moreover, if x_1,x_2,x_3 are arbitrary points in X and for $n \in \mathbb{N}$, $x_{n+2} = f(x_n,x_{n+1})$, then the sequence $\{x_n\}$ is convergent and

$$\lim_{n\to+\infty} x_n = f\left(\lim_{n\to+\infty} x_n, \lim_{n\to+\infty} x_n\right).$$

4. Conclusions

In summary, this survey paper consolidates research articles focusing on Prešić-type mappings since 1965, while also introducing Prešić–Menger and Prešić–Ćirić–Menger contractions. It serves as a comprehensive resource for young researchers in functional and nonlinear analysis, highlighting the ongoing relevance of Prešić's method, which expands upon Banach's theorem. The paper underscores the growing importance of Prešić's approach in metric fixed point theory and establishes the existence and uniqueness of fixed points in Menger spaces. By addressing gaps in the existing literature, it contributes to the advancement of knowledge in this specialized area. All pertinent works related to

Prešić's approach have been referenced for further exploration. Meanwhile, it is important to note three open problems for further exploration:

1. Investigate whether the outcomes presented in [12,37] can be demonstrated solely under the assumption of property F1 for the function F, particularly in the context of F-contractions, as discussed in a recent review paper by N. Fabiano et al. [63].
2. Define and formulate the Ćirić–Prešić–Meir–Keeler contraction according to the $x, y; y, z$ rule. Disprove or prove the formulated theorem, thereby contributing to the ongoing discourse in this area.
3. In most works on fixed point metric theory and Prešić's approach, we encounter the Prešić–Picard sequence given by $x_{n+k} = T(x_n, x_{n+1}, \ldots, x_{n+k-1})$, which demonstrates that the defined sequence x_n is Cauchy. If the mapping T is continuous, the existence of a point u from X such that $u = T(u, \ldots, u)$ directly follows. In many works where Prešić's approach has been considered, the continuity of the mapping T is not assumed. The natural question arises: Can we find an example of a metric space (X, d) and a mapping T from X to itself that is not continuous?

Addressing these open problems could potentially yield valuable insights and advancements in the field of functional and nonlinear analysis, building upon the foundation laid out in this survey paper. Additionally, these inquiries offer avenues for future research and exploration within the domain of Prešić-type mappings and related contractions.

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