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Multi-Objective Optimization for Finding Main Design Factors of a Two-Stage Helical Gearbox with Second-Stage Double Gear Sets Using the EAMR Method

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Abstract: When optimizing a mechanical device, the symmetry principle provides important guidance. Minimum gearbox mass and maximum gearbox efficiency are two single objectives that need to be achieved when designing a gearbox, and they are not compatible. In order to address the multi-objective optimization (MOO) problem with the above single targets involved in building a two-stage helical gearbox with second-stage double gear sets, this work presents a novel application of the multi-criteria decision-making (MCDM) method. This study's objective is to identify the best primary design elements that will increase the gearbox efficiency while lowering the gearbox mass. To carry this out, three main design parameters were selected: the first stage's gear ratio and the first and second stages' coefficients of wheel face width (CWFW). Furthermore, a study focusing on two distinct goals was carried out: the lowest possible gearbox mass and the highest possible gearbox efficiency. Furthermore, the two stages of the MOO problem are phase 1 and phase 2, respectively. Phase 2 solves the single-objective optimization issue to minimize the difference between variable levels and the MOO problem to determine the optimal primary design factors. To solve the MOO problem, the EAMR (Evaluation by an Area-based Method of Ranking) method was also chosen. The following are important features of this study: First, a MCDM method (EAMR technique) was successfully applied to solve a MOO problem for the first time. Secondly, this work explored the power losses during idle motion to calculate the efficiency of a two-stage helical gearbox with second-stage double gear sets. This study's findings were used to identify the optimal values for three important design variables to design a two-stage helical gearbox with second-stage double gear sets.

Keywords: gearbox; two-stage helical gearbox; gear ratio; multi-objective optimization; EAMR method

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1. Introduction

One of the most important parts of the drive is the gearbox. It can lower the torque and speed transfer from the motor shaft to the working shaft. Of all the gearbox types, the helical gearbox is the most widely used. This is because the structure of the helical gearbox is straightforward. Its pricing is also reasonable because neither its fabrication nor its design are complex. This is the reason why many scholars are trying to optimize the helical gearbox.

A variety of methods have been used to solve the gearbox MOO problem. Using the NSGA-II (Non-Dominated Sorting Genetic Algorithm II) method, Tudose L. et al. [1]

conducted a MOO study for designing helical gears. The goal of the work was to lower both the gearing mass and the flank adhesive wear speed. The MOO of a two-stage helical gear train was solved by R. C. Sanghvi et al. [2] using three different approaches: the MATLAB optimization toolbox, genetic algorithms (GA), and NSGA-II. The optimization of volume and load-carrying capacity were two of the study's goals. The results' comparison indicated that, with regard to both objectives, the NSGA-II approach yielded a better outcome than the other methods. Kalyanmoy D. and Sachin J. [3] carried out a multi-speed gearbox design optimization problem which had four conflicting objectives of design using the NSGA-II technique. It was found from the study that to obtain the same output speed requirement, a larger module is needed for larger delivered power. Also, for low-powered gearboxes, the wear stress failure is more critical than bending stress failure; for high-powered gearboxes, the opposite is true. Edmund S. M. and Rajesh A. [4] used the NSGA-II and the TOPSIS (Technique for Order of Preference by Similarity to Ideal Solution) approach to solve a MOO by taking three objectives into consideration: the gearbox volume, the power output, and the center distance. The study's findings provide insights into the design of small gearboxes. A two-stage helical gearbox was the subject of a MOO by M. Patil et al. [5], with two objective functions: the lowest gearbox volume and the smallest gearbox total power loss. Several tribological and design limitations were used for this investigation. It has been observed that the multi-objective technique reduces the gearbox's overall power loss by half and that solutions derived from single-objective minimization without tribological constraints have a significant probability of wear failure. A. Parmar et al. [6] also used the NSGA-II method to solve an optimization study of a planetary gearbox while accounting for significant regular mechanical and tribological constraints. Utilizing the study's conclusions resulted in a significant reduction in weight and power loss when comparing the outcomes of single-objective optimization with and without tribological limitations. In addition, the method has been applied in [7] to enhance the hypoid gears' operational features and in [8] to reduce the power loss and the vibrational excitation caused by meshing.

An auto encoder and bidirectional long short-term memory (BLSTM) are used in a neural network-based model presented by Sreenatha, M. and P. Mallikarjuna [9] to categorize the state of the gearbox for wind turbines into excellent or bad (broken tooth) condition. To assess the trade-off between three functions—axle stiffness, assemblability score, and overall mass—a MOOP is performed in [10]. By creating the Pareto front in this work, a precise and effective trade-off between the gearbox design's objectives may be made, enabling one to choose the optimum gearbox design in a logical manner. A. Kumar et al. [11] conducted a study on optimization of a three-stage wind turbine gearbox with two objectives: minimizing weight and minimizing power loss. In the study, the standard mechanical design restrictions as well as tribological constraints were considered and various synthetic-based ISO VG PAO (Polyalphaolefin) oils were used. It was reported that PAO 320 oil performs better than the other two grades (PAO 680 and 1000). Also, the power loss is significantly reduced with tribological restriction for the selected model. A spur gear set design optimization technique was established by Jawaz Alam and Sumanta Panda [12] to decrease gear weight, contact stress, and ideal film thickness at the contact site. This work combined particle swarm optimization, particle swarm optimization-based teaching learning optimization, and Jaya methods to ensure a significant decrease in weight and contact stress of a profile-modified spur gear set with sufficient film thickness at the site of contact. The study's conclusions show that, compared to traditional designs, the gear design with optimal addendum coefficient values inside the design space is significantly better. G. Istenes and J. Polák [13] conducted research to cooperatively optimize an electric motor and a gearbox in order to construct a drive system for electric automobiles. Reducing the weight of the driving system and total energy waste was the aim of this work. The optimization results were compared with previous research to emphasize the added possibilities of cooperative optimization. It was reported that increasing the gear ratio boosts the system's overall efficiency if the overall drive system is adjusted.

The multi-objective design of transmission using helical gear pairs is investigated by Sabarinath P. et al. [14]. Gear volumes and the opposing number of overlap ratio are indications of the objective functions. The optimization issue in this study was solved using the Parameter Adaptive Harmony Search Algorithm (PAHS). In [15], an optimal multi-objective study of a cycloid pin gear planetary reducer is described. Using Pareto optimal solutions, the reducer volume, turning arm bearing force, and pin maximum bending stress were examined with the aim of reducing all three of these objectives. According to the study's findings, the updated algorithm can produce Pareto optimum solutions that are superior to those produced by the routine design. In [16], the optimization of tooth modifications for spur and helical gears was solved using a mono-objective self-adaptive algorithm technique. This strategy is based on particle swarm optimization (PSO) technology. The maximal contact pressures and root mean square values of the transmission error signal were improved with the multi-objective optimization. The multi-objective design of transmission using helical gear pairs is investigated in [16]. The Taguchi and Grey relation analysis (GRA) methods were recently used by X.H. Le and N.P. Vu [17] to investigate the MOO problem of building a two-stage helical gearbox. Two goals were chosen for this study: the lowest gearbox mass and the highest gearbox efficiency. The study's findings were used to determine the ideal values for the five key design elements that encompass creating a two-stage helical gearbox. In order to maximize the gearbox efficiency and minimize the gearbox volume, the optimal primary design parameters for a two-stage bevel helical gearbox were also determined in [18] using a combination of Taguchi and GRA approaches. Moreover, these methods were applied to solve the optimization of a two-stage helical gearbox with second-stage double gear sets in [19] to increase the efficiency and reduce the mass of the gearbox.

Analysis shows that numerous investigations into the MOO problem of helical gearbox have been conducted up to this point. Power loss in gears has been the subject of numerous studies ([2,4,5,17,18], etc.). But the study previously stated did not take into consideration the power loss that happens while a gear is idling or when it is immersed in a lubricant during bath lubrication. In addition, a range of methods have been used to solve MOO problems, such as the NSGA-II method [1–8], Parameter Adaptive Harmony Search Algorithm (PAHS) [14], PSO method [16], Taguchi and GRA [17–19], etc. Among them, the NSGA-II approach is more frequently employed to solve the MOO problem. Nevertheless, a set of a lot of solutions is typically obtained when the MOO problem is solved using the NSGA-II approach (for instance, 389 Pareto optimum solutions [2]). As a result, to obtain the final results, it is required to combine the NSGA-II approach with another method, like TOPSIS (as in [4]).

While helical gearbox MOO has been extensively studied, MCDM's technique has not been used to find the optimal primary design parameters for these gearboxes. This paper presents the results of a MOO study conducted on a two-stage helical gearbox with double gear sets in the second stage. The two main objectives of this optimization effort are to reduce the gearbox mass and increase the gearbox efficiency. Additionally, the first stage's gear ratio and the CWFW for both stages—three optimal fundamental design characteristics for the gearbox—were looked at. Furthermore, the optimization task was approached using the EAMR method, and the weights of the criteria were determined using the Entropy method. One of the main findings of this research is the suggestion to apply an MCDM technique to solve MOO problems in conjunction with two-step problem solving, tackling single- and multi-objective problems. Moreover, the problem's solutions are more effective than those of earlier studies.

2. Optimization Problem

In this part, the gearbox mass and efficiency are first calculated in order to build the optimization problem. Next, the specified objective functions and constraints are given. To facilitate calculations, the nomenclature used in the optimization problem are presented in Table 1.

Table 1. The nomenclature for optimization problem.

Parameter	Nomenclature	Units
Allowable contact stress of stages *i* (*i* = 1 ÷ 2)	AS_i	Mpa
Allowable shear stress of shaft material	$[\tau]$	MPa
Arc of approach on *i* stage	β_{ai}	-
Arc of recess on *i* stage	β_{ri}	-
Base circle radius of the pinion	R_{01i}	mm
Base circle radius of the gear	R_{02i}	mm
Center distance of stage 1	a_{w1}	mm
Center distance of stage 2	a_{w2}	mm
Coefficient of wheel face width of stage 1	X_{ba1}	-
Coefficient of wheel face width of stage 2	X_{ba2}	-
Coefficient of gear material	k_a	$Mpa^{1/3}$
Contacting load ratio for pitting resistance	$k_{H\beta}$	-
Diameter of shaft *i*	d_{si}	mm
Efficiency of a helical gearbox	η_{hb}	-
Efficiency of the *i* stage of the gearbox	η_{gi}	-
Efficiency of a helical gear unit	η_{hg}	-
Efficiency of a rolling bearing pair	η_b	-
Friction coefficient	f	-
Friction coefficient of bearing	f_b	-
Gearbox ratio (or total gearbox ratio)	u_{gb}	
Gear ratio of stage 1	u_1	-
Gear ratio of stage 2	u_2	-
Gear width of stage 1	b_{w1}	mm
Gear width of stage 2	b_{w2}	mm
Gearbox mass	m_{gb}	kg
Gear mass	m_g	kg
Shaft mass	m_s	kg
Gearbox housing mass	m_{gh}	kg
Gear mass of stage 1	m_{g1}	kg
Gear mass of stage 2	m_{g2}	kg
Hydraulic moment of power losses	T_H	Nm
ISO Viscosity Grade number	VG_{40}	-
Length of shaft *i*	l_{si}	mm
Load of bearing *i*	Fi	N
Mass density of gearbox housing materials	ρ_{gh}	kg/m^3
Mass of shaft *j* (*j* = 1 ÷ 3)	m_{sj}	kg
Mass density of shaft material	ρ_s	kg/m^3
Outside radius of the pinion	R_{e1i}	mm
Outside radius of the gear	R_{e2i}	mm
Output torque	T_{out}	Nmm
Pitch diameter of the pinion of stage 1	d_{w11}	mm
Pitch diameter of the gear of stage 2	d_{w21}	mm
Pitch diameter of the pinion of stage 2	d_{w12}	mm
Pitch diameter of the gear of stage 2	d_{w22}	mm
Power loss in the gears	Plg	Kw

Table 1. *Cont.*

Parameter	Nomenclature	Units
Power loss in the bearings	Plb	Kw
Power loss in the seals	Pls	Kw
Power loss in the idle motion	Pzo	Kw
Pressure angle	α	rad.
Peripheral speed of bearing	v_b	m/s
Sliding velocity of gear	v	m/s
Total power loss in the gearbox	Pl	-
Torque on the pinion of stage i (i = 1 ÷ 2)	T_{1i}	Nmm
Volume coefficients of the pinion	e_1	-
Volume coefficients of the gear	e_2	-
Volume of gearbox housing	V_{gh}	dm^3
Volumes of bottom housing A	V_A	dm^3
Volumes of bottom housing B	V_B	dm^3
Volumes of bottom housing B	V_C	dm^3
Weight density of gear materials	ρ_g	kg/m^3

2.1. Calculation of Gearbox Mass

The gearbox mass m_{gb}, is calculated with

$$m_{gb} = m_g + m_{gh} + m_s \tag{1}$$

in which m_g, m_{gh}, and m_s can be found in detail as follows:

(+) Determining m_g:

$$m_g = m_{g1} + 2 \cdot m_{g2} \tag{2}$$

in which

$$m_{g1} = \rho_g \cdot \left(\frac{\pi \cdot e_1 \cdot d_{w11}^2 \cdot b_{w1}}{4} + \frac{\pi \cdot e_2 \cdot d_{w21}^2 \cdot b_{w1}}{4} \right) \tag{3}$$

$$m_{g2} = \rho_g \cdot \left(\frac{\pi \cdot e_1 \cdot d_{w12}^2 \cdot b_{w2}}{4} + \frac{\pi \cdot e_2 \cdot d_{w22}^2 \cdot b_{w2}}{4} \right) \tag{4}$$

$$b_{w1} = X_{ba1} \cdot a_{w1} \tag{5}$$

$$b_{w2} = X_{ba2} \cdot a_{w2} \tag{6}$$

$$d_{w1i} = 2 \cdot a_{wi} / (u_i + 1) \tag{7}$$

$$d_{w2i} = 2 \cdot a_{wi} \cdot u_i \cdot / (u_i + 1) \tag{8}$$

In the above Equations, i = 1 ÷ 2; $\rho_g = 7800$ (kg/m^3) because the material of the gears is steel; $e_1 = 1$ and $e_2 = 0.6$ [20]; and a_{wi} can be found with [20]

$$a_{wi} = k_a \cdot (u_i + 1) \cdot \sqrt[3]{T_{1i} \cdot k_{H\beta} / \left([AS_i]^2 \cdot u_i \cdot X_{bai} \right)} \tag{9}$$

where T_{1i} (i = 1 ÷ 2) is determined by the following equations:

$$T_{11} = T_{out} / \left(u_{gb} \cdot \eta_{hg}^2 \cdot \eta_b^3 \right) \tag{10}$$

$$T_{12} = T_{out} / \left(2 \cdot u_2 \cdot \eta_{hg} \cdot \eta_{be}^2 \right) \tag{11}$$

(+) Determining m_{gh}:

For the gearbox, m_{gh} can be calculated by

$$m_{gh} = \rho_{gh} \cdot V_{gh} \tag{12}$$

in which V_{gh} can be found by (see Figure 1)

$$V_{gh} = 2 \cdot V_A + 2 \cdot V_B + 2 \cdot V_C \tag{13}$$

wherein

$$V_A = L \cdot H \cdot S_G \tag{14}$$

$$V_B = L \cdot B_1 \cdot 1.5 \cdot S_G \tag{15}$$

$$V_C = B_2 \cdot H \cdot S_G = (B_1 - 2 \cdot S_G) \cdot H \cdot S_G \tag{16}$$

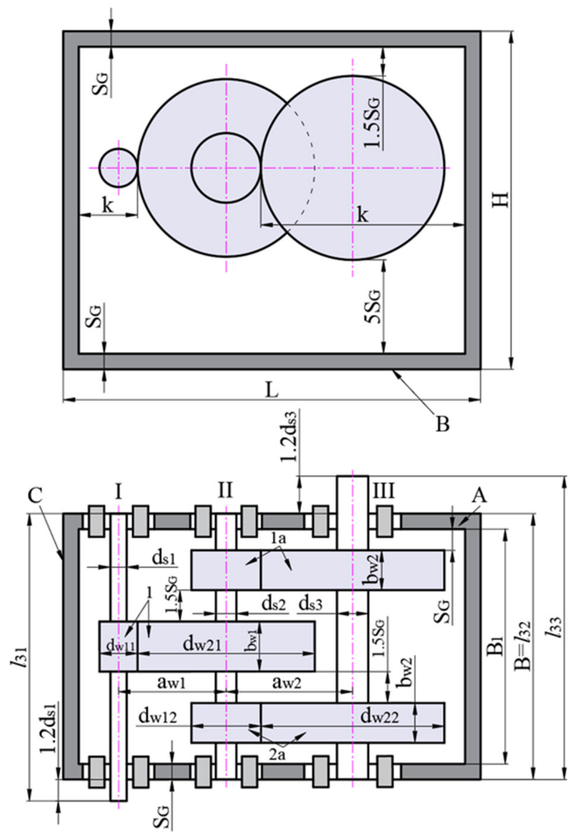


Figure 1. Calculated schema.

In the above Equations, L, H, B_1, and S_G are calculated by [21]

$$L = (d_{w11} + d_{w21}/2 + d_{w12}/2 + d_{w22}/2 + 22.5)/0.975 \tag{17}$$

$$H = max(d_{w21}; d_{w22}) + 8.5 \cdot S_G \tag{18}$$

$$B_1 = b_{w1} + 2 \cdot b_{w2} + 5 \cdot S_G \tag{19}$$

$$S_G = 0.005 \cdot L + 4.5 \tag{20}$$

(+) Determining m_s:
For the gearbox, m_s can be determined by

$$m_s = \sum_{j=1}^{3} m_{sj} \tag{21}$$

where

$$m_{sj} = \rho_s \cdot \pi \cdot d_{sj}^2 \cdot l_{sj}/4 \quad (22)$$

In (22), l_{sj} is determined by (see Figure 1)

$$l_{s1} = B_1 + 1.2 \cdot d_{s1} \quad (23)$$

$$l_{s2} = B_1 \quad (24)$$

$$l_{s3} = B_1 + 1.2 \cdot d_{s3} \quad (25)$$

The diameter of the shaft j (j = 1 ÷ 3) can be found by [20]

$$d_{sj} = \left[T_{1j}/(0.2 \cdot [\tau])\right]^{1/3} \quad (26)$$

In the above Equations, $\rho_g = \rho_s = 7800\ (\text{kg/m}^3)$ as the gear and shaft materials are steel; $[\tau]$ = 17 (Mpa) [20].

2.2. Calculation of Gearbox Efficiency

For the gearbox, η_{gb} is determined by

$$\eta_{gb} = 100 - \frac{100 \cdot P_l}{P_{in}} \quad (27)$$

wherein P_l can be calculated by [22]

$$P_l = P_{lg} + P_{lb} + P_{ls} + P_{Z0} \quad (28)$$

in which P_{lg}, P_{lb}, P_{ls}, and P_{zo} can be found by

(+) Determining P_{lg}:

$$P_{lg} = \sum_{i=1}^{2} P_{lgi} \quad (29)$$

in which

$$P_{lgi} = P_{gi} \cdot (1 - \eta_{gi}) \quad (30)$$

where η_{gi} can be determined by [23]

$$\eta_{gi} = 1 - \left(\frac{1 + 1/u_i}{\beta_{ai} + \beta_{ri}}\right) \cdot \frac{f_i}{2} \cdot \left(\beta_{ai}^2 + \beta_{ri}^2\right) \quad (31)$$

In (31), β_{ai} and β_{ri} are found by [23]

$$\beta_{ai} = \frac{\left(R_{e2i}^2 - R_{02i}^2\right)^{1/2} - R_{2i} \cdot \sin\alpha}{R_{01i}} \quad (32)$$

$$\beta_{ri} = \frac{\left(R_{e1i}^2 - R_{01i}^2\right)^{1/2} - R_{1i} \cdot \sin\alpha}{R_{01i}} \quad (33)$$

where f is calculated by the following Equations [17]:

- If $v \leq 0.424$ (m/s):

$$f = -0.0877 \cdot v + 0.0525 \quad (34)$$

- If $v > 0.424$ (m/s):

$$f = 0.0028 \cdot v + 0.0104 \quad (35)$$

(+) Determining P_{lb} [22]:

$$P_{lb} = \sum_{i=1}^{6} f_b \cdot F_i \cdot v_i \quad (36)$$

in which i = 1 ÷ 6 and $f_b = 0.0011$ (the radical ball bearings with angular contact were selected) [14].

(+) Determining P_s [22]:

$$P_s = \sum_{i=1}^{2} P_{si} \quad (37)$$

where i is the ordinal number of seal (i = 1 ÷ 2), and P_{si} can be found by

$$P_{si} = [145 - 1.6 \cdot t_{oil} + 350 \cdot loglog(VG_{40} + 0.8)] \cdot d_s^2 \cdot n \cdot 10^{-7} \quad (38)$$

(+) Determining P_{zo} [22]:

$$P_{Z0} = \sum_{i=1}^{k} T_{Hi} \cdot \frac{\pi \cdot n_i}{30} \quad (39)$$

in which k = 2 is the total number of gear pairs of the gearbox; n is the number of revolutions of a driven gear; T_{Hi} is calculated by [22]

$$T_{Hi} = C_{Sp} \cdot C_1 \cdot e^{\frac{C_2 \cdot v}{v_{t0}}} \quad (40)$$

where $C_{Sp} = 1$ in the case of stage 1 when the involved oil has to pass until the mesh; in another instance (for stage 2) (Figure 2), C_{Sp} can be determined by

$$C_{Sp} = \left(\frac{4 \cdot e_{max}}{3 \cdot h_C}\right)^{1.5} \cdot \frac{2 \cdot h_C}{l_{hi}} \quad (41)$$

wherein l_{hi} is determined by [22]

$$l_{hi} = (1.2 \div 2.0) \cdot d_{a2i} \quad (42)$$

In (40), C_1 and C_2 are calculated by [22]

$$C_1 = 0.063 \cdot \left(\frac{e_1 + e_2}{e_0}\right) + 0.0128 \cdot \left(\frac{b}{b_0}\right) \quad (43)$$

$$C_2 = \frac{e_1 + e_2}{80 \cdot e_0} + 0.2 \quad (44)$$

in which e_0 = b_0 = 10 (mm).

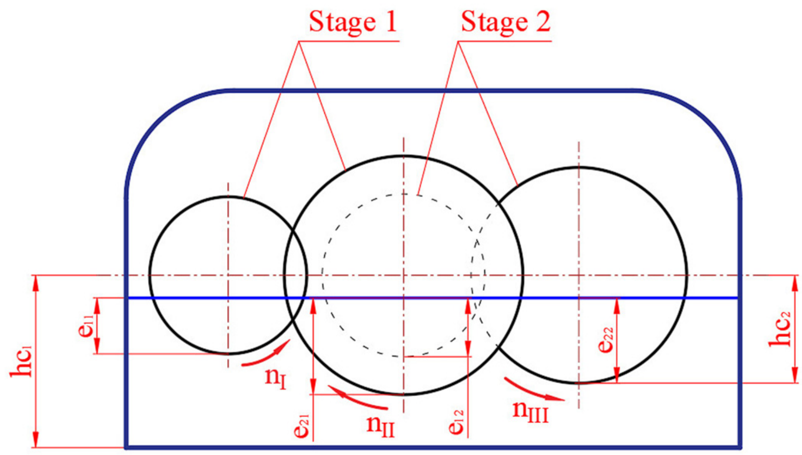


Figure 2. Determining factors of bath lubrication.

2.3. Objective Functions and Constrains

2.3.1. Objectives Functions

The MOO problem in this paper has two single objectives:

- Minimizing the mass of the gearbox:

$$\min f_1(X) = m_{gb} \tag{45}$$

- Maximizing the efficiency of the gearbox:

$$\min f_2(X) = \eta_{gb} \tag{46}$$

where the vector representing the design variables is denoted by X. There are three primary design factors for a two-stage helical gearbox with second-stage double gear sets: u_1, $X_{ba}1$, $X_{ba}2$, AS_1, and AS_2. Furthermore, it was shown that AS_1 and AS_2's maximum values correspond to their ideal values [17]. As a result, the three primary design aspects in this work, u_1, $X_{ba}1$, and $X_{ba}2$, were chosen as the variables for the optimization problem, and the result is

$$X = \{u_1, Xba_1, Xba_2\} \tag{47}$$

2.3.2. Constrains

The following limitations must be met by the multi-objective function:

$$1 \leq u_1 \leq 9 \text{ and } 1 \leq u_2 \leq 9 \tag{48}$$

$$0.25 \leq Xba_1 \leq 0.4 \text{ and } 0.25 \leq Xba_2 \leq 0.4 \tag{49}$$

3. Methodology

3.1. Method to Solve the Multi-Objective Optimization

Three primary design factors are chosen as variables for the MOO problem, as mentioned in Section 2.3. Table 2 lists these variables along with their minimum and maximum values. In reality, it is challenging to solve the MOO problem using an MCDM technique. The reason is because there are a lot of options or potential solutions when it comes to dealing with a MOO problem. To ensure the accuracy of the parameters and avoid missing the optimization problem's solution, the three parameters in this work have limits, as shown in Table 2, and the difference between variables is 0.02. As a result, the number of options (or experimental runs) that must be determined and compared is $(9-1)/0.02 \cdot (0.4-0.25)/0.02 \cdot (0.4-0.25)/0.02 = 22{,}500$ (runs). The OMO problem cannot be solved directly with the MCDM method due to the enormous number of options. In order to determine the ideal values for the three primary design variables, the MOO issue in this work was approached using the EAMR method. The two objectives were minimum gearbox mass and maximum gearbox efficiency. To solve the MOO problem for a two-stage helical gearbox with second-stage double gear sets, a simulation experiment was built. Moreover, because this is a simulation experiment, the number of experiments can be raised without a consideration of the budget for each experiment by utilizing the full factorial design. Because there are three experimental variables (as previously specified) and five levels for each variable, the result will be 5^3 = 125 experiments. However, Table 2 indicates that u_1 has the broadest spread among the three specified variables (ranging from 1 to 9). As a result, even with five levels, there was still a significant disparity between the levels of this variable (in this case, ((9 − 1)/4 = 2). To close this gap, reduce time, and improve the accuracy of the outcomes, a strategy for resolving multi-objective issues was proposed (Figure 3). The two parts of this procedure are as follows: phase 1 factors solve the MOO problem to identify the optimal primary design, and phase 2 factors solve the single-objective optimization problem to minimize the gap between levels. Additionally, in the process of addressing the multi-objective problem, the EAMR issue will be rerun using the smaller distance between two levels of the u_1 if the variable's levels are not sufficiently close to one another or if the best answer is not appropriate for the requirement (see Figure 3).

Table 2. Input parameters.

Parameter	Symbol	Lower Limit	Upper Limit
Gearbox ratio of first stage	u_1	1	9
CWFW of stage 1	X_{ba1}	0.25	0.4
CWFW of stage 2	X_{ba2}	0.25	0.4

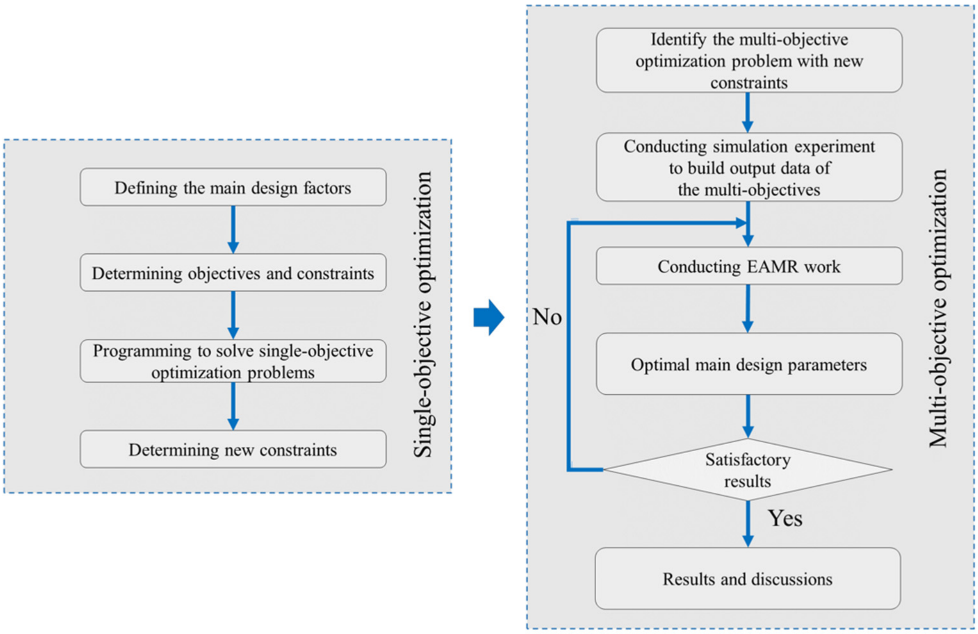


Figure 3. The procedure of solving multi-objective problems.

3.2. Method to Solve MCDM Problem:

The EAMR technique is implemented in the following stages [24]:

- Step 1: Creating the decision-making matrix:

$$X_d = \begin{bmatrix} x_{11}^d & \cdots & x_{1n}^d \\ x_{21}^d & \cdots & x_{21}^d \\ \vdots & \cdots & \vdots \\ x_{m1}^d & \cdots & x_{mn}^d \end{bmatrix} \tag{50}$$

where $1 \le d \le k$, the decision maker's number is k, and their indication is denoted by d.

- Step 2: For each criterion, ascertain the mean value of each possibility by

$$\overline{x}_{ij} = \frac{1}{k}\left(x_{ij}^1 + x_{ij}^2 + \cdots + x_{ij}^k\right) \tag{51}$$

- Step 3: Determine the weights of creation:
- Step 4: Find each criterion's weighted average:

$$\overline{w}_j = \frac{1}{k}\left(w_j^1 + w_j^2 + \cdots + w_j^k\right) \tag{52}$$

- Step 5: Calculate n_{ij} using

$$n_{ij} = \frac{\overline{x}_{ij}}{e_j} \tag{53}$$

in which e_j is determined by

$$e_j = max_{i\in\{1,\ldots,m\}}\left(\overline{x}_{ij}\right) \tag{54}$$

- Step 6: Find the normalized weight using

$$v_{ij} = n_{ij}\cdot\overline{w}_j \tag{55}$$

- Step 7: Determine the criteria's normalized score:
(+) When criteria j is greater as better:

$$G_i^+ = v_{i1}^+ + v_{i2}^+ + \cdots + v_{im}^+ \tag{56}$$

(+) When criteria j is smaller as better:

$$G_i^- = v_{i1}^- + v_{i2}^- + \cdots + v_{im}^{-+} \tag{57}$$

- Step 8: Calculate the ranking values (RVs) from ${G_i}^+$ and ${G_i}^-$:
- Step 9: Calculate the alternatives' evaluation score using

$$S_i = \frac{RV\left(G_i^+\right)}{RV\left(G_i^-\right)} \tag{58}$$

The best option is the one with the largest S_i.

3.3. Method to Find the Weight of Criteria:

In this paper, the Entropy technique was used to establish the weights of the criteria. The actions listed below can be used to put this strategy into practice [25].

- Calculate indicator normalized values as follows:

$$p_{ij} = \frac{x_{ij}}{m + \sum_{i=1}^m x_{ij}^2} \tag{59}$$

- Determine the Entropy for each indicator as follows:

$$me_j = -\sum_{i=1}^m\left[p_{ij}\times ln\left(p_{ij}\right)\right] - \left(1-\sum_{i=1}^m p_{ij}\right)\times ln\left(1-\sum_{i=1}^m p_{ij}\right) \tag{60}$$

- Find the weight of each indicator as follows:

$$w_j = \frac{1-me_j}{\sum_{j=1}^m\left(1-me_j\right)} \tag{61}$$

4. Single-Objective Optimization

In this study, a direct search strategy was used to solve the single-objective optimization problem. Furthermore, an Excel computer program was created to solve two single-objective problems: reducing gearbox mass and optimizing gearbox efficiency. The following are some of the program findings' figures and observations (calculated with u_{gb} = 20). Figure 4 shows the relationship between η_{gb} and u_1. It is evident that η_{gb} achieves its maximum value at an optimal value of u_1. Figure 5 shows how u_1 and m_{gb} are related. When u_1 is at its optimal value, m_{gb} reaches its lowest value (Figure 4). Figures 6 and 7 show the associations between X_{ba1} and X_{ba2} as well as η_{gb} and m_{gb}, respectively. These results (Figures 6a and 7a) demonstrate that as X_{ba1} and X_{ba2} rise, η_{gb} will fall. However, as X_{ba1} and X_{ba2} rise, m_{gb} also rises (Figures 6b and 7b). Figure 8 illustrates the link between the ideal gear ratio, u_1, for the first stage and the overall gearbox ratio, u_t. Moreover,

Table 3 displays newly computed restrictions for the variable u_1 based on the outcomes of single-objective problems.

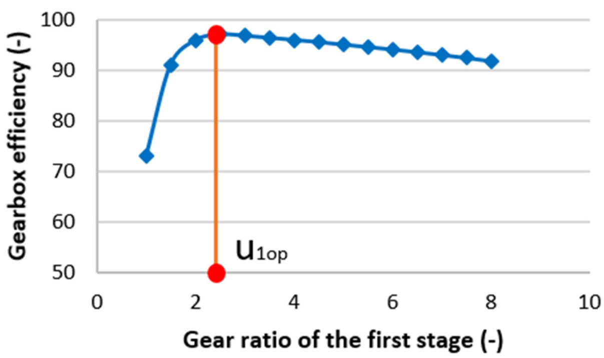


Figure 4. Gearbox efficiency versus first-stage gear ratio.

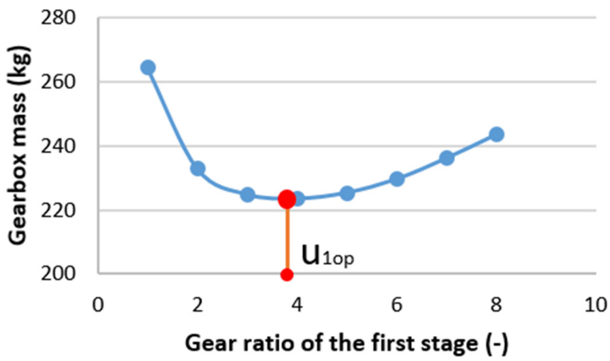
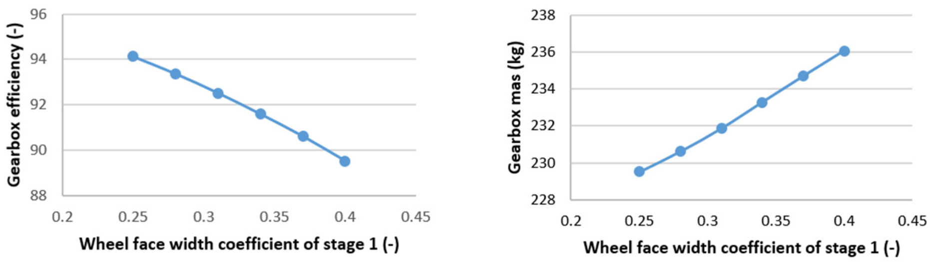
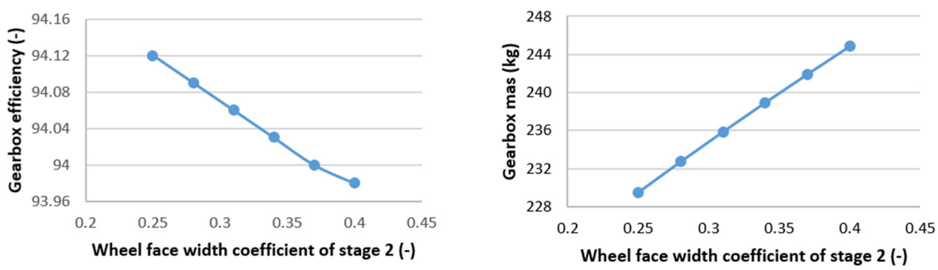


Figure 5. Gearbox mass versus first-stage gear ratio.



(**a**) (**b**)

Figure 6. Relation between X_{ba1} and gearbox efficiency (**a**) and gearbox mass (**b**).



(**a**) (**b**)

Figure 7. Relation between X_{ba2} and gearbox efficiency (**a**) and gearbox mass (**b**).

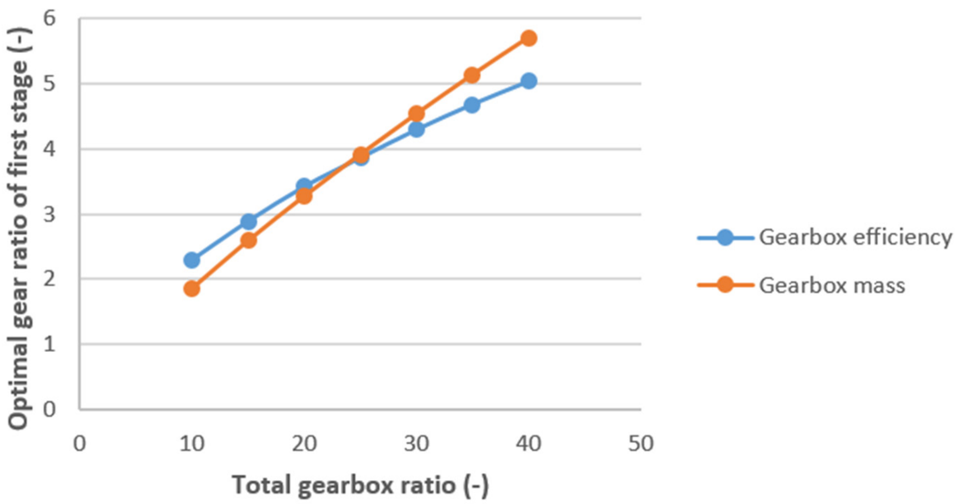


Figure 8. Optimal gear ratio of stage 1 versus total gearbox ratio.

Table 3. New constraints of u_1.

u_t	u_1	
	Lower Limit	**Upper Limit**
10	1.76	2.4
15	2.49	2.99
20	3.17	3.52
25	3.76	4.01
30	4.19	4.63
35	4.58	5.23
40	4.93	5.80

5. Multi-Objective Optimization

A computer program was created based on the optimization (in Section 2) to carry out the simulation experiment. The gearbox ratios of 10, 15, 20, 25, 30, 35, and 40 were all included for the analysis. This problem, with u_{gb} = 30, has the answers displayed below. This total gearbox ratio was used for the 125 initial testing cycles (as specified in Section 3.1). The experiment's output values, the gearbox mass and efficiency, will be used as input parameters by EAMR to resolve the MOO issue. Figure 9 illustrates the procedure for determining the optimal major design values when using the EAMR technique. The distance between the two levels of each variable will decrease with each EAMR's step. For instance, in step 1, u_1 increases from 4.19 to 4.63 when u_{gb} = 30 (Table 3). As a result, (4.63–4.19)/4 = 0.11 is the distance between the two levels of u_1. This procedure will be repeated until there is less than 0.02 separating the two levels of u_1. The primary design parameters and output responses for u_{gb} = 30 in the fourth and final iteration of the EAMR experiment are shown in Table 4. The criteria's weights were established using the Entropy technique (see Section 3.3) as follows: First, use Equation (59) to obtain the normalized values of p_{ij}. Use Equation (60) to determine each indicator me_j's Entropy value. Finally, use Equation (61) to find the weight of the criteria w_j. The weights of m_{gb} and η_{gb} for the most recent EAMR experiment were determined to be 0.4886 and 0.5114, respectively. Guidelines for using the EAMR technique in multi-objective decision making are given in Section 3.2. After that, the decision matrix should be assembled using Formula (50), considering the fact that k = 1 and there is only one result set. Determine the mean of the choices for each criterion using Equation (51), bearing in mind that $\overline{x}_{ij} = x_{ij}$ since k = 1. The average weighted values can then be obtained using Formula (52) while noting that $\overline{w}_j = w_j$ because k = 1. Utilizing Formula (53) and the definition of e_j given by (54), obtain n_{ij}. Next, use Formula (55) to compute v_{ij}. Use Equation (56) for gearbox efficiency and

Equation (57) for gearbox mass to calculate the values of G_i. Finally, calculate the S_i value using Formula (58). Table 5 shows the outcomes of the option ranking and the EAMR approach's computation of various parameters (for the final run of the EAMR). Out of all the possibilities provided, option 26 is the most ideal one, according to the table. The best values for the main design elements are therefore u_1 = 4.31, X_{ba1} = 0.25, and X_{ba2} = 0.25 (see Table 4).

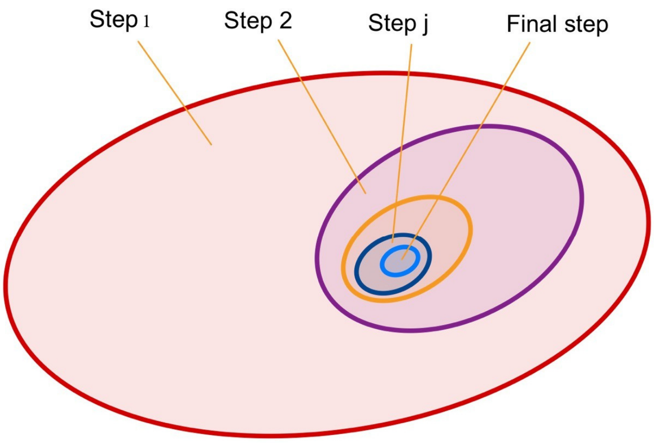


Figure 9. Strategy to find the best main design factors by EAMR.

Table 4. Main design parameter and output results for u_t = 30 in the 3rd run of EAMR.

Trial.	u_1	X_{ba1}	X_{ba2}	m_{gb} (kg)	η_{gb} (%)
1	4.29	0.25	0.25	222.74	96.04
2	4.29	0.25	0.29	226.18	95.97
3	4.29	0.25	0.33	229.60	95.92
4	4.29	0.25	0.36	232.98	95.86
5	4.29	0.25	0.40	236.32	95.79
6	4.29	0.29	0.25	223.95	95.45
...					
25	4.29	0.40	0.40	240.91	93.04
26	4.31	0.25	0.25	222.69	96.02
27	4.31	0.25	0.29	226.13	95.97
...					
51	4.34	0.25	0.25	222.65	96.00
52	4.34	0.25	0.29	226.08	95.95
53	4.34	0.25	0.33	229.50	95.88
...					
76	4.36	0.25	0.25	222.60	95.98
77	4.36	0.25	0.29	226.04	95.93
78	4.36	0.25	0.33	229.45	95.86
...					
101	4.38	0.25	0.25	222.56	95.95
102	4.38	0.25	0.29	225.99	95.91
103	4.38	0.25	0.33	229.41	95.84
...					
123	4.38	0.40	0.33	234.09	93.01
124	4.38	0.40	0.36	237.45	92.95
125	4.38	0.40	0.40	240.77	92.88

The color describes the best option.

Table 5. Calculated results and ranking of options by EAMR method for u_t = 30.

Trial.	n_{ij}		v_{ij}		G_{ij}		Ri	Rank
	m_{gb}	η_{gb}	m_{gb}	η_{gb}	m_{gb}	η_{gb}		
1	0.9246	1.0000	0.4517	0.5114	0.4517	0.5114	1.1322	3
2	0.9389	0.9993	0.4587	0.5111	0.4587	0.5111	1.1142	15
3	0.9531	0.9988	0.4656	0.5108	0.4656	0.5108	1.0970	29
4	0.9671	0.9981	0.4725	0.5105	0.4725	0.5105	1.0804	48
5	0.9809	0.9974	0.4793	0.5101	0.4793	0.5101	1.0644	75
6	0.9296	0.9939	0.4542	0.5083	0.4542	0.5083	1.1192	7
...								
25	1.0000	0.9688	0.4886	0.4955	0.4886	0.4955	1.0141	121
26	0.9244	0.9998	0.4516	0.5113	0.4516	0.5113	1.1322	1
27	0.9386	0.9993	0.4586	0.5111	0.4586	0.5111	1.1144	12
...								
51	0.9242	0.9996	0.4515	0.5112	0.4515	0.5112	1.1322	4
52	0.9384	0.9991	0.4585	0.5110	0.4585	0.5110	1.1144	11
53	0.9526	0.9983	0.4654	0.5106	0.4654	0.5106	1.0970	27
...								
76	0.9240	0.9994	0.4514	0.5111	0.4514	0.5111	1.1322	2
77	0.9383	0.9989	0.4584	0.5108	0.4584	0.5108	1.1144	14
78	0.9524	0.9981	0.4653	0.5105	0.4653	0.5105	1.0970	26
...								
101	0.9238	0.9991	0.4514	0.5110	0.4514	0.5110	1.1320	5
102	0.9381	0.9986	0.4583	0.5107	0.4583	0.5107	1.1144	13
103	0.9523	0.9979	0.4652	0.5104	0.4652	0.5104	1.0970	30
...								
123	0.9717	0.9685	0.4747	0.4953	0.4747	0.4953	1.0433	100
124	0.9856	0.9678	0.4816	0.4950	0.4816	0.4950	1.0279	115
125	0.9994	0.9671	0.4883	0.4946	0.4883	0.4946	1.0129	125

Table 6 shows the optimal values for the main design parameters that correspond to the remaining u_{gb} values of 10, 20, 25, 30, 35, and 40, being a continuation of the previous discussion. The following conclusions can be drawn using the information in this table:

Table 6. Optimum values of main design parameters.

No.	u_t						
	10	**15**	**20**	**25**	**30**	**35**	**40**
u_1	2.04	2.74	3.36	3.85	4.31	4.76	5.16
X_{ba1}	0.25	0.25	0.25	0.25	0.25	0.25	0.25
X_{ba2}	0.25	0.25	0.25	0.25	0.25	0.25	0.25

The lowest values that correspond to the optimal values for X_{ba1} and X_{ba2} are X_{ba1} = 0.25 and X_{ba2} = 0.25. This result is also consistent with the observations stated in [20]. This is due to the fact that in order to achieve the intended minimum gearbox mass, the coefficients X_{ba1} and X_{ba2} must be as small as possible. Lowering these coefficients will result in a decrease in the gear widths (represented by Equations (5) and (6)) and, in turn, the gear mass (represented by Equations (3) and (4)).

Figure 10 shows that there is a definite first-order relationship between the ideal values of u_1 and u_{gb}. Additionally, it was found that the following regression equation (with R^2 = 0.9901) can be used to calculate the optimal values of u_1:

$$u_1 = 0.1025 \cdot u_t + 1.1832 \quad (62)$$

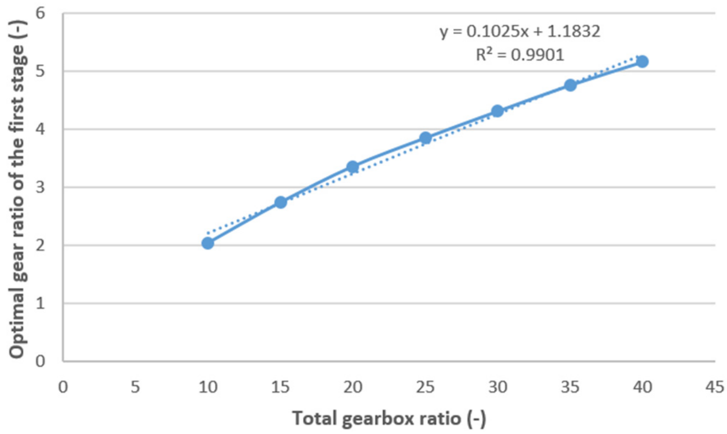


Figure 10. Optimum gear ratio of the first stage versus total gearbox ratio. (The solid line if the values of optimum gear ratio of the first stages with different total gearbox ratio. The dashed line describes the regression equation for that.)

After determining u_1, the optimal value of u_2 can be determined via the formula below:

$$u_2 = u_t / u_1 \quad (63)$$

To evaluate the model's outcomes for determining the ideal values when calculated using the EAMR method (new method), the findings of this study are compared with those acquired using the Taguchi and Gray Relational Analysis method (old method) in [20]. The ideal values of u_1 corresponding to different u_{gb} generated by the two approaches were compared and are shown in Figure 11. Additionally, Figures 12 and 13 show the gearbox mass and efficiency data derived from the old and new techniques, respectively. The results presented show that in comparison to the calculations made with the old method, the new approach produces a significantly lower gearbox mass (from 4.9 to 21.6%) and significantly improved gearbox efficiency (from 40.7 to 0.5%) when u_{gb} changes from 15 to 40.

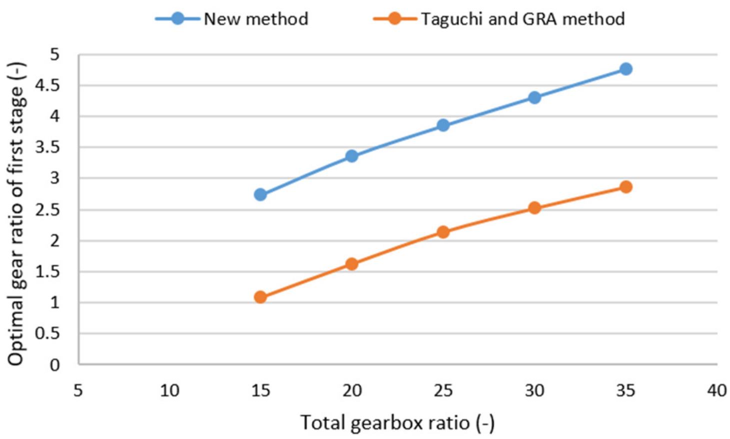


Figure 11. Optimum values of u_1 calculated by old and new methods.

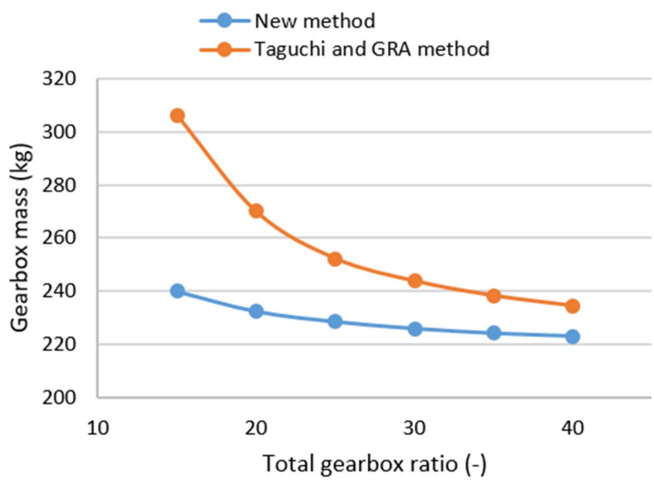


Figure 12. Minimum gearbox mass values calculated by old and new methods.

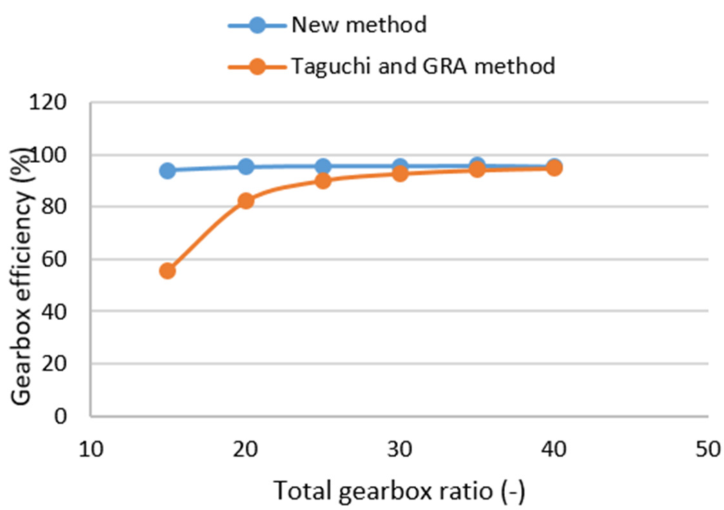


Figure 13. Maximum gearbox efficiency values calculated by old and new methods.

6. Conclusions

The EAMR approach was utilized in this study to solve the MOO problem related to the design of a two-stage helical gearbox with a second-stage double gear set. The study's goal was to identify the best critical design parameters that maximize the gearbox efficiency while reducing the gearbox mass. To carry this out, three essential design components were chosen: the CWFW for the first and second stages, and the first-stage gear ratio. In addition, there were two steps in the MOO problem solution process. Phase 1 was dedicated to solving the single-objective optimization problem of reducing the difference between variable values, whereas phase 2 was concerned with determining the optimal primary design factors. The following findings were drawn from this work:

- The single-objective optimization problem speeds up and simplifies the resolution of the MOO problem by bridging the gap between variable levels.
- Equation (62) and Table 6 present the optimal values for the three main design parameters of a two-stage helical gear gearbox with second-stage double gear sets based on this study's findings.
- Two single targets were assessed concerning the principal design parameters.
- By using the EAMR technique repeatedly until the required results are attained, the MOO problem can be solved more precisely (u_1 has an accuracy of less than 0.02).
- The experimental data's extraordinary degree of concordance with the proposed model of u_1 verifies their reliability.

- The results show that the novel approach to the MOO issue outperforms the prior method (the Taguchi and GRA approaches) in terms of yielding superior results.
- The proposed method of utilizing the MCDM method to solve the MOOP can be applied for the design of a gearbox when teaching mechanical students and for industry applications.
- The limitation of this study is that a statistical analysis was not conducted on the experimental data. Therefore, the proposed further research direction is to use statistical methods to analyze the experimental data.

Author Contributions: N.-P.V. first proposed the idea of this study, based on which the other authors agreed to conduct it; N.-P.V. solved the optimization problem with help from T.-T.D.; V.-T.D. and D.V. carried out the simulation's design; V.-T.D., H.-D.T., D.-B.V. and N.-P.V. conducted the experimental data analysis, and the simulation's experimental result analysis; with help from T.-T.D. and N.-P.V. wrote the manuscript. All authors have read and agreed to the published version of the manuscript.

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