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Adaptive Fuzzy Fixed-Time Control for Nonlinear Systems with Unmodeled Dynamics

Rongzheng Luo [1], Lu Zhang [1], You Li [2,]* and Jiwei Shen [3]

[1] Institute of Remote Sensing Satellite, China Academy of Space Technology, Beijing 100094, China
[2] School of Aerospace Science and Technology, Xidian University, Xi'an 710126, China
[3] School of Information, Changchun Polytechnic, Changchun 130033, China; shenjiwei2024@126.com
* Correspondence: liyou@xidian.edu.cn

Abstract: This article concentrates on the problem of fixed-time tracking control for a certain class of nonlinear systems with unmodeled dynamics. Unmodeled dynamics are prevalent in practical engineering systems, such as axially symmetric systems like robotic arms, spacecraft, and missiles. In this paper, the fuzzy-logic systems (FLSs) are implemented to address the challenge of accurately approximating the unknown nonlinear terms that arise during the derived control algorithm process. By employing fixed-time command filters (FTCF), the "explosion of complexity" issues encountered in traditional backstepping methods will be effectively resolved. Moreover, error compensation mechanisms are derived to effectively mitigate the filtering errors that may arise from the FTCFs. The computational burden associated with FLSs is reduced through the utilization of the weight vector estimation method based on the maximal norm and an adaptive approach. A fixed-time adaptive fuzzy tracking controller is developed within the backstepping control framework to ensure the boundedness of all signals and achieve fixed-time convergence of the tracking error for the controlled system. Illustrative examples are conducted to illustrate the viability of the derived controller.

Keywords: unmodeled dynamics; command filters; fixed-time control

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1. Introduction

Substantial advancements have been achieved in the application of backstepping-based control to a wide range of systems, leading to substantial development in this field. Several important results have been established as a result of these advancements [1–3]. However, the applicability of backstepping-based control schemes becomes limited when nonlinear systems contain unknown functions. Unfortunately, imposing the presumption that the analyzed nonlinear system is fully understood proved overly limiting for a broad spectrum of systems encountered in the field of engineering. To surmount the limitation of assuming complete knowledge of the nonlinear system, the function approximation methods, such as the utilization of FLSs and neural networks (NNs), have been comprehensively applied [4,5]. In [4], based on the anti-stepping method, a consensus fuzzy controller was developed in a distributed manner to ensure synchronization of output signals among all followers and the leader, and then the algorithm was applied to a second-order Lagrange symmetric system. In [5], a learning strategy based on NNs was proposed for nonlinear systems with strict feedback and full-state constraints, where the directions of gain are unknown. The efficiency of the proposed algorithm was confirmed by a case study of an axisymmetric robotic manipulator system. These methods are highly effective in handling unknown terms because of their remarkable capability to approximate complex functions [6–8]. Adaptive backstepping control protocols incorporating FLSs have been widely and fruitfully applied in different axisymmetric systems, such as satellite clusters and unmanned aerial vehicles [9,10]. In [9], they devised an adaptive fuzzy backstepping control approach to tackle the attitude stabilization problem under consideration. where the adaptive fuzzy logic technique was utilized for approximate the nonlinearities arising from the coupling between rigidity and flexibility in the spacecraft system.

It is noteworthy that the practical application of traditional adaptive backstepping approaches necessitates recursive differentiation of virtual control strategies at each stage. However, this can lead to a significant increase in complexity, which may result in higher computational demands or even a decline in control performance. To overcome this drawback, a solution was implemented by incorporating command filters to evaluate the differential coefficients of the virtual control laws during the derivation stage of the backstepping controller. In recent publications, notable research breakthroughs have been made regarding adaptive backstepping command filter-based control for a specific category of nonlinear systems and these achievements can be found in [11–13]. In [11], an adaptive neural network (NN) fault-tolerant controller was developed utilizing the command filter approach, backstepping algorithm, and average dwell time method. The aim was to minimize tracking errors and guarantee the constraint on all signals within the closed-loop system, enabling precise tracking of a reference signal. In [12], a innovative finite-time command filtered backstepping control approach was introduced, which applied new virtual control signals and altered error compensation signals. This approach retained the benefits of conventional command-filtered backstepping control while guaranteeing finite-time convergence. The method was demonstrated on an axially symmetric electromechanical system. In [13], a robust adaptive neural network tracking control scheme was introduced for a category of strict-feedback nonlinear system featuring unknown nonlinearity and external interference, while also considering input saturation. The command filter backstepping control method was employed to address the limitations of other methods such as dynamic surface control and tracking-differentiator-based control. On the other hand, it is important to acknowledge that unmodeled dynamics are inherent in real-world systems owing to various factors such as modeling inaccuracies, external disturbances, and measurement noise [14–18]. These unmodeled dynamics can significantly degrade system performance and even cause instability. As a result, considerable research endeavors have been devoted to investigating and addressing the issue of unmodeled dynamics [19–21]. More significantly, existing works on unmodeled dynamic systems control have focused on ensuring asymptotic stability.

It is widely recognized that achieving a high convergence rate is essential when dealing with tracking issues for nonlinear systems. Finite-time control algorithms offer several advantages compared to asymptotic control algorithms [22–25]. They not only improve disturbance rejection capabilities, achieve faster convergence rate, and enhance tracking accuracy, but they also guarantee that the control objective can be achieved within a finite time [26–29]. It is crucial to note that the determination of settling time in finite-time control algorithms is assessed based on the initial states of the systems [30–32]. However, meeting the initial condition requirement is not always possible as the initial conditions are always unknown in applied settings. Hence, the fixed-time control approach has emerged as a progression of the finite-time control algorithm, specifically designed to calculate the settling time of uncertain nonlinear systems, regardless of the initial conditions [33–36]. So far, considerable research has been dedicated to exploring fixed-time control techniques suitable for intricate engineering systems, such as airplanes, racing vehicles, and multiagent systems [37–42]. In [37], two innovative adaptive fault-tolerant fixed-time control schemes were introduced: the distributed fixed-time control (DFTC) method and the priority-based fixed-time control (PFTC) method. These methods were developed to ensure practical fixed-time stability of tracking errors and prevent violations of error constraints. Despite the notable progress that has been reported, a fundamental challenge persists in the domain of fixed-time tracking control for nonlinear system featuring unknown function and unmodeled dynamic, demanding further attention and resolution. This issue adds an additional layer of complexity and difficulty to the control design process, particularly when incorporating command filters.

Building upon the insights from the aforementioned statements, this article seeks to tackle the challenge of fixed-time control for nonlinear systems affected by unmodeled dynamics and unknown functions. Through the utilization of fuzzy systems and adaptive techniques, the challenges posed by unrepresented dynamics and unknown functions are effectively managed. Additionally, the issue of "explosion of complexity," arising from the derivatives of virtual controllers, is mitigated using the command filter. Finally, we recommend a adaptive fuzzy fixed-time control scheme. The developed protocol demonstrates the capability to achieve practical fixed-time stability of the tracking error and ensure the boundedness of all signals within the controlled systems. The superiority of this article is threefold, as follows: (1) Compared to recent studies that focus purely on the asymptotic or finite-time stability of controlled systems [19–21], this article derives a novel fixed-time controller for unmodeled dynamical nonlinear systems. The proposed scheme ensures that the tracking error remains bounded within fixed time. (2) Unlike the fixed-time control algorithm discussed in [38], this paper takes into account the impact of unmodeled dynamics. The challenge posed by the presence of unmodeled dynamics is effectively resolved by the derived fixed-time controller. (3) To mitigate the challenge of the "explosion of complexity" inherent in traditional backstepping methods, the controller design incorporates the use of the command filter technique. Furthermore, to compensate filtering errors, error compensation mechanisms are constructed.

This article follows this structure: Section 2 presents the problem statement and some preliminary information, while Section 3 offers a thorough review of the key findings regarding the design of the fixed-time control protocol. Moreover, the detailed design of control protocol and stability analysis are also displayed. Section 4 contains an demonstrative illustration for explanation purposes, and the article is concluded in Section 5.

2. Problem Formulation and Preliminaries

2.1. System Description

We examine a category of uncertain nonlinear systems featuring unaccounted dynamics and dynamic disturbances, described as follows: [43–46]:

$$\begin{cases} \dot{\varphi} = q(\varphi, x) \\ \dot{x}_1 = x_2 + f_1(x_1) + \Delta_1(x, \varphi, t), \\ \dot{x}_i = x_{i+1} + f_i(\bar{x}_i) + \Delta_i(x, \varphi, t), \quad 2 \le i \le n-1 \\ \dot{x}_n = f_n(x) + u + \Delta_n(x, \varphi, t), \\ y = x_1 \end{cases} \tag{1}$$

where $\bar{x}_i = [x_1, \ldots, x_i]^T \in \mathbb{R}^i$, $x = [x_1, \ldots, x_n]^T \in \mathbb{R}^n$ symbolizes state, $u \in \mathbb{R}$ symbolizes control input, $y \in \mathbb{R}$ symbolizes output. The φ-dynamics in (1) is the unmeasurable unmodeled dynamics, $\varphi \in \mathbb{R}$ is the unmeasured part of the state. $\Delta_i(\cdot)$ represents the dynamic disturbances, $q(\cdot)$ and $f_i(\cdot)$ denote unknown nonlinear functions, $i = 1, \ldots, n$.

Remark 1. *The primary focus of this article is on a category of nonlinear systems. Owing to the complexity of real-world engineering systems, their outputs are not always proportional to changes in inputs. For example, in hypersonic flight vehicles, the highly coupled and rapidly changing nature of the system's variables makes linear models no longer applicable. Therefore, nonlinear systems are often required to describe such engineering systems. Nonlinear systems are ubiquitous in engineering applications, such as spacecraft, nuclear power stations, and hypersonic flight vehicles. Overall, the study of nonlinear systems is essential for understanding and controlling complex systems in engineering applications. Alternatively, it is important to acknowledge that dynamic disturbances and unmodeled dynamics are inherent in real-world systems owing to various factors among others modeling inaccuracies, external disturbances, and measurement noise. For instance, the flight conditions encountered by hypersonic vehicles are intricate and cannot be fully replicated in a wind tunnel, thus rendering the constructed model incapable of precisely capturing the complex nonlinear dynamics of hypersonic flight vehicles. The existence of dynamic disturbances and unmodeled dynamics can significantly degrade system performance and even cause instability.*

Thus, it is essential to consider dynamic disturbances and unmodeled dynamics when designing the controller. Hence, for a more precise representation of nonlinear system with dynamic disturbances and unmodeled dynamics, we introduce $\Delta_i(\cdot)$ and φ dynamics into the nonlinear system model as (1). Moreover, this paper will address the control problem of nonlinear systems characterized by unmodeled dynamics and dynamic disturbances, as described in Equation (1).

Control objective: Developing a fixed-time adaptive fuzzy controller for systems (1) with unmodeled dynamics, aiming to achieve fixed-time stability and ensure convergence of the tracking error $y - y_d$ to a small region within a fixed time. Furthermore, all signals within the advanced system converge to a compact region near the origin within a fixed time.

Lemma 1 ([47]). *For $z \in R$ and $x \in R$, if $\imath_1 > 0$, $\jmath > 0$, and $\imath_2 > 0$, one has*

$$|z|^{\imath_1}|x|^{\imath_2} \leq \frac{\imath_1 \jmath}{\imath_1 + \imath_2}|z|^{\imath_1+\imath_2} + \frac{\imath_2 \jmath^{-\imath_1/\imath_2}}{\imath_1 + \imath_2}|x|^{\imath_1+\imath_2} \tag{2}$$

Lemma 2 ([48]). *If $\varrho_0 > 0$, we have*

$$0 \leq |\varrho_0| - \varrho_0 \tanh(\frac{\varrho_0}{\varsigma}) \leq 0.2785\varsigma \tag{3}$$

where $\varsigma > 0$.

Lemma 3 ([48]). *For $\zeta \in R$ and $x \in R$, we obtain*

$$\zeta x \leq \frac{l^{\varrho_1}}{\varrho_1}|\zeta|^{\varrho_1} + \frac{l}{b l^{\varrho_2}}|x|^{\varrho_2} \tag{4}$$

where $\varrho_1 > 1$, $l > 0$, $\varrho_2 > 1$, and $(\varrho_1 - 1)(\varrho_2 - 1) = 1$.

Lemma 4 ([49,50]). *If $x_i \in R$, $0 < s \leq 1$, $i = 1, 2, \ldots, n$*

$$\Big(\sum_{i=1}^{n}|x_i|\Big)^s \leq \sum_{i=1}^{n}|x_i|^s \leq n^{1-s}\Big(\sum_{i=1}^{n}|x_i|\Big)^s \tag{5}$$

Lemma 5 ([51]). *If $\vartheta \in R^+$ and $\vartheta > 1$, for $z, x \in R$, one has $|z + x|^{\vartheta} \leq 2^{\vartheta-1}|z^{\vartheta} + x^{\vartheta}|$.*

Lemma 6 ([52,53]). *For the system*

$$\dot{y} = h(y(t)), \;\; y(0) = 0, \;\; h(0) = 0 \tag{6}$$

Let us assume the existence of a Lyapunov function $V(y)$ and scalars ς_1, ς_2, m, $n \in \mathbb{R}^+$, $\varsigma_2 k > 1$, $\varsigma_1 k < 1$, and $0 < \vartheta < \infty$ such that

$$\dot{V}(y) \leq -(mV(y)^{\varsigma_1} + nV(y)^{\varsigma_2})^k + \vartheta, \quad y \in U_o \tag{7}$$

Following, the trajectory of system (6) is fixed-time steady and the solution satisfies

$$\Big\{\lim_{t \to T} y|V(y) \leq \min\big\{m^{-\frac{1}{\varsigma_1}}\big(\frac{\vartheta}{1-\theta^k}\big)^{\frac{1}{\varsigma_1 k}}, n^{-\frac{1}{\varsigma_2}}\big(\frac{\vartheta}{1-\theta^k}\big)^{\frac{1}{\varsigma_2 k}}\big\}\Big\} \tag{8}$$

where $0 < \theta < 1$. The settling time is defined by

$$T \leq \frac{1}{m^k \theta^k (1 - \varsigma_1 k)} + \frac{1}{n^k \theta^k (\varsigma_2 k - 1)} \tag{9}$$

Assumption 1. *We assume that both the tracking signal y_d and its nth order derivatives are continuous and have bounded magnitudes.*

Assumption 2. *The dynamic uncertainty satisfying*

$$|\Delta_i(x,\varphi,t)| \leq \psi_{i1}(|\bar{x}_i|) + \psi_{i2}(|\varphi|) \tag{10}$$

where $\psi_{i1}(\cdot)$ denotes smooth unknown functions, and $\psi_{i2}(\cdot)$ denotes strictly increasing functions, $i = 1,\ldots,n$.

Assumption 3. *For $\dot{\varphi} = q(\varphi, x)$, let us assume the existence of $V_\varphi(\varphi)$ such that*

$$\psi_1(|\varphi|) \leq V_\varphi(\varphi) \leq \psi_2(|\varphi|)\frac{\partial V_\varphi}{\partial \varphi} q(\varphi,x) \leq -c_0 V_\varphi(\varphi) + \Upsilon(|x_1|) + d_0 \tag{11}$$

where $c_0 > 0$ and $d_0 > 0$. $\Upsilon(\cdot)$ denotes a known function. Furthermore, $V_\varphi(\varphi)$ is a Lyapunov function which guarantees practical input-to-state stability.

The dynamic signal is formed by

$$\dot{\delta} = -\bar{c}\delta + \bar{\Upsilon}(|x_1|) + d_0, \delta(0) = \delta_0 \tag{12}$$

where $\bar{\Upsilon}(|x_1|) \geq \Upsilon(|x_1|)$, $c_0 > 0$, and $\bar{c} \in (0, c_0)$.

Lemma 7 ([54]). *Based on Assumption 3 and (12), one obtains*

$$V_z(z) \leq \delta(t) + B(t) \tag{13}$$

for all $t \geq 0$, where $B(t)$ denotes a function that is not negative and $B(t) = 0$ for $t \geq T_0$ with $T_0 = T_0(\bar{c}, \delta_0)$ is finite time.

Moreover, we have

$$\dot{\delta} = -\bar{c}\delta + x_1^2\Upsilon(x_1^2) + d_0, \delta(0) = \delta_0 \tag{14}$$

where $\bar{\Upsilon}(|x_1|) = x_1^2\Upsilon(x_1^2)$.

2.2. Fuzzy-Logic Systems

In this paper, FLSs are applied to estimate the unidentified nonlinear term. The FLSs with the following IF-THEN rules are constructed as

$$R^\imath : \text{if } x_1 \text{ is } F_1^\imath, \ x_2 \text{ is } F_2^\imath, \ \ldots, \text{ and } x_n \text{ is } F_n^\imath, \text{ then } y \text{ is } G^\imath \tag{15}$$

where $x = [x_1,\ldots,x_n]^T \in \mathbb{R}^n$ and $y \in \mathbb{R}$ represent the input and output of the FLSs, respectively. $F_i^\imath$ and $G^\imath$ denote the fuzzy sets associated with the membership functions $\mu_{F_i^\imath}(x_i)$ and $\mu_{G^\imath}(y)$, respectively, where $\imath = 1,2,\ldots,l$ and l is the number of rules. The FLSs can characterize

$$y(x) = \frac{\sum_{\imath=1}^{l}\theta_\imath \prod_{i=1}^{n}\varphi_{F_i^\imath}(x_i)}{\sum_{\imath=1}^{l}\prod_{i=1}^{n}\varphi_{F_i^\imath}(x_i)} \tag{16}$$

where $\theta_\imath = \max_{y\in\mathbb{R}}\varphi_{G^\imath}(y)$. Define the fuzzy basis functions

$$\varphi_\imath(x) = \frac{\prod_{i=1}^{n}\varphi_{F_i^\imath}(x_i)}{\sum_{\imath=1}^{l}\left[\prod_{i=1}^{n}\varphi_{F_i^\imath}(x_i)\right]} \tag{17}$$

Denote $\varphi^T(x) = [\varphi_1, \varphi_2, \ldots, \varphi_l]$. According to (17), we have $\varphi^T(x)\varphi(x) < 1$. FLSs may be reformulated as

$$y(x) = \varpi^T \varphi(x) \tag{18}$$

where $\varpi^T = [\theta_1, \theta_2, \ldots, \theta_l]$ is the adaptive parameter vector.

Furthermore, let $f(x)$ be a continuous function defined over a compact set. We have

$$\sup_{x \in \mathfrak{A}} |f(x) - \varpi^T \varphi(x)| \leq \varepsilon \tag{19}$$

where $\varepsilon > 0$.

Remark 2. *Referring to Section 2.2, it becomes evident that FLSs possess universal approximation capability. This implies that FLSs can estimate continuous function with desired level of accuracy. As a result, FLSs have been widely used to address the uncertainties in diverse control systems, as demonstrated in [6,7,37]. In this paper, we will use FLSs to estimate the unknown nonlinear terms. By using FLSs to estimate unknown nonlinear terms, we can avoid the need to know the exact form of these nonlinear terms. This makes our control scheme more robust to uncertainties in the system dynamics. Moreover, the implementation of FLSs is relatively straightforward, enhancing the practicality of our control scheme.*

3. Control Law Design

In this section, we will develop an adaptive fixed-time control strategy for nonlinear systems subject to unmodeled dynamics and uncertain functions. This will be accomplished by employing the command filtered technique, backstepping control approach, and fixed-time control method. By combining the command filter with the backstepping control approach, we can design an adaptive fixed-time control law that both robust to unmodeled dynamics and uncertain functions, and also avoids the "explosion of complexity" issue.

To streamline the control protocol development, the coordinate transformations are expressed as follows:

$$\begin{aligned} \rho_1 &= x_1 - y_d \\ \rho_i &= x_i - \pi_i, \quad i = 2, \ldots, n \end{aligned} \tag{20}$$

where y_d represents the desired tracking signal, while π_i represents the output of the fixed-time command filter (FTCF) and the virtual controllers a_i serving as the input.

The FTCF is designed as follows:

$$\begin{aligned} \dot{p}_{i,1} &= -b_{i,1} l_{i,1}(\sigma_i) + p_{i,2} \\ \dot{p}_{i,2} &= -b_{i,2} l_{i,2}(\sigma_i) \end{aligned} \tag{21}$$

where $l_{i,1}(\sigma_i) = \phi_i \text{sig}(\sigma_i)^{\beta_{i,1}} + (1 - \phi_i)\text{sig}(\sigma_i)^{\beta_{i,2}}$, $\text{sig}(\sigma_i)^{\beta_{i,1}} = \text{sign}(\sigma_i)|\sigma_i|^{\beta_{i,1}}$, $\text{sig}(\sigma_i)^{\beta_{i,2}} = \text{sign}(\sigma_i)|\sigma_i|^{\beta_{i,2}}$, $\text{sign}(\cdot)$ denotes the sign function, $\sigma_i = p_{i,1} - a_i$, $i = 1, \ldots, n$, $l_{i,2}(\sigma_i) = \phi_i \text{sig}(\sigma_i)^{2\beta_{i,1}-1} + (1 - \phi_i)\text{sig}(\sigma_i)^{2\beta_{i,2}-1}$, $b_{i,2} > 0$, $b_{i,1} > 0$, $\beta_{i,2} \in (1, 1+q)$, $0.5 < \beta_{i,1} < 1$, q is a small positive constant, a_i denotes the virtual controllers. Let $\pi_{i+1} = p_{i,1}$ and $\dot{\pi}_{i+1} = p_{i,2}$.

Lemma 8 ([55]). *For the FTCF defined by (21), if a_i satisfies $|\ddot{a}_i| \leq \nu$ with $\nu > 0$ for all $t \geq 0$, then for $\Im_{ri} > 0$, there exist $r_{i,2}, r_{i,1} > 0$ such that $|\pi_{i+1} - a_i| \leq \Im_{ri}$ is attained within fixed time. Furthermore, the $\dot{p}_{i,1}$, $\ddot{p}_{i,1}$, and $\ddot{p}_{i,2}$ are bounded within fixed time.*

Remark 3. *It is widely recognized that the issue of "explosion of complexity" may arise when using the traditional backstepping control technique. This problem arises from the fact that multiple differential operators are imposed on the virtual control signal a_{i-1} at each step of the backstepping method. This leads to a rapid increase in the complexity of the overall control law, making it difficult to implement and analyze. Fixed-time command filter is utilized to address this issue. The signal π_i*

is produced through a series of terms with the virtual control signal a_{i-1} serving as the input, which will be employed in the control strategy. Therefore, the frequent differentiation can be effectively mitigated. To counterbalance the filtering errors induced by FTCFs, a function transformation $\xi_i = \rho_i - \chi_i$ will be introduced for the design of the updated fixed-time controller. Therefore, the use of FTCFs in our control scheme allows to avoid the issue of "explosion of complexity" and design a control law that is both effective and implementable.

During the process of designing the fixed-time command filtered backstepping control, construct the virtual signals a_i as follows:

$$\begin{aligned} a_1 = &- \frac{\xi_1 \hat{\bar{\omega}}_1^2}{\|\xi_1 \varphi_1^T\| \hat{\bar{\omega}}_1 + \epsilon_{1,1}^*} - \frac{\xi_1 \hat{\bar{\varsigma}}_1^2}{|\xi_1| \hat{\bar{\varsigma}}_1 + \epsilon_{1,2}^*} - c_{1,1}\rho_1 + \dot{y}_d - c_{1,2}\text{sgn}^{\gamma_1}(\xi_1) \\ &- c_{1,3}\text{sgn}^{\gamma_2}(\xi_1) \end{aligned} \tag{22}$$

$$\begin{aligned} a_i = &- \frac{\xi_i \hat{\bar{\omega}}_i^2}{\|\xi_i \varphi_i^T\| \hat{\bar{\omega}}_i + \epsilon_{i,1}^*} - \frac{\xi_i \hat{\bar{\varsigma}}_i^2}{|\xi_i| \hat{\bar{\varsigma}}_i + \epsilon_{i,2}^*} - c_{i,1}\rho_i + \dot{\pi}_i - c_{i,2}\text{sgn}^{\gamma_1}(\xi_i) \\ &- c_{i,3}\text{sgn}^{\gamma_2}(\xi_i) - \xi_{i-1} \end{aligned} \tag{23}$$

$$\begin{aligned} a_n = &- \frac{\xi_n \hat{\bar{\omega}}_n^2}{\|\xi_n \varphi_n^T\| \hat{\bar{\omega}}_n + \epsilon_{n,1}^*} - \frac{\xi_n \hat{\bar{\varsigma}}_n^2}{|\xi_n| \hat{\bar{\varsigma}}_n + \epsilon_{n,2}^*} - c_{n,1}\rho_n + \dot{\pi}_n - \xi_{n-1} \\ &- c_{n,2}\text{sgn}^{\gamma_1}(\xi_n) - c_{n,3}\text{sgn}^{\gamma_2}(\xi_n) \end{aligned} \tag{24}$$

where $0 < \gamma_1 < 1$, $\gamma_2 > 1$, $i = 1, 2, \ldots, n$, $\text{sgn}^{\gamma_1}(\xi_i) = \text{sign}(\xi_i)|\xi_i|^{\gamma_1}$, $\text{sgn}^{\gamma_2}(\xi_i) = \text{sign}(\xi_i)|\xi_i|^{\gamma_2}$, $\text{sign}(\cdot)$ denotes the sign function, $\epsilon > 0$, $\epsilon_{i,1}^* = \epsilon * \text{sign}(|\xi_i| \hat{\bar{\varsigma}}_i)$, $\epsilon_{i,1}^* = \epsilon * \text{sign}(\|\xi_i \varphi_i^T\| \hat{\bar{\omega}}_i)$, $c_{i,2} > 1$, $c_{i,2} > 0$, and $c_{i,3} > 0$. The ω_i and φ_i are vectors related to FLSs, with definitions provided later. $\|\omega_i\| \leq \bar{\omega}_i$ with $\hat{\bar{\omega}}_i$ is the estimate of the parameter $\bar{\omega}_i$. $\hat{\bar{\varsigma}}_i$ is the estimate of the parameter $\bar{\varsigma}_i$. The definition of $\bar{\varsigma}_i$ will be given later. The parameter update laws of $\hat{\bar{\omega}}_i$ and $\hat{\bar{\varsigma}}_i$ are defined as follows:

$$\dot{\hat{\bar{\omega}}}_i = -2\lambda_{i,1}\hat{\bar{\omega}}_i - \ell_{i,1}\lambda_{i,1}\hat{\bar{\omega}}_i^{\gamma_2} + \lambda_{i,1}\|\xi_i \varphi_i^T\| \tag{25}$$

$$\dot{\hat{\bar{\varsigma}}}_i = -2\lambda_{i,2}\hat{\bar{\varsigma}}_i - \ell_{i,2}\lambda_{i,2}\hat{\bar{\varsigma}}_i^{\gamma_2} + \lambda_{i,2}|\xi_i| \tag{26}$$

where $\lambda_{i,1}$, $\lambda_{i,2}$, $\ell_{i,1}$, and $\ell_{i,1}$ are positive constants. The ξ_i signal represents the compensated tracking error and is described as

$$\xi_i = \rho_i - \chi_i \tag{27}$$

and χ_i is the error compensation signal and is described as

$$\dot{\chi}_1 = (\pi_2 - \alpha_1) - c_{1,1}\chi_1 + \chi_2 \tag{28}$$

$$\dot{\chi}_i = (\pi_{i+1} - \alpha_i) - c_{i,1}\chi_i + \chi_{i+1} \tag{29}$$

$$\dot{\chi}_n = -c_{n,1}\chi_n \tag{30}$$

with $\chi_i(0) = 0$.

Next, we will introduce the primary findings of this paper, and Figure 1 illustrates the schematic diagram of the proposed fixed-time scheme.

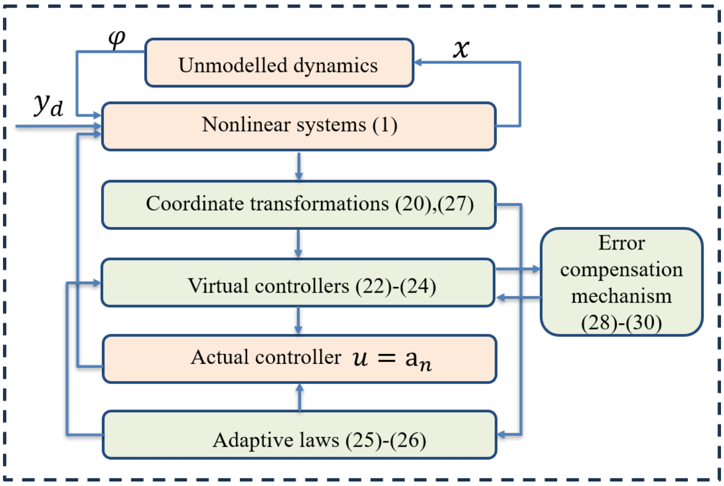

Figure 1. Schematic diagram of the fixed-time scheme.

Theorem 1. *Consider the nonlinear system (1) under Assumptions 1–3, the action of controller $u = a_n$, which is related to the virtual controllers (22)–(24) and the adaptive laws (25) and (26), resulting in practically fixed-time stable closed-loop system and the tracking error converging to a region within fixed-time.*

Proof. The proof will consist of two parts. Part A will establish the proof of the fixed-time tracking problem. Part B will demonstrate that χ_i remains bounded at all times.

Part A: The main results are derived through the following n recursive steps.

Step $i = 1$: The definition of the compensated tracking error signal ξ_1 is expressed as $\xi_1 = \rho_1 - \chi_1$. In view of (20), we have

$$\begin{aligned}\dot{\xi}_1 =& x_2 + \Delta_1(\varphi, x, t) + f_1(x_1) - \dot{y}_d - \dot{\chi}_1 \\ =& \pi_2 + \rho_2 + \Delta_1(\varphi, x, t) + f_1(x_1) - \dot{y}_d - \dot{\chi}_1\end{aligned} \tag{31}$$

Design the Lyapunov function as

$$V_1 = \frac{1}{2}\xi_1^2 + \frac{1}{2\lambda_{1,1}}\tilde{\bar{\varpi}}_1^2 + \frac{1}{2\lambda_{1,2}}\tilde{\bar{\varsigma}}_1^2 \tag{32}$$

where $\xi_1 = \rho_1 - \chi_1$ denotes the compensated tracking error for ρ_1, χ_1 denotes the compensation signal. $\tilde{\bar{\varpi}}_1 = \bar{\varpi}_1 - \hat{\bar{\varpi}}_1$, $\tilde{\bar{\varsigma}}_1 = \bar{\varsigma}_1 - \hat{\bar{\varsigma}}_1$ with $\hat{\bar{\varpi}}_1$ and $\hat{\bar{\varsigma}}_1$ being the estimates of $\bar{\varpi}_1$ and $\bar{\varsigma}_1$, respectively. $\bar{\varpi}_1$ and $\bar{\varsigma}_1$ will be defined later. $\lambda_{1,1} > 0$ and $\lambda_{1,2} > 0$ are constants.

Then, we have

$$\dot{V}_1 = \xi_1\dot{\xi}_1 + \frac{1}{\lambda_{11}}\tilde{\bar{\varpi}}_1\dot{\hat{\bar{\varpi}}}_1 + \frac{1}{\lambda_{12}}\tilde{\bar{\varsigma}}_1\dot{\hat{\bar{\varsigma}}}_1 \tag{33}$$

Invoking (31), $\xi_1\dot{\xi}_1$ is calculated as

$$\xi_1\dot{\xi}_1 = \xi_1(\pi_2 + \rho_2 + \Delta_1(\varphi, x, t) + f_1(x_1) - \dot{\chi}_1 - \dot{y}_d) \tag{34}$$

Subsequently, we obtain

$$\dot{V}_1 = \xi_1(\pi_2 + \rho_2 + \Delta_1(\varphi, x, t) - \dot{y}_d + f_1(x_1) - \dot{\chi}_1) + \frac{1}{\lambda_{11}}\tilde{\bar{\varpi}}_1\dot{\hat{\bar{\varpi}}}_1 + \frac{1}{\lambda_{12}}\tilde{\bar{\varsigma}}_1\dot{\hat{\bar{\varsigma}}}_1 \tag{35}$$

According to Assumption 2, the $\xi_1\Delta_{i,1}(\varphi, x, t)$ may be expressed as

$$\xi_1\Delta_1(\varphi, x, t) \le |\xi_1|\psi_{11}(|x_1|) + |\xi_1|\psi_{12}(|\varphi|) \tag{36}$$

According to Lemma 2, we have

$$|\xi_1|\psi_{11}(|x_1|) \leq \xi_1\hat{\psi}_{11}(x_1,\xi_1) + \acute{\tau}_{11} \tag{37}$$

where the terms $\hat{\psi}_{11}(x_1,\xi_1)$ and $\acute{\tau}_{11}$ is defined as

$$\hat{\psi}_{11}(x_1,\xi_1) = \psi_{11}(|x_1|)\tanh(\frac{\xi_1\psi_{11}(|x_1|)}{\tau_{11}}), \quad \acute{\tau}_{11} = 0.2785\tau_{11} > 0 \tag{38}$$

For the term $|\xi_1|\psi_{12}(|\varphi|)$, we have

$$\begin{aligned} |\xi_1|\psi_{12}(|\varphi|) \leq& |\xi_1|\psi_{12}(\eta_1^{-1}(\delta + B)) \\ \leq& |\xi_1|\psi_{12}(\eta_1^{-1}(2\delta)) + |\xi_1|\psi_{12}(\eta_1^{-1}(2B)) \\ \leq& \xi_1\hat{\psi}_{12}(\xi_1,\delta) + \acute{\tau}_{12} + \frac{1}{4}\xi_1^2 + d_1(t) \end{aligned} \tag{39}$$

with

$$\begin{aligned} &\acute{\tau}_{12} = 0.2785\tau_{12} > 0, \quad d_1(t) = \left(\psi_{12}(\eta_1^{-1}(2B))\right)^2 \\ &\hat{\psi}_{12}(\xi_1,\delta) = \psi_{12}(\eta_1^{-1}(2\delta))\tanh(\frac{\xi_1\psi_{12}(\eta_1^{-1}(2\delta))}{\tau_{12}}) \end{aligned} \tag{40}$$

Substituting (36)–(39) into (35), we have

$$\begin{aligned} \dot{V}_1 \leq& \xi_1(\pi_2 + \rho_2 + f_1(x_1) - \dot{y}_d - \dot{\chi}_1) + \frac{1}{\lambda_{11}}\tilde{\varpi}_1\dot{\hat{\varpi}}_1 + \frac{1}{\lambda_{12}}\tilde{\xi}_1\dot{\hat{\xi}}_1 \\ &+ \xi_1\hat{\psi}_{11}(x_1,\xi_1) + \acute{\tau}_{11} + \xi_1\hat{\psi}_{12}(\xi_1,\delta) + \acute{\tau}_{12} + \frac{1}{4}\xi_1^2 + d_1(t) \\ \leq& \xi_1(\pi_2 - \alpha_1 + \alpha_1 + \rho_2 + \Phi_1(X_1) - \dot{y}_d - \dot{\chi}_1) + \frac{1}{\lambda_{11}}\tilde{\varpi}_1\dot{\hat{\varpi}}_1 + \frac{1}{\lambda_{12}}\tilde{\xi}_1\dot{\hat{\xi}}_1 \\ &+ \acute{\tau}_{11} + \acute{\tau}_{12} + d_1(t) \end{aligned} \tag{41}$$

where $\Phi_1(X_1) = \hat{\psi}_{11}(x_1,\xi_1) + \hat{\psi}_{12}(\xi_1,\delta) + f_1(x_1) + \frac{1}{4}\xi_1^2$, $X_1 = [x_1,\xi_1,\delta]^T + \frac{1}{4}\xi_1$.

The FLSs function as an estimator to identify $\Phi_1(X_1)$ in such a way that

$$\Phi_1(X_1) = \varphi_1^T(X_1)\varpi_1 + \varepsilon_1 \tag{42}$$

where ε_1 denotes the approximation error, with $\bar{\xi}_1 \geq |\varepsilon_1|$ as its upper bound.

Then, we have

$$\begin{aligned} \dot{V}_1 \leq& \xi_1(\pi_2 - \alpha_1 + \alpha_1 + \rho_2 + \varphi_1^T(X_1)\varpi_1 + \varepsilon_1 - \dot{y}_d - \dot{\chi}_1) + \frac{1}{\lambda_{11}}\tilde{\varpi}_1\dot{\hat{\varpi}}_1 + \frac{1}{\lambda_{12}}\tilde{\xi}_1\dot{\hat{\xi}}_1 \\ &+ \acute{\tau}_{12} + \acute{\tau}_{11} + d_1(t) \end{aligned} \tag{43}$$

Substituting (22), (25), (26), and (28) into (43), we have

$$\begin{aligned} \dot{V}_1 \leq& \xi_1(\varphi_1^T\varpi_1 + \varepsilon_1 + \rho_2 - \chi_2 - c_{1,1}\xi_1 - \frac{\xi_1\hat{\varpi}_1^2}{\|\xi_1\varphi_1^T\|\hat{\varpi}_1 + \epsilon_{1,1}^*} - \frac{\xi_1\hat{\xi}_1^2}{|\xi_1|\hat{\xi}_1 + \epsilon_{1,2}^*} - c_{1,2}\mathrm{sgn}^{\gamma_1}(\xi_1) \\ &- c_{1,3}\mathrm{sgn}^{\gamma_2}(\xi_1)) + \frac{1}{\lambda_{1,1}}\tilde{\xi}_1\dot{\hat{\xi}}_1 + \frac{1}{\lambda_{1,2}}\tilde{\varpi}_1\dot{\hat{\varpi}}_1 + \acute{\tau}_{11} + \acute{\tau}_{12} + d_1(t) \\ \leq& \|\xi_1\varphi_1^T\|\varpi_1 + |\xi_1||\varepsilon_1| + \xi_1(\rho_2 - \chi_2) - \frac{\xi_1^2\hat{\varpi}_1^2}{\|\xi_1\varphi_1^T\|\hat{\varpi}_1 + \epsilon_{1,1}^*} - \frac{\xi_1^2\hat{\xi}_1^2}{|\xi_1|\hat{\xi}_1 + \epsilon_{1,2}^*} \\ &- c_{1,2}\mathrm{sgn}^{\gamma_1+1}(\xi_1) - c_{1,3}\mathrm{sgn}^{\gamma_2+1}(\xi_1) + 2\tilde{\varpi}_1\hat{\varpi}_1 - \tilde{\varpi}_1\|\xi_1\varphi_1^T\| + 2\tilde{\xi}_1\hat{\xi}_1 - \tilde{\xi}_1|\xi_1| \\ &- c_{1,1}\xi_1^2 + \ell_{1,1}\tilde{\varpi}_1\hat{\varpi}_1^{\gamma_2} + \ell_{1,2}\tilde{\xi}_1\hat{\xi}_1^{\gamma_2} + \acute{\tau}_{11} + \acute{\tau}_{12} + d_1(t) \end{aligned} \tag{44}$$

By using the property $\varphi_1^T\varphi_1 \leq 1$, we can obtain

$$\|\xi_1\varphi_1^T\|\bar{\omega}_1 - \frac{\xi_1^2\hat{\bar{\omega}}_1^2}{\|\xi_1\varphi_1^T\|\hat{\bar{\omega}}_1 + \epsilon_{1,1}^*} - \tilde{\bar{\omega}}_1\|\xi_1\varphi_1^T\| = \frac{\|\xi_1\varphi_1^T\|\hat{\bar{\omega}}_1\epsilon_{1,1}^*}{\|\xi_1\varphi_1^T\|\hat{\bar{\omega}}_1 + \epsilon_{1,1}^*} \leq \epsilon \tag{45}$$

Similar to (45), we have

$$|\xi_1||\varepsilon_1| - \frac{\xi_1^2\hat{\bar{\varsigma}}_1^2}{|\xi_1|\hat{\bar{\varsigma}}_1 + \epsilon_{1,2}^*} - \tilde{\bar{\varsigma}}_1|\xi_1| \leq \frac{\xi_1\hat{\bar{\varsigma}}_1\epsilon_{1,2}^*}{|\xi_1|\hat{\bar{\varsigma}}_1 + \epsilon_{1,2}^*} \leq \epsilon \tag{46}$$

Combine (45) and (46) with (44), it yields

$$\begin{aligned}\dot{V}_1 \leq &\xi_1(\rho_2 - \chi_2) - c_{1,2}\mathrm{sgn}^{\gamma_1+1}(\xi_1) - c_{1,3}\mathrm{sgn}^{\gamma_2+1}(\xi_1) + 2\tilde{\bar{\varsigma}}_1\hat{\bar{\varsigma}}_1 + 2\tilde{\bar{\omega}}_1\hat{\bar{\omega}}_1 \\ &+ \ell_{1,1}\tilde{\bar{\omega}}_1\hat{\bar{\omega}}_1^{\gamma_i} + \ell_{1,2}\tilde{\bar{\varsigma}}_1\hat{\bar{\varsigma}}_1^{\gamma_i} - c_{1,1}\xi_1^2 + 2\epsilon + \acute{\tau}_{11} + \acute{\tau}_{12} + d_1(t)\end{aligned} \tag{47}$$

Step $i = 2, \ldots, n-1$: Choose the Lyapunov function candidate as

$$V_i = V_{i-1} + \frac{1}{2}\xi_i^2 + \frac{1}{2\lambda_{i,1}}\tilde{\bar{\omega}}_i^2 + \frac{1}{2\lambda_{i,2}}\tilde{\bar{\varsigma}}_i^2 \tag{48}$$

where $\xi_i = \rho_i - \chi_i$ denotes the compensated tracking error for ρ_i, χ_i denotes the compensation signal. $\tilde{\bar{\omega}}_i = \bar{\omega}_i - \hat{\bar{\omega}}_i$, $\tilde{\bar{\varsigma}}_i = \bar{\varsigma}_i - \hat{\bar{\varsigma}}_i$, where $\hat{\bar{\omega}}_i$ and $\hat{\bar{\varsigma}}_i$ are the estimates of $\bar{\omega}_i$ and $\bar{\varsigma}_i$. $\bar{\omega}_i$, and $\bar{\varsigma}_i$ will be defined later. $\lambda_{i,1} > 0$ and $\lambda_{i,2} > 0$ are constants. Based on (48), we have

$$\begin{aligned}\dot{V}_i =&\dot{V}_{i-1} + \frac{1}{\lambda_{i,1}}\tilde{\bar{\omega}}_i\dot{\hat{\bar{\omega}}}_i + \xi_i\dot{\xi}_i + \frac{1}{\lambda_{i,2}}\tilde{\bar{\varsigma}}_i\dot{\hat{\bar{\varsigma}}}_i \\ =&\dot{V}_{i-1} + \xi_i(\pi_{i+1} + \rho_{i+1} + f_i(\bar{x}_i) + \Delta_i(\varphi, x, t) - \dot{\pi}_i - \dot{\chi}_i) + \frac{1}{\lambda_{i,1}}\tilde{\bar{\varsigma}}_i 2\dot{\hat{\bar{\varsigma}}}_i + \frac{1}{\lambda_{i,2}}\tilde{\bar{\omega}}_i\dot{\hat{\bar{\omega}}}_i\end{aligned} \tag{49}$$

According to Assumption 2, the $\xi_i\Delta_i(\varphi, x, t)$ may be expressed as

$$\xi_i\Delta_i(\varphi, x, t) \leq |\xi_i|\psi_{i1}(|x_i|) + |\xi_i|\psi_{i2}(|\varphi|) \tag{50}$$

According to Lemma 2, we have

$$|\xi_i|\psi_{i1}(|x_i|) \leq \xi_i\hat{\psi}_{i1}(x_i, \xi_i) + \acute{\tau}_{i1} \tag{51}$$

where

$$\hat{\psi}_{i1}(x_i, \xi_i) = \psi_{i1}(|x_i|)\tanh(\frac{\xi_i\psi_{i1}(|x_i|)}{\tau_{i1}}), \ \ \acute{\tau}_{i1} = 0.2785\tau_{i1} > 0 \tag{52}$$

For the term $|\xi_i|\psi_{i2}(|\varphi|)$, one obtains

$$\begin{aligned}|\xi_i|\psi_{i2}(|\varphi|) \leq&|\xi_i|\psi_{i2}(\eta_1^{-1}(\delta + B)) \\ \leq&|\xi_i|\psi_{i2}(\eta_1^{-1}(2\delta)) + |\xi_i|\psi_{i2}(\eta_1^{-1}(2B)) \\ \leq&\xi_i\hat{\psi}_{i2}(\xi_i, \delta) + \acute{\tau}_{i2} + \frac{1}{4}\xi_i^2 + d_i(t)\end{aligned} \tag{53}$$

with

$$\begin{aligned}d_i(t) = &\left(\psi_{i2}(\eta_1^{-1}(2B))\right)^2, \ \ \acute{\tau}_{i2} = 0.2785\tau_{i2} > 0 \\ \hat{\psi}_{i2}(\xi_i, \delta) = &\psi_{i2}(\eta_1^{-1}(2\delta))\tanh(\frac{\xi_i\psi_{i2}(\eta_1^{-1}(2\delta))}{\tau_{i2}})\end{aligned} \tag{54}$$

Substituting (50)–(53) into (49), we have

$$\begin{aligned}\dot{V}_i \leq & \dot{V}_{i-1} + \xi_i(\pi_{i+1} + \rho_{i+1} + f_i(\bar{x}_i) - \dot{\chi}_i - \dot{\pi}_i) + \frac{1}{\lambda_{i1}}\tilde{\bar{\omega}}_i\dot{\hat{\bar{\omega}}}_i + \frac{1}{\lambda_{i2}}\tilde{\bar{\xi}}_i\dot{\hat{\xi}}_i \\ & + \xi_i\hat{\psi}_{i1}(x_i,\xi_i) + \acute{\tau}_{i1} + \xi_i\hat{\psi}_{i2}(\xi_i,\delta) + \acute{\tau}_{i2} + \frac{1}{4}\xi_i^2 + d_i(t) \\ \leq & \dot{V}_{i-1} + \xi_i(\pi_{i+1} - \alpha_i + \alpha_i + \rho_{i+1} + \Phi_i(X_i) - \dot{\chi}_i - \dot{\pi}_i) + \frac{1}{\lambda_{i1}}\tilde{\bar{\omega}}_i\dot{\hat{\bar{\omega}}}_i + \frac{1}{\lambda_{i2}}\tilde{\bar{\xi}}_i\dot{\hat{\xi}}_i \\ & + \acute{\tau}_{i1} + \acute{\tau}_{i2} + d_i(t)\end{aligned} \tag{55}$$

where $\Phi_i(X_i) = \hat{\psi}_{i1}(x_i,\xi_i) + \hat{\psi}_{i2}(\xi_i,\delta) + f_i(\bar{x}_i) + \frac{1}{4}\xi_i$, $X_i = [x_i,\xi_i,\delta]^T$.

The FLSs function as an estimator to ascertain $\Phi_i(X_i)$ in such a way that

$$\Phi_i(X_i) = \varphi_i^T(X_i)\varpi_i + \varepsilon_i \tag{56}$$

where ε_i is the approximate error with $\bar{\varsigma}_i \geq |\varepsilon_i|$ as its upper bound.

Then, we have

$$\begin{aligned}\dot{V}_i \leq & \dot{V}_{i-1} + \xi_1(\pi_{i+1} - \alpha_i + \alpha_i + \rho_{i+1} + \varphi_i^T(X_i)\varpi_i + \varepsilon_i - \dot{\pi}_i - \dot{\chi}_i) + \frac{1}{\lambda_{i1}}\tilde{\bar{\omega}}_i\dot{\hat{\bar{\omega}}}_i \\ & + \frac{1}{\lambda_{i2}}\tilde{\bar{\xi}}_i\dot{\hat{\xi}}_i + \acute{\tau}_{i1} + \acute{\tau}_{i2} + d_i(t)\end{aligned} \tag{57}$$

Substituting (23), (25), (26), (29) into (57), one obtains

$$\begin{aligned}\dot{V}_i = & \dot{V}_{i-1} + \xi_i(\varphi_i^T\varpi_i + \varepsilon_i + \rho_{i+1} - \chi_{i+1} - c_{i,1}\xi_i - \frac{\xi_i\hat{\bar{\omega}}_i^2}{\|\xi_i\varphi_i^T\|\hat{\bar{\omega}}_i + \epsilon_{i,1}^*} - \frac{\xi_i\hat{\xi}_i^2}{|\xi_i|\hat{\xi}_i + \epsilon_{i,2}^*} \\ & - c_{i,2}\mathrm{sgn}^{\gamma_1}(\xi_i) - c_{i,3}\mathrm{sgn}^{\gamma_2}(\xi_i)) - \xi_{i-1}\xi_i + \frac{1}{\lambda_{i,1}}\tilde{\bar{\xi}}_i\dot{\hat{\xi}}_i \\ & + \frac{1}{\lambda_{i,2}}\tilde{\bar{\omega}}_i\dot{\hat{\bar{\omega}}}_i + \acute{\tau}_{i1} + \acute{\tau}_{i2} + d_i(t) \\ \leq & \dot{V}_{i-1} + \|\xi_i\varphi_i^T\|\bar{\omega}_i + |\xi_i||\varepsilon_i| + \xi_i(\rho_{i+1} - \chi_{i+1}) - \frac{\xi_i^2\hat{\bar{\omega}}_i^2}{\|\xi_i\varphi_i^T\|\hat{\bar{\omega}}_i + \epsilon_{i,1}^*} \\ & - \frac{\xi_i^2\hat{\xi}_2^i}{|\xi_i|\hat{\xi}_i + \epsilon_{i,2}^*} - c_{i,2}\mathrm{sgn}^{\gamma_1+1}(\xi_i) - c_{i,3}\mathrm{sgn}^{\gamma_2+1}(\xi_i) + 2\tilde{\bar{\omega}}_i\hat{\bar{\omega}}_i \\ & - \tilde{\bar{\omega}}_i\|\xi_i\varphi_i^T\| + 2\tilde{\bar{\xi}}_i\hat{\xi}_i - \tilde{\bar{\xi}}_i|\xi_i| - c_{i,1}\xi_2^2 - \xi_{i-1}\xi_i + \ell_{i,1}\tilde{\bar{\omega}}_i\hat{\bar{\omega}}_i^{\gamma_2} \\ & + \ell_{i,2}\tilde{\bar{\xi}}_i\hat{\xi}_i^{\gamma_2} + \acute{\tau}_{i1} + \acute{\tau}_{i2} + d_i(t)\end{aligned} \tag{58}$$

By using the property $\varphi_i^T\varphi_i \leq 1$, we can obtain

$$\|\xi_i\varphi_i^T\|\bar{\omega}_i - \frac{\xi_i^2\hat{\bar{\omega}}_i^2}{\|\xi_i\varphi_i^T\|\hat{\bar{\omega}}_i + \epsilon_{i,1}^*} - \tilde{\bar{\omega}}_i\|\xi_i\varphi_i^T\| = \frac{\|\xi_i\varphi_i^T\|\hat{\bar{\omega}}_i\epsilon_{i,1}^*}{\|\xi_i\varphi_i^T\|\hat{\bar{\omega}}_i + \epsilon_{i,1}^*} \leq \epsilon \tag{59}$$

Similar to (59), we have

$$|\xi_i||\varepsilon_i| - \frac{\xi_i^2\hat{\xi}_i^2}{|\xi_i|\hat{\xi}_i + \epsilon_{i,2}^*} - \tilde{\bar{\xi}}_i|\xi_i| \leq \frac{\xi_i\hat{\xi}_i\epsilon_{i,2}^*}{|\xi_i|\hat{\xi}_i + \epsilon_{i,2}^*} \leq \epsilon \tag{60}$$

Combine (59) and (60) with (58), we have

$$\begin{aligned}\dot{V}_i \leq & \dot{V}_{i-1} + \xi_i(\rho_{i+1} - \chi_{i+1}) - c_{i,2}\text{sgn}^{\gamma_1+1}(\xi_i) - c_{i,3}\text{sgn}^{\gamma_2+1}(\xi_i) + 2\tilde{\bar{\xi}}_i\hat{\bar{\xi}}_i + 2\tilde{\bar{\varpi}}_i\hat{\bar{\varpi}}_i \\ & - c_{i,1}\xi_i^2 + 2\epsilon - \xi_{i-1}\xi_i + \ell_{i,1}\tilde{\bar{\varpi}}_i\hat{\bar{\varpi}}_i^{\gamma_2} + \ell_{i,2}\tilde{\bar{\xi}}_i\hat{\bar{\xi}}_i^{\gamma_2} + \acute{\tau}_{i1} + \acute{\tau}_{i2} + d_i(t) \\ \leq & - c_{i,2}\sum_{i=1}^{n-1}\text{sgn}^{\gamma_1+1}(\xi_i) - c_{i,3}\sum_{i=1}^{n-1}\text{sgn}^{\gamma_2+1}(\xi_i) + 2\sum_{i=1}^{n-1}\tilde{\bar{\xi}}_i\hat{\bar{\xi}}_i + 2\sum_{i=1}^{n-1}\tilde{\bar{\varpi}}_i\hat{\bar{\varpi}}_i \\ & - c_{i,1}\sum_{i=1}^{n-1}\xi_i^2 + \xi_i(\rho_{i+1} - \chi_{i+1}) + 2i\epsilon + \ell_{i,1}\sum_{i=1}^{n-1}\tilde{\bar{\varpi}}_i\hat{\bar{\varpi}}_i^{\gamma_2} \\ & + \ell_{i,2}\sum_{i=1}^{n-1}\tilde{\bar{\xi}}_i\hat{\bar{\xi}}_i^{\gamma_2} + \sum_{i=1}^{n-1}\acute{\tau}_{i1} + \sum_{i=1}^{n-1}\acute{\tau}_{i2} + \sum_{i=1}^{n-1}d_i(t)\end{aligned} \tag{61}$$

Step n: The Lyapunov function is defined as

$$V_n = V_{n-1} + \frac{1}{2}\xi_n^2 + \frac{1}{2\lambda_{n,1}}\tilde{\bar{\varpi}}_n^2 + \frac{1}{2\lambda_{n,2}}\tilde{\bar{\xi}}_n^2 \tag{62}$$

where $\xi_n = \rho_n - \chi_n$ denotes the compensated tracking error for z_n, χ_n denotes the compensation signal. $\tilde{\bar{\varpi}}_n = \bar{\varpi}_n - \hat{\bar{\varpi}}_n$, $\tilde{\bar{\xi}}_n = \bar{\varsigma}_n - \hat{\bar{\xi}}_n$, where $\hat{\bar{\varpi}}_n$ and $\hat{\bar{\xi}}_n$ represent the estimates of $\bar{\varpi}_n$ and $\bar{\varsigma}_n$. $\bar{\varpi}_n$, and $\bar{\varsigma}_n$ will be provided at a later stage, $\lambda_{n,1} > 0$, and $\lambda_{n,2} > 0$ are constants. Furthermore, we have

$$\begin{aligned}\dot{V}_n = & \dot{V}_{n-1} + \xi_n\dot{\xi}_n + \frac{1}{\lambda_{n,2}}\tilde{\bar{\xi}}_n\dot{\hat{\bar{\xi}}}_n + \frac{1}{\lambda_{n,1}}\tilde{\bar{\varpi}}_n\dot{\hat{\bar{\varpi}}}_n \\ = & \dot{V}_{n-1} + \xi_n(f_n(x) + \Delta_n(\varphi, x, t) + u - \dot{\chi}_n - \dot{\pi}_n) + \frac{1}{\lambda_{n,1}}\tilde{\bar{\xi}}_n\dot{\hat{\bar{\xi}}}_n + \frac{1}{\lambda_{n,2}}\tilde{\bar{\varpi}}_n\dot{\hat{\bar{\varpi}}}_n\end{aligned} \tag{63}$$

According to 2, we have

$$\xi_n\Delta_n(\varphi, x, t) \leq |\xi_n|\psi_{n1}(|x_n|) + |\xi_n|\psi_{n2}(|\varphi|) \tag{64}$$

According to Lemma 2, we have

$$|\xi_n|\psi_{n1}(|x_n|) \leq e_n\hat{\psi}_{n1}(x_n, \xi_n) + \acute{\tau}_{n1} \tag{65}$$

where

$$\hat{\psi}_{n1}(x_n, e_n) = \psi_{n1}(|x_n|)\tanh(\frac{e_n\psi_{n1}(|x_n|)}{\tau_{n1}}), \quad \acute{\tau}_{n1} = 0.2785\tau_{n1} > 0 \tag{66}$$

For the term $|\xi_n|\psi_{n2}(|\varphi|)$, one obtains

$$\begin{aligned}|\xi_n|\psi_{n2}(|\varphi|) \leq & |\xi_n|\psi_{n2}(\eta_1^{-1}(\delta + B)) \\ \leq & |\xi_n|\psi_{n2}(\eta_1^{-1}(2\delta)) + |\xi_n|\psi_{n2}(\eta_1^{-1}(2B)) \\ \leq & e_n\hat{\psi}_{n2}(\xi_n, \delta) + \acute{\tau}_{n2} + \frac{1}{4}\xi_n^2 + d_n(t)\end{aligned} \tag{67}$$

with

$$\begin{aligned}d_n(t) &= \left(\psi_{n2}(\eta_1^{-1}(2B))\right)^2, \quad \acute{\tau}_{n2} = 0.2785\tau_{n2} > 0 \\ \hat{\psi}_{n2}(\xi_n, \varphi) &= \psi_{n2}(\eta_1^{-1}(2\delta))\tanh(\frac{\xi_n\psi_{n2}(\eta_1^{-1}(2\delta))}{\tau_{n2}})\end{aligned} \tag{68}$$

Then,

$$\begin{aligned}\dot{V}_n \leq & \dot{V}_{n-1} + \xi_n(u + f_n(\xi_n) - \dot{\rho}_n - \dot{\pi}_n) + \frac{1}{\lambda_{n2}}\tilde{\bar{\xi}}_n\dot{\hat{\bar{\xi}}}_n + \frac{1}{\lambda_{n1}}\tilde{\bar{\varpi}}_n\dot{\hat{\bar{\varpi}}}_n \\ & + \xi_n\hat{\psi}_{n1}(x_n, e_n) + \dot{\tau}_{n1} + \xi_n\hat{\psi}_{n2}(\xi_n, \delta) + \dot{\tau}_{n2} + \frac{1}{4}\xi_n^2 + d_n(t) \\ \leq & \dot{V}_{n-1} + \xi_n(u + \Phi_n(X_n) - \dot{\pi}_n - \dot{\rho}_n) + \frac{1}{\lambda_{n1}}\tilde{\bar{\varpi}}_n\dot{\hat{\bar{\varpi}}}_n \\ & + \dot{\tau}_{n2} + \dot{\tau}_{n1} + d_n(t) + \frac{1}{\lambda_{n2}}\tilde{\bar{\xi}}_n\dot{\hat{\bar{\xi}}}_n\end{aligned} \tag{69}$$

where $\Phi_n(X_n) = \hat{\psi}_{n1}(x_n, \xi_n) + \hat{\psi}_{n2}(\xi_n, \delta) + f_n(x_n) + \frac{1}{4}\xi_n$, $X_n = [x_n, \xi_n, \delta]^T$.

The FLSs function as an estimator to ascertain $\Phi_n(X_n)$ in such a way that

$$\Phi_n(X_n) = \varphi_n^T(X_n)\varpi_n + \varepsilon_n \tag{70}$$

where ε_n is the approximate error with $\bar{\xi}_n \geq |\varepsilon_n|$ as its upper bound.

Based on (70), we can obtain

$$\begin{aligned}\dot{V}_n \leq & \dot{V}_{n-1} + \xi_n(\varphi_n^T\varpi_n + \varepsilon_n + u - \dot{\chi}_n - \dot{\pi}_n) + \frac{1}{\lambda_{n,1}}\tilde{\bar{\xi}}_n\dot{\hat{\bar{\xi}}}_n + \frac{1}{\lambda_{n,2}}\tilde{\bar{\varpi}}_n\dot{\hat{\bar{\varpi}}}_n \\ & + \dot{\tau}_{n2} + \dot{\tau}_{n1} + d_n(t) + \frac{1}{4}e_n^2\end{aligned} \tag{71}$$

Substituting (24), (25), (26), (30) into (71), one has

$$\begin{aligned}\dot{V}_n = & \dot{V}_{n-1} + \xi_n\Big(\varphi_n^T\varpi_n + \varepsilon_n - \xi_{n-1} - c_{n,1}\xi_n - \frac{\xi_n\hat{\bar{\varpi}}_n^2}{\|\xi_n\varphi_n^T\|\hat{\bar{\varpi}}_n + \epsilon_{n,1}^*} - \frac{\xi_n\hat{\bar{\xi}}_n^2}{|\xi_n|\hat{\bar{\xi}}_n + \epsilon_{n,2}^*} \\ & - c_{n,2}\mathrm{sgn}^{\gamma_1}(\xi_n) - c_{n,3}\mathrm{sgn}^{\gamma_2}(\xi_n)\Big) + \frac{1}{\lambda_{n,1}}\tilde{\bar{\xi}}_n\dot{\hat{\bar{\xi}}}_n + \frac{1}{\lambda_{2,2}}\tilde{\bar{\varpi}}_n\dot{\hat{\bar{\varpi}}}_n \\ \leq & \dot{V}_{n-1} + |\xi_n||\varepsilon_n| - \xi_n\xi_{n-1} - \frac{\xi_n^2\hat{\bar{\varpi}}_n^2}{\|\xi_n\varphi_n^T\|\hat{\bar{\varpi}}_n + \epsilon_{n,1}^*} - \frac{\xi_n^2\hat{\bar{\xi}}_n^2}{|\xi_n|\hat{\bar{\xi}}_n + \epsilon_{n,2}^*} \\ & - c_{n,2}\mathrm{sgn}^{\gamma_1+1}(\xi_n) - c_{n,3}\mathrm{sgn}^{\gamma_2+1}(\xi_n) + \tilde{\bar{\varpi}}_n\hat{\bar{\varpi}}_n - \tilde{\bar{\varpi}}_n\|\xi_n\varphi_n^T\| + \tilde{\bar{\xi}}_n\hat{\bar{\xi}}_n \\ & - \tilde{\bar{\xi}}_n|\xi_n| - c_{n,1}\xi_n^2 + \ell_{n,1}\tilde{\bar{\varpi}}_n\hat{\bar{\varpi}}_n^{\gamma_2} + \ell_{n,2}\tilde{\bar{\xi}}_n\hat{\bar{\xi}}_n^{\gamma_2} + \|\xi_n\varphi_n^T\|\bar{\varpi}_n \\ \leq & \dot{V}_{n-1} - \xi_n\xi_{n-1} - c_{n,2}\mathrm{sgn}^{\gamma_1+1}(\xi_n) - c_{n,3}\mathrm{sgn}^{\gamma_2+1}(\xi_n) + 2\tilde{\bar{\xi}}_n\hat{\bar{\xi}}_n \\ & + 2\tilde{\bar{\varpi}}_n\hat{\bar{\varpi}}_n - c_{n,1}\xi_n^2 + 2\epsilon + \ell_{n,1}\tilde{\bar{\varpi}}_n\hat{\bar{\varpi}}_n^{\gamma_2} + \ell_{n,2}\tilde{\bar{\xi}}_n\hat{\bar{\xi}}_n^{\gamma_2} \\ \leq & - c_{i,2}\sum_{i=1}^n \mathrm{sgn}^{\gamma_1+1}(\xi_i) - c_{i,3}\sum_{i=1}^n \mathrm{sgn}^{\gamma_2+1}(\xi_i) + \sum_{i=1}^n 2\tilde{\bar{\xi}}_i\hat{\bar{\xi}}_i + \sum_{i=1}^n 2\tilde{\bar{\varpi}}_i\hat{\bar{\varpi}}_i \\ & - \sum_{i=1}^n c_{i,1}\xi_i^2 + 2n\epsilon + \sum_{i=1}^n \ell_{i,1}\tilde{\bar{\varpi}}_i\hat{\bar{\varpi}}_i^{\gamma_2} + \sum_{i=1}^n \ell_{i,2}\tilde{\bar{\xi}}_i\hat{\bar{\xi}}_i^{\gamma_2}\end{aligned} \tag{72}$$

Noting that

$$\|\xi_n\varphi_n^T\|\varpi_n - \frac{\xi_n^2\hat{\bar{\varpi}}_n^2}{\|\xi_n\varphi_n^T\|\hat{\bar{\varpi}}_n + \epsilon_{n,1}^*} - \tilde{\bar{\varpi}}_n\|\xi_n\varphi_n^T\| = \frac{\|\xi_n\varphi_n^T\|\hat{\bar{\varpi}}_n\epsilon_{n,1}^*}{\|\xi_n\varphi_n^T\|\hat{\bar{\varpi}}_n + \epsilon_{n,1}^*} \leq \epsilon \tag{73}$$

$$|\xi_n||\varepsilon_n| - \frac{\xi_n^2\hat{\bar{\xi}}_n^2}{|\xi_n|\hat{\bar{\xi}}_n + \epsilon_{n,2}^*} - \tilde{\bar{\xi}}_n|\xi_n| \leq \frac{\xi_n\hat{\bar{\xi}}_n\epsilon_{n,2}^*}{|\xi_n|\hat{\bar{\xi}}_n + \epsilon_{n,2}^*} \leq \epsilon \tag{74}$$

where the property $\varphi_n^T\varphi_n \leq 1$ is applied.

For $\bar{o}_i > 1/2$ and $o_i > 1/2$, one has

$$\tilde{\bar{\varpi}}_i\hat{\bar{\varpi}}_i \leq -\frac{2o_i - 1}{2o_i}\tilde{\bar{\varpi}}_i^2 + \frac{o_i}{2}\bar{\varpi}_i^2, \;\; \tilde{\bar{\xi}}_i\hat{\bar{\xi}}_i \leq -\frac{2\bar{o}_i - 1}{2\bar{o}_i}\tilde{\bar{\xi}}_i^2 + \frac{\bar{o}_i}{2}\bar{\xi}_i^2 \tag{75}$$

From (72) and (75), one has

$$\begin{aligned}\dot{V}_n \leq & -c_{i,2}\sum_{i=1}^{n}|\xi_i|^{\gamma_1+1} - c_{i,3}\sum_{i=1}^{n}|\xi_i|^{\gamma_2+1} - \sum_{i=1}^{n}\frac{2o_i-1}{o_i}\tilde{\bar{\omega}}_i^2 + \sum_{i=1}^{n}o_i\bar{\omega}_i^2 \\ & - \sum_{i=1}^{n}\frac{2\bar{o}_i-1}{\bar{o}_i}\tilde{\bar{\xi}}_i^2 + \sum_{i=1}^{n}\bar{o}_i\bar{\xi}_i^2 + \sum_{i=1}^{n}\ell_{i,1}\tilde{\bar{\omega}}_i\hat{\bar{\omega}}_i^{\gamma_2} + \sum_{i=1}^{n}\ell_{i,2}\tilde{\bar{\xi}}_i\hat{\bar{\xi}}_i^{\gamma_2} + 2n\epsilon \\ \leq & -c_{i,2}\sum_{i=1}^{n}|\xi_i|^{\gamma_1+1} - c_{i,3}\sum_{i=1}^{n}|\xi_i|^{\gamma_2+1} - \aleph\Big\{\sum_{i=1}^{n}\frac{1}{2\lambda_{i,1}}\tilde{\bar{\omega}}_i^2 + \sum_{i=1}^{n}\frac{1}{2\lambda_{i,2}}\tilde{\bar{\xi}}_i^2\Big\} \\ & + \sum_{i=1}^{n}o_i\bar{\omega}_i^2 + \sum_{i=1}^{n}\bar{o}_i\bar{\xi}_i^2 + \sum_{i=1}^{n}\ell_{i,1}\tilde{\bar{\omega}}_i\hat{\bar{\omega}}_i^{\gamma_2} + \sum_{i=1}^{n}\ell_{i,2}\tilde{\bar{\xi}}_i\hat{\bar{\xi}}_i^{\gamma_2} + 2n\epsilon\end{aligned} \tag{76}$$

where $\aleph = \min\{\frac{2\lambda_{i,2}(2\bar{o}_i-1)}{\bar{o}_i}, \frac{2\lambda_{i,1}(2o_i-1)}{o_i}\}$.

Based on Lemma 3, Let $x = 1$, $z = \sum_{i=1}^{n}\frac{1}{2\lambda_{i,1}}\tilde{\bar{\omega}}_i^2$, $a = 1 - \frac{\gamma_1+1}{2}$, $b = \frac{\gamma_1+1}{2}$, and $\jmath = \frac{\gamma_1+1}{2}^{(\frac{\gamma_1+1}{2}/1-\frac{\gamma_1+1}{2})}$, we have

$$\Big(\sum_{i=1}^{n}\frac{1}{2\lambda_{i,1}}\tilde{\bar{\omega}}_i^2\Big)^{\frac{\gamma_1+1}{2}} \leq (1-\frac{\gamma_1+1}{2})\jmath + \sum_{i=1}^{n}\frac{1}{2\lambda_{i,1}}\tilde{\bar{\omega}}_i^2 \tag{77}$$

Similar to (77), one has

$$\Big(\sum_{i=1}^{n}\frac{1}{2\lambda_{i,1}}\tilde{\bar{\xi}}_i^2\Big)^{\frac{\gamma_1+1}{2}} \leq (1-\frac{\gamma_1+1}{2})\jmath + \sum_{i=1}^{n}\frac{1}{2\lambda_{i,1}}\tilde{\bar{\xi}}_i^2 \tag{78}$$

Moreover, in accordance with [37], the following inequalities are satisfied

$$\begin{aligned}\ell_{i,1}\tilde{\bar{\omega}}_i\hat{\bar{\omega}}_i^{\gamma_2} &\leq \frac{\ell_{i,1}}{1+\gamma_2}(2\bar{\omega}_i^{1+\gamma_2} - \tilde{\bar{\omega}}_i^{1+\gamma_2}) \\ \ell_{i,2}\tilde{\bar{\xi}}_i\hat{\bar{\xi}}_i^{\gamma_2} &\leq \frac{\ell_{i,2}}{1+\gamma_2}(2\bar{\xi}_i^{1+\gamma_2} - \tilde{\bar{\xi}}_i^{1+\gamma_2})\end{aligned} \tag{79}$$

From (77) to (79), one has

$$\begin{aligned}\dot{V}_n \leq & -c_{i,2}\sum_{i=1}^{n}|\xi_i|^{\gamma_1+1} - c_{i,3}\sum_{i=1}^{n}|\xi_i|^{\gamma_2+1} - \aleph\Big(\sum_{i=1}^{n}\frac{1}{2\lambda_{i,1}}\tilde{\bar{\omega}}_i^2\Big)^{\frac{\gamma_1+1}{2}} - \aleph\Big(\sum_{i=1}^{n}\frac{1}{2\lambda_{i,1}}\tilde{\bar{\xi}}_i^2\Big)^{\frac{\gamma_1+1}{2}} \\ & + 2\aleph(1-\frac{\gamma_1+1}{2})\jmath + \sum_{i=1}^{n}o_i\bar{\omega}_i^2 + \sum_{i=1}^{n}\frac{\ell_{i,1}}{1+\gamma_2}2\bar{\omega}_i^{1+\gamma_2} - \sum_{i=1}^{n}\frac{\ell_{i,1}}{1+\gamma_2}\tilde{\bar{\omega}}_i^{1+\gamma_2} \\ & + \sum_{i=1}^{n}\bar{o}_i\bar{\xi}_i^2 + \frac{\ell_{i,2}}{1+\gamma_2}2\bar{\xi}_i^{1+\gamma_2} - \sum_{i=1}^{n}\frac{\ell_{i,2}}{1+\gamma_2}\tilde{\bar{\xi}}_i^{1+\gamma_2} + 2n\epsilon\end{aligned} \tag{80}$$

According to Lemmas 4 and 5, we have

$$\begin{aligned}\dot{V}_n \leq & -\mu_1 V_n^{\frac{\gamma_1+1}{2}} - \mu_2\Big(\frac{1}{2^{\frac{\gamma_2+1}{2}}-1}\Big)^n V_n^{\frac{\gamma_2+1}{2}} + 2\aleph(1-\frac{\gamma_1+1}{2})\jmath + \sum_{i=1}^{n}o_i\bar{\omega}_i^2 \\ & + \sum_{i=1}^{n}\frac{\ell_{i,1}}{1+\gamma_2}2\bar{\omega}_i^{1+\gamma_2} + \sum_{i=1}^{n}\bar{o}_i\bar{\xi}_i^2 + \frac{\ell_{i,2}}{1+\gamma_2}2\bar{\xi}_i^{1+\gamma_2} + 2n\epsilon\end{aligned} \tag{81}$$

Then, we have

$$\dot{V}_n \leq -\mu_1 V_n^{\frac{\gamma_1+1}{2}} - \mu_2\Big(\frac{1}{2^{\frac{\gamma_2+1}{2}}-1}\Big)^n V_n^{\frac{\gamma_2+1}{2}} + v \tag{82}$$

where $v = 2\aleph(1-\frac{\gamma_1+1}{2})J + \sum_{i=1}^{n} o_i\bar{\omega}_i^2 + \sum_{i=1}^{n} \frac{\ell_{i,1}}{1+\gamma_2}2\bar{\omega}_i^{1+\gamma_2} + \sum_{i=1}^{n} \bar{o}_i\bar{\xi}_i^2 + \frac{\ell_{i,2}}{1+\gamma_2}2\bar{\xi}_i^{1+\gamma_2} + 2n\epsilon$, $\mu_1 = \min\{c_{2\min}2^{\frac{\gamma_1+1}{2}}, \aleph\}$, $c_{2\min} = \min\{c_{i,2}\}$, $\mu_2 = \min\{c_{3\min}2^{\frac{\gamma_2+1}{2}}, \ell_{\min}^{\frac{\gamma_2+1}{2}}2^{\frac{\gamma_2+1}{2}}, \bar{\ell}_{\min}^{\frac{\gamma_2+1}{2}}2^{\frac{\gamma_2+1}{2}}\}$, $c_{3\min} = \min\{c_{i,3}\}$, $\ell_{\min} = \min\{\frac{\ell_{i,1}}{1+\gamma_2}\}$, $\bar{\ell}_{\min} = \min\{\frac{\ell_{i,2}}{1+\gamma_2}\}$. Drawing upon Lemma 6 and [53], we can infer the closed-loop systems are practically stable in fixed-time, ξ_i, $\tilde{\bar{\omega}}_i$, $\tilde{\bar{\xi}}_i$ are bounded. Moreover, the ξ_i, $\tilde{\bar{\omega}}_i$, and $\tilde{\bar{\xi}}_i$ will converge into

$$\Pi = \left\{ \lim_{t\to T_s} \xi_i, \tilde{\bar{\omega}}_i, \tilde{\bar{\xi}}_i | V(x) \leq \min\{\mu_1^{-\frac{2}{\gamma_1+1}}(\frac{v}{1-\hbar})^{\frac{2}{\gamma_1+1}}, \mu_2^{-\frac{2}{\gamma_2+1}}(\frac{v}{1-\hbar})^{\frac{2}{\gamma_2+1}}\} \right\} \tag{83}$$

in a settling time

$$T_s \leq \frac{1}{\mu_1\hbar(1-\frac{\gamma_1+1}{2})} + \frac{1}{\mu_2(\frac{1}{2^{\frac{\gamma_2+1}{2}}-1})^n\hbar(\frac{\gamma_2+1}{2}-1)} \tag{84}$$

where $0 < \hbar < 1$.

According to (83), we have

$$|\xi_i| \leq 2V_n^{\frac{1}{2}} \leq \min\{2\mu_1^{-\frac{1}{\gamma_1+1}}(\frac{v}{1-\hbar})^{\frac{2}{\gamma_1+1}}, 2\mu_2^{-\frac{1}{\gamma_2+1}}(\frac{v}{1-\hbar})^{\frac{2}{\gamma_2+1}}\} \tag{85}$$

Part B: The demonstration of the boundedness of χ_i for all time will be provided.

For the compensating systems (28), (29), and (30), considering the Lyapunov function

$$\bar{V} = \frac{1}{2}\sum_{i=1}^{n}\chi_i^2 \tag{86}$$

Then, we have

$$\begin{aligned}
\dot{\bar{V}} =& \chi_1\dot{\chi}_1 + \chi_2\dot{\chi}_2 + \cdots + \chi_{n-1}\dot{\chi}_{n-1} + \chi_n\dot{\chi}_n \\
=& \chi_1(-c_{1,1}\chi_1 + (\pi_2 - \alpha_1) + \chi_2) + \chi_2(-c_{2,1}\chi_2 + (\pi_3 - \alpha_2) + \chi_3) + \ldots \\
& + \chi_{n-1}(-c_{n-1,1}\chi_{n-1} + (\pi_n - \alpha_{n-1}) + \chi_n) + \chi_n(-c_{n,1}\chi_n) \\
=& -c_{1,1}\chi_1^2 + \chi_1(\pi_2 - \alpha_1) + \chi_1\chi_2 - c_{1,2}\chi_2^2 + \chi_2(\pi_3 - \alpha_2) + \chi_2\chi_3 + \ldots \\
& - c_{n-1,1}\chi_{n-1}^2 + \chi_{n-1}(\pi_n - \alpha_{n-1}) + \chi_{n-1}\chi_n - c_{n,1}\chi_n^2
\end{aligned} \tag{87}$$

Young's inequality yields

$$\chi_i\chi_{i+1} \leq \frac{1}{2}\chi_i^2 + \frac{1}{2}\chi_{i+1}^2, i = 1, 2, \ldots, n-1 \tag{88}$$

Moreover, one has

$$\begin{aligned}
\dot{\bar{V}} \leq & -(c_{1,1} - \frac{1}{2})\chi_1^2 - (c_{2,1} - 1)\chi_2^2 - \cdots - (c_{n-1,1} - 1)\chi_{n-1}^2 \\
& -(c_{n,1} - \frac{1}{2})\chi_n^2 + \sum_{i=1}^{n-1}\chi_i(\pi_{i+1} - \alpha_i) \\
\leq & -\underline{\mu}\bar{V} + \sum_{i=1}^{n-1}\chi_i(\pi_{i+1} - \alpha_i)
\end{aligned} \tag{89}$$

Based on Lemma 8, there exists $|\pi_{i+1} - \alpha_i| \leq \Im_{ri}$, we have

$$\begin{aligned}
\dot{\bar{V}} \leq & -\underline{\mu}\bar{V} + (\sum_{i=1}^{n-1}\Im_{ri}|\chi_i| + |\chi_n|) \\
\leq & -\underline{\mu}\bar{V} + \sqrt{2n}\bar{\mu}\bar{V}^{\frac{1}{2}}
\end{aligned} \tag{90}$$

where $\underline{\mu} = \min\{2(c_{1,1} - \frac{1}{2}), 2(c_{2,1} - 1), \ldots, 2(c_{n-1,1} - 1), 2(c_{n,1} - \frac{1}{2})\}$, $\bar{\mu} = \max\{\Im_{ri}, 1\}$.

Furthermore, it has

$$|\chi| \leq e^{-\frac{\mu}{2}t}|\chi(0)| + 2\frac{\sqrt{n}\bar{\mu}}{\underline{\mu}}(1 - e^{-\frac{\mu}{2}t}) \tag{91}$$

where $\chi(0)$ denotes the initial condition of $\chi(t)$. Due to $\chi_i(0) = 0,\ i = 1, 2, \ldots, n$, one has $|\chi| \leq 2\frac{\sqrt{n}\bar{\mu}}{\underline{\mu}}$ for all $t > 0$, χ_i is bounded for all the time.

Due to $\xi_i = \rho_i - \chi_i$, according to (91) and (85), we conclude that ρ_i is bounded. Since $\rho_i = x_i - \pi_i$, $|\pi_{i+1} - \alpha_i| \leq \Im_{ri}$, and α_i is bounded, we know x_i is bounded. Note that $\xi_1 = \rho_1 - \chi_1$ and ξ_1 is bounded, then we can obtain the tracking error ρ_1 is bounded. Furthermore, according to (85), it is known that the tracking error ρ_1 converges to a region with the radius $\min\{2\mu_1^{-\frac{1}{\gamma_1+1}}(\frac{v}{1-\hbar})^{\frac{2}{\gamma_1+1}}, 2\mu_2^{-\frac{1}{\gamma_2+1}}(\frac{v}{1-\hbar})^{\frac{2}{\gamma_2+1}}\}$. □

Remark 4. *This article discusses fixed-time adaptive fuzzy control for a particular class of unmodeled dynamical systems, which is a relatively emerging field of study. Unmodeled dynamics are prevalent in practical engineering system, such as axially symmetric systems like robotic arms, spacecraft, and missiles. These systems are frequently defined by intricate nonlinear dynamics that pose challenges for accurate modeling. Unmodeled dynamics can arise from a variety of sources, such as friction, inertia coupling, and external disturbances. In practical engineering systems, unaccounted dynamics can considerably influence the system's performance and stability.. It is noteworthy that there have been scarce efforts to address this particular problem up until now, especially in the case of guaranteeing fixed-time convergence of the controlled systems. This article centers on a type of systems that encompass unmodeled dynamics and unknown nonlinear functions. By taking these factors into consideration, the research presents a more realistic scenario and introduces novel challenges for control scheme derivation. Moreover, the $\Phi(\cdot)$ discussed in this article is identified using FLSs. By employing a combination of update laws (25) and (26), the challenges posed by the uncertainties $\Phi(\cdot)$ in nonlinear control systems can be effectively addressed. Moreover, this paper introduces a fixed-time tracking controller integrating adaptation laws. This approach guarantees that all signals within the controlled system, as well as the tracking error, remain bounded within a fixed time.*

The design procedure for the proposed fixed-time tracking controller is outlined in Algorithm 1.

Algorithm 1 Algorithm to design fixed-time fuzzy controller

Input: The parameters $b_{i,1}$, $b_{i,2}$, $\beta_{i,1}$, and $\beta_{i,2}$ in fixed-time command filter (21); the parameters γ_1, γ_2, $c_{i,1}$, $c_{i,2}$, $c_{i,3}$, and ϵ in virtual control laws (22), (23), and (24); the parameters $\lambda_{i,1}$, $\lambda_{i,2}$, $\ell_{i,1}$, and $\ell_{i,2}$ in adaptive laws (25) and (26);
Output: The controller u.
Begin:
Step 1: Derive the system model (1).
Step 2: Establish the correlative variables (20).
Step 3: Establish the fixed-time command filter (21).
Step 4: Establish adaptive laws (25) and (26),virtual controllers (22)–(24), and error compensation mechanism (28)–(30).
Step 5: Establish actual control law $u = a_n$.
Step 6: Select appropriate design parameters.
end

4. Illustrative Examples

This section includes a practical illustration to illustrate the effectiveness of the proposed control algorithm. A pulled car system is considered, which is modeled as [38]:

$$\begin{aligned}\dot{\varphi} &= q(\varphi, x) \\ \dot{x}_1 &= x_2 \\ \dot{x}_2 &= \frac{1}{M}(-k_0 e^{-x_1} x_1 - F_d x_2 + u) + \Delta_2(x, \varphi, t) \\ y &= x_1\end{aligned} \tag{92}$$

where $q(\varphi, x) = -2\varphi + x_1^2 + 0.3$, x_1 represents the displacement of the car relative to its equilibrium position., and x_2 denotes the velocity of the car. The symbol $M = 1$ kg represents the vehicle's mass, and $F_d = 1.1$ Ns/m represents the damping factor. $K = k_0 e^{-x_1}$ represents the stiffness of the spring, $k_0 = 0.33$ N/m. $\Delta_2(x, \varphi, t) = \varphi x_1 \cos(\mathrm{x}_2)$ denotes dynamic uncertainty. Let dynamic signal $\dot{\delta} = -\delta + 2x_1^4 + 1$.

Based on Theorem 1, the action of the controller is defined as

$$u = -\frac{\xi_2 \hat{\hat{\omega}}_2^2}{\|\xi_2 \varphi_2^T\| \hat{\hat{\omega}}_2 + \epsilon_{2,1}^*} - \frac{\xi_2 \hat{\hat{\varsigma}}_2^2}{|\xi_2| \hat{\hat{\varsigma}}_2 + \epsilon_{2,2}^*} + c_{2,1}\rho_n + \dot{\pi}_2 - \xi_1 - c_{2,2}\mathrm{sgn}^{\gamma_1}(\xi_2) - c_{2,3}\mathrm{sgn}^{\gamma_2}(\xi_2) \tag{93}$$

with

$$\dot{\hat{\hat{\omega}}}_2 = -2\lambda_{2,1}\hat{\hat{\omega}}_2 - \ell_{2,1}\lambda_{2,1}\hat{\hat{\omega}}_2^{\gamma_2} + \lambda_{2,1}\|\xi_n \varphi_2^T\|, \quad \dot{\hat{\hat{\varsigma}}}_2 = -2\lambda_{2,2}\hat{\hat{\varsigma}}_2 - \ell_{2,2}\lambda_{2,2}\hat{\hat{\varsigma}}_2^{\gamma_2} + \lambda_{2,2}|\xi_2| \tag{94}$$

The error compensation signals are designed as

$$\begin{aligned}\dot{\chi}_1 &= (\pi_2 - \alpha_1) - c_{1,1}\chi_1 + \chi_2 \\ \dot{\chi}_2 &= -c_{2,1}\chi_2\end{aligned} \tag{95}$$

where

$$a_1 = -\frac{\xi_1 \hat{\hat{\omega}}_1^2}{\|\xi_1 \varphi_1^T\| \hat{\hat{\omega}}_1 + \epsilon_{1,1}^*} - \frac{\xi_1 \hat{\hat{\varsigma}}_1^2}{|\xi_1| \hat{\hat{\varsigma}}_1 + \epsilon_{1,2}^*} - c_{1,1}\rho_1 + \dot{y}_d - c_{1,2}\mathrm{sgn}^{\gamma_1}(\xi_1) - c_{1,3}\mathrm{sgn}^{\gamma_2}(\xi_1) \tag{96}$$

with

$$\dot{\hat{\hat{\omega}}}_1 = -2\lambda_{1,1}\hat{\hat{\omega}}_1 - \ell_{1,1}\lambda_{1,1}\hat{\hat{\omega}}_1^{\gamma_2} + \lambda_{1,1}\|\xi_1 \varphi_1^T\|, \quad \dot{\hat{\hat{\varsigma}}}_1 = -2\lambda_{1,2}\hat{\hat{\varsigma}}_1 - \ell_{1,2}\lambda_{1,2}\hat{\hat{\varsigma}}_1^{\gamma_2} + \lambda_{1,2}|\xi_1| \tag{97}$$

Let fuzzy the membership functions $\mu_{F^l}(x_1, x_2, \delta) = \exp[-(x_1 - 4 + l)^2/4] \times \exp[-(x_2 - 4 + l)^2/16] \times \exp[-(\delta - 4 + l)^2/16]$, where $l = 1, 2, \ldots, 7$. Then, we have

$$\varphi^l(x_1, x_2, \delta) = \frac{\mu_{F^l}(x_1, x_2, \delta)}{\sum_{l=1}^{7} \mu_{F^l}(x_1, x_2, \delta)} \tag{98}$$

where $\varphi = [\varphi^1, \varphi^2, \ldots, \varphi^7]^T$.

Case 1. The design parameters are $\gamma_1 = \frac{3}{5}$, $\gamma_2 = \frac{21}{19}$, $\lambda_{1,1} = 0.002$, $c_{1,1} = 1.01$, $c_{1,2} = 0.01$, $c_{1,3} = 0.01$, $c_{2,1} = 1.01$, $c_{2,2} = 0.01$, $c_{2,3} = 0.01$, $\lambda_{1,2} = 0.02$, $\lambda_{2,1} = 0.002$, $\lambda_{2,2} = 0.02$, $[x_1(0), x_2(0), \varphi(0), \delta(0)] = [-2.5, -0.5, 0, 0]^T$, $y_d = 0.2\cos(0.05t)$, $\hat{\hat{\varsigma}}_1(0) = 0.01$, $\hat{\hat{\theta}}_2(0) = 0.01$, $\hat{\hat{\omega}}_1(0) = 0.01$, $\hat{\hat{\omega}}_2(0) = 0.01$, $\epsilon = 0.1$, $\ell_{1,1} = 0.01$, $\ell_{1,2} = 0.01$, $\ell_{2,1} = 0.01$, $\ell_{2,2} = 0.01$, $b_{2,1} = 0.1$, $b_{2,2} = 0.1$, $\beta_{2,1} = \frac{3}{5}$, $\beta_{2,2} = \frac{21}{19}$.

Case 2. The parameters are designed as $\gamma_1 = \frac{3}{5}$, $\gamma_2 = \frac{21}{19}$, $\lambda_{1,1} = 0.002$, $c_{1,1} = 1.01$, $c_{1,2} = 0.01$, $c_{1,3} = 0.01$, $c_{2,1} = 1.01$, $c_{2,2} = 0.01$, $c_{2,3} = 0.01$, $\lambda_{1,2} = 0.02$, $\lambda_{2,1} = 0.002$, $\lambda_{2,2} = 0.02$, $[x_1(0), x_2(0), \varphi(0), \delta(0)] = [-3, -0.2, 0, 0]^T$, $y_d = 0.4\sin(0.01t)$, $\hat{\hat{\varsigma}}_1(0) = 0.01$, $\hat{\hat{\theta}}_2(0) = 0.01$, $\hat{\hat{\omega}}_1(0) = 0.01$, $\hat{\hat{\omega}}_2(0) = 0.01$, $\epsilon = 0.1$, $\ell_{1,1} = 0.01$, $\ell_{1,2} = 0.01$, $\ell_{2,1} = 0.01$, $\ell_{2,2} = 0.01$, $b_{2,1} = 0.1$, $b_{2,2} = 0.1$, $\beta_{2,1} = \frac{3}{5}$, $\beta_{2,2} = \frac{21}{19}$.

Simulation results are depicted in Figures 2–9. For Case 1, Figures 2 and 3 depict the response trajectories of the states x_1, y_d, and x_2. The tracking error trajectory is illustrated in Figure 4. It is evident from the simulation results that the output variable y is able to accurately track the desired signal y_d within 20 s. Figure 5 presents the control signal curve. For Case 2, the simulation results can be seen in Figures 6–9. Figures 6 and 7 display the response trajectories of the states x_1, y_d, and x_2 for Case 2, respectively. The tracking error curve is depicted in Figure 8. It is evident from the simulation results that the output variable y is able to accurately track the desired signal y_d within 20 s for case 2. The control signal curve is depicted in Figure 9.

From the results, it is evident that the developed fixed-time control strategy handles unknown functions and unmodeled dynamics. Moreover, the adaptive fixed-time fuzzy control protocol ensures good tracking performance within fixed-time for both cases. To further validate the effectiveness of the algorithm proposed in this article, we conducted a set of comparative simulation verifications in the revised manuscript. To ensure a fair comparison, we have designed the same parameters for the system model (92) in both the fixed-time controller presented in [38] and this paper proposed fixed-time controller. The simulation results obtained from the fixed-time controller proposed in [38] and this article introduced fixed-time controller are illustrated in Figures 6, 7, 10 and 11.

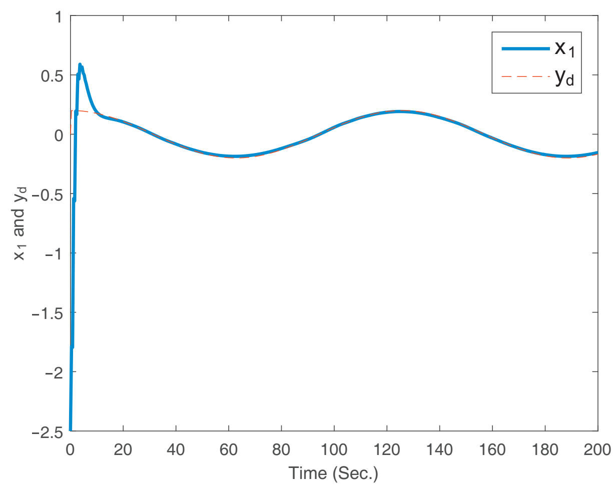

Figure 2. The trajectories of the state x_1 and y_d for Case 1.

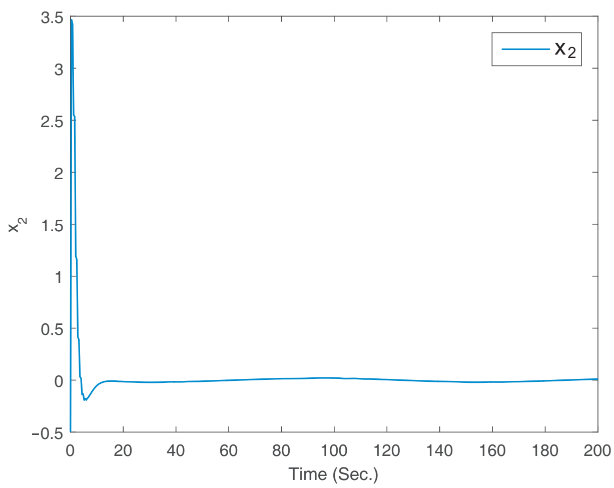

Figure 3. The trajectory of the state x_2 for Case 1.

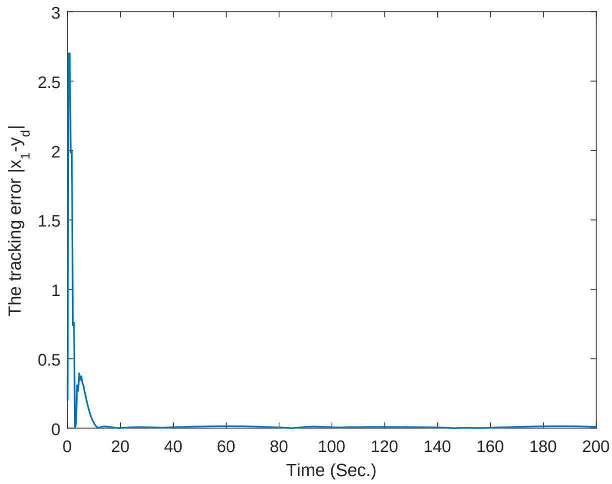

Figure 4. The trajectory of tracking error $|x_1 - y_d|$ for Case 1.

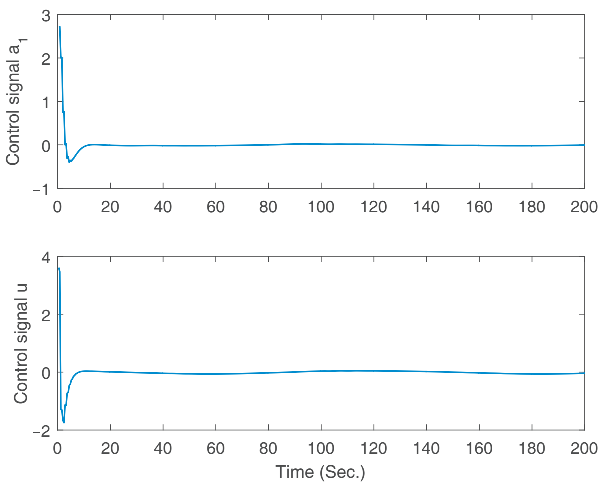

Figure 5. The trajectories of control signal for Case 1.

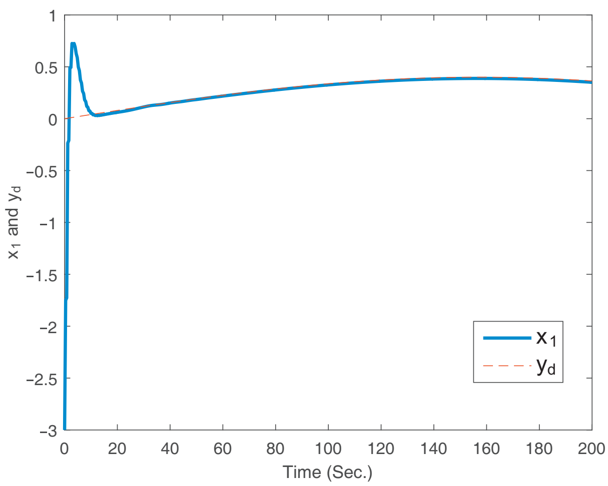

Figure 6. The trajectories of the state x_1 and y_d for Case 2.

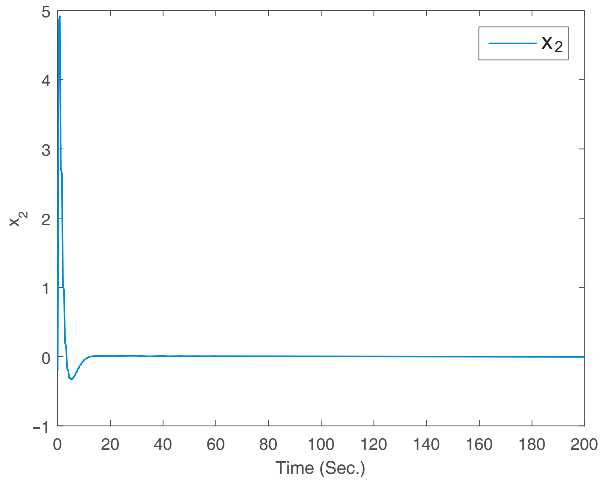

Figure 7. The trajectory of the state x_2 for Case 2.

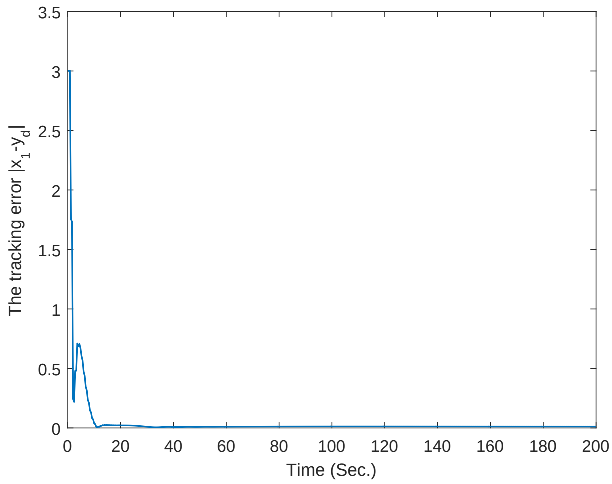

Figure 8. The trajectory of tracking error $|x_1 - y_d|$ for Case 2.

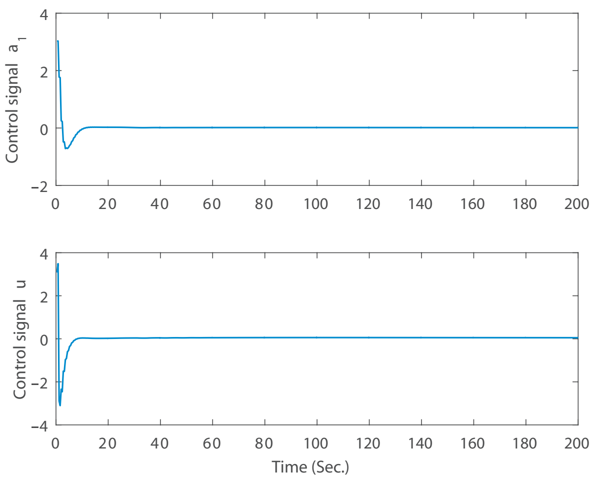

Figure 9. The trajectories of control signal for Case 2.

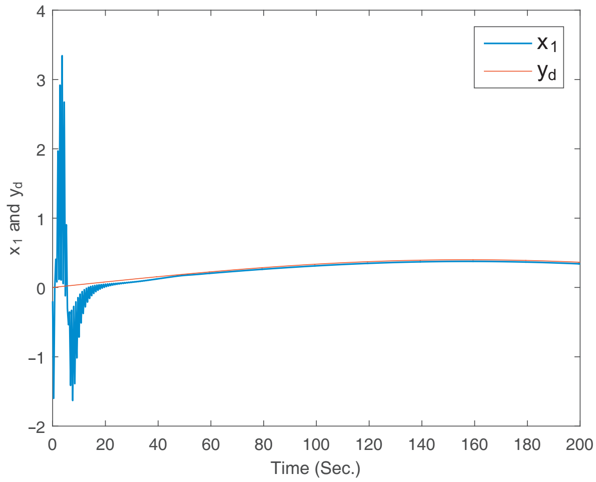

Figure 10. The trajectory of state x_1 under the fixed-time controller in [38].

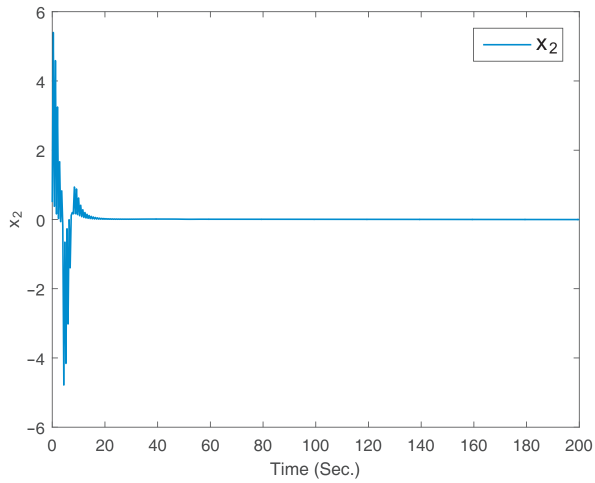

Figure 11. The trajectory of state x_2 under the fixed-time controller in [38].

Based on Figures 6, 7, 10 and 11, it is apparent that the system state x_1 can track the desired signal within 50 s and 20 s, respectively, with the fixed-time controller proposed in [38] and the proposed fixed-time controller. From Figures 6 and 10, it is evident that the system state can track the desired signal within 50 s using the fixed-time controller in reference [38], and within 20 s using our controller. Additionally, the comparison of simulation results indicates that the disturbances in the trajectories of states x_1 and x_2 are more pronounced when utilizing the fixed-time controller from reference [38], resulting in decreased control accuracy compared to our fixed-time controller in this study. Consequently, drawing upon the simulation results mentioned earlier, we can conclude that our proposed controller ensures swift convergence and enhanced control precision for the closed-loop system.

5. Conclusions

This paper addresses the topic of adaptive fixed-time fuzzy control applied to nonlinear systems exhibiting unmodeled dynamics disturbances. We process incorporates FLSs to represent the nonlinear terms that emerge as unknown factors. The control scheme is designed using FTCFs to address the challenge of complexity explosion typically related to traditional backstepping. Additionally, mechanisms for error compensation are devised to eliminate filtering errors that may arise from the use of FTCFs. A fixed-time fuzzy control algorithm has been formulated for nonlinear system, integrating the adaptive backstepping approach and fixed-time control strategy. Within the established fixed-time controller, the resultant system achieves practical fixed-time stability, and the tracking error remains bounded within fixed time. Simulation examples illustrate the effectiveness of the obtained controller. There are several practical yet complex issues that warrant consideration. For instance, future investigations will delve into nonlinear systems incorporating event-triggered mechanisms, predefined-time predefined-bounded tracking control, time-delays, and the symmetric full-state constrained problem.

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