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Review

Thermodynamics and Decay of de Sitter Vacuum

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Abstract: We discuss the consequences of the unique symmetry of de Sitter spacetime. This symmetry leads to the specific thermodynamic properties of the de Sitter vacuum, which produces a thermal bath for matter. de Sitter spacetime is invariant under the modified translations, $\mathbf{r} \to \mathbf{r} - e^{Ht}\mathbf{a}$, where H is the Hubble parameter. For $H \to 0$, this symmetry corresponds to the conventional invariance of Minkowski spacetime under translations $\mathbf{r} \to \mathbf{r} - \mathbf{a}$. Due to this symmetry, all the comoving observers at any point of the de Sitter space perceive the de Sitter environment as the thermal bath with temperature $T = H/\pi$, which is twice as large as the Gibbons–Hawking temperature of the cosmological horizon. This temperature does not violate de Sitter symmetry and, thus, does not require the preferred reference frame, as distinct from the thermal state of matter, which violates de Sitter symmetry. This leads to the heat exchange between gravity and matter and to the instability of the de Sitter state towards the creation of matter, its further heating, and finally the decay of the de Sitter state. The temperature $T = H/\pi$ determines different processes in the de Sitter environment that are not possible in the Minkowski vacuum, such as the process of ionization of an atom in the de Sitter environment. This temperature also determines the local entropy of the de Sitter vacuum state, and this allows us to calculate the total entropy of the volume inside the cosmological horizon. The result reproduces the Gibbons–Hawking area law, which is attributed to the cosmological horizon, $S_{\rm hor} = 4\pi KA$, where $K = 1/(16\pi G)$. This supports the holographic properties of the cosmological event horizon. We extend the consideration of the local thermodynamics of the de Sitter state using the $f(\mathcal{R})$ gravity. In this thermodynamics, the Ricci scalar curvature $\mathcal{R}$ and the effective gravitational coupling K are thermodynamically conjugate variables. The holographic connection between the bulk entropy of the Hubble volume and the surface entropy of the cosmological horizon remains the same but with the gravitational coupling $K = df/d\mathcal{R}$. Such a connection takes place only in the $3+1$ spacetime, where there is a special symmetry due to which the variables K and $\mathcal{R}$ have the same dimensionality. We also consider the lessons from de Sitter symmetry for the thermodynamics of black and white holes.

Keywords: de Sitter expansion; local temperature; local entropy; holography; de Sitter decay; cosmological constant

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Contents

1. Introduction

We consider the local thermodynamics of the de Sitter stage of the expansion of the universe. The term "local" means that we consider the de Sitter vacuum as the thermal state, which is characterized by the local temperature. This consideration is based on the observation that matter immersed in the de Sitter vacuum feels this vacuum as a heat bath with the local temperature $T = H/\pi$, where H is the Hubble parameter. This temperature has no relation to the cosmological horizon or the Hawking radiation from the cosmological horizon. However, it is exactly twice the Gibbons–Hawking temperature, $T_{\rm GH} = \frac{H}{2\pi}$. The reason for such a relation is the symmetry of de Sitter spacetime with respect to the combined translations. In the Minkowski vacuum, this symmetry becomes the conventional invariance of under-translations.

The existence of the local temperature suggests the existence of the other local thermodynamic variables that participate in the local thermodynamics of the de Sitter state. In addition to the local entropy density, s, and the local vacuum energy density, ϵ, there are also the local thermodynamic variables related to the gravitational degrees of freedom.

Although the local temperature is twice as large as the Gibbons–Hawking temperature assigned to the cosmological horizon, there is a certain connection between the local entropy, s, and the global entropy usually attributed to the event horizon. It appears that the total entropy of the de Sitter state in the volume V_H bound by the cosmological horizon coincides with Gibbons–Hawking entropy, which is proportional to the area, A, of the horizon, $sV_H = \frac{A}{4G}$. Such holographic bulk–surface correspondence takes place only in $3+1$ spacetime. Although the peculiarity of $3+1$ spacetime may be related to the special symmetry that connects the local thermodynamic variables, the origin of the holography is still not very clear. That is why we extended the consideration of the local thermodynamics to the $f(\mathcal{R})$ gravity and checked the bulk–surface correspondence in this modified version of general relativity.

The $f(\mathcal{R})$ gravity in terms of the Ricci scalar $\mathcal{R}$ is one of the simplest geometrical models that describe the dark energy and de Sitter expansion of the universe [1–7]. It was used to construct an inflationary model of the early universe—Starobinsky inflation, which is controlled via the $\mathcal{R}^2$ contribution to the effective action. This class of models, $f(\mathcal{R}) \propto \mathcal{R} - \mathcal{R}^2/M^2$, was also reproduced in so-called q-theory [8,9], where q is the 4-form field introduced by Hawking [10] for the phenomenological description of the physics of the deep (ultraviolet) vacuum (here, the sign convention for $\mathcal{R}$ is opposite to that in Ref. [2]). The Starobinsky model is in good agreement with these observations. However, despite the observational success, the theory of Starobinsky inflation is still phenomenological. Due to a rather small mass scale M compared with the Planck scale, it is difficult to embed the model into a UV-complete theory [11–19]. But we used this model only for the generalization of de Sitter thermodynamics and the consideration of the validity of the holographic principle.

The $f(\mathcal{R})$ theory demonstrates that the effective gravitational coupling K (it is the inverse Newton constant, $K = \frac{1}{16\pi G}$) and the scalar curvature $\mathcal{R}$ are connected via equation $K = df/d\mathcal{R}$. This suggests that K and $\mathcal{R}$ are the thermodynamically conjugate variables [20,21]. This pair of the non-extensive gravitational variables is similar to the pair of the electrodynamic variables: the electric field $\mathbf{E}$ and electric induction $\mathbf{D}$, which participate in the thermodynamics of dielectrics. They are also similar to the pair of magnetic thermodynamic variables: magnetic induction $\mathbf{B}$ and the magnetic field $\mathbf{H}$.

In de Sitter spacetime, the local temperature does not depend on the theories of general relativity and, thus, has the same value, $T = H/\pi$. Using the local thermodynamics with this temperature, we obtained the general result for the total entropy of the Hubble volume, $S_{\rm bulk} = sV_H = 4\,\pi K A = A/4\,G = S_{\rm hor}$, where $K = df/d\mathcal{R}$ is the effective gravitational coupling, and $\mathcal{R} = -12\,H^2$. This supports the holographic bulk–surface correspondence in $3+1$ spacetime.

Note that $3+1$ spacetime has a special symmetry that is absent from the other dimensions. The thermodynamically conjugate variables have the same dimensionality: all of them have dimensionality of the square of the inverse length: $[K] = [\mathcal{R}] = [1/l^2]$, $[\mathbf{B}] = [\mathbf{H}] = [1/l^2]$

and $[\mathbf{D}] = [\mathbf{E}] = [1/l^2]$. It appears that this symmetry is important for the validity of the holographic correspondence.

Since the de Sitter state has thermal behavior, it serves as the thermal bath for matter. The matter violates the de Sitter symmetry; as a result, the energy exchange between vacuum and matter leads to the decay of the de Sitter state.

The format of this paper is as follows.

In Section 2, we show that the de Sitter vacuum serves as the thermal bath for matter immersed in the de Sitter environment. Our approach differs from the traditional consideration of the vacua of the quantum fields in de Sitter spacetime, which uses the Euclidean action method. An example of the influence of the de Sitter vacuum on immersed matter is an atom in the de Sitter environment (Section 2.1). As distinct from the atom in flat space, the atom in the de Sitter vacuum has a certain probability of ionization. The rate of ionization is similar to the rate of ionization of an atom in flat spacetime in the presence of the thermal bath with temperature $T = H/\pi$ [22–26]. In Section 2.2, it is shown that the same temperature determines the other activation processes that are energetically forbidden in Minkowski spacetime but allowed in the de Sitter background. Examples are the splitting of the heavy particle with mass m to two particles with masses $m_1 + m_2 > m$ [27,28] and the radiation of electron–positron pairs via the electron at rest. In Section 2.4, it is shown that the local temperature, T, also determines the Gibbons–Hawking temperature of radiation from the cosmological horizon $T_{\rm GH} = T/2$ without using the Euclidean action.

Section 3 is devoted to the local thermodynamics of the de Sitter state, which is determined by the local temperature, $T = H/\pi$. The local temperature leads to the local entropy of the de Sitter thermal bath (Section 3.2) that, when integrated over the Hubble volume, reproduces the Gibbons–Hawking entropy of the cosmological horizon (Section 3.4).

Section 4 describes the attempt to obtain the de Sitter thermal states from the general principles of thermodynamics of many-body systems. We consider multi-metric gravity, which can be viewed as an ensemble of the sub-universes, each being described using its own metric, $g_{\mu\nu(n)}$ (or tetrads, $e^a_{\mu(n)}$), by its own gravitational coupling, K_n, and by the cosmological constant Λ_n. The heat exchange between the sub-universes leads to their thermalization—the formation of the universe in which the sub-universes have the common Hubble parameter and, thus, a common temperature.

Section 5 is devoted to the thermodynamics of de Sitter in $f(\mathcal{R})$ gravity. $f(\mathcal{R})$ gravity contains the pair of thermodynamically conjugate variables K and $\mathcal{R}$ (Section 5.1). These variables, together with the local temperature and local entropy, provide the generalization of the Gibbs–Duhem relation for the de Sitter state (Section 5.2). The confirmation of the holographic result for the total entropy of the Hubble volume in the $f(\mathcal{R})$ gravity is obtained in Section 5.3: $S_{\rm bulk} = sV_H = 4\pi KA = S_{\rm hor}$. The quadratic gravity and its symmetry are discussed in Section 5.4.

Section 6 is devoted to de Sitter decay due to the thermalization of matter via the de Sitter heat bath and the thermal fluctuations of the de Sitter state. These two mechanisms lead to different power laws of decay, which may correspond to two different epochs.

Sections 7 and 8 demonstrate how the local entropy of the de Sitter state allows us to consider the thermodynamics of the Schwarzschild black hole. The starting point of our consideration in Section 7.3 is that the black hole can be obtained from the relaxation of the gravastar object—a black hole that has the de Sitter core. It is important that the de Sitter interior of the gravastar is represented by the contracting de Sitter state. The contracting de Sitter in Section 7.2 has a negative Hubble parameter, $H < 0$, and, thus, the negative temperature $T = H/\pi < 0$ and negative entropy. The contracting and expanding de Sitter states can be considered two phases obtained from the symmetric state of the Minkowski vacuum via the spontaneous breaking of the time-reversal symmetry. These phases transform into each other under the time reversal, $t \to -t$. In this sense, the Hubble parameter, H, can be considered the order parameter of the symmetry-breaking phase transition.

Since, in the considered gravastar object, the cosmological horizon and the black hole horizon cancel each other, the gravastar has zero entropy. In the process of relaxation of the gravastar to the black hole, the de Sitter core, with its negative entropy, shrinks. This results in the Hawking–Bekenstein entropy of the black hole horizon (Section 7.4). The negative entropy of the white hole is considered in Section 7.5. The heat exchange between black holes in multi-metric gravity is discussed in Section 7.8. Section 8 provides the alternative derivation of the thermodynamics of black and white holes using macroscopic quantum tunneling.

Finally, the conclusion is presented in Section 9.

2. de Sitter State as Heat Bath for Matter

2.1. Atom in de Sitter Environment as Thermometer

We consider the de Sitter thermodynamics using the Painlevé–Gullstrand (PG) form [29,30], whose metric is

$$ds^2 = -dt^2 + (d\mathbf{r} - \mathbf{v}(\mathbf{r})dt)^2 \,. \tag{1}$$

Here, $\mathbf{v}(\mathbf{r})$ is the shift velocity, which in condensed matter plays the role of the velocity of the inviscid superfluid component of the liquid (the velocity of the "superfluid quantum vacuum" [31]). In the de Sitter expansion, the velocity of the "vacuum" is $\mathbf{v}(\mathbf{r}) = H\mathbf{r}$, and the metric is

$$ds^2 = -dt^2 + (dr - Hrdt)^2 + r^2 d\Omega^2 \,. \tag{2}$$

The PG metric is stationary, i.e., it does not depend on time, and it does not have the unphysical singularity at the cosmological horizon. That is why it is appropriate for consideration of the local thermodynamics both inside and outside the horizon. It also allows us to consider two different phases of the vacuum with broken time-reversal symmetry. These are the expanding de Sitter universe with $H > 0$ and the contracting de Sitter universe with $H < 0$. These two phases transform to each other under the time reversal, $t \to -t$. In this sense, the Hubble parameter, H, can be considered the order parameter of the symmetry-breaking phase transition from the symmetric state of the Minkowski vacuum.

Now, let us consider an atom at the origin $r = 0$. The atom is the external object in de Sitter spacetime that violates de Sitter symmetry. It plays the role of the detector (or the role of the static observer) in this spacetime. The electron bound to an atom may absorb the energy from the gravitational field of the de Sitter background and escape from the electric potential barrier. If the ionization potential is much smaller than the electron mass but much larger than the Hubble parameter, $H \ll \epsilon_0 \ll m$, one can use the nonrelativistic quantum mechanics to estimate the tunneling rate through the barrier.

Let us consider an electron on the n-th level in the hydrogen atom. Under the de Sitter gravitational field, this electron can escape from the atom with the conservation of energy, which in the classical limit given via the classical equation

$$\frac{p_r^2}{2m} + p_r v(r) = -E_n \;,\; E_n = \frac{me^4}{2\hbar^2}\frac{1}{n^2} \,. \tag{3}$$

Here, p_r is the momentum of the electron in the radial direction, $v(r) = Hr$, and $p_r v(r)$ is the Doppler shift, which allows for the electron to reach the negative energy $-E_n$ when it escapes from the atom. The corresponding radial trajectory $p_r(r)$ for the escape of an electron from the atom is as follows:

$$p_r(r) = -mv(r) + \sqrt{m^2 v^2(r) - 2mE_n} \,. \tag{4}$$

The integral of $p_r(r)$ over the classically forbidden region, $0 < r < r_n = \sqrt{2E_n/mH^2}$, gives the ionization rate in the semiclassical WKB approximation:

$$w \sim \exp(-2\,\mathrm{Im}\,S) = \\ = \exp\left(-2\int_0^{r_n} dr\sqrt{2mE_n - m^2H^2r^2}\right) = \exp\left(-\frac{\pi E_n}{H}\right) = \exp\left(-\frac{E_n}{T}\right). \quad (5)$$

This ionization rate is equivalent to the rate of ionization in the flat Minkowski spacetime in the presence of the heat bath with temperature $T = H/\pi$.

This heat bath temperature is twice the Gibbons–Hawking temperature, $T_{\rm GH}$, usually attributed to the cosmological horizon, $T = 2\,T_{\rm GH}$. Moreover, the electron trajectory is well inside the horizon since $r_n \ll r_H = 1/H$, and thus, this process has no relation to Hawking radiation. However, in Section 2.4, we show that the relation between the temperature of ionization and the Gibbons–Hawking temperature, $T = 2\,T_{\rm GH}$, is the property of the de Sitter symmetry.

Since the process of ionization takes place well inside the cosmological horizon, one can use the conventional static metric with the gravitational potential $U(r) = -mH^2r^2/2$ [26]. Then, the bound state decays via quantum tunneling from the point $r = 0$ to the point $r = r_n$, at which the electron level $-E_n$ matches the de Sitter gravitational potential, $U(r_n) = -mH^2r_n^2/2$. The radial trajectory, $p_r(r)$, is obtained from the classical equation

$$\frac{\mathbf{p}^2}{2m} - \frac{1}{2}mH^2r^2 = -E_n\,, \quad (6)$$

which gives, for the radial trajectory, the following:

$$p_r(r) = \sqrt{m^2H^2r^2 - 2mE_n}\,. \quad (7)$$

The imaginary part of the action at this trajectory again gives Equation (5) for the WKB tunneling rate.

2.2. Decay of Composite Particles in de Sitter Spacetime

The same local temperature, $T = H/\pi$, describes the process of the splitting of the composite particle with mass m into two components with the larger total mass $m \to m_1 + m_2 > m$, which is also not allowed in the Minkowski vacuum [23,32–34].

It is instructive to derive the rate of this process using the semiclassical tunneling picture also. For simplicity, we consider the creation of particles with equal masses, $m_1 = m_2 > m/2$. The trajectory of each of the two particles with mass m_1 moving in the radial direction from the origin at $r = 0$ is obtained from equation

$$E(p_r, r) = \sqrt{p_r^2 + m_1^2} + p_r H r = \frac{m}{2}\,. \quad (8)$$

We took into account the fact that each of the two particles carries one half of the energy of the original particle, i.e., $E = m/2$. The momentum along the trajectory is

$$p_r(r) = \frac{1}{1 - H^2r^2}\left[-\frac{m}{2}Hr + \sqrt{\frac{m^2}{4} - m_1^2 + m_1^2H^2r^2}\right]. \quad (9)$$

This momentum is imaginary in the classically forbidden region $r < r_0$, where $r_0 = \sqrt{1 - m^2/4m_1^2}/H$. This gives the following imaginary contribution to the action:

$$\mathbf{Im}S = \mathbf{Im}\int dr\; p_r(r) = \frac{m_1}{H}\int_0^{r_0} dr\frac{\sqrt{r_0^2 - r^2}}{r_H^2 - r^2} = \frac{\pi}{4}(2m_1 - m). \quad (10)$$

where, as before, $r_H = 1/H$ is the position of the de Sitter horizon. We must take into account that, due to momentum conservation, the two particles tunnel simultaneously in opposite directions. Such a coherent process of co-tunneling adds a multiplier of 2 to the argument of the exponent. As a result, one obtains the rate of the decay of the composite particle into two particles, which is again governed by the temperature $T = H/\pi$:

$$\Gamma(1 \to 2) \propto \exp(-4\,\mathbf{Im}S) = \exp\left(-\frac{\pi(2m_1 - m)}{H}\right). \quad (11)$$

In a general case, when $m_2 \neq m_1$, the rate of the decay of the composite particle in the limit $m \gg H$ is determined by the same temperature:

$$\Gamma(1 \to 2) \sim \exp\left(-\frac{\pi(m_1 + m_2 - m)}{H}\right) = \exp\left(-\frac{m_1 + m_2 - m}{T}\right). \quad (12)$$

2.3. Triplication of Particles in de Sitter Spacetime

In the same way, massive fermions can be reproduced in the de Sitter environment. For example, the electron at $r = 0$ can create an electron-positron pair, with electron and positron moving in opposite directions.In this process, the fermion is triplicated with the rate given by the following:

$$\Gamma(1 \to 3) \sim \exp\left(-\frac{\pi(3m - m)}{H}\right) = \exp\left(-\frac{2m}{T}\right). \quad (13)$$

Since the created electron and positron move in opposite directions, they are detected via two different detectors. Each of the two detectors can detect only a single particle, and thus, for each detector, the radiation rate looks thermal with the Gibbons–Hawking temperature $T_{\rm GH} = T/2$:

$$\Gamma(1 \to 3) \sim \exp\left(-\frac{2m}{T}\right) = \exp\left(-\frac{m}{T/2}\right) = \exp\left(-\frac{m}{T_{\rm GH}}\right). \quad (14)$$

However, in this process, two particles are created simultaneously and coherently (the so-called co-tunneling process). That is why the temperature, which describes such a co-tunneling process of the radiation of the pair, is twice the Gibbons–Hawking temperature. The same connection between the local temperature and the temperature of the Hawking radiation from the cosmological horizon is discussed in Section 2.4.

It is important that each of the three particles (the original electron + the electron–positron pair, which is created in the process of Equation (13)) is able to create the other three particles, and then the triplication process continues further. After n replications, there will be 3^n fermions ($(3^n + 1)/2$ electrons and $(3^n - 1)/2$ positrons). Such a creation of multiple particles from a single particle immersed in the de Sitter environment demonstrates that the de Sitter vacuum is unstable towards the creation of matter. The consequence of such instability is discussed in Section 6.

Similar processes take place in the so-called Cosmological Collider [27,28], where the new particle created by Hawking radiation plays the role of the external object that produces the heavy particles. In this case, we have two different physical processes: the Hawking radiation from the cosmological horizon and the further local process—the splitting of the created particles, which is determined by the local temperature.

2.4. Connection between the Local and Hawking Temperatures

The local temperature $T = H/\pi$ also determines the process of the Hawking radiation from the cosmological horizon and the Gibbons–Hawking temperature $T_{\rm GH} = T/2 = \frac{H}{2\pi}$. The reason is that, in the Hawking process, two particles are created coherently (this is the analog of cotunneling): one particle is created inside the horizon, while its partner is simultaneously created outside the horizon [35]. The rate of the coherent cotunneling of two particles, each with energy E, is $w \propto \exp(-\frac{2E}{T})$. However, the observer who uses the Unruh–DeWitt detector can detect only the particle created inside the horizon. For this observer with limited information, the creation rate $w \propto \exp(-\frac{2E}{T})$ is perceived as

$$w \propto \exp\left(-\frac{E}{T/2}\right) = \exp\left(-\frac{E}{T_{\rm GH}}\right), \tag{15}$$

with the Gibbons–Hawking temperature $T_{\rm GH} = T/2 = \frac{H}{2\pi}$.

The similar scenario of doubling the Hawking temperature was suggested for the black hole horizon [36,37]. In this case, the partner of the created particle is in the mirror image of the spacetime that replaces the coordinate singularity. Although the existence of such a quantum clone of the black hole is problematic, the consideration of the coherent creation of two partners on the two sides of the cosmological horizon is applicable to the de Sitter vacuum.

On the contrary, in the local process of the decay of an atom, which is not related to the cosmological horizon, only a single particle (electron) is radiated from the atom. This process is fully determined by the local temperature, $w \propto \exp(-\frac{E_n}{T})$.

2.5. Two Detectors: Exited Atom vs. Ionized Atom

Note the main difference between the temperature measured by the observer using the Unruh–DeWitt detector (see, e.g., Ref. [38] and references therein) and the temperature measured by the observer using the ionization of an atom. The ionization process is possible because the radiated electron moves far away from the atom to position $r = r_0$, where its negative gravitational energy compensates the ionization potential. In the Unruh–DeWitt detector, which corresponds to the two-level atom interacting with a quantum field, the electron in the atom is excited but remains in the same position in the same atom. That is why such excitation of the electron may only come either via the Hawking radiation or via the radiation of photons; see Section 2.6.

However, in the de Sitter case, there is no difference in temperatures measured via the two detectors. If, in the Unruh–DeWitt detector experiments, the observer properly interprets the result of the observations according to Equation (15), the measured temperature of the Hawking radiation is also $T = 2T_{\rm GH}$. So, in spite of different physical principles, both detectors in the de Sitter environment show the same physical temperature, $T = 2T_{\rm GH}$. This also demonstrates the uniqueness of the de Sitter state with its symmetry under generalized translations.

2.6. Radiation of Photons via Atom in de Sitter Environment

Instead of the radiation of the electron from the atom, one can consider the radiation of the photon via the same atom. In both cases, the escape of the photon or electron from the atom provides the negative gravitational energy. That is why these processes, which are prohibited in Minkowski spacetime, are energetically possible in the de Sitter environment. Let us consider the atom in the excited state with energy $E_0 + \epsilon$, where E_0 is the ground state energy of the atom. In the de Sitter environment, the radiation of the photon is possible even if its energy, cp, exceeds the energy difference between the excited and the ground state level, i.e., when $cp > \epsilon$.

For the relativistic photon, the energy conservation Equation (3) takes the following form (here, we use $c = 1$):

$$\sqrt{p_r^2 + p_\perp^2} + p_r H r = \epsilon\,. \tag{16}$$

This gives the following trajectory of the photon:

$$p_r(r) = \frac{1}{1 - H^2 r^2}\left[-\epsilon H r \pm \sqrt{\epsilon^2 + p_\perp^2 (H^2 r^2 - 1)}\right]. \tag{17}$$

For $cp_\perp > \epsilon$, the integration over the classically forbidden region, $r < r_0$, where $H^2 r_0^2 = 1 - \frac{\epsilon^2}{c^2 p_\perp^2}$, gives the following radiation rate of photon via the excited atom:

$$w \sim \exp\left(-2\,\mathrm{Im}\,S\right) = \exp\left(-\frac{\pi(cp_\perp - \epsilon)}{H}\right)\;,\; cp_\perp > \epsilon\,. \tag{18}$$

Since $r_0 < 1/H$, the trajectory of the photon in this process is inside the cosmological horizon, and the radiation rate is again determined by the local temperature $T = H/\pi$.

Equation (18) demonstrates that radiation takes place even if the atom is in the ground state, i.e., when $\epsilon \to 0$. In this case, Equation (18) describes the process of radiation of a single photon in the de Sitter environment:

$$w \sim \exp\left(-\frac{cp}{T}\right)\;,\; T = \frac{H}{\pi}\,. \tag{19}$$

In this process, the atom in its ground state serves as an external object, which violates the de Sitter symmetry and, thus, provides the nonzero matrix element for the radiation of the photon. This process is certainly different from the Hawking process of radiation.

2.7. Accelerating Detector: Is There a Connection between the Local Process and Unruh Radiation?

In the same way as described in Sections 2.5 and 2.6, one may consider different independent processes related to the accelerating detector. One of them is the Unruh radiation measured via the Unruh–DeWitt detector, which could be linked to the apparent event horizon in Rindler spacetime and the corresponding Unruh temperature $T_U = \frac{a}{2\pi}$, where a is acceleration [39–41]. The other processes, such as the local process of ionization of the accelerating atom [42], have no relation to the Rindler horizon.

Let us consider the process of ionization of an atom moving with acceleration. Similar to Equation (6), the trajectory $p_x(x)$ of the radiated electron is obtained from the classical equation

$$\frac{\mathbf{p}^2}{2m} - max = -\epsilon\,. \tag{20}$$

Here, we used the equivalence between the acceleration of the reference frame and the constant gravitational field, $g = a$; and ϵ is the ionization potential.

In the WKB approximation and in the limit of slow acceleration, $a \ll \epsilon \ll m$, the classically forbidden trajectory of the electron escaping from the atom gives the radiation rate [42], which is similar to the radiation rate in the electric field [43]:

$$w \sim \exp\left(-\frac{4\sqrt{2}}{3}\frac{\epsilon}{a}\left(\frac{\epsilon}{m}\right)^{1/2}\right). \tag{21}$$

In principle, one can introduce the effective temperature, which is similar to the effective temperature $T_{\rm eff} \sim El$ of hopping electrons in a strong electric field, E, where l is the localization length [44,45]. In our case, the analog of the localization length is $l = 1/\sqrt{m\epsilon}$. But otherwise, the local process of ionization of the accelerated atom is certainly non-thermal. Since $\epsilon \ll m$, the rate of ionization in Equation (21) essentially exceeds the ionization rate in any thermal process that involves acceleration, $w_{\rm thermal} \sim \exp(-\gamma\epsilon/a)$. This is very different from the ionization of the atom in the de Sitter environment, which looks thermal.

With the same approach, one can also calculate the Unruh radiation of photons using different arrangements of detectors. The state of the art in the theoretical understanding of Unruh radiation can be found in Refs. [46–48] and the references therein. Our examples demonstrate that the rate of Unruh radiation depends on the details of the considered processes, and it is not necessarily thermal. The thermal behavior of the local and global processes in the de Sitter environment is the result of the special de Sitter symmetry.

3. Thermodynamics of the de Sitter State

3.1. de Sitter Symmetry and de Sitter Heat Bath

As distinct from the Unruh effect, different arrangements of detectors in the de Sitter environment show the same temperature. This demonstrates the uniqueness of de Sitter spacetime in producing a thermal bath with a local temperature. The reason for that is that de Sitter spacetime is homogeneous under the combination of translation and the proper conformal transformations [49,50]. In the PG metric, it is the invariance of the de Sitter state under the modified translations $\mathbf{r} \to \mathbf{r} - e^{Ht}\mathbf{a}$, which, for $H \to 0$, corresponds to the conventional invariance under translations $\mathbf{r} \to \mathbf{r} - \mathbf{a}$ in Minkowski spacetime. Due to this combined translational symmetry, all the comoving observers at any point of the de Sitter space observe the same temperature, $T = H/\pi$. That is why one may conclude that the de Sitter state is the heat bath produced by gravity.

The uniqueness of the de Sitter thermal state lies in the fact that its temperature does not violate de Sitter symmetry and, thus, does not require the preferred reference frame. This is distinct from the thermal state of matter, which always has a preferred reference frame where matter is at rest. As a result, thermal matter violates de Sitter symmetry, which leads to the heat exchange between the two thermal subsystems, gravity and matter, and finally to the de Sitter decay.

3.2. From Local Temperature to Local Entropy

The gravity subsystem—the de Sitter quantum vacuum—has its own temperature and entropy; see also Ref. [51]. On the other hand, de Sitter spacetime serves as the thermal bath for matter. Then, the quasi-equilibrium states of the expanding universe can be described with two different temperatures: the temperature of the gravitational vacuum (temperature of dark energy) and the temperature of the matter's degrees of freedom [52]. In this section, we discuss the pure de Sitter vacuum without the excited matter, ignoring for the moment the thermally activated creation of matter from the vacuum. The excitation and thermalization of matter via the de Sitter thermal bath are discussed in Section 6.

If the vacuum thermodynamics is determined by the local activation temperature $T = H/\pi$, then, in Einstein gravity with a cosmological constant, the vacuum energy density is quadratic in temperature:

$$\epsilon_{\rm vac} = \frac{3}{8\,\pi G}H^2 = \frac{3\,\pi}{8\,G}T^2\,. \tag{22}$$

This leads to the free energy density of the de Sitter vacuum, $F = \epsilon_{\rm vac} - Ts_{\rm vac}$, which is also quadratic in T, and thus, the entropy density $s_{\rm vac}$ in the de Sitter vacuum is linear in T:

$$s_{\rm vac} = -\frac{\partial F}{\partial T} = \frac{3\,\pi}{4\,G}T = 12\pi^2 KT\,. \tag{23}$$

Here, as before, $K = \frac{1}{16\pi G}$ is the gravitational coupling.

The temperature, T, and the entropy density, $s_{\rm vac}$, are the local quantities that can be measured by the local static observer.

3.3. de Sitter Vacuum, Fermi Liquid and Cosmological Constant Problem

Equation (23) demonstrates that the thermal properties of the de Sitter state are similar to that of the non-relativistic Fermi liquid, where the entropy density is also linear in temperature:

$$s_{\rm FL} = \frac{p_F^2}{3v_F}T\,. \tag{24}$$

The Fermi velocity, v_F, and the Fermi momentum, p_F, of this cosmological analog of the Fermi liquid are on the order of the speed of light and the inverse Planck length, correspondingly $v_F \sim c$ and $p_F \sim M_{\rm Pl} = 1/l_{\rm Pl}$. Of course, this is only a parallel, and one should not identify the de Sitter vacuum with a real Fermi liquid, although the real Fermi surface may form inside the black hole horizon [53,54].

Since the thermodynamics of the de Sitter state with the thermal energy $\epsilon_{\rm vac} \propto T^2$ is similar to the thermodynamics of the Fermi liquid, let us try to exploit this connection. One of the directions is the Sommerfeld law in Fermi liquid, which states that the entropy per one atom of Fermi liquid is

$$S = \frac{s_{\rm FL}}{n_{\rm FL}} \sim \frac{T}{E_F}\,, \tag{25}$$

where $n_{\rm FL} \sim p_F^3$ is the density of atoms in the Fermi liquid, and E_F is the Fermi energy.

We do not know what the “atoms of the vacuum” are, but from Equation (23), it follows that the entropy density of the vacuum $s_{\rm vac} \sim T/l_{\rm Pl}^2 \sim (T/M_{\rm Pl})/l_{\rm Pl}^3$, where $l_{\rm Pl}$ is the Planck length, and $M_{\rm Pl}$ is the Planck energy. This suggests that the density of the “atoms of the vacuum” is $n_{\rm Pl} \sim 1/l_{\rm Pl}^3$, and the entropy per “atom of the vacuum” is as follows:

$$S = \frac{s_{\rm vac}}{n_{\rm Pl}} \sim s_{\rm vac} l_{\rm Pl}^3 \sim \frac{T}{M_{\rm Pl}}\,. \tag{26}$$

Equation (26) is the full analog of the Sommerfeld law for Fermi liquid. This analogy also suggests that the corresponding density of states in the quantum vacuum (the analog of the density of states at the Fermi level $N_F \sim m p_F$ in Fermi liquids) is $N_{\rm Pl} \sim M_{\rm Pl}^2$. For bosonic and fermionic degrees of freedom of this quantum vacuum, the density of states is $N_{\rm Pl} = \frac{9}{4\pi G}$ and $N_{\rm Pl} = \frac{9}{2\pi G}$, correspondingly, which can be compared with the value $N_{\rm Pl} = \frac{3\pi}{G}$ suggested in Ref. [55]. This huge density of states leads to the very large entropy of the de Sitter state even at a very low temperature of the vacuum.

So, the quantum vacuum looks like some specific form of the relativistic Fermi liquid. However, some consequences are the same as for the non-relativistic Fermi liquid. In particular, in the full equilibrium at $T = 0$ and in the absence of external pressure, $P = 0$, the energy density of the non-relativistic Fermi liquid is exactly zero, $\epsilon(n) - \mu n = Ts - P = 0$. This follows solely from thermodynamics, and it is valid for any macroscopic system in its ground state. In all such systems, the contribution of the zero-point energies of collective modes (phonons, fermionic quasiparticles, magnions, etc.) is exactly compensated for with the microscopic (atomic) degrees of freedom. This thermodynamic property does not depend on microscopic physics, and it is easily checked in condensed-matter systems for which we know both the micro and macro physics. That is why it is not surprising that, for the relativistic quantum vacuum, the vacuum energy density is zero if the following conditions are fulfilled: there is full thermodynamic equilibrium; the temperature is $T = 0$; there is no matter (no quasiparticles); and there is no external pressure, $\epsilon_{\rm vac}(H = 0, T = 0) = -P_{\rm vac} = 0$. This demonstrates that thermodynamics, which is insensitive to the microscopic structure of the vacuum (ground state), solves the main cosmological constant problem. The other cosmological constant problems are discussed in Section 6.6.

3.4. Hubble Volume Entropy vs. Entropy of the Cosmological Horizon

Using the entropy density in Equation (23), one may find the total entropy of the Hubble volume V_H—the volume surrounded by the cosmological horizon with radius $R = 1/H$:

$$S_{\rm bulk} = s_{\rm vac} V_H = \frac{4\,\pi R^3}{3} s_{\rm vac} = \frac{\pi}{GH^2} = \frac{A}{4\,G} = S_{\rm hor}\,, \tag{27}$$

where A is the horizon area.

The Hubble-volume entropy coincides with the entropy attributed to the cosmological horizon, as suggested by Gibbons and Hawking. However, Equation (27) demonstrates that the thermodynamic entropy comes from the local entropy of the de Sitter quantum vacuum, rather than from the hypothetical horizon degrees of freedom. In this hypothesis, it is assumed that there are horizon microstates, which are concentrated in the region of the Planck length $l_{\rm Pl} = 1/M_{\rm Pl}$ in the vicinity of the horizon, and this leads to the area law for the total entropy inside the horizon; see, e.g., Ref. [56]. The local thermodynamics in the de Sitter state leads to the same area law but without assumptions about the spatial distribution of the degrees of freedom.

The connection between the bulk entropy of the region inside the horizon due to the local vacuum thermodynamics and the surface entropy of the horizon may have a holographic origin when the horizon is considered a null surface [56]. This may be the reason why the presence of the horizon allows us to know the total entropy of the Hubble volume without knowledge of the distribution of the local entropy inside the horizon.

However, it appears that the bulk–surface correspondence in Equation (27) is valid only in $(3+1)$-dimensional spacetime. In the general $d+1$ dimension of spacetime, the same approach gives the factor $(d-1)/2$ in the relation between the entropy of the Hubble volume and the Gibbons–Hawking entropy of the cosmological horizon, $S_{\rm bulk} = \frac{d-1}{2} S_{\rm hor}$. This may add to the peculiarities of the $d=3$ space dimension [57] where, in particular, the mass dimension of the gravitational coupling, $[K] = d-1$, coincides with the mass dimension of curvature, $[\mathcal{R}] = 2$. The same concerns such pair of thermodynamically conjugate variables as an electric field with the mass dimension $[E] = 2$ and electric induction with the mass dimension $[D] = d-1$. Their dimensions also coincide only for $d=3$. A discussion of the natural dimensions of physical quantities in $d=3$ can be found in Ref. [58].

4. Thermodynamics from the Heat Transfer in the Multi-Metric Gravity Ensemble

4.1. Multi-Metric Gravity

As we know, the main source of emergent thermodynamics is the heat exchange between bodies or between systems; see also Ref. [59]. Following this rule, we can consider de Sitter thermodynamics from the point of view of the heat transfer between different cosmological objects or different universes.

The heat exchange can be discussed in the frame of so-called multi-metric gravity; see Ref. [60] and the references therein. The corresponding model action of the whole system can be written as the sum of the actions of the sub-systems in the same coordinate spacetime:

$$S = -\int d^4x \sum_{n=1}^{N} \mathcal{L}_n \ , \ \mathcal{L}_n = \sqrt{-g_{(n)}}\Big(K_n \mathcal{R}\{g_{\mu\nu(n)}\} + \Lambda_n\Big) . \tag{28}$$

Then, the universe can be seen as the system of N sub-universes, each with its own gravitational coupling, K_n, cosmological constant, Λ_n, and metric, $g_{\mu\nu(n)}$.

Following Froggatt and Nielsen [61], one can introduce N independent tetrad fields $e_\mu^{a(n)}$ for N fermionic species. In this multi-tetrad gravity, one has the ensemble of the gravitational actions,

$$\mathcal{L}_n = e_{abcd}\Big(K_n\, R^{ab(n)} \wedge e^{c(n)} \wedge e^{d(n)} + \Lambda_n\, e^{a(n)} \wedge e^{b(n)} \wedge e^{c(n)} \wedge e^{d(n)}\Big) , \tag{29}$$

and the corresponding ensemble of actions for the fermionic species,

$$S_M = e_{abcd} \int \sum_n \Theta^{a(n)} \wedge e^{b(n)} \wedge e^{c(n)} \wedge e^{d(n)} , \tag{30}$$

$$\Theta^{a(n)} = \frac{i}{2}\Big[\bar\Psi^{(n)}\gamma^a D_\mu \Psi^{(n)} - D_\mu \bar\Psi^{(n)}\gamma^a \Psi^{(n)}\Big] dx^\mu . \tag{31}$$

This can be extended to multi-fünfbein gravity, where, instead of the tetrad fields, the Dirac fermions are described by the rectangular vielbein (fünfbein) [62].

Gravity with multiple tetrad fields may also come from Akama– Diakonov– Wetterich theory [63–71], where the tetrads are formed as composite objects—the bilinear combinations of the fundamental fermionic fields:

$$e_\mu^{a(n)} = \left\langle \Theta^{a(n)} \right\rangle . \quad (32)$$

In this approach, the metric is the quartet of fermions. In principle, so-called vestigial gravity can be realized, in which the bilinear combination of fermions in Equation (32) is zero, $e_\mu^{a(n)} = 0$, while the metric—the quartet of fermions—is nonzero [72]:

$$g_{\mu\nu(n)} = \eta_{ab} \left\langle \Theta_\mu^{a(n)} \Theta_\nu^{b(n)} \right\rangle . \quad (33)$$

On the levels of particles, the vestigial gravity acts with different strengths on fermions and bosons. In principle, it is not excluded that such gravity can be formed at an early stage of the development of the universe.

4.2. Heat Exchange in Multi-Metric Gravity

The heat exchange between the sub-universes leads to their equilibration with the formation of the common expansion rate and, thus, the common temperature. We consider first the system of two sub-universes, assuming that, in each of them, the entropy of the horizon obeys the area law, and we show that the maximum entropy corresponds to the situation in which both states acquire the same expansion rate.

In the de Sitter state, which is determined by the cosmological constant, the equation of the state for the vacuum energy is $\epsilon_{\rm vac} = -P_{\rm vac}$. The total vacuum energy is proportional to the volume, V, of the system if we assume that the volume, V, is much larger than the Hubble volume, $V \gg V_H$, so that the boundary terms are not important. Then, we have

$$E_V = \epsilon_{\rm vac} V = 6KH^2 V . \quad (34)$$

Let us assume that the bulk–surface correspondence is valid, i.e., the entropy of the Hubble volume, V_H, is equal to the Gibbons–Hawking entropy of cosmological horizon $S_{V_H} = S_{\rm hor} = 4\,\pi K A$. Then, the total entropy, S_V, in the volume $V \gg V_H$ can be obtained from the entropy of the Hubble volume, V_H:

$$S_V = S_{\rm hor} \frac{V}{V_H} = 12\pi K H V . \quad (35)$$

Let us now consider two de Sitter sub-states with different values of the gravitational coupling, K_1 and K_2, and different values of the Hubble parameter, H_1 and H_2:

$$S = -\int d^4x \sqrt{-g_{(1)}} \Big(K_1 \mathcal{R}\{g_{\mu\nu(1)}\} + \Lambda_1 \Big) - \int d^4x \sqrt{-g_{(2)}} \Big(K_2 \mathcal{R}\{g_{\mu\nu(2)}\} + \Lambda_2 \Big) . \quad (36)$$

This corresponds to the higher dimensional analog of the bilayer graphene [73], where two 2 + 1 dimensional universes are in the neighboring layers of 3 + 1 spacetime. In this interpretation, we have two 3 + 1 dimensional universes in the neighboring layers in the 4 + 1 space.

The total energy and total entropy of the two layers are (if the interaction between the layers is neglected) as follows:

$$E_V = E_1 + E_2 = 6(K_1 H_1^2 + K_2 H_2^2) V , \quad (37)$$

$$S_V = S_1 + S_2 = 12\pi (K_1 H_1 + K_2 H_2) V . \quad (38)$$

Let us now allow for the energy exchange (the heat exchange) between these two sub-universes (analogs of the two layers of graphene). This exchange can be realized via the matter field, which interacts with both metrics. It leads to the variations in the Hubble parameters H_1 and H_2 at a fixed E_V. If we ignore the thermalization of matter via the de Sitter environment, the heat exchange will finally produce the equilibrium state with the maximum entropy S, in which the Hubble parameters become equal:

$$H_1^2 = H_2^2 = \frac{E_V}{6(K_1 + K_2)V} \equiv H^2 \, . \tag{39}$$

The equilibration of the Hubble parameters demonstrates that the Hubble parameter (with some numerical factor) plays the role of the temperature of the de Sitter universe.

The temperature of the de Sitter universe can be obtained via a variation of the Hubble parameter:

$$\begin{aligned} \frac{1}{T_1} &= \frac{dS_1}{dE_1} = \frac{dS_1/dH_1}{dE_1/dH_1} = \frac{\pi}{H_1} \, , \\ \frac{1}{T_2} &= \frac{dS_2}{dE_2} = \frac{dS_2/dH_2}{dE_2/dH_2} = \frac{\pi}{H_2} \, , \end{aligned} \tag{40}$$

with $T_1 = T_2 = H/\pi$ in equilibrium.

In the case of the arbitrary number N of sub-universes, the heat exchange between them leads to the equilibrium state of the universe, in which all the sub-universes coherently expand with the same rate, H, i.e., with the same de Sitter metric in all subsystems. In this equilibrium universe, the gravitational coupling K is equal to the sum of the individual couplings in the sub-universes and the vacuum energy density is equal to the sum of the energy densities of subsystems:

$$E_V = 6\, KH^2 V \ , \ K = \sum_n K_n \ , \ \Lambda = \sum_n \Lambda_n \, . \tag{41}$$

All the substates acquire the same temperature in equilibrium, $T_n = T = H/\pi$.

4.3. Thermodynamics from the Multi-Metric Ensemble

Let us recall that, in the above approach, we used the bulk–horizon correspondence $S_{V_H} = S_{\rm hor} = 4\,\pi K A$, which finally led to the equilibrium universe with the temperature $T = H/\pi$. Let us now consider the thermodynamics of the whole de Sitter system without any assumption about the entropy of the cosmological horizon. For that, we consider the statistical ensemble of N de Sitter sub-universes with random Hubble parameters H_n. This is the extension of multi-metric gravity to the statistical ensemble with the randomly distributed parameters K_n and Λ_n.

For a large N, the random distribution of the parameters results in the exponential behavior of the distribution functions, $w_n \propto \exp(-E_n/T)$, with the same parameter, T, for all subsystems. As in the statistical ensemble of atoms, where the temperature of the system is determined by physical processes, the temperature of the ensemble of the sub-universes is also determined by physical processes. In our case, it is the behavior of matter (atom) in the de Sitter environment that gives $T = H/\pi$. This connection between T and H is rather natural. Both the parameter T, which plays the role of temperature, and the Hubble parameter H are the quantities that, in equilibrium, become common for all the subsystems in the ensemble, and they have the same dimension of inverse time, $[T] = [H] = [1/t]$.

The physical temperature, in turn, gives rise to the total entropy and the local entropy, $S_V = \sum_n S_n = 12\,\pi KHV = s_{\rm loc} V$. So, in this scenario, de Sitter entropy comes from a set of many randomly distributed subsystems with the expansion rates H_n. Due to the heat exchange at the fixed total energy E_V, these states are organized in the equilibrium thermal state, which corresponds to the coherent de Sitter expansion of the whole system. The coherence due to thermalization may explain the horizon problem, i.e., why the causally disconnected regions of the CMB are in thermal equilibrium.

4.4. Regularization vs. Thermalization

The multi-metric ensemble may include the ensemble of N species of Weyl or Dirac fermions. At a large N, all tetrads in the random ensemble approach the same value, $e_\mu^{a(n)} \rightarrow e_\mu^a$, and thus, in the equilibrium state, all fermionic species experience the same geometry. In Refs. [61,74], the formation of the common Lorentz invariance for different fermionic species was also considered. But this was achieved via the renormalization group effect in the infrared limit instead of thermalization. This suggests the possible connection between renormalization and thermalization.

One may expect that the heat exchange between subsystems leads not only to the coherence of the de Sitter states but also to the general coherence of the metric fields when the metric fields $g_{\mu\nu(n)}$ of the subsystems become equal, thus forming the common metric $g_{\mu\nu}$. If this is true, this could be a kind of thermodynamic gravity but without using the holographic principle.

4.5. Coherence vs. Thermalization

At first glance, this formation of the coherent de Sitter expansion from the ensemble of the random microstates looks similar to the formation of the Bose–Einstein condensate (BEC) of magnons [75]. The magnon BEC represents the coherent precession of all spins, which results from the incoherent precessions of the individual spins with random frequencies ω_n in the local magnetic fields. The originally random frequencies correspond to the random Hubble parameters H_n in different sub-universes. The coherence of precession develops due to the spin currents between the regions of the local precessions—the analog of the heat exchange between the sub-universes. The formed common frequency ω of the coherent precession corresponds to the formed common Hubble parameter H of the large universe. Moreover, they have the same dimension of the inverse time, $[H] = [\omega] = [1/t]$, while the dimensionless number of magnons (or the dimensionless spin projection S_z, which is the same) corresponds to the total entropy of de Sitter, which is also dimensionless.

However, the source of the coherence is the exchange of spins instead of the exchange of energies. As a result, the formation of the coherent state of spin precession is due to the minimization of the total energy, E, at the fixed projection S_z of the total spin on the magnetic field. This leads to the common frequency of precession. In the magnon BEC interpretation, the common frequency plays the role of the chemical potential μ for magnons, which becomes constant in space due to the exchange of magnons between regions. That is why this process is quite the opposite of the formation of the common temperature, where the total energy is fixed, while the total entropy reaches its maximum value.

Anyway, in all cases, all the parameters, which are the same in all subsystems in equilibrium—common frequency, ω, common chemical potential, μ, common temperature, T, common angular velocity, $\mathbf{\Omega}$, and now also the common Hubble parameter H—have the same dimension of inverse time, $[T] = [H] = [\mu] = [\omega] = [\Omega] = [1/t]$. In Section 4.6, it is shown that the Hubble parameter H enters the chiral anomaly effects, together with the other thermodynamic variables: T, μ, and $\mathbf{\Omega}$. All of this supports the thermodynamic nature of the Hubble parameter.

4.6. de Sitter Contribution to Chiral Anomaly

As the thermodynamic quantity, the Hubble parameter H participates in different thermodynamical effects. We consider this using as an example the chiral vortical effect—the appearance of the chiral current in fermionic systems in the presence of rotation. For Dirac fermions in flat spacetime, the chiral current $\mathbf{j}_A$ contains the following contributions from the temperature, chemical potential, and angular velocity [76–80]:

$$\mathbf{j}_A = \left(\frac{T^2}{6} + \frac{\mu^2}{2\,\pi^2} + \frac{\Omega^2}{24\,\pi^2}\right)\mathbf{\Omega}\,. \tag{42}$$

In the de Sitter state, the gravitational thermodynamical variable—the curvature $\mathcal{R}$—is added [81,82]. According to Ref. [82], the gradient expansion provides the following contribution to the chiral current, which is very similar to the contributions of other thermodynamic quantities in Equation (42):

$$\mathbf{j}_A = \frac{H^2}{8\,\pi^2}\mathbf{\Omega}\,. \tag{43}$$

The same term is obtained for anti-de Sitter spacetime, where $H^2 < 0$ [83]. On the other hand, in Ref. [81], the curvature term comes from the shift in the fermionic mass gap, $-M^2 \to -(M^2 + H^2)$, and this leads to Equation (43) with the opposite sign.

A comparison of Equation (43) with Equation (42) demonstrates that there is a difference between the contribution $T^2/6$ in Equation (42), which comes from the temperature of matter, and the contribution $T^2/8$ from the temperature $T = H/\pi$ of the de Sitter environment in Equation (43). However, it is possible that the more rigorous calculations (beyond the gradient expansion, with the proper conservation laws and with the proper limit cases) can modify the coefficient in Equation (43). It is not excluded that these two contributions may cancel each other when the matter and the gravitational background are in equilibrium and, thus, have the same temperature. Such a cancellation of the currents generated via two different mixed gravitational anomalies in rotating the chiral liquid under the full equilibrium was found in Ref. [84]. This supports the Bloch theorem concerning the absence of the total current in equilibrium; see also Refs. [85–87].

Anyway, the participation of the curvature $\mathcal{R} = -12\, H^2$ in the thermodynamics of the chiral anomaly demonstrates the uniqueness of the de Sitter state in providing the contributions of gravitational variables to different thermodynamic effects together with the traditional thermodynamic variables, such as the temperature, chemical potential, angular velocity, and electric and magnetic fields.

5. Thermodynamics of de Sitter State and $f(R)$ Gravity

5.1. Thermodynamic Variables in $f(R)$ Gravity

Here, we consider the thermodynamics of the de Sitter state in terms of the gravitational variables and the corresponding modification of the thermodynamic Gibbs–Duhem relation for the quantum vacuum.

The conventional vacuum pressure $P_{\rm vac}$ obeys the equation of state $w = -1$ and enters the energy-momentum tensor of the vacuum medium in the following form:

$$T^{\mu\nu} = \Lambda g^{\mu\nu} = {\rm diag}(\epsilon_{\rm vac}, P_{\rm vac}, P_{\rm vac}, P_{\rm vac}) \ , \ P_{\rm vac} = -\epsilon_{\rm vac} \,. \tag{44}$$

In the de Sitter state, the vacuum pressure is negative, $P_{\rm vac} = -\epsilon_{\rm vac} < 0$.

Due to the linear dependence of the de Sitter entropy density on temperature, this pressure, $P_{\rm vac}$, does not satisfy the standard thermodynamic Gibbs–Duhem relation, $Ts_{\rm vac} = \epsilon_{\rm vac} + P_{\rm vac}$, because the right-hand side of this equation is zero. The reason for that is that, in this equation, we did not take into account the gravitational degrees of freedom of the quantum vacuum. Earlier, it was shown that gravity contributes to thermodynamics with a pair of thermodynamically conjugate variables: the gravitational coupling $K = \frac{1}{16\,\pi G}$ and the scalar Riemann curvature $\mathcal{R}$; see Refs. [9,88,89]. The contribution of the term $K\mathcal{R}$ to thermodynamics is similar to the work density [90–93].

The quantities K and $\mathcal{R}$ can be considered the local thermodynamic variables that, in condensed matter physics, are similar to the temperature, pressure, chemical potential, number density, spin density, etc. Indeed, since de Sitter spacetime is maximally symmetric, its local structure is characterized by the scalar curvature alone, while all the other components of the Riemann curvature tensor are expressed via $\mathcal{R}$:

$$R_{\mu\nu\alpha\beta} = \frac{1}{12}\left(g_{\mu\alpha}g_{\nu\beta} - g_{\mu\beta}g_{\nu\alpha}\right)\mathcal{R}\,. \tag{45}$$

That is why the scalar Riemann curvature, as the covariant quantity, naturally serves as one of the thermodynamical characteristics of macroscopic matter [94,95].

Another argument is related to the so-called Larkin–Pikin effect [96]. This is the jump in the number of degrees of freedom when the fully homogeneous state is considered. One has extra parameters, which are space-independent but participate in thermodynamics [97–99]. The same concerns the constant electric and magnetic fields *in vacuo* that, together, add three more degrees of freedom. These constant fields are mutually independent, in contrast to the spacetime-dependent fields connected via the Maxwell equations [99]. The scalar curvature

$\mathcal{R}$ in the de Sitter vacuum, which is constant in spacetime, also serves as such a thermodynamic parameter. Then, the gravitational coupling $K = df/d\mathcal{R}$ serves as the analog of the chemical potential, which is constant in full equilibrium.

5.2. Gibbs–Duhem Relation in $f(R)$ Gravity

The new thermodynamic variables, K and $\mathcal{R}$, which come from gravity and Equation (23) for entropy density, allow us to restore the Gibbs–Duhem relation for the de Sitter vacuum in the following form:

$$Ts_{\rm vac} = \epsilon_{\rm vac} + P_{\rm vac} - K\mathcal{R}\,. \tag{46}$$

This equation is obeyed, as can be checked using an example of Einstein gravity, where $Ts_{\rm vac} = 12\,\pi^2 KT^2 = 12\,KH^2$, using the equations $\epsilon_{\rm vac} + P_{\rm vac} = 0$ and $\mathcal{R} = -12\,H^2$. This supports the earlier proposal that K and $\mathcal{R}$ can be considered the thermodynamically conjugate variables [88,89].

Equation (46) can also be written using the effective vacuum pressure, which absorbs the gravitational degrees of freedom:

$$P = P_{\rm vac} - K\mathcal{R}\,. \tag{47}$$

Then, the conventional Gibbs–Duhem relation is restored:

$$Ts_{\rm vac} = \epsilon_{\rm vac} + P\,. \tag{48}$$

Equation (48) is just another form of writing the Gibbs–Duhem relation (46). But it allows for a different interpretation of the de Sitter vacuum state. The introduced effective de Sitter pressure, P, is positive, $P = \epsilon_{\rm vac} > 0$, and it satisfies the equation of state $w = 1$, which is similar to matter with the same equation of state. As a result, due to the gravitational degrees of freedom, the de Sitter state has many common properties with the non-relativistic Fermi liquid, where the thermal energy is proportional to T^2, and also with the relativistic stiff matter with $w = 1$ introduced by Zel'dovich [100].

5.3. Entropy of the Cosmological Horizon in Terms of Effective Gravitational Coupling

Let us now show that the holographic bulk–surface correspondence also remains valid in $f(\mathcal{R})$ gravity, i.e., that the entropy of the Hubble volume coincides with the entropy attributed to the cosmological horizon, $S_{\rm hor} = 4\,\pi KA$.

In the $f(\mathcal{R})$ gravity, the action is as follows:

$$S = -\int d^4x\sqrt{-g}f(\mathcal{R})\,. \tag{49}$$

In the equilibrium de Sitter state, the curvature is determined by the Einstein equations obtained via a variation of the action (49):

$$2f(\mathcal{R}) = \mathcal{R}\frac{df}{d\mathcal{R}}\,. \tag{50}$$

The corresponding vacuum energy density $\epsilon_{\rm vac} = -P_{\rm vac}$ in the $f(\mathcal{R})$ gravity is as follows:

$$\epsilon_{\rm vac} = f(\mathcal{R}) - K\mathcal{R}\;,\; K = \frac{df}{d\mathcal{R}}\,. \tag{51}$$

This shows that the gravitational coupling K is the natural definition of the variable, which is thermodynamically conjugate to the curvature $\mathcal{R}$, while $\epsilon_{\rm vac}$ serves as the corresponding thermodynamic potential. The Gibbs–Duhem relation for the de Sitter states in $f(\mathcal{R})$ gravity has the following conventional thermodynamic form:

$$Ts_{\rm vac} = \epsilon_{\rm vac} + P_{\rm vac} - K\mathcal{R}\,. \tag{52}$$

Let us now use the fact that the local temperature, $T = H/\pi$, of the equilibrium de Sitter state is the geometric property of de Sitter, being fully determined by the PG metric in Equation (1). This temperature, which in particular regulates the process of the ionization of an atom in the de Sitter environment, does not depend on the function $f(\mathcal{R})$. Using this local temperature, one obtains from Equation (52) the local entropy, $s_{\rm vac} = -K\mathcal{R}/T$, where $\mathcal{R} = -12\,H^2 = -12\,\pi^2 T^2$. Then, the total entropy of the Hubble volume, V_H, is given via the same Equation (27) as in the Einstein gravity:

$$S_{\rm bulk} = s_{\rm vac} V_H = 4\,\pi K A = S_{\rm hor}\,. \quad (53)$$

But now, K is the effective gravitational coupling in Equation (51).

This generalization of Gibbons–Hawking entropy was discussed in Refs. [9,101–103]. But here, it was obtained using the local thermodynamics of the de Sitter vacuum. This demonstrates that the local thermodynamics of the de Sitter vacuum is also valid for the $f(\mathcal{R})$ gravity. The effective gravitational coupling K serves as one of the thermodynamic variables of the local thermodynamics. This quantity plays the role of chemical potential, which is thermodynamically conjugate to the curvature $\mathcal{R}$, and it is constant in the thermodynamic equilibrium state of de Sitter spacetime.

5.4. Example of Quadratic Gravity

For illustration, we consider the simple example of the $f(\mathcal{R})$ gravity and the corresponding modification of the gravitational coupling K in the de Sitter environment. In the conventional Einstein gravity, where $f(\mathcal{R}) = K_0\mathcal{R} + \Lambda$, the de Sitter state has the equilibrium value of the curvature, $\mathcal{R}_0 = -2\Lambda/K_0 = -12H^2$. Let us add the quadratic term to the Einstein action [9,101]:

$$f(\mathcal{R}) = K_0\mathcal{R} - \frac{1}{2}\epsilon_G \mathcal{R}^2 + \Lambda\,. \quad (54)$$

Here, ϵ_G is the dimensionless parameter. In electrodynamics, this parameter corresponds to such parameters as the dielectric constant, magnetic permeability, and inverse fine-structure constant. These parameters contain the logarithm $\ln \frac{M_{\rm Pl}^4}{\mathbf{B}^2}$. This is the running coupling, in which the ultraviolet cut-off is given via the Planck mass, while the infrared cut-off is provided via the magnetic field $\mathbf{B}$. The coefficient of the logarithmic term depends on the number of massless fermionic and bosonic species. In gravity, there is also the logarithmic correction $\ln \frac{M_{\rm Pl}^4}{\mathcal{R}^2}$ to ϵ_G, where the infrared cut-off is provided via the Hubble parameter. But here, we ignore the logarithmic contributions for simplicity.

In the analogy with electrodynamics, the parameter K_0 corresponds to the spontaneous magnetization or the spontaneous polarization, which breaks the corresponding discrete T and P symmetries. In gravity, the corresponding broken discrete symmetry is the symmetry with respect to transformation, $\mathcal{R} \to -\mathcal{R}$. The possible origin of such discrete symmetry [104] is the symmetry under the coordinate transformation $x^\mu \to ix^\mu$ (the complex metric was also considered in Ref. [105]). Under this operation, the de Sitter state is transformed to anti-de Sitter; see also Ref. [106]. This is different from the time-reversal symmetry operation, which connects the black and white holes in Section 8.5, and the expanding and contracting de Sitter states in Section 7.2.

The equilibrium curvature $\mathcal{R}_0$ in the $f(\mathcal{R})$ gravity in Equation (54) can be obtained from Equation (50):

$$\mathcal{R}_0 = -2\frac{\Lambda}{K_0} = -12H^2\,, \quad (55)$$

It is the same as in Einstein gravity because the quadratic terms in Equation (50) are canceled. The equilibrium value of the effective gravitational coupling K is as follows:

$$K = \frac{df}{d\mathcal{R}}\Big|_{\mathcal{R}=\mathcal{R}_0} = K_0 + 2\epsilon_G\frac{\Lambda}{K_0}\,. \quad (56)$$

This modified gravitational coupling K determines the local entropy $s_{\rm vac}$, which follows from Equation (52). As a result, the entropy of the Hubble volume in Equation (57), which we identify with the entropy attributed to the horizon $S_{\rm hor}$, is also determined by the modified coupling K:

$$S_{\rm hor} = s_{\rm vac} V_H = 4\,\pi K A\,. \tag{57}$$

The local and global entropies change signs for $K < 0$, while the cosmological expansion is still described by the de Sitter metric. However, the negative K requires the negative parameter $\epsilon_G < 0$, which marks the instability of such a de Sitter vacuum [101].

6. From de Sitter Thermodynamics to de Sitter Decay

6.1. de Sitter State as Thermal Bath for Matter

The validity of the holographic connection between the bulk and surface entropies in the extension of the thermodynamics to $f(\mathcal{R})$ gravity also supports the idea that the de Sitter vacuum is the thermal state with the local temperature $T = H/\pi$. Such a gravitational temperature, which is twice the Hawking temperature, has also been obtained in the particular de Sitter limit, when the relativistic and non-relativistic matter tends to zero (see footnote 2 on page 4 in Ref. [107]). This demonstrates that, in the conventional approaches to de Sitter thermodynamics, where Euclidean time is used, the results may depend on the choice of the order of limits.

The nonzero local temperature of the gravitational vacuum shows that the de Sitter vacuum is locally unstable towards the creation of matter if some matter (such as an atom or electron) is originally present, see Section 2.3. This is distinct from the mechanism of creation of the pairs of particles via Hawking radiation from the cosmological horizon, which may or may not lead to the decay of the vacuum energy. There are still controversies concerning the stability of the de Sitter vacuum caused by Hawking radiation; see, e.g., refs. [98,108,109] and the references therein.

6.2. de Sitter Decay Due to Thermalization of Matter via de Sitter Heat Bath

To describe the decay of the vacuum due to the creation and further thermalization of matter, the extension of the Starobinsky analysis of vacuum decay [110–113] is needed. Particularly, the revolutionary stochastic inflation approach pioneered by Starobinsky [114,115] is extremely useful, although it requires some modifications [116–119]. This also includes the so-called separate universe approach, which is somewhat similar to the multi-metric gravity discussed in Section 4. Here, we consider the simple phenomenological scenario based on the energy exchange between the thermal de Sitter vacuum and the created thermal matter. This phenomenological description does not depend on the details of microscopic (UV) theory, and it requires only the slow-roll condition, i.e., the slow variation in the Hubble parameter, $|\dot{H}| \ll H^2$, or $|\dot{T}| \ll T^2$, which is the same.

The thermal exchange between the de Sitter heat bath and the excited matter generates the thermal relativistic gas. The temperature of relativistic gas tends to approach the temperature $T = H/\pi$ of the de Sitter background. Correspondingly, the energy density of this matter, ϵ_M, tends to approach the (quasi)equilibrium value at this temperature, $\epsilon_M(T) \sim T^4$. In terms of the Hubble parameter, one has $\epsilon_M(H) \to bH^4$, where the dimensionless parameter b depends on the number of massless relativistic fields—for example, $b = 7N_F/120\,\pi^2$ for N_F species of massless Weyl fermions.

The energy exchange between the vacuum heat bath and matter can be described by the following dynamical modification of the Friedmann equations [120], where the dissipative Hubble friction equation $\partial_t \epsilon_M = -4\,H\epsilon_M$ is extended to

$$\partial_t \epsilon_M = -4\,H(\epsilon_M - bH^4)\,. \tag{58}$$

This equation describes the tendency of matter to approach the local temperature of the vacuum, $T = H/\pi$. The extra gain in the matter energy, $4\,bH^5$, is compensated for with the corresponding loss of the vacuum energy:

$$\partial_t \epsilon_{\rm vac} = -4\, bH^5\,, \tag{59}$$

Since the vacuum energy density is $\epsilon_{\rm vac} = 6KH^2$, one obtains from Equation (59) the following time dependence of the Hubble parameter and the energy densities of the vacuum and matter:

$$H(t) = b^{-1/3} M_{\rm Pl} \left(\frac{t_{\rm Pl}}{t+t_0}\right)^{1/3}, \tag{60}$$

$$\epsilon_{\rm vac}(t) = 6\, b^{-2/3} M_{\rm Pl}^4 \left(\frac{t_{\rm Pl}}{t+t_0}\right)^{2/3}, \tag{61}$$

$$\epsilon_M(t) = bH^4 = b^{-1/3} M_{\rm Pl}^4 \left(\frac{t_{\rm Pl}}{t+t_0}\right)^{4/3}. \tag{62}$$

Here, $M_{\rm Pl}$ is the Planck mass, $M_{\rm Pl}^2 = K$, and $t_{\rm Pl} = 1/M_{\rm Pl}$ is the Planck time. We assume that $t_0 \gg t_{\rm Pl}$, and thus, $|\dot{H}| \ll H^2$.

Thus, the thermal character of the de Sitter state determines the process of its decay. The obtained power-law decay of H in Equation (60) was also found in Refs. [121–126], though using different approaches. In the Padmanabhan model [121,122], the de Sitter horizon is considered the photosphere with the Gibbons–Hawking temperature and the radiative luminosity $dE/dt \propto T^4 A_H$, where $A_H = 4\,\pi/H^2$ is the area of the horizon. Since the energy of the Hubble volume is $E \sim M_{\rm Pl}^2/H$, and $T^4 A_H \sim H^2$, this leads to the power law for the vacuum energy density in Equation (61). As was mentioned by Padmanabhan [122], in his model, the late-time cosmological constant is independent of its initial value; see Equation (61) at $t \gg t_0$.

In Refs. [123–125], the Starobinsky stochastic inflation approach was used. The parameter b therein is proportional to the number, N, of conformal fields, and the parameter t_0 is related to the initial value of the Hubble parameter at the beginning of inflation at $t = 0$:

$$H(t=0) = b^{-1/3} M_{\rm Pl} \left(\frac{t_{\rm Pl}}{t_0}\right)^{1/3} \ll M_{\rm Pl}\,. \tag{63}$$

This $H(t = 0)$ corresponds to the scaleron mass M in Starobinsky inflation. The time $t_0 \sim E_{\rm Pl}^2/H_{t=0}^3$ is called the quantum breaking time of spacetimes with a positive cosmological constant [127,128].

All of this demonstrates that the phenomenological scenario of the thermalization of matter via the de Sitter heat bath in Equations (58) and (59) is rather natural. It produces the inflation in terms of two phenomenological parameters, b and t_0, which determine the decay of the vacuum energy density in Equation (61). However, in Section 6.4, we consider another phenomenological approach that gives a different power law for de Sitter decay.

6.3. Connection to Holographic Principle

The evolution of the gravitational (dark) energy in Equation (61) and of the energy of the relativistic matter in Equation (62) allows us to consider another proposal made by Padmanabhan [129] (see also Refs. [130,131] and the references therein). It is the holographic postulate that connects the expansion of the Hubble volume with the difference between the number, $N_{\rm hor}$, of microstates on the surface of the horizon (one degree of freedom per Planck area) and the number, $N_{\rm bulk}$, of the degrees of freedom in bulk:

$$\frac{dV_H}{dt} = G\,(N_{\rm hor} - N_{\rm bulk})\,. \tag{64}$$

This postulate suggests that the expansion of the universe is being driven towards the holographic equipartition so that in the equilibrium de Sitter state that all the bulk degrees of freedom inside the horizon can be expressed via the horizon degrees of freedom, $N_{\rm bulk} = N_{\rm hor}$.

Instead of the speculative horizon degrees of freedom, we consider here the corresponding thermodynamic entropy. We already found that, according to Equation (27), the gravitational entropy of the Hubble volume is equal to the entropy attributed to the cosmological horizon, $S_{\rm bulk} = S_{\rm hor}$. Let us consider what happens in the presence of matter. On the one hand, matter adds its contribution, S_M, to the bulk entropy, and thus, $S_{\rm bulk} = S_M + S_{\rm hor}$. On the other hand, matter violates the de Sitter symmetry, which leads to the time dependence of the Hubble volume.

The entropy density of matter $s_M(t)$ can be obtained from the matter energy density in Equation (62):

$$s_M(t) = \frac{4\,\pi}{3} b H^3(t)\,. \tag{65}$$

Then, the total entropy of matter S_M in the Hubble volume is time-independent:

$$S_M = s_M(t) V_H(t) = \left(\frac{4\,\pi}{3}\right)^2 b\,. \tag{66}$$

The time derivative of the Hubble volume dV_H/dt, which is obtained from Equation (60), is also time-independent:

$$\frac{dV_H}{dt} = \frac{4\,\pi}{3} b l_{\rm Pl}^2\,. \tag{67}$$

Comparing Equation (67) with Equation (66), one has

$$\frac{dV_H}{dt} = \frac{3}{4\,\pi} l_{\rm Pl}^2 S_M\,. \tag{68}$$

Then, using equation $S_M = S_{\rm bulk} - S_{\rm hor}$, one obtains the general relation between the expansion in the Hubble volume and the difference between the bulk entropy and its holographic surface value:

$$\frac{dV_H}{dt} = 12\,G\,(S_{\rm bulk} - S_{\rm hor})\,. \tag{69}$$

Up to the sign and numerical coefficient, this coincides with Equation (64), which supports the Padmanabhan holographic conjecture [129], but without using the speculative degrees of freedom of the horizon.

6.4. de Sitter Decay and Zel'dovich Stiff Matter

As was mentioned by Padmanabhan, his photosphere model [122] leads to a late-time cosmological constant in Equation (61), which is independent of the initial value, but its value is still far too large. Can we fix this? In Section 5.2, we obtained indication that the thermodynamics of the de Sitter thermal bath also has the properties of the Zel'dovich stiff matter with $w = 1$. Let us try the stiff-matter scenario using our phenomenological approach.

We assume that the dynamics of the decaying de Sitter state can be considered as a kind of two-fluid hydrodynamics of superfluid liquid. We have the (superfluid) vacuum component with $w = -1$, which has de Sitter symmetry, and the (normal) stiff matter component with $w = 1$, which violates this symmetry. These two components tend to approach the common temperature. Such two-fluid behavior of the de Sitter state may also come from the observation in Section 6.5 that thermal fluctuations of the energy density in the Hubble volume are on the order of the vacuum energy density itself, $< (\Delta\epsilon_{\rm vac})^2 > / < \epsilon_{\rm vac} >^2 \sim 1$. So, in these speculations, the de Sitter state behaves as a mixture of dark energy (the de Sitter vacuum) and dark matter (the stiff matter or the thermal fluctuations of de Sitter).

For the (dark) matter with $w = 1$, the dissipative Hubble friction equation is $\partial_t \epsilon_{\rm DM} = -6\,H\epsilon_{\rm DM}$. Due to the energy exchange between the gravitational (dark energy) component and the dark matter component, the temperature of dark matter tends to approach the heat bath temperature $T = H/\pi$. Then, instead of Equations (58) and (59), one obtains

$$\partial_t \epsilon_{\rm DM} = -6\, H(\epsilon_{\rm DM} - \tilde{b} H^2)\,, \tag{70}$$

and

$$\partial_t \epsilon_{\rm vac} = -6\, \tilde{b} H^3\,. \tag{71}$$

Here, $\tilde{b}$ is the phenomenological dimensionless parameter on the order of unity whose microscopic origin is to be found. Similar equations with the corresponding phenomenological dimensionless parameter γ were suggested in Ref. [132], where the Polyakov scenario [109] of the infrared instability of the de Sitter space was discussed, and the pressureless matter was considered with $w = 0$.

Equation (71) gives the following power-law decay of dark energy and dark matter:

$$\epsilon_{\rm vac}(t) \sim \epsilon_{\rm DM}(t) \sim M_{\rm Pl}^4 \left(\frac{t_{\rm Pl}}{t + t_0}\right)^2 . \tag{72}$$

For a large $t \gg t_0$, this gives the reasonable order of magnitude of the vacuum energy density and of the energy density of matter in the present time:

$$\epsilon_{\rm vac}(t_{\rm present}) \sim \epsilon_{\rm DM}(t_{\rm present}) \sim \frac{M_{\rm Pl}^2}{t_{\rm present}^2} \sim 10^{-120} M_{\rm Pl}^4\,. \tag{73}$$

The same behavior of the vacuum energy density was obtained using the Hawking four-form field [9,133]. In this case, the role of dark matter is played by the oscillations of the four-form field during decay [134]. Note that, in our approach, both dark energy and dark matter come from the gravitational degrees of freedom. In this sense, it has relations to Refs. [135–137] and references therein, where the role of the gravitational degrees of freedom is discussed.

6.5. Thermal Fluctuations in de Sitter State

Here, we consider thermal fluctuations in the de Sitter thermal state, which may also serve as the source of the two-fluid behavior of de Sitter dynamics. The de Sitter thermal state represents the excited state of the Minkowski quantum vacuum. In addition, the deep Minkowski quantum vacuum experiences thermal fluctuations, which may play the role of dark matter.

According to Landau– Lifshitz [138], the thermal fluctuations are determined by the compressibility of the system and the considered volume, V. In the case of the fluctuating relativistic vacuum, one has the following [133]:

$$\left\langle (\Delta \epsilon_{\rm vac})^2 \right\rangle = \left\langle (\Delta P_{\rm vac})^2 \right\rangle = \frac{T}{V \chi_{\rm vac}}\,. \tag{74}$$

Here, $\chi_{\rm vac}$ is the vacuum compressibility [99]—the compressibility of the equilibrium Minkowski vacuum with $\epsilon_{\rm vac} = -P_{\rm vac} = 0$.

Note the main difference between the thermal fluctuations and quantum fluctuations. The contribution of the quantum fluctuations of the relativistic quantum fields to the vacuum energy density is typically in the order of $M_{\rm Pl}^4$, where $M_{\rm Pl}$ is the Planck mass. But in the full equilibrium vacuum state, this contribution is canceled due to the ultraviolet trans-Planckian degrees of freedom via the thermodynamic Gibbs–Duhem relation [99,133]. This cancellation is universal, being valid both for the relativistic vacuum states and for the non-relativistic ground states of the condensed matter systems. But the contribution of thermal fluctuations to vacuum energy is in the range of the applicability of infrared physics, where it is expressed in terms of the temperature, T, and the compressibility of the vacuum.

The value of the vacuum compressibility is determined by ultraviolet physics [99] with its Planck energy scale, $\chi_{\rm vac}^{-1} \sim M_{\rm Pl}^4$. This is similar to the gravitational coupling, which is also determined by the Planck scale, $K \sim M_{\rm Pl}^2$. On the other hand, the temperature corrections to $\chi_{\rm vac}^{-1}$ and K, as well as the Casimir corrections, are within the range of the applicability of infrared physics; see Ref. [139] for the universal temperature correction to the gravitational coupling K.

The contribution to compressibility via infrared physics was discussed in Refs. [140–143]. The negative contributions to compressibility obtained in these papers do not violate the stability of the quantum vacuum since these contributions represent corrections that are small compared to the main value of the vacuum compressibility, $\chi_{\rm vac}^{-1} \sim M_{\rm Pl}^4$.

Taking into account that the excited vacuum (the de Sitter state) has the temperature $T = H/\pi$ and the energy density $< \epsilon_{\rm vac} > \sim M_{\rm Pl}^2 H^2$, it follows from Equation (74) that the thermal fluctuations of the energy density in the Hubble volume are in the order of thermal energy density:

$$\left\langle (\Delta\epsilon_{\rm vac})^2 \right\rangle_{V=V_H} \sim \langle \epsilon_{\rm vac} \rangle^2 \,. \tag{75}$$

This can be the reason for the two-fluid behavior of the de Sitter state discussed in Section 6.4. The thermal fluctuations of dark energy play the role of dark matter in the same way as the oscillations of the dark energy in Ref. [134], and they lead to the same power-law decay in Equation (72) and to the present values of dark energy and dark matter in Equation (73).

6.6. Cosmological Constant Problems

In principle, the phenomenological approaches to the dynamics of vacuum energy density may produce different power-law decay. Examples are Equations (72) and (61). However, it is not excluded that these two asymptotic laws may correspond to different epochs.

So, if this speculative approach in Section 6.4 works, Equation (73) may solve all the cosmological constant problems:

(1) Why the cosmological constant is not large;
(2) Why the dark energy is on the order of magnitude of dark matter;
(3) Why they have the present value.

7. From de Sitter to Black Hole Thermodynamics

7.1. de Sitter vs. Black Hole

The thermodynamics of the de Sitter state is very different from the thermodynamics of black holes. A black hole is a compact object. The temperature of the Hawking radiation, T_H, from the black hole horizon is well determined, which is also supported by condensed matter analogs [144]. On the other hand, the origin of black hole entropy is still not clear, although it can be determined from the equation $dM = T_H dS$, assuming that the laws of thermodynamics are applicable to this compact object.

On the contrary, the de Sitter state is not a compact object. It is a homogeneous vacuum state without boundaries that has the homogeneous energy density as the local thermodynamic variable. The local energy density allows us to also introduce the local temperature $T = H/\pi$. This local temperature can be measured via any detector that is stationary with respect to the shift velocity and that measures, for example, the rate of the ionization of an atom, $w \propto \exp(-E/T)$.

7.2. Entropy of Expanding, Contracting, and Static de Sitter

However, the de Sitter state allows us to probe the origin of black hole entropy. For that, we must consider three different states of the de Sitter quantum vacuum: the expanding de Sitter state with $H > 0$, the contracting de Sitter state with $H < 0$, and the de Sitter state with the fully static metric. Let us note that the metrics of the expanding and contracting de Sitter states are stationary but not static since their shift velocities are non-zero.

Let us start with the fully static de Sitter metric:

$$ds^2 = -(1 - H^2r^2)dt^2 + \frac{dr^2}{1 - H^2r^2} + r^2d\Omega^2 \,. \tag{76}$$

This metric has singularity at the cosmological horizon, which requires the proper attention. In one approach, this static configuration can be considered the intermediate symmetric state between the two states with broken time-reversal symmetry—the expanding and contracting states. The same consideration applies to the static hole, that can be viewed as the intermediate state between the black and white holes [88].

The contracting de Sitter vacuum has a negative Hubble parameter, $H < 0$. That is why its local temperature is negative, $T = H/\pi < 0$, and the local entropy is also negative, $s_{\rm contracting} = 3\,H/4\,G = -3/(4\,Gr_H) < 0$. The total entropy in the Hubble volume of the contracting de Sitter state is

$$S_{\rm contracting} = s_{\rm contracting} V_H = -\frac{A}{4\,G} \,, \tag{77}$$

where A is the area of the cosmological horizon. That is why the entropy of the static de Sitter state in Equation (76), as the intermediate state between the states with positive and negative entropies, is zero. Correspondingly, the temperature of this state must be infinite, which is consistent with the singularity at the horizon; see also Ref. [145].

7.3. Gravastar—Black Hole with de Sitter Core

The connection between the black hole and the de Sitter state appears when we consider the black hole obtained through the deformation of the gravastar [20]. The gravastar is an object that contains a de Sitter vacuum inside a black hole horizon [146–148]. We consider the gravastar in which the black hole horizon coincides with the de Sitter horizon, $r_{\rm bh} = r_H$ (the metric in the state with the critical value of the mass parameter $m = 2\,MG|H| = 1$, at which two horizons merge, is illustrated in Figure 1 of Ref. [149]). Since, in such a gravastar, the two horizons cancel each other, there is no Hawking radiation, and the entropy of the gravastar is zero. In the Painlevé–Gullstrand form, the metric of such a gravastar is given via Equation (1) with the following shift velocity:

$$v(r) = -\sqrt{\frac{r_H}{r}} \;,\; r > r_H \,, \tag{78}$$

$$v(r) = -\frac{r}{r_H} \;,\; r < r_H \,. \tag{79}$$

Here, $r_H = 1/|H| = r_{\rm bh}$, where $r_{\rm bh} = 2MG$, and $M = 1/(2\,G|H|)$ is the mass of the black hole, which is formed by the de Sitter core.

The shift velocity $v(r)$ is continuous across the surface, $r = r_H$, while the gradient of the shift velocity dv/dr experiences a jump at this surface. It is important that the shift velocity $v(r)$ is negative everywhere. Since it is negative in the de Sitter core, this means that de Sitter spacetime in the core of this gravastar is contracting, $v(r) = Hr = -r/r_H < 0$; i.e., the Hubble parameter is negative, $H = -1/r_H < 0$.

7.4. Entropy of Black Hole from Negative Entropy of Contracting de Sitter Core

According to Equation (77) the region of the contracting de Sitter core of the gravastar has negative entropy, $S_{\rm contracting} = -A/4\,G$. That is why the gravastar is unstable towards the shrinking of the de Sitter region since this leads to the increase in the entropy of the whole system due to the decrease in the negative entropy of the contracting de Sitter state. Due to energy conservation, the shrinking of the volume of the de Sitter core leads to the formation of the singularity at $r = 0$, where the mass becomes concentrated. In the final state—the black hole—the de Sitter region with negative entropy fully disappears by shrinking to the singularity with mass M. The resulting black hole with mass M acquires positive entropy, $A/4\,G$:

$$S_{\rm BH} = S_{\rm gstar} - S_{\rm contracting} = 0 - s_{\rm contracting} V_H = \frac{A}{4\,G}\,. \tag{80}$$

The black hole entropy is also concentrated in the singularity, together with the curvature $\mathcal{R}$ and mass M [20]. This again supports the holographic connection between the entropy of bulk, which is concentrated in the singularity, and the surface entropy of the black hole horizon.

Equation (80) allows the interpretation of the zero value of the entropy of the gravastar in terms of the cancelation of the entropies of two horizons. In the initial gravastar state, the entropy of the contracting de Sitter horizon fully compensates for the entropy of the black hole horizon:

$$S_{\rm gstar} = S_{\rm contracting} + S_{\rm BH} = -\frac{A}{4\,G} + \frac{A}{4\,G} = 0\,. \tag{81}$$

7.5. White Hole and Anti-Gravastar

In the same way, the anti-gravastar can be obtained as the white hole with the de Sitter core. This object has the following shift velocities:

$$v(r) = \sqrt{\frac{r_H}{r}}\ ,\ r > r_H\,, \tag{82}$$

$$v(r) = \frac{r}{r_H}\ ,\ r < r_H\,. \tag{83}$$

It is obtained from the pure white hole, which has the negative horizon entropy $S_{\rm WH} = -A/4\,G$ [88,89], by growing the de Sitter state in its core with positive entropy. In the anti-gravastar state, the entropies of two horizons cancel each other in the same way as in the gravastar:

$$S_{\rm antigstar} = S_{\rm expanding} + S_{\rm WH} = \frac{A}{4\,G} - \frac{A}{4\,G} = 0\,. \tag{84}$$

It is important that the coordinate singularities in the metric of the gravastar and in the metric of the anti-gravastar can be smoothly removed via small deformations. That is why these objects do not depend on the choice of the coordinate systems, and thus, they are equivalent to the fully static black hole with the fully static de Sitter core:

$$ds^2 = -(1 - 2\,GM/r)dt^2 + \frac{dr^2}{1 - 2\,GM/r} + r^2 d\Omega^2\ ,\ r > 2\,MG = 1/H\,, \tag{85}$$

$$ds^2 = -(1 - H^2 r^2)dt^2 + \frac{dr^2}{1 - H^2 r^2} + r^2 d\Omega^2\ ,\ r < 2\,MG = 1/H\,. \tag{86}$$

This again supports the zero entropy of these gravastars and also the zero entropy of the fully static de Sitter state with the metric in Equation (76).

7.6. Gibbs–Duhem and Black Hole Thermodynamics

Let us show that the modified Gibbs–Duhem relation in Equation (46) is also applicable to the thermodynamics of black holes. As distinct from the de Sitter state, the black hole is a compact object, and its thermodynamics operates with global parameters, such as mass, M, the entropy of the horizon, the total electric charge, Q, and the total angular momentum, J. This global thermodynamics can be described by the integral form of the Gibbs–Duhem relation in Equation (46). The curvature here comes from the central singularity [150]:

$$\mathcal{R} = 8\,\pi MG\,\delta(\mathbf{r})\,. \tag{87}$$

Since the energy density here is $\epsilon = M\delta(\mathbf{r})$, the integration of the right-hand side of Equation (46) over space gives the following relation for the Schwarzschild black hole:

$$T_{\rm BH}S_{\rm BH} = M - \int d^3r\sqrt{-g}K\mathcal{R} = \frac{M}{2}\,. \tag{88}$$

This agrees with the global thermodynamics of the black hole with the Hawking temperature $T_{\rm BH} = \frac{1}{8\,\pi MG}$ and the Bekenstein– Hawking entropy $S_{\rm BH} = \frac{A}{4G}$. Equation (88) is also valid for the white hole, where the temperature and entropy are opposite to that of the black hole with the same mass [88], $T_{\rm WH}(M) = -T_{\rm BH}(M)$ and $S_{\rm WH}(M) = -S_{\rm BH}(M)$.

In principle, the central singularity in the black hole may spontaneously lose spherical symmetry, forming, for example, a kind of rigid top. This, in turn, may influence the shape of the event horizon. This is similar to the deformations of the cosmological horizon via masses concentrated on vertices of Platonic solids deep within the Hubble volume [151].

7.7. Entropy of the Schwarzschild– de Sitter Cosmological Horizon

Let us consider the possible application of the modified Gibbs–Duhem relation to the Schwarzschild–de Sitter (SdS) black hole. We discuss the simple case of the Nariai limit, when the black hole horizon approaches the cosmological horizon, $r_b \to r_0 - 0$ and $r_c \to r_0 + 0$, where

$$r_0 = (GM/H^2)^{1/3} = \frac{1}{\sqrt{3}\,H}\,. \tag{89}$$

In this limit, the temperatures of the horizons approach the Bousso–Hawking value [152]:

$$T_b = T_c = \frac{\sqrt{3}}{2\,\pi}H = \frac{1}{6\,\pi GM}\,. \tag{90}$$

The entropy of the cosmological horizon $S_{\rm c}$ can be obtained via the integration of the right-hand side of Equation (46) over the volume inside the cosmological horizon with radius r_0:

$$T_{\rm c}S_{\rm c} = M - \int_{r<r_0} d^3r\sqrt{-g}K\mathcal{R}\,. \tag{91}$$

Since we discuss here the Nariai limit, one obtains the following:

$$T_{\rm c}S_{\rm c} = M - \int_{r<r_0} d^3r\sqrt{-g}K\mathcal{R} = M - \frac{M}{2} + \frac{3\,H^2}{4\,\pi}\,\frac{4\,\pi r_0^3}{3} = \frac{3}{2}M. \tag{92}$$

From this equation, one obtains the entropy of the cosmological horizon:

$$S_{\rm c} = \frac{3\,M}{2\,T_{\rm c}} = \frac{\pi r_0^2}{G} = \frac{A}{4\,G}\,. \tag{93}$$

This again agrees with the Gibbons–Hawking entropy attributed to the horizon, although this holographic connection is valid only in the Narai limit.

7.8. Heat Exchange between Black Holes in the Multi-Metric Ensemble

The thermodynamic character of the black hole objects can be tested using the multi-metric ensemble in Section 4. Let us consider an ensemble of n sub-universes in the same coordinate spacetime, $S = -\int d^4x \sum_n \mathcal{L}_n$, but with with different metrics, $g_{\mu\nu(n)}$, different gravitational couplings, K_n, and the corresponding individual black holes with masses M_n. If one introduces the energy exchange between these sub-universes, then the masses M_n will be varied at the fixed total mass $M = \sum_n M_n$ of the whole universe. In the thermal equilibrium, which corresponds to the maximum of the total entropy, the individual metrics approach the common metric, and the whole system corresponds to the universe with the gravitational coupling $K = \sum_n K_n$ and with the black hole of mass M. The equilibrium masses, M_n, of the individual substate approach the following values:

$$M_n = M\,\frac{K_n}{K}\,. \tag{94}$$

In this equilibrium state, the Hawking temperature of the black hole with mass M in the whole ensemble coincides with the Hawking temperatures of the individual black holes in each gravitational sub-state:

$$\frac{1}{T_n} = \frac{dS_n}{dM_n} = \frac{dS}{dM} = \frac{1}{T_H}\,. \tag{95}$$

This is the effect of thermalization.

8. Black and White Holes Entropy from Macroscopic Quantum Tunneling

8.1. Collective Canonically Conjugate Variables for Schwarzschild Black Hole

Considering the thermodynamics of gravity systems, we used thermodynamically conjugate variables—the gravitational coupling $K = \frac{1}{16\pi G}$ and the scalar curvature $\mathcal{R}$, which, as the covariant quantity, may serve as one of the thermodynamical characteristics of macroscopic matter [94]. These variables are the local thermodynamic variables that are similar to the temperature, pressure, chemical potential, number density, etc., in condensed-matter physics.

The gravitational coupling K is determined by UV microscopic physics, but in the description of gravity as a macroscopic phenomenon in the IR limit, it is the collective variable. In the superfluid ^{3}He-A with Weyl fermionic quasiparticles, both the coupling K in the effective gravity and the fine structure "constant" α in the effective electrodynamics are determined by physics on the microscopic (atomic) level [31]. In microscopic theory, one obtains $K \propto \Delta_0^2$ and $1/\alpha \propto \ln(\Delta_0/T)$, where Δ_0 is the gap amplitude, and T is the temperature of the liquid. The gap amplitude Δ_0 plays the role of the ultraviolet cut-off $M_{\rm Pl}$, while T provides the infrared cut-off. For the relativistic quantum fields with massless particles, the infrared cut-off is either the temperature T or the strength of the fields. In the inhomogeneous superfluid (inhomogeneous vacuum), both K and α depend on coordinates. It is not surprising that, in the relativistic quantum vacuum, there is also a connection between the gravitational coupling K and the coupling α in quantum electrodynamics, as suggested in Refs. [153–159].

Now, for the discussion of the quantum-mechanical tunneling of the macroscopic objects, we need the collective dynamical variables instead of the thermodynamic variables. The canonically conjugate dynamical variables that are relevant to the black hole are the gravitational coupling K and the horizon area $A = 4\,\pi R^2$. Bekenstein [160] proposed that A is an adiabatic invariant and, thus, can be quantized according to the Ehrenfest principle that classical adiabatic invariants may correspond to observables with a discrete spectrum. So, the area of the horizon, A, is a proper candidate for the quantum mechanics of the black hole.

8.2. Modified First Law of Black Hole Thermodynamics

Let us first consider how these variables enter black hole thermodynamics. For that, it is convenient to use the redefined gravitational coupling $\tilde{K} = 4\,\pi K = \frac{1}{4G}$. In terms of this coupling, the Hawking temperature of the Schwarzschild black hole and its Bekenstein entropy are as follows:

$$T_{\rm BH} = \frac{\tilde{K}}{2\,\pi M}\ ,\ \ S_{\rm BH} = \frac{\pi M^2}{\tilde{K}} = A\tilde{K}\,. \tag{96}$$

If $\tilde{K}$ is a global thermodynamic variable, one obtains the following modification of the first law of black hole thermodynamics,

$$dS_{\rm BH} = d(A\tilde{K}) = \pi d(M^2/\tilde{K}) = -\pi\frac{M^2}{\tilde{K}^2}d\tilde{K} + 2\,\pi\frac{M}{\tilde{K}}dM \tag{97}$$

or

$$dS_{\rm BH} = -Ad\tilde{K} + \frac{dM}{T_{\rm BH}}\,. \tag{98}$$

This modification is similar to the modification in terms of the moduli fields [161]. But in our case, the thermodynamic variable, which is conjugate to the thermodynamic variable $\tilde{K}$, is the product of the black hole area and the black hole temperature, $AT_{\rm BH}$. On the other hand, in dynamics, $\tilde{K}$ and A are canonically conjugate variables; see Section 8.4.

In general, the variable $\tilde{K}$ is local and depends on a space coordinate, but in the same way as for the moduli fields [161], the black hole thermodynamics is determined by the asymptotic value of K at spatial infinity. In Equation (98), $\tilde{K} \equiv \tilde{K}(\infty)$ is the global quantity that characterizes the quantum vacuum in full equilibrium, i.e., far from the black hole.

Also, the variable $\tilde{K}$ allows us to study the transition to the vacuum without gravity, i.e., to the vacuum where $\tilde{K} \to \infty$ and, thus, $G \to 0$; see Section 8.4.

8.3. Adiabatic Change in Coupling K and Adiabatic Invariant

Let us change the coupling, $\tilde{K}$, and the black hole mass, M, adiabatically, i.e., at the constant entropy of the black hole. Then, the equation $dS_{\rm BH} = 0$ gives

$$\frac{dM}{d\tilde{K}} = AT_{\rm BH} = \frac{M}{2\,\tilde{K}}\,. \tag{99}$$

This shows that $M^2/\tilde{K} = {\rm const}$ is the adiabatic invariant for the spherical, electrically neutral black hole. Thus, according to the Bekenstein conjecture [160], it can be quantized in quantum mechanics:

$$\frac{M^2}{\tilde{K}} = aN\,. \tag{100}$$

Here, N is an integer, and a is some fundamental dimensionless parameter of order unity. If this conjecture is correct, one obtains the quantization of the entropy of the Schwartzschild black hole:

$$S_{\rm BH}(N) = \pi\frac{M^2}{\tilde{K}} = \pi a N\,. \tag{101}$$

The Bekenstein idea of the role of adiabatic invariants in the quantization of the black hole requires further consideration; see some approaches to that in Refs. [162–167]. In particular, the similarity between the energy levels of the Schwarzschild black hole and the hydrogen atom has been suggested [168,169]. We leave this problem for the future. This consideration should be supported with microscopic theory; see, e.g., ref. [170].

In this respect, the condensed matter analogs can be useful since, in condensed-matter systems, the physics is known both on macro and micro (atomic) levels. One example is provided via a consideration of the quantum nucleation of the vortex ring in moving superfluids—the vortex instanton. It shows that the corresponding entropy that determines the nucleation process, $\exp(-S_{\rm ring})$, is quantized:

$$S_{\rm ring}(N) = 2\,\pi N\,. \tag{102}$$

Here, N is the number of atoms involved in the process of vortex instanton; see Section 26.4 and Equations (26.20) and (26.21) in Ref. [31]. This would correspond to the parameter $a = 2$ in Equation (100).

8.4. A and K as Canonically Conjugate Variables and Black-Hole–White-Hole Quantum Tunneling

The canonically conjugate variables A and K allow us to consider the quantum mechanical tunneling from the black hole to the white hole—the process discussed in Refs. [171–180] and in the references therein. In the semiclassical description of this macroscopic quantum tunneling, the trajectory in the $(\tilde{K}, A)$ phase space must be considered, which, in the complex plane, connects the black and white holes.

Note the difference from the consideration in Section 8.3, where $\tilde{K}$ varies in the adiabatic regime, i.e., at a fixed entropy, $S_{\rm BH}$, while the area, A, temperature, $T_{\rm BH}$, and mass, M, follow the variation of $\tilde{K}$. In the dynamic regime, which is relevant for the description of quantum tunneling, the parameter $\tilde{K}$ varies at a fixed energy (fixed mass M of the black hole), while the area, A, temperature, $T_{\rm BH}$, and entropy, $S_{\rm BH}$, follow the variation in $\tilde{K}$.

As in the case of the semiclassical consideration of Hawking radiation in terms of quantum tunneling, we shall use the Painleve–Gullstrand coordinate system with the following metric:

$$ds^2 = -dt^2(1 - \mathbf{v}^2) - 2dt\, d\mathbf{r} \cdot \mathbf{v} + d\mathbf{r}^2 \,, \tag{103}$$

where, for the Schwartzschild black hole, one has

$$\mathbf{v}(\mathbf{r}) = \mp\hat{\mathbf{r}}\sqrt{\frac{R}{r}} = \mp\hat{\mathbf{r}}\sqrt{\frac{M}{2r\tilde{K}}} = \mp\hat{\mathbf{r}}\sqrt{\frac{2MG}{r}} \,, \tag{104}$$

where R is the radius of the horizon, the minus sign corresponds to the black hole, and the plus sign describes the white hole. In the theory with variable gravitational coupling, the sign changes at the singularity $\tilde{K} = \infty$ (i.e., at $G = 0$), when the black hole shrinks to a point and then expands as a white hole. The point $\tilde{K} = \infty$, where gravity disappears, serves as the branch point, where the velocity of the freely falling observer changes signs.

The vector $\mathbf{v}$ is normal to the surface of the horizon. When $\mathbf{v}$ changes sign, the horizon area, A, also changes sign: it crosses zero at $\tilde{K} = \infty$ and becomes negative on the white-hole side of the process, $A \to -A$. This also could mean that, due to the connection between the area and entropy, the white hole may have negative entropy. This is what we discuss in Section 8.5.

The quantum tunneling exponent is usually determined by the imaginary part of the action on the trajectory, which transforms the black hole into a white hole. In terms of Euclidean action, one has the following:

$$p_{\rm BH\to WH} \propto \exp\left(-I_{\rm BH\to WH}\right) \,,\; I_{\rm BH\to WH} = \int_C A(\tilde{K}')d\tilde{K}' \,. \tag{105}$$

Here, the semiclassical trajectory C is at $M = {\rm const}$, and thus, $A(\tilde{K}') = \pm\pi M^2/(\tilde{K}')^2$. Along this trajectory, the variable $\tilde{K}'$ changes from $\tilde{K}$ to the branch point at $\tilde{K}' = \infty$ and then from $\tilde{K}' = \infty$ to $\tilde{K}' = \tilde{K}$ along the other branch, where the area $A(\tilde{K}') < 0$. The integral gives the tunneling exponent of the transition to the white hole

$$I_{\rm BH\to WH} = 2\,\pi M^2 \int_{\tilde{K}}^{\infty} \frac{d\tilde{K}'}{\tilde{K}'^2} = 2\pi\frac{M^2}{\tilde{K}} \,. \tag{106}$$

The tunneling exponent in Equation (106) can be expressed in terms of the black hole entropy in Equation (96), and for the probability of transition, one obtains the following:

$$p_{\rm BH\to WH} \propto \exp\left(-2\,\pi M^2/\tilde{K}\right) = \exp\left(-2\,S_{\rm BH}\right) . \tag{107}$$

It is important that Equation (107) contains exactly twice the black hole entropy. This allows us to consider again the entropy of the white hole.

8.5. Negative Entropy of White Hole

The connection between the probability of the transition and the thermodynamic fluctuations [181] is also applicable to the transition between the black and white holes. The total change in the entropy in this process is $\Delta S = S_{\rm WH} - S_{\rm BH}$. According to Equation (107), this change is equal to $-2\,S_{\rm BH}$. Then, from equation $S_{\rm WH} - S_{\rm BH} = -2\,S_{\rm BH}$, one obtains that the entropy of the white hole is equal to the opposite sign of the entropy of the black hole with the same mass:

$$S_{\rm WH}(M) = -S_{\rm BH}(M) \,. \tag{108}$$

This means that the white hole, which is obtained via quantum tunneling from the black hole and, thus, has the same mass, M, as the black hole, has the negative temperature $T_{\rm WH} = -T_{\rm BH}$ and the negative area $A_{\rm WH} = -A_{\rm BH}$. According to the first law in Equation (98), applied now to the white hole, this gives the negative entropy $S_{\rm WH} = -S_{\rm BH}$.

Such anti-symmetry between the black and white holes is similar to the anti-symmetry between the expanding and contracting de Sitter states discussed in Section 7.2.

The discussed transition from the black hole to the white hole with the same mass, M, is not the thermodynamic transition. It is the quantum process of tunneling between the two quantum states. It is one of many routes of black hole evaporation, including the formation of a small white hole in the late stage of decay [171–180]. The uniqueness of this particular route, the hidden information, and the (anti)symmetry between the black and white holes are combined to produce the negative entropy of the white hole.

8.6. Black-Hole-to-White-Hole Transition as a Series of Hawking Radiation Co-Tunneling

Let us show that the result (107), where the probability of the quantum tunneling between the black and white holes is determined by twice the black hole entropy, is supported by a consideration of the conventional Hawking radiation of particles from the black hole. The tunneling exponent of radiation can be expressed in terms of the change in the entropy of the black hole after radiation, $p \propto e^{\Delta S_{\rm BH}}$; see Refs. [182–185]. This demonstrates that the quantum process of Hawking radiation can be considered as thermodynamic fluctuation [181].

We consider the process of co-tunneling, in which the particle escapes the black hole via quantum tunneling, and then this particle tunnels to the white hole through the white hole horizon (this is the analog to electron tunneling via an intermediate virtual state in electronic systems [186,187]). This process takes place at the fixed total mass M. The tunneling exponent needed for this process to occur is $e^{(\Delta S_{\rm BH}+\Delta S_{\rm WH})}$. A summation of all the processes of the tunneling of matter from the black hole to the formed white hole finally gives Equation (107):

$$p \propto e^{\sum(\Delta S_{\rm BH}+\Delta S_{\rm WH})} = e^{2\sum \Delta S_{\rm BH}} = \exp\left(-2S_{\rm BH}\right). \tag{109}$$

Here, we took into account the (anti)symmetry in the dynamics of black and white holes in the process of quantum tunneling, $\sum \Delta S_{\rm BH} = \sum \Delta S_{\rm WH}$.

8.7. Emission of Small Black Holes vs. Hawking Radiation

The principle that macroscopic quantum tunneling can be considered a thermodynamic fluctuation can also be applied to the process of the creation of pairs of black holes [188], to the process of the splitting of the black hole into two or several smaller black holes with the same total mass (see, e.g., Ref. [189]), and to other processes with macroscopic objects. For example, the probability of the decay of the black hole with mass M into two black holes with $M_1 + M_2 = M$ is as follows:

$$p(M \to M_1 + M_2) \propto \tag{110}$$

$$e^{S_{\rm BH}(M_1)+S_{\rm BH}(M_2)-S_{\rm BH}(M_1+M_2)} = e^{-8\pi G M_1 M_2}. \tag{111}$$

In the particular limit case $m = M_1 \ll M_2 \approx M$, this channel of the black hole decay describes the emission of the small black hole with mass m via the large black hole with mass M:

$$p(m, M) \propto \exp\left(-\frac{m}{T_{\rm BH}(M)}\right), \; m \ll M. \tag{112}$$

This shows that the macroscopic tunneling process of the emission of a small black hole via a large black hole is governed by the same Hawking temperature $T_{\rm BH}(M) = \frac{1}{8\pi GM}$ as the Hawking radiation of a particle, which tunnels across the horizon.

However, there is a difference. In Equation (112), the quadratic term m^2 is neglected. In a general case, one obtains from Equation (111) the following:

$$p(m, M - m) \propto \exp\left(-8\,\pi G m(M - m)\right). \tag{113}$$

This equation demonstrates the effect of a back reaction—the correction to the Hawking radiation caused by the reduction in the black hole mass after the radiation of a small black hole. A similar correction due to the back reaction in the process of the radiation of particles was obtained by Parikh and Wilczek [185] (see also Ref. [182]):

$$p(\omega, M - \omega) \propto \exp\left(-8\,\pi G \omega\left(M - \frac{\omega}{2}\right)\right), \tag{114}$$

where ω is the energy of the emitted particle. As distinct from Equation (113), for the emission of a small black hole, Equation (114) contains the factor $1/2$. The reason for such a difference is that, in the case of the emission of a small black hole, the probability of emission contains the extra term compared to the emission of particles—the entropy of the emitted black hole, $S(m) = 4\,\pi G m^2$:

$$p(m, M - m) \propto \exp\left(-8\,\pi G m\left(M - \frac{m}{2}\right) + 4\,\pi G m^2\right). \tag{115}$$

As a result, Equation (113) is restored. This also coincides with the result of the radiation of the self-gravitating shell [190].

Equation (111) can be extended to the emission of several black holes. For example, the probability of the emission of N black holes with masses $m = M/N$ is as follows:

$$p(M \to mN) \propto \exp\left(-4\,\pi G M^2\left(1 - \frac{1}{N}\right)\right). \tag{116}$$

For a large N, this corresponds to the probability of the destruction of the black hole via an explosion within the particular channel:

$$p_{N\to\infty} \propto \exp(-S_{\mathrm{BH}}). \tag{117}$$

There are many channels for the destruction of the black hole in quantum tunneling. The black hole entropy S_{BH} can be estimated using the probability of a black hole explosion in a single quantum event. Then, Equation (117) suggests that the entropy of the black hole is determined by the number of possible channels leading to the destruction of the black hole.

The discussed scenario of the destruction of black holes via an explosion into small-Planck-scale objects also explains the origin of the negative entropy of the white hole. The explosion is a rare process in which the entropy is reduced to zero. However, after the explosion occurs, an even rarer process may follow when the small-Planck-scale objects are collected back, forming the white hole. The rarity of this process reduces the entropy even further, which leads to the negative entropy of the formed white hole. This is the reason why the white hole has negative entropy. The latter is also supported by the consideration of the super-rare process of quantum tunneling from the black hole to the white hole of the same mass in Section 8.5. The decrease in the entropy in this super-rare process corresponds to the loss of the double entropy of the black hole, which results in the negative entropy of the white hole.

The negative temperature of the white hole is also not very surprising. In general, the negative absolute temperatures are consistent with equilibrium thermodynamics. All the thermodynamic properties, such as thermometry, the thermodynamics of cyclic transformations, ensemble equivalence, fluctuation–dissipation relations, response theory, the transport processes, and symmetry breaking-phase transitions, can be reformulated to include negative temperatures [191–193].

9. Conclusions

The starting point of our consideration was that matter immersed in the de Sitter vacuum perceives this vacuum as a heat bath with the local temperature $T = H/\pi$, where H is the Hubble parameter. This temperature has no relation to the cosmological horizon or the Hawking radiation from the cosmological horizon. However, it is exactly twice the Gibbons–Hawking temperature, $T_{\rm GH} = \frac{H}{2\pi}$. The reason for such a relation is the specific symmetry of de Sitter spacetime, which is similar to the invariance under translations in the Minkowski vacuum.

There are also discrete symmetries that are important in considerations of the thermodynamics of de Sitter. The expanding de Sitter universe represents one of the two degenerate states formed via broken time-reversal symmetry. These states are the expanding de Sitter state and the contracting de Sitter state. These states are obtained from each other via the time reversal transformation $t \to -t$. Another broken discrete symmetry corresponds to the reversal of the sign of the scalar curvature, $\mathcal{R} \to -\mathcal{R}$. This symmetry operation transforms the de Sitter state into the anti-de Sitter state. This symmetry is spontaneously broken by the term linear in $\mathcal{R}$ in the the Einstein action, $\sqrt{-g}K\mathcal{R}$, where the gravitational coupling K plays the role of the order parameter in this scenario of symmetry breaking.

The local thermodynamics of the de Sitter state in Einstein gravity gives rise to the Gibbons–Hawking area law for the total entropy inside the cosmological horizon. We extended the consideration of local thermodynamics to $f(\mathcal{R})$ gravity and obtained the same area law but with the modified gravitational coupling $K = df/d\mathcal{R}$. The agreement with the traditional global thermodynamics of de Sitter supports the suggestion that the de Sitter vacuum is the thermal state with the local temperature $T = H/\pi$ and that the local thermodynamics is based on the thermodynamically conjugate gravitational variables K and $\mathcal{R}$. This pair of non-extensive gravitational variables is similar to the pair of electrodynamic variables, the electric field, $\mathbf{E}$, and the electric induction, $\mathbf{D}$, which participate in the thermodynamics of dielectrics. The gravitational variables modify the thermodynamic Gibbs–Duhem relation, due to which the thermal properties of the de Sitter state become similar to those of Zel'dovich stiff matter and the non-relativistic Fermi liquid.

The local temperature $T = H/\pi$ leads to the creation of multiple particles even if only a single electron is introduced to the de Sitter vacuum. This results in the thermal instability of the de Sitter state towards the formation of matter and to the further thermalization of this matter via the de Sitter heat bath. The process of the thermalization of matter via the de Sitter heat bath, which takes place without the effects from the cosmological horizon, leads to the decay of the vacuum energy density. Distinct from this process, the possibility of instability in the de Sitter state due to Hawking radiation from the cosmological horizon is still controversial.

We considered two scenarios of vacuum decay that give two different power laws of decay. One of them reproduces the result of the Padmanabhan model [122]. The second one is based on the thermal fluctuations in the de Sitter heat bath. It leads to the reasonable values of dark energy and dark matter in the present time, and these values are not sensitive to the initial state of the universe. This scenario suggests the simultaneous solution of three cosmological constant problems: why the cosmological constant is not large; why the dark energy is in the order of dark matter; and why they have the present order of magnitude.

Using local thermodynamics with the local temperature $T = H/\pi$, we obtained the connection between the bulk entropy of the Hubble volume and the surface entropy of the cosmological horizon $S_{\rm hor} = \frac{A}{4G}$. This suggests a kind of bulk–surface correspondence that may have a holographic origin [194–196]. It would be interesting to check this correspondence using the more general extensions of Einstein gravity and also different types of generalized entropy [93,197–200]. It is important that such a connection takes place only in $3+1$ spacetime, where there is special symmetry due to which both gravitational variables, K and $\mathcal{R}$, have the mass dimension of 2, the same as the electrodynamic variables, the electric field $\mathbf{E}$, and electric induction $\mathbf{D}$. It will be interesting to extend these considerations to another system with similar symmetry properties—the Einstein static universe, which also has a constant curvature.

We also discussed the thermodynamics of de Sitter in the frame of the statistical ensemble of multi-metric gravities. The heat exchange between different "sub-Universes" in the ensemble leads to the common de Sitter expansion with the common temperature $T = H/\pi$. The application of local thermodynamics to the entropy of the Schwarzschild black hole was also considered. We obtained the Bekenstein–Hawking entropy of the black hole from the negative entropy of the contracting de Sitter core of the gravastar object.

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