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A Multi-Criteria Method Integrating Distances to Ideal and Anti-Ideal Points

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Abstract: Multi-criteria decision-making methods based on reference points and distances from them are essential for evaluating alternatives across multiple criteria. These methods provide structured approaches to comparing and ranking alternatives relative to specified reference points. The main objective of this paper is to present the Multi-Criteria Method Integrating Distances to Ideal and Anti-ideal Points (MIDIA), which, through a weighted system, allows for the consideration of balance and asymmetry in assessing alternatives based on their distances from the ideal and anti-ideal points. As a multi-criteria algorithm, MIDIA is user-friendly and reflects the human mind's natural tendency to assess objects based on fundamental concepts—comparison with the ideal solution and the anti-ideal solution—that are familiar from everyday experiences and provide valuable insights from a behavioral perspective. Moreover, the proposed method can be seen as an extension of Hellwig's approach, designed to facilitate the ranking of alternatives based on two reference points: the ideal point and the anti-ideal point, measuring the distance between the alternative and the ideal point and the distance between the ideal and anti-ideal points. The MIDIA method integrates elements from both TOPSIS and VIKOR, by incorporating the structure of TOPSIS and the compromise perspective of VIKOR, offering a balanced approach to multi-criteria decision-making by focusing on the distances from ideal and anti-ideal points. Illustrative examples are given to demonstrate the usability of the proposed tool in situations where the decision-maker has asymmetrical preferences concerning the importance of ideal and anti-ideal points in ranking alternatives. Moreover, the MIDIA method is applied to one of the Sustainable Development Goals, in the area of education (SDG4), to obtain the rankings of EU member countries in 2022. The results obtained using the MIDIA method were compared with those obtained using the TOPSIS and VIKOR approaches. The study concludes that the ranking of alternatives depends on the coefficients of the importance of the distances to reference points and the data setup.

Keywords: multi-criteria methods; ideal point; anti-ideal point; coefficient of the importance of the distance to reference point; asymmetrical data; TOPSIS; VIKOR; MIDIA

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1. Introduction

Multi-Criteria Decision-Making (MCDM) is a field of study rooted in operations research that focuses on evaluating and making decisions when multiple conflicting criteria or objectives are involved [1–4]. It provides methodologies and tools to systematically compare and prioritize different alternatives based on various criteria. MCDM is widely used in areas such as economics, engineering, environmental management, and business to support decision-making processes where trade-offs between different factors must be carefully considered. The field encompasses a range of techniques, from simple scoring methods to complex mathematical models, to aid decision-makers in solving various problems, such as ranking alternatives, sorting them into predefined categories, and classifying them based on multiple criteria.

Within MCDM, methods based on distances from reference points are powerful tools specifically designed to evaluate and rank alternatives in complex decision scenarios involving multiple conflicting criteria [5–12]. The reference points can be internal or external to the set of compared alternatives. The internal reference point is defined as the combination of the performances of all alternatives under consideration. On the contrary, the external ones are simply examples arbitrarily and subjectively defined by decision-makers or experts [10,12]. Examples of internal reference points are the ideal and anti-ideal points. The ideal point or ideal solution is represented by the maximum values of the benefit criteria and the minimum values of the cost criteria. Conversely, the anti-ideal point or anti-ideal solution is represented by the minimum values of the benefit criteria and maximum values of the cost criteria. In an MCDM environment, the ideal point represents a state where all criteria are optimized. Additionally, both the ideal and anti-ideal points can closely align with the perceived ideal or anti-ideal concept, which underpins the rationale behind human choice [13,14].

The use of the ideal solution to obtain a compromise solution to multi-criteria problems was first suggested by Zelany [15]. Among the most important and frequently used in practice multi-criteria methods based on distance from the reference points are TOPSIS (Technique for Ordering Preferences by Similarity to Ideal Solution), VIKOR (VlseKriterijuska Optimizacija I Komoromisno Resenje) [9], the Hellwig method [5,6], or BIPOLAR [8].

The objective of this work is to present the Multi-Criteria Method Integrating Distances to Ideal and Anti-ideal Points (MIDIA), which, through a weighted system in assessing alternatives based on their importance of distance to the ideal (α) and anti-ideal ($1-\alpha$), allows for the consideration of balance and asymmetry in evaluating alternatives. In particular, when α equals 1, the formula yields Hellwig's measure [5].

The MIDIA method was influenced by the decision-maker (DM) evaluation approach, where subjective judgments include both positive and negative information. Therefore, the scales used in MCDM could be bipolar, reflecting the human mind's natural tendency to assess objects based on both positive (ideal point) and negative (anti-ideal point) considerations [14,16]. The MIDIA method is also inspired by one of the most widely used rules in decision-making under uncertainty, proposed by Hurwicz [17]. The Hurwicz rule, employing a "coefficient of optimism", serves as a foundation for simultaneously considering the best and worst possible outcomes, which generalizes the most optimistic Maximax and the most pessimistic Maximin criteria into a unified framework. Similarly, the MIDIA measure can be viewed as a weighted average of the realizations of the importance of preferences to distances from the ideal or anti-ideal solutions.

The MIDIA method integrates elements from TOPSIS and VIKOR by incorporating the structure of TOPSIS and the compromise perspective of VIKOR, offering a balanced approach to multi-criteria decision-making by focusing on the distances from ideal and anti-ideal points. The MIDIA method consists of nine main steps: (1) determination of the decision matrix; (2) determination of the system of weights; (3) determination of the ideal and anti-ideal points; (4) determination of the normalized decision matrix to make criteria comparable; (5) determination of the normalized weighted decision matrix; (6) calculation of the Euclidean distance of each alternative from the ideal and anti-ideal solutions; (7) computation of the relative closeness of each alternative to the ideal and anti-ideal solution; (8) building the aggregation measure using coefficients of distance importance to ideal and anti-ideal solutions; (9) ranking the alternatives based on their relative closeness to the aggregated measure.

The rest of the paper is organized as follows. Section 2 presents a short overview of multi-criteria methods based on reference points. Section 3 introduces the new Multi-Criteria Methods Integrating Distances to Ideal and Anti-ideal Points. In Section 4, illustrative examples of the use and effectiveness of the proposed MIDIA method are presented. Moreover, sensitivity analysis is conducted by varying how changes in coefficient assignments impact the rankings of alternatives. Section 5 presents a practical application of the MIDIA method for a set of EU Sustainable Development Goals indicators in the area of education (Goal 4), including a comparison of the results obtained using the MIDIA, TOPSIS, and VIKOR methods. In Section 6, a discussion is conducted comparing the MIDIA method with the TOPSIS and VIKOR approaches. Finally, the conclusions and future research are presented in Section 7.

2. Multi-Criteria Methods Based on Reference Points—A Short Overview

Multi-criteria methods based on reference points are essential for effective, balanced, and transparent decision-making in complex scenarios. In such methods, each alternative's proximity to the ideal and anti-ideal solutions is quantified using distance metrics, such as Euclidean distance or other norms. By calculating these distances, the decision-maker (DM) gains insights into how well each alternative performs relative to these reference points. MCDM methods based on the distance from the ideal and anti-ideal solutions, such as TOPSIS and VIKOR, and the Hellwig method offer robust frameworks for making complex decisions. By evaluating alternatives based on their proximity to the best and worst possible outcomes, these methods help decision-makers arrive at rational and balanced decisions.

TOPSIS, introduced by Hwang and Yoon [7] in 1981, is a widely recognized multi-criteria decision-making ranking method for discrete decision problems that rely on reference points. The methodology involves identifying both ideal and anti-ideal solutions to determine the best alternative. Specifically, the selected alternative should be as close as possible to the ideal solution while being as far as possible from the anti-ideal solution. The process of calculating TOPSIS begins by constructing a decision matrix, where each alternative's performance is evaluated against various criteria. To ensure comparability across different units, the matrix is normalized. Next, the criteria are weighted according to their importance, resulting in a weighted normalized decision matrix. Once the matrix is prepared, the ideal solution (representing the best possible performance for each criterion—maximum for benefit criteria and minimum for cost criteria) and the negative-ideal solution (representing the worst possible performance—minimum for benefit criteria and maximum for cost criteria) are identified. The alternatives are then evaluated by calculating their Euclidean distances to both the ideal solution, known as the separation measure S^+ and the negative-ideal solution, referred to as the separation measure S^-. Next, the relative closeness of each alternative to the ideal solution T_i is computed as $T_i = \frac{S^-}{S^- + S^+}$, which serves as the basis for ranking the alternatives. Finally, the alternatives are ranked based on their T_i values, with a higher value indicating closer proximity to the ideal solution (for details, see [7]). This method has been widely applied in various socio-economic and decision-making scenarios. Numerous adaptations and hybrid versions of the TOPSIS method have been developed, utilizing diverse data sources for their analyses. Recent advancements and applications of the TOPSIS technique have been extensively reviewed in the literature [18–21].

The VIKOR method (VlseKriterijumska Optimizacija I Kompromisno Resenje) [9] is based on the concept of compromise programming and a compromise ranking index that balances the closeness to the ideal solution and the distance from the anti-ideal solution. This index helps to identify alternatives that are closest to the ideal solution while being as far as possible from the anti-ideal solution. It follows a similar initial process as TOPSIS, starting with the building of a decision matrix and the normalization of the data. The best and worst values for each criterion are then identified. Unlike TOPSIS, VIKOR does not measure the distances to ideal and anti-ideal solutions. Instead, VIKOR determines two additional measures: the utility measure S_i which reflects the overall distance from the worst criterion values, and the regret measure R_i which captures the maximum deviation from the best criterion values. These measures are combined into a VIKOR index $Q_i = vS_i + (1 - v)R_i$ that balances utility and regret, using a weight v that reflects the decision-maker's preference for either a majority rule or a compromise approach. The alternatives are ranked based on the index Q_i, with the lowest value indicating the most balanced compromise solution (for details see [9]). The application of the VIKOR technique can be found in Yazdani and Graeml [22].

Hellwig's method [6], known as the Taxonomic Measure of Development, is very similar to the TOPSIS procedure, but it uses only the concept of the distance to the ideal solution. Hellwig's method has also been extended to address various problems and scenarios. A systematic review of Hellwig applications can be found in [23–28]. A less-known variant of the classical Hellwig method in the aggregation procedure considers both the ideal point and the anti-ideal [5]. It compares the distance of each alternative to the ideal point with the distance between the ideal and the anti-ideal. This variant of the Hellwig method has been applied in several papers [29–32]. The algorithm of this variant of the Hellwig method is a special case of the proposed MIDIA method (see Section 3).

The BIPOLAR method, proposed by Konarzewska-Gubała [8], is an outranking technique based on the concept of a synthesizing preference relational system, applicable in sorting and ranking problems. Unlike the TOPSIS, Hellwig's, and VIKOR techniques, the BIPOLAR method does not rely on single reference alternatives (ideal and anti-ideal). Instead, it uses the concept of two bipolar sets of reference alternatives: a set of good (desirable) solutions and a set of bad (non-acceptable) solutions. The BIPOLAR procedure, similar to other outranking methods, involves steps such as determining the outranking relations for each alternative in relation to each reference alternative, checking concordance and discordance, identifying types of preference relations, and performing BIPOLAR-sorting and BIPOLAR-ranking to establish a final ranking of all alternatives. The result of the BIPOLAR ranking is a partial ordering of alternatives. Through BIPOLAR-sorting, alternatives are categorized based on the reference space and then ranked within each category, although indifference between alternatives may occur. This method does not provide information on the strength of preferences (for details, see [8]). Several modifications of the BIPOLAR method have been proposed for ranking and sorting problems [33–36]. The BIPOLAR method has been applied to the ranking of European projects [37], selecting projects applying for co-financing from the European Union [38], project realization [39], the selection of logistics service providers [40], and evaluation negotiation offers [12].

3. A Multi-Criteria Method Integrating Distances to Ideal and Anti-Ideal Points

Let us assume that we have a set of m alternatives $A = \{A_1, A_2, \ldots, A_m\}$ and set of n decision criteria $C = \{C_1, C_2, \ldots, C_n\}$, where I and J are the set of benefit and cost criteria, respectively $(C = I \cup J)$. Benefit criteria refer to those where higher values are better, while cost criteria refer to those where lower values are preferred. Let x_{ij} denote the criteria value of A_i on C_j $(i = 1, 2, \ldots, m;\ j = 1, 2, \ldots, n)$.

The procedure MIDIA consists of the following steps:

Step 1. Determination of the decision matrix:

$$D = [x_{ij}], \tag{1}$$

where x_{ij} is the value of the j-th criterion for i-th alternative $i = 1, \ldots, m$, $j = 1, \ldots, n$.

Step 2. Determination of the vector of weights:

$$w = [w_1, \ldots, w_n] \tag{2}$$

where $w_j > 0$ $(j = 1, \ldots, n)$ is the weight of the criterion C_j and $\sum_{j=1}^{n} w_j = 1$.

Weights in MIDIA, similarly as in other multi-criteria methods, are used to indicate the relative importance of each criterion. They help prioritize certain criteria over others, ensuring that the final decision reflects the decision-maker's preferences. They establish that criteria with higher importance have a greater impact on the outcome. It should be noted that the literature offers several approaches to determining weights. Da Silva et al. [41] reviewed and categorized 56 methods for determining criteria weights published from 1949 to 2020. For a thorough description and comparison of weighting techniques, refer to sources [41–47]. Tzeng et al. [47] categorized weighting methods as objective when computed based on outcomes, subjective when derived solely from DMs preferences, and a combination of both of them. The objective methods included the entropy method [7], SD (standard deviation) [48], CRITIC (Criteria Importance Through Intercriteria Correlation) [48], the maximizing deviation method [49], and the ideal point method [50], among others. The subjective methods included rank ordering methods [51], the tradeoff method [52], the ratio method [53], SWING [54], SMARTS (Simple MulTi attribute Rating Technique) [55], SMARTER [55], DR (Direct Rating) [56,57], PA (Point Allocation) [58], AHP (Analytic Hierarchy Process) [59], DEMATEL (Decision-making Trial and Evaluation Laboratory) [60], FITradeoff (Flexible and Interactive Tradeoff) method [61], and the Simons method [62], among others.

Step 3a. Building the ideal point (ideal solution):

$$I = \left[x_1^+, \ldots, x_n^+\right] \tag{3}$$

where:

$$x_j^+ = \begin{cases} \max\limits_i x_{ij} & \text{for benefit criterion} \\ \min\limits_i x_{ij} & \text{for cost criterion} \end{cases} \tag{4}$$

for $j = 1, \ldots, n$.

In other words, an ideal solution is a theoretical best-case scenario where each criterion achieves its maximum possible value for benefit criteria and minimum possible value for cost criteria. It represents the most desirable outcome in terms of all the evaluated criteria.

Step 3b. Building the anti-ideal point (anti-ideal solution):

$$\mathrm{AI} = \left[x_1^-, \ldots, x_n^-\right] \tag{5}$$

where:

$$x_j^- = \begin{cases} \max\limits_i x_{ij} & \text{for cost criterion} \\ \min\limits_i x_{ij} & \text{for benefit criterion.} \end{cases} \tag{6}$$

for $j = 1, \ldots, n$.

An anti-ideal solution, as opposed to an ideal solution, is a theoretical worst-case scenario where each criterion achieves its minimum possible value for benefit criteria or maximum possible value for cost criteria. It represents the least desirable outcome in terms of all evaluated criteria.

Step 4. Determination of the normalized matrix

$$\overline{D} = \left[z_{ij}\right]$$

using the standardization formula:

$$z_{ij} = \frac{x_{ij} - \bar{x}_j}{S_j} \tag{7}$$

where:

$$\bar{x}_{ij} = \frac{1}{m}\sum\nolimits_{i=1}^{m} x_{ij},\ S_j = \sqrt{\frac{1}{m}\sum\nolimits_{i=1}^{m}\left(x_{ij} - \bar{x}_{ij}\right)^2}\ (i = 1, \ldots, m, j = 1, \ldots, n). \tag{8}$$

Normalization standardizes the range of different criteria on a uniform scale, enabling each criterion to be compared and combined in the overall evaluation. It is worth noting that, in the literature, various other normalization methods can be found. Among the popular ones are Vector normalization, Max-Min, or the Sum method [63,64].

Step 5. Building the weighted normalized matrix:

$$\widetilde{D} = [\tilde{x}_{ij}], \tag{9}$$

where

$$\tilde{x}_{ij} = w_j \overline{x}_{ij} \tag{10}$$

Step 6a. Calculating the distances of i-th alternative A_i from the ideal I by using Euclidean distance measure

$$dI_i(A_i, I) = \mathrm{E}(\tilde{A}_i, \tilde{\mathrm{I}}) = \sqrt{\sum_{j=1}^{n} \left(\tilde{x}_{ij} - \tilde{x}_j^+ \right)^2} \tag{11}$$

where $\tilde{x}_{ij}, \tilde{x}_j^+$ are weighted normalized values x_{ij} and x_j^+, respectively.

Step 6b. Calculating the distances of i-th alternative A_i from the anti—ideal AI by using Euclidean distance measure

$$dAI_i(A_i, AI) = \mathrm{E}(\tilde{A}_i, \widetilde{AI}) = \sqrt{\sum_{j=1}^{n} \left(\tilde{x}_{ij} - \tilde{x}_j^- \right)^2} \tag{12}$$

where $\tilde{x}_{ij}, \tilde{x}_j^-$ are weighted normalized values x_{ij} and x_j^-, respectively.

Originally, well-known multi-criteria methods such as TOPSIS, Hellwig, and VIKOR employed Euclidean distances to calculate the distances of alternatives from the ideal or anti-ideal points. Consequently, we also utilized Euclidean distances in our approach. However, other distance measures can be used in multi-criteria methods [65–68]. It is worth noticing that Euclidean distance is a particular case of Minkowski's L_p metrics in an n-dimensional space, defined as follows: $L_p(x, y) = \sqrt[p]{\sum_{j=1}^{n} |x_j - y_j|^p}$, where $x = [x_1, x_2, \ldots, x_n]$, $y = [y_1, y_2, \ldots, y_n]$, $p \geq 1$. For $p = 1$, we have Manhattan (city block) distance, $p = 2$ Euclidean distance, and $p = \infty$ Chebyshev distance.

Step 7a. Calculating the closeness distance measure DI_i to the ideal solution for the i-th alternative using the formula

$$DI_i = 1 - \frac{dI_i}{d^{+-}} \tag{13}$$

where $d^{+-} = \sqrt{\sum_{j=1}^{n} (\tilde{x}_j^+ - \tilde{x}_j^-)^2}$ is a distance between ideal and anti-ideal solutions.

Let us note that $DI_i \in [0,1]$. Moreover, $DI_i = 1$, for ideal solution $(dI_i = 0)$, and $DI_i = 0$, for anti-ideal solution $(dI_i = d^{+-})$.

Step 7b. Calculating the closeness measure DAI_i to the anti-ideal solution for the i-th alternative using the formula

$$DAI_i = \frac{dAI_i}{d^{+-}} \tag{14}$$

where $d^{+-} = \sqrt{\sum_{j=1}^{n} (\tilde{x}_j^+ - \tilde{x}_j^-)^2}$ is a distance between ideal and anti-ideal solutions.

Let us note that $DAI_i \in [0,1]$. Moreover, $DAI_i = 1$, for ideal solution $(dAI_i = d^{+-})$, and $DAI_i = 0$, for anti-ideal solution $(dAI_i = 0)$.

In this step, the measures DI_i and DAI_i are normalized to a range of 0 to 1 to make comparison and interpretation easier.

Step 8. Calculating the measure $D(\alpha)_i$ for the i-th alternative using the formula:

$$D(\alpha)_i = \alpha\, DI_i + (1 - \alpha) DAI_i, \tag{15}$$

where $\alpha \in [0,1]$—is the coefficient of the importance of the distance to the ideal solution (simply the coefficient of importance or weight), and $1 - \alpha$ is the coefficient of the importance of the distance to the anti-ideal solution in the aggregation procedure.

The coefficients α and $1 - \alpha$ are used to combine the importance of distances from both the ideal and anti-ideal solutions to balance these two aspects.

Step 9. Ranking of objects according to descending $D(\alpha)_i$ values.

A higher value of the $D(\alpha)_i$ measurement corresponds to a higher ranking position for the respective alternative.

Let us note that the coefficient of importance denoted by α, ranges between 0 and 1 and reflects the decision-maker's importance of the distance to the ideal solution. It should be observed [69] that evaluating alternatives based on their distances from ideal and anti-ideal points reflects two distinct decision-making approaches. Assessing the distance from the ideal point encourages more aggressive decisions, where the decision-maker aims to get as close as possible to the optimal outcome. On the other hand, evaluating the distance from the anti-ideal point follows a more conservative approach, focusing on maximizing the distance from the worst outcome. The MIDIA measure can be viewed as a weighted average of the realizations of the simultaneous importance of preferences to distances from the ideal or anti-ideal solution. Consequently, the coefficient α can be seen as a crucial factor in representing the aggressive-conservative mix decision-making strategy.

Assigning $\alpha = 1$ means that only the distance to the ideal solution is included in evaluating alternatives, which means a pure aggressive strategy ($D(1) = DI$). Conversely, assigning $\alpha = 0$ means that only the distance to the anti-ideal solution is included in evaluating alternatives, which means a pure conservative strategy ($D(0) = DAI$). Values of α between 0 and 1 represent a mix of the importance of distance to the ideal and anti-ideal solutions. For each alternative, the measure $D(\alpha)$ computes a weighted average of DI and DAI. If both aspects are equally important, α = 0.5.

4. Numerical Examples

In this section, we present and compare the results obtained using the $D(\alpha)$ method with coefficients $\alpha \in \{0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1\}$. We examine the influence of these coefficients on ranking alternatives for different datasets. The task involves evaluating ten alternatives based on five benefit criteria. We kept the weights equal across all analyses to focus exclusively on the coefficient. The sensitivity analysis is conducted by varying how changes in coefficient assignments impact the rankings of alternatives. To validate the $D(\alpha)$ method and assess the impact of coefficients, we employ both Spearman and Pearson correlation coefficients. We consider the following situations with the consistency of the ranking measure $D(\alpha)$ for different coefficients α: Example 1 with full consistency, Example 2 with partial consistency, and Example 3 without consistency.

4.1. Full Consistency of Ranking for D(α) and Different Coefficient

Example 1. *(full consistency of ranking for D(α) and different coefficients α).*

Table 1 contains data and descriptive statistics for Example 1.

Table 1. Data and descriptive statistics for Example 1.

Alternative	C1	C2	C3	C4	C5
A1	16	50	98	20	64
A2	11	50	97	23	68
A3	17	29	90	23	70
A4	8	55	98	22	54
A5	11	43	91	18	56
A6	9	58	86	10	50
A7	18	44	82	5	49
A8	4	25	85	14	66
A9	6	35	80	8	59
A10	3	38	78	5	58
Maximum	18	58	98	23	70
Minimum	3	25	78	5	49
Median	10.30	42.70	88.50	14.80	59.40
Standard Deviation	5.06	10.37	7.10	6.97	7.00
Coefficient of variation	49.13	24.29	8.03	47.08	11.79

The ideal and anti-ideal based on max and min values (see Formulas (3)–(6)) have the form:

$$I = [18, 58, 98, 23, 70], AI = [3, 25, 78, 5, 49].$$

After normalization (see Formulas (7) and (8)), the Euclidean distances between the alternatives and the ideal or anti-ideal solutions are calculated using Formulas (11) and (12). Next, the values of DI and DAI measures are calculated by applying Formulas (13) and (14). Finally, the values of the $D(\alpha)$ measure are determined by Formula (15) for $\alpha = 0$, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1. The results of the calculation are presented in Table 2.

Table 2. The values of $D(\alpha)$ measure for different coefficients α (Example 1).

Alternative	$D(\alpha)$ Values for Different Coefficients α											Max-Min
	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	
A1	0.834	0.831	0.828	0.824	0.821	0.818	0.815	0.811	0.808	0.805	0.802	0.032
A2	0.833	0.825	0.817	0.809	0.801	0.793	0.785	0.776	0.768	0.760	0.752	0.081
A3	0.787	0.762	0.737	0.712	0.687	0.662	0.637	0.612	0.587	0.562	0.537	0.250
A4	0.747	0.726	0.705	0.683	0.662	0.641	0.619	0.598	0.577	0.555	0.534	0.213
A5	0.560	0.557	0.554	0.550	0.547	0.544	0.541	0.537	0.534	0.531	0.528	0.033
A6	0.560	0.540	0.519	0.499	0.478	0.458	0.437	0.417	0.396	0.376	0.355	0.205
A7	0.541	0.514	0.487	0.460	0.433	0.406	0.379	0.352	0.325	0.298	0.271	0.270
A8	0.449	0.430	0.412	0.393	0.374	0.356	0.337	0.318	0.300	0.281	0.262	0.187
A9	0.290	0.286	0.282	0.278	0.273	0.269	0.265	0.260	0.256	0.252	0.248	0.043
A10	0.275	0.264	0.252	0.241	0.229	0.217	0.206	0.194	0.182	0.171	0.159	0.116

Table 3 presents the rankings of alternatives for Example 1 by applying the MIDIA approach.

Table 3. The rankings of alternatives obtained by the $D(\alpha)$ measure for different coefficients α (Example 1).

Alternative	The Range for $D(\alpha)$ Measure and Different Coefficients α										
	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
A1	1	1	1	1	1	1	1	1	1	1	1
A2	2	2	2	2	2	2	2	2	2	2	2
A3	3	3	3	3	3	3	3	3	3	3	3
A4	4	4	4	4	4	4	4	4	4	4	4
A5	5	5	5	5	5	5	5	5	5	5	5
A6	6	6	6	6	6	6	6	6	6	6	6
A7	7	7	7	7	7	7	7	7	7	7	7
A8	8	8	8	8	8	8	8	8	8	8	8
A9	9	9	9	9	9	9	9	9	9	9	9
A10	10	10	10	10	10	10	10	10	10	10	10

Figure 1 presents a graphical representation of the evaluation of alternatives obtained for different coefficients of importance for the MIDIA method. Figure 1 illustrates how MIDIA values change with different α values for the analyzed alternatives. It also allows for assessing whether these changes in values have affected the ranking.

Table 3 and Figure 1 show that the rankings for all coefficients α of the $D(\alpha)$ measures are identical. Moreover, for each alternative, the measure values decrease, reaching their lowest values for the coefficient of importance 1. However, the rate of change varies and depends on the alternative. The smallest decline in values was observed for alternatives A1, A2, A5, and A9, while the largest decline was seen for A3 and A7 (see Table 2, Figure 1). Interesting situations were observed for alternatives A5, A6, and A7, where the distances from the anti-ideal ($D(0)$ values) differed only slightly, but much greater variation was noted when considering the distances from the ideals ($D(1)$ values). A similar trend, though to a lesser extent, was observed for alternatives A9 and A10. On the other hand, for alternatives A1 and A2, the $D(0)$ measure values were found to be similar (difference 0.01), with only slight differences in the $D(1)$ values (0.05) and minor differences between the $D(0)$ and $D(1)$ measures (0.032 for A1 and 0.082 for A2). In the case of alternatives A3 and A4, small differences were obtained for the $D(0)$ (0.040) and $D(1)$ (0.003) measures, but greater differences were observed between the $D(0)$ and $D(1)$ measures for each alternative (0.250 for A3 and 0.213 for A4). The Pearson correlation coefficient is presented in Table 4.

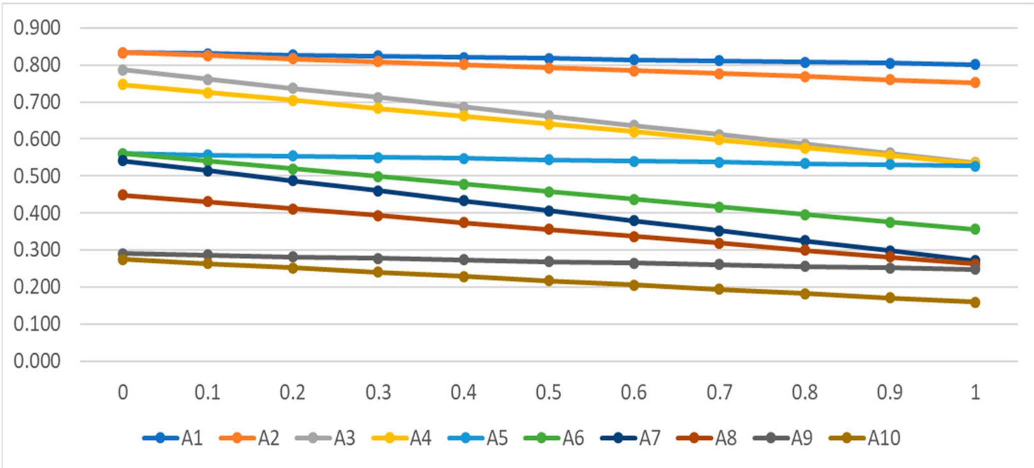

Figure 1. Graphical representation of evaluation of alternatives obtained for different coefficients for MIDIA method in Example 1.

Table 4. The Pearson correlation coefficient for $D(\alpha)$ and different coefficients α (Example 1).

Pearson Coefficient	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.0	1.000										
0.1	0.999	1.000									
0.2	0.996	0.999	1.000								
0.3	0.991	0.996	0.999	1.000							
0.4	0.984	0.991	0.996	0.999	1.000						
0.5	0.976	0.984	0.991	0.996	0.999	1.000					
0.6	0.965	0.976	0.984	0.991	0.996	0.999	1.000				
0.7	0.953	0.965	0.976	0.985	0.991	0.996	0.999	1.000			
0.8	0.939	0.954	0.966	0.977	0.985	0.992	0.996	0.999	1.000		
0.9	0.924	0.940	0.954	0.967	0.977	0.986	0.992	0.997	0.999	1.000	
1.0	0.908	0.926	0.942	0.956	0.968	0.978	0.986	0.992	0.997	0.999	1.000

High values of the Pearson correlation were observed for the $D(\alpha)$ measure and coefficients α, differing by 0.1. As the difference between these coefficients increases, the Pearson correlation value slightly, but systematically, decreases, reaching its lowest value (0.908) when compared to the $D(0)$ and $D(1)$ measures.

4.2. Partial Consistency of Ranking for D(α) and Different Coefficients α

Example 2. *(partial consistency of ranking for D(α) and different coefficients α).*

Table 5 contains data and descriptive statistics for Example 2.

The ideal and anti-ideal points based on max and min values (see Formulas (3)–(6)) have the form:

$$I = [20, 100, 100, 50, 80],\ AI = [2, 3, 76, 4, 5].$$

Table 5. Data and descriptive statistics for Example 2.

Alternative	C1	C2	C3	C4	C5
A1	20.00	3.00	91.00	50.00	80.00
A2	16.00	50.00	100.00	11.00	62.00
A3	11.00	50.00	97.00	22.00	68.00
A4	10.00	100.00	85.00	10.00	50.00
A5	10.00	55.00	98.00	10.00	54.00
A6	12.00	43.00	92.00	18.00	56.00
A7	8.00	35.00	84.00	6.00	59.00
A8	4.00	30.00	89.00	15.00	43.00
A9	2.00	36.00	76.00	5.00	53.00
A10	13.00	45.00	79.00	4.00	5.00
Max	20	100	100	50	80
Min	2	3	76	4	5
Median	10.60	44.70	89.10	15.10	53.00
Standard Deviation	5.00	23.13	7.67	12.85	18.69
Coefficient of variation	47.21	51.74	8.61	85.09	35.27

In Tables 6 and 7, the values and the ranking alternatives for Example 2 have been calculated by applying the MIDIA approach, respectively. Graphical representation is also shown in Figure 2.

Table 6. The values of the $D(\alpha)$ measure for different coefficients α (Example 2).

Alternative	$D(\alpha)$ Values for Different Coefficients α											Max-Min
	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	
A1	0.812	0.779	0.745	0.712	0.678	0.644	0.611	0.577	0.544	0.510	0.477	0.336
A2	0.673	0.658	0.644	0.629	0.615	0.600	0.586	0.571	0.557	0.542	0.528	0.145
A3	0.638	0.630	0.623	0.615	0.608	0.600	0.593	0.585	0.578	0.570	0.563	0.075
A4	0.631	0.614	0.597	0.580	0.563	0.546	0.529	0.512	0.495	0.478	0.461	0.169
A5	0.575	0.565	0.554	0.544	0.533	0.523	0.512	0.501	0.491	0.480	0.470	0.106
A6	0.537	0.533	0.529	0.525	0.521	0.517	0.513	0.508	0.504	0.500	0.496	0.041
A7	0.430	0.420	0.410	0.400	0.390	0.381	0.371	0.361	0.351	0.341	0.331	0.099
A8	0.366	0.360	0.355	0.350	0.344	0.339	0.333	0.328	0.323	0.317	0.312	0.054
A9	0.353	0.337	0.322	0.306	0.290	0.275	0.259	0.243	0.228	0.212	0.196	0.157
A10	0.346	0.332	0.317	0.303	0.289	0.274	0.260	0.246	0.231	0.217	0.203	0.143

Table 7. The rankings of alternatives obtained by the $D(\alpha)$ measure for different coefficients α (Example 2).

Alternative	The Range for $D(\alpha)$ Measure and Different Coefficients α										
	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
A1	1	1	1	1	1	1	1	2	3	3	4
A2	2	2	2	2	2	3	3	3	2	2	2
A3	3	3	3	3	3	2	2	1	1	1	1
A4	4	4	4	4	4	4	4	4	5	6	6
A5	5	5	5	5	5	5	6	6	6	5	5
A6	6	6	6	6	6	6	5	5	4	4	3
A7	7	7	7	7	7	7	7	7	7	7	7
A8	8	8	8	8	8	8	8	8	8	8	8
A9	9	9	9	9	9	9	10	10	10	10	10
A10	10	10	10	10	10	10	9	9	9	9	9

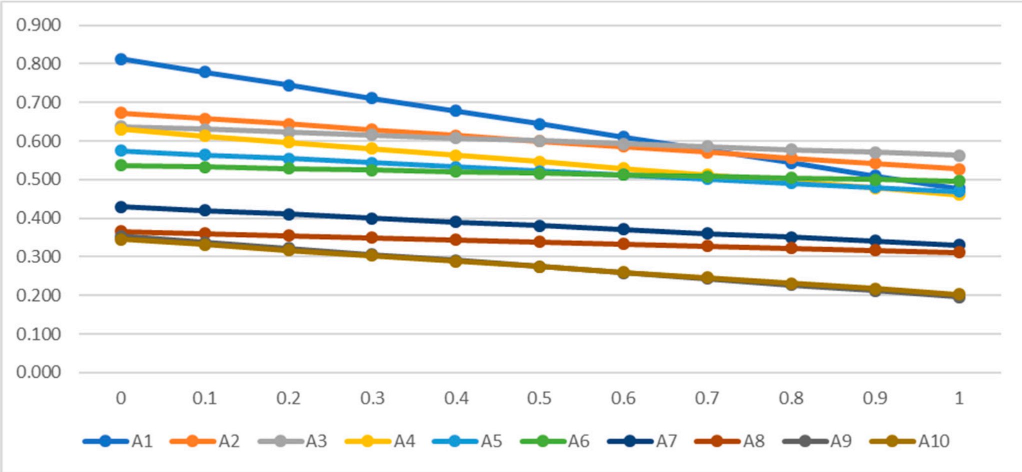

Figure 2. Graphical representation evaluation of alternatives obtained for different coefficients for the MIDIA method in Example 2.

The smallest decrease in value was observed for alternatives A3, A5, and A7, while the largest decrease was for A1 (see Table 6). Such a significant drop in the measure's value caused this alternative to change its position from first to fourth place. Interesting relationships were observed for alternatives A8, A9, and A10, for which the $D(0)$ measure values differed slightly. The differences in the $D(1)$ measure values for alternatives A9 and A10 were also insignificant. At the same time, much greater variability was observed in the $D(1)$ measure values for A8 compared to A9 and A10 (see Table 6 and Figure 2).

Table 7 and Figure 2 show that, for α values 0, 0.1, 0.2, 0.3, and 0.4, the rankings for the $D(\alpha)$ measurements are the same, while for others, there is a change in the ranking positions of the alternatives. It is noteworthy that the changes in the positions of the individual alternatives occurred in different ways. With the increase in the α parameter value, albeit with varying intensity, the positions of alternatives A3, A6, and A10 systematically decrease, while the positions of alternatives A1 and A4 systematically increase. In contrast, the positions of alternatives A2 and A5 changed in different directions. Finally, two alternatives did not change their ranking positions: A7 (7th position) and A8 (8th position). The most significant changes occurred for alternative A1, which moved from first position for the $D(0)$ measure to fourth position for the $D(1)$ measure, and for alternative A6, which changed from sixth position for the $D(0)$ measure to third position for the $D(1)$ measure.

This observation confirms the Spearman rank correlation coefficients between the values of the $D(\alpha)$ measure for different α values (Table 8). The lowest Spearman rank correlation coefficient, equal to 0.830, was observed for the measures $D(0)$ and $D(0.1)$, $D(0)$ and $D(0.2)$, $D(0)$ and $D(0.3)$, and $D(0)$ and $D(0.4)$ (see Table 8).

Table 8. The Spearman coefficient for $D(\alpha)$ and different coefficients α (Example 2).

Spearman Coefficient	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.0	1.000										
0.1	1.000	1.000									
0.2	1.000	1.000	1.000								
0.3	1.000	1.000	1.000	1.000							
0.4	1.000	1.000	1.000	1.000	1.000						
0.5	0.988	0.988	0.988	0.988	0.988	1.000					
0.6	0.964	0.964	0.964	0.964	0.964	0.976	1.000				
0.7	0.939	0.939	0.939	0.939	0.939	0.964	0.988	1.000			
0.8	0.903	0.903	0.903	0.903	0.903	0.915	0.952	0.976	1.000		
0.9	0.891	0.891	0.891	0.891	0.891	0.903	0.927	0.952	0.988	1.000	
1.0	0.830	0.830	0.830	0.830	0.830	0.842	0.879	0.915	0.976	0.988	1.000

Similar to Example 1, the Pearson correlation values remained high for $D(\alpha)$ when the α coefficients had a difference of 0.1 (Table 9). However, as the difference between these coefficients grew, the Pearson correlation value gradually decreased, with the lowest value (0.848) observed between the $D(0)$ and $D(1)$ measures.

Table 9. The Pearson coefficient for $D(\alpha)$ and different coefficients α (Example 2).

Pearson Coefficient	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.0	1.000										
0.1	0.999	1.000									
0.2	0.996	0.999	1.000								
0.3	0.990	0.995	0.999	1.000							
0.4	0.980	0.988	0.995	0.999	1.000						
0.5	0.968	0.978	0.987	0.994	0.998	1.000					
0.6	0.952	0.965	0.976	0.986	0.994	0.998	1.000				
0.7	0.932	0.947	0.962	0.974	0.985	0.993	0.998	1.000			
0.8	0.908	0.926	0.943	0.959	0.973	0.984	0.993	0.998	1.000		
0.9	0.880	0.901	0.921	0.939	0.956	0.971	0.983	0.992	0.998	1.000	
1.0	0.848	0.872	0.894	0.916	0.936	0.954	0.970	0.983	0.992	0.998	1.000

4.3. No Consistency of Ranking D(α) Measure for Different Coefficients α

Example 3. *(no consistency of ranking D(α) measure for different coefficients α).*

Table 10 contains data and descriptive statistics for Example 3.

Table 10. Data and descriptive statistics for Example 3.

Alternative	C1	C2	C3	C4	C5
A1	20.00	3.00	91.00	50.00	80.00
A2	16.00	50.00	100.00	11.00	62.00
A3	11.00	50.00	97.00	25.00	68.00
A4	10.00	100.00	85.00	10.00	50.00
A5	11.00	60.00	98.00	10.00	54.00
A6	20.00	35.00	84.00	6.00	59.00
A7	12.00	43.00	92.00	18.00	56.00
A8	2.00	36.00	76.00	5.00	53.00
A9	13.00	45.00	79.00	4.00	5.00
A10	4.00	20.00	80.00	15.00	43.00
Max	20	100	100	50	80
Min	2	3	76	4	5
Median	11.90	44.20	88.20	15.40	53.00
Standard Deviation	5.61	24.22	8.15	13.04	18.69
Coefficient of variation	47.16	54.80	9.24	84.67	35.27

The ideal and anti-deal points based on maximum and minimum values (see Formulas (3)–(6)) have the form:

$$I = [20, 100, 100, 50, 80], AI = [2, 3, 76, 4, 5].$$

In Table 11, the values, and in Table 12, the rankings of alternatives for Example 3, have been calculated using the MIDIA approach. A graphical representation is also provided in Figure 3.

Table 11. The values of the $D(\alpha)$ measure for different coefficients α (Example 3).

Alternative	$D(\alpha)$ Values for Different Coefficients α											Max-Min
	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	
A1	0.815	0.782	0.748	0.714	0.681	0.647	0.613	0.580	0.546	0.513	0.479	0.336
A2	0.667	0.652	0.637	0.623	0.608	0.593	0.579	0.564	0.549	0.535	0.520	0.147
A3	0.651	0.644	0.637	0.631	0.624	0.617	0.610	0.603	0.597	0.590	0.583	0.068
A4	0.631	0.614	0.597	0.580	0.562	0.545	0.528	0.511	0.494	0.477	0.460	0.171
A5	0.595	0.584	0.573	0.563	0.552	0.541	0.530	0.520	0.509	0.498	0.487	0.108
A6	0.580	0.561	0.542	0.523	0.504	0.485	0.465	0.446	0.427	0.408	0.389	0.191
A7	0.537	0.533	0.529	0.525	0.520	0.516	0.512	0.508	0.504	0.499	0.495	0.042
A8	0.365	0.349	0.333	0.317	0.302	0.286	0.270	0.254	0.239	0.223	0.207	0.158
A9	0.332	0.317	0.303	0.288	0.274	0.260	0.245	0.231	0.217	0.202	0.188	0.144
A10	0.300	0.294	0.289	0.283	0.278	0.273	0.267	0.262	0.256	0.251	0.245	0.054

Table 12. The ranks of alternatives obtained by the $D(\alpha)$ measure for different coefficients α (Example 3).

Alternative	Range for $D(\alpha)$ Measure and Different Coefficients α										
	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
A1	1	1	1	1	1	1	1	2	3	3	5
A2	2	2	2	3	3	3	3	3	2	2	2
A3	3	3	3	2	2	2	2	1	1	1	1
A4	4	4	4	4	4	4	5	5	6	6	6
A5	5	5	5	5	5	5	4	4	4	5	4
A6	6	6	6	7	7	7	7	7	7	7	7
A7	7	7	7	6	6	6	6	6	5	4	3
A8	8	8	8	8	8	8	8	9	9	9	9
A9	9	9	9	9	10	10	10	10	10	10	10
A10	10	10	10	10	9	9	9	8	8	8	8

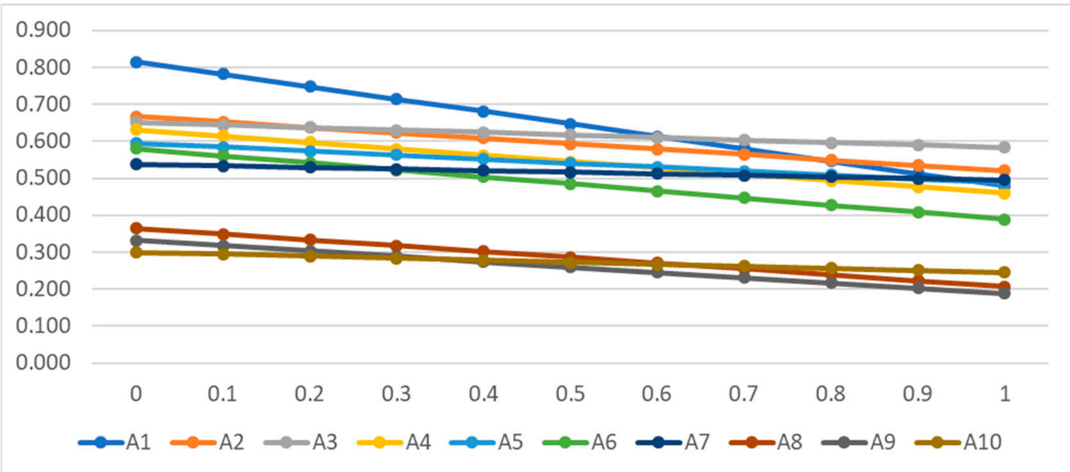

Figure 3. Graphical representation evaluation of alternatives obtained for different coefficients for the MIDIA method in Example 3.

The smallest decrease in value was observed for alternatives A3, A7, and A10. Conversely, the largest decrease was for A1 (see Table 12), which resulted in changing its position from first to fifth place. The alternatives A2, A3, and A4 obtained similar values with respect to $D(0)$ (0.631 for A4, 0.651 for A3, 0.667 for A2) and were different for the $D(1)$ measure (0.460 for A4, 0.583 for A3, 0.520 for A2). At the same time, the differences between $D(0)$) and $D(1)$ were quite different (0.068 for A2, 0.171 for A3, and 0.108 for A4) (see Table 10 and Figure 3).

Table 12 and Figure 3 show that rankings for the $D(0), D(0.1), D(0.2)$ measurements are identical, but not for others. First, let us note that there are no alternatives with stable positions. As previously mentioned, the changes in the positions of the individual alternatives occurred in different ways. With the increase in the α, albeit with varying intensity, the positions of alternatives A1, A4, A6, A8, and A9 systematically decrease, while the positions of alternatives A1, A3, and A7 systematically increase. The positions of the remaining two alternatives, A2 and A5, changed in different directions. The greatest changes occurred for alternative A1. The Spearman rank correlation coefficients between the values of the $D(\alpha)$ measure for different α values are presented in Table 13. The lowest Spearman rank correlation coefficient, equal to 0.709, was observed while comparing ranking obtained by $D(1)$ and $D(0)$), $D(1)$ and $D(0.1)$, and $D(1)$ and $D(0.2)$ (see Table 13).

Table 13. The Spearman coefficient for $D(\alpha)$ and different coefficients α (Example 3).

Spearman Coefficient	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.0	1.000										
0.1	1.000	1.000									
0.2	1.000	1.000	1.000								
0.3	0.976	0.976	0.976	1.000							
0.4	0.964	0.964	0.964	0.988	1.000						
0.5	0.964	0.964	0.964	0.988	1.000	1.000					
0.6	0.952	0.952	0.952	0.976	0.988	0.988	1.000				
0.7	0.903	0.903	0.903	0.939	0.964	0.964	0.976	1.000			
0.8	0.855	0.855	0.855	0.891	0.915	0.915	0.939	0.976	1.000		
0.9	0.830	0.830	0.830	0.879	0.903	0.903	0.915	0.952	0.988	1.000	
1.0	0.709	0.709	0.709	0.770	0.794	0.794	0.818	0.879	0.952	0.964	1.000

Similarly to previous examples, the Pearson correlation values were high for $D(\alpha)$ measures, differing by 0.1 for the $D(\alpha)$ coefficients. The lowest Pearson coefficient value (0.858) occurred when comparing the $D(0)$ and $D(1)$ measures (Table 14)

Table 14. The Pearson coefficient for $D(\alpha)$ and different coefficients α (Example 3).

Pearson Coefficient	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.0	1.000										
0.1	0.999	1.000									
0.2	0.996	0.999	1.000								
0.3	0.990	0.995	0.999	1.000							
0.4	0.981	0.989	0.995	0.999	1.000						
0.5	0.970	0.980	0.988	0.994	0.999	1.000					
0.6	0.954	0.967	0.978	0.987	0.994	0.998	1.000				
0.7	0.936	0.951	0.964	0.976	0.986	0.994	0.998	1.000			
0.8	0.913	0.931	0.947	0.962	0.975	0.985	0.993	0.998	1.000		
0.9	0.888	0.907	0.926	0.944	0.960	0.973	0.985	0.993	0.998	1.000	
1.0	0.858	0.880	0.902	0.922	0.941	0.958	0.972	0.984	0.993	0.998	1.000

Table 15 summarizes the results obtained from the examples. Three examples based on differently structured numerical data demonstrate the impact parameter α for the evaluation and ranking of alternatives obtained using the MIDIA approach. The results confirm the usefulness of the $D(\alpha)$ method based on a combination of distances to the ideal and anti-ideal in the analysis.

Table 15. Comparison results obtained in the examples.

Example	Properties
Example 1	Full consistency of alternative positions in ranking for $D(\alpha)$ and different coefficients α. The rankings for all coefficients α of the $D(\alpha)$ measures are identical. The Pearson correlation coefficient between the measures decreases as the difference between the α coefficients increases. The lowest Pearson coefficient (0.908) is between $D(0)$ and $D(1)$.
Example 2	Partial consistency of alternative positions in ranking for $D(\alpha)$ and different coefficients α. Regardless of the α coefficient, some alternatives maintain a fixed position in the ranking, while the positions of others fluctuate. Specifically, alternatives A7, A8, A9, and A10 have constant positions, whereas the positions of the remaining alternatives vary across α. Notably, A1 can occupy positions 1, 2, 3, or 4, depending on the value of α. The lowest Spearman coefficient (0.830) and Pearson coefficient (0.848) is between $D(0)$ and $D(1)$.
Example 3	No consistency of alternative positions for $D(\alpha)$ and different coefficients α. No alternative retains a constant position regardless of the α parameter. Notably, A1 can occupy positions 1, 2, 3, and 5 while A3 positions 3, 2, and 1 depending on the value of α. The lowest Spearman coefficient (0.709) and Pearson coefficient (0.84) is between $D(0)$ and $D(1)$ and between $D(0)$ and $D(0.1)$. The lowest Pearson coefficient (0.858) is between $D(0)$ and $D(1)$.

5. Application of the MIDIA Method in the Education Area

In this section, we present and compare the results obtained using the MIDIA method in a practical application related to one of the areas of sustainable development. By applying the $D(\alpha)$ measure with $\alpha \epsilon \{0,\ 0.1, 0.2,\ 0.3,\ 0.4,\ 0.5,\ 0.6,\ 0.7,\ 0.8,\ 0.9,\ 1\}$, we compare the resulting rankings of the European Union (EU) countries.

The Sustainable Development Goals (SDGs) are a set of 17 global objectives adopted by all United Nations Member States in 2015 as part of the 2030 Agenda for Sustainable Development. These goals are vital to the broader sustainable development agenda and aim to address a wide range of global challenges, promoting a more sustainable, equitable, and prosperous world by 2030 [70]. Among the seventeen SDGs, SDG 4 is a pivotal agenda item, focusing on education-related matters [28,71]. Referred to as "Quality Education", its full title is:

Goal 4: Ensure Inclusive and Equitable Quality Education and Promote Lifelong Learning Opportunities for All.

SDG 4 highlights the transformative role of education in advancing sustainable development [72]. Education is not only a basic human right but also an essential driver of reducing poverty, enhancing health, stimulating economic growth, and fostering peace and social cohesion. Achieving this goal involves not just expanding access to education but also improving its quality, relevance, and inclusiveness. Governments, organizations, and communities around the world are committed to fulfilling the vision of SDG 4, striving to ensure quality education and lifelong learning opportunities for all individuals. Consequently, examining the educational performance of European countries should be regarded as a significant area of interest [73]. To evaluate and measure progress toward achieving the SDGs, in particular SDG 4, multi-criteria methods provide a comprehensive framework that incorporates various dimensions [74–76]. These methods are useful for conducting comparisons of countries by considering multiple indicators and assessment criteria, as well as for monitoring progress in specific countries. Additionally, the outcomes of multi-criteria analyses can be used to inform and adjust national policies, thereby enhancing progress and ensuring the achievement of SDG 4.

5.1. Problem and Data Description

In this study, we used data from Eurostat for 2022, focusing on the Sustainable Development indicators related to education (SDG 4) for EU member states [77]. Table 16 lists eight indicators that monitor the progress towards SDG 4 in EU countries in 2022. The set of SDG 4 indicators encompasses key aspects designed to measure progress across different educational levels and domains.

Table 16. Indicators measuring progress towards SDG 4 in the EU in 2022.

Indicator	Criterion Type	Description
C1: Early leavers from education and training (%) [sdg_04_10]	cost	C1 measures the share of the population aged 18 to 24 with at most lower secondary education who were not involved in any education or training during the four weeks preceding the survey.
C2: Tertiary educational attainment (%) [sdg_04_20]	benefit	C2 measures the share of the population aged 25–34 who have successfully completed tertiary studies (e.g., university, higher technical institution, etc.).
C3: Participation in early childhood education (%) [sdg_04_31] *	benefit	C3 measures the share of the children between the age of three and the starting age of compulsory primary education who participated in early childhood education and care (ECEC), which can be classified as ISCED level 0 according to the International Standard Classification for Education (ISCED 2011).
C4: Adult participation in learning (%) [sdg_04_60]	benefit	C4 measures the share of people aged 25 to 64 who stated that they received formal or non-formal education and training in the four weeks preceding the survey (numerator). The denominator consists of the total population of the same age group, excluding those who did not answer the question on 'participation in education and training'.
C5: Share of individuals having at least basic digital skills (%) [sdg_04_70]	benefit	C5 measures the share of people aged 16 to 74 who have at least basic digital skills. It is a composite indicator based on selected activities performed by individuals on the internet in specific areas: information and data literacy, communication and collaboration, digital content creation, safety, and problem solving.
C6: Low achieving 15-year-olds in reading (%) [sdg_04_40] **	cost	C6-C8 measure the share of 15-year-old students failing to reach level 2 ('basic skills level') on the PISA scale for the three core school subjects of reading, mathematics, and science. The data stem from the Programme for International Student Assessment (PISA), a triennial international survey that aims to evaluate education systems by testing the skills and knowledge of 15-year-old students.
C7: Low achieving 15-year-olds in mathematics (%) [sdg_04_40] **	cost	
C8: Low achieving 15-year-olds in science (%) [sdg_04_40] **	cost	

Source: Eurostat [77]. (*) data for Greece were estimated; (**) data for Luxembourg were estimated.

The set of SDG 4 indicators in Table 16 adequately describes the education phenomenon by introducing an appropriate amount of information into the analysis and sufficiently differentiating the classified objects. Although we found a strong correlation between underachievement in reading, maths, and science, we used all the indicators in our study. The dataset for 28 EU countries in 2022, consisting of 8 indicators marked by symbols from C1 to C8, along with descriptive statistics, is presented in Table 17. It is worth noting that the EU countries show the most differentiation concerning adult participation in learning (C4), and the least in participation in early childhood education (C3).

Table 17. The data of the 27 EU countries for education area and descriptive statistics.

EU Country	C1	C2	C3	C4	C5	C6	C7	C8
Belgium	6.40	51.40	98.30	10.30	54.23	25.30	25.00	22.40
Bulgaria	10.30	34.00	80.40	1.60	31.18	52.90	53.60	48.00
Czechia	6.20	34.60	85.30	9.40	59.69	21.30	25.50	19.90
Denmark	10.00	49.00	97.10	27.90	68.65	19.00	20.40	19.50
Germany	12.70	36.70	93.10	8.10	48.92	25.50	29.50	22.90
Estonia	10.80	43.90	91.90	21.10	56.37	13.80	15.00	10.10
Ireland	3.70	63.00	93.20	11.80	70.49	11.40	19.00	15.60
Greece	4.10	45.20	68.80	3.50	52.48	37.60	47.20	37.30
Spain	13.90	50.20	96.70	15.20	64.16	24.40	27.30	21.30
France	7.60	50.40	100.00	13.30	61.96	26.90	28.80	23.80
Croatia	2.30	35.50	83.50	4.40	63.37	22.70	32.90	22.40
Italy	11.50	29.20	92.70	9.60	45.60	21.40	29.60	23.90

Table 17. *Cont.*

EU Country	C1	C2	C3	C4	C5	C6	C7	C8
Cyprus	8.10	59.20	84.40	10.50	50.21	60.60	53.20	51.80
Latvia	6.70	45.90	95.50	9.70	50.80	22.80	22.20	16.50
Lithuania	4.80	58.20	96.70	8.50	48.84	24.90	27.80	21.80
Luxembourg	8.20	61.00	90.50	18.10	63.79	29.30	27.20	26.80
Hungary	12.40	31.90	92.60	7.90	49.09	25.90	29.50	22.90
Malta	10.30	42.50	87.50	13.00	61.23	36.30	32.60	30.30
Netherlands	5.60	56.40	92.00	26.40	78.94	34.60	27.40	27.30
Austria	8.40	43.10	90.60	15.80	63.33	25.30	24.90	22.70
Poland	4.70	41.70	92.40	7.80	42.93	22.20	23.00	18.60
Portugal	6.50	42.50	96.30	13.30	55.31	23.10	29.70	21.80
Romania	15.60	24.70	74.80	5.40	27.82	41.70	48.60	44.00
Slovenia	4.00	47.30	92.70	22.30	49.67	26.10	24.60	17.80
Slovakia	7.40	39.10	78.60	12.80	55.18	35.40	33.20	30.60
Finland	8.40	40.70	89.00	25.20	79.18	21.40	24.90	18.00
Sweden	8.80	52.40	96.10	36.20	66.60	24.30	27.20	23.70
Max	15.60	63.00	100.00	36.20	79.18	60.60	53.60	51.80
Min	2.30	24.70	68.80	1.60	27.82	11.40	15.00	10.10
Median	8.13	44.80	90.03	13.67	56.30	28.00	29.99	25.25
Standard Deviation	3.27	9.74	7.40	8.09	11.88	10.54	9.57	9.57
Coefficient of variation	40.00	21.74	8.22	59.18	21.10	37.65	31.91	37.90

Source: Eurostat [77].

5.2. Results

For comparative analysis, we employed the MIDIA method with 10 variants $\alpha \epsilon \{0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1\}$, with equal weights. Applying equal weights simplifies the analysis and is suitable when no clear rationale exists for assigning different weights to the criteria.

Firstly, according to Step 3 (see Formulas (3)–(6)), the ideal and anti-ideal points based on maximum and minimum values taking into account criterion type are determined:

$$I = [2.30, 63.00, 100.00, 36.20, 79.18, 11.40, 15.00, 10.10],$$
$$AI = [15.60, 24.70, 68.80, 1.60, 27.82, 60.60, 53.60, 51.80].$$

Following Steps 4 and 5 of the MIDIA procedure, the data from Table 17 were normalized and weighted using Formulas (7)–(10). In Step 6, the Euclidean distances between the alternatives and the ideal or anti-ideal solutions are calculated using Formulas (11) and (12). Next, the values of DI and DAI measures for all EU countries are calculated by applying Formulas (13) and (14). Finally, the values of the $D(\alpha)$ measure are determined by Formula (15) for $\alpha = 0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1$. The results of the calculation are presented in Table 18. The rankings of EU countries based on the $D(\alpha)$ measure are shown in Table 19.

Figure 4 shows a graphical representation of the values of the $D(\alpha)$ measure and the rankings of countries in the area of education in 2022 obtained using different values of coefficients α using the MIDIA method.

Table 19 and Figure 4 show that seven EU countries maintained the same ranking positions regardless of the α coefficient value used. These countries were Ireland, Denmark, and Sweden, which occupied the top three positions in the ranking, as well as Greece, Cyprus, Bulgaria, and Romania, which were consistently ranked in the bottom four positions. The indicator values for Ireland were very close to the values of the ideal vector, and in two cases (for C2 and C6), they exactly matched its values. The $D(\alpha)$ measure for Ireland, regardless of the alpha value, was significantly higher than the values determined for other countries. On the other hand, the positions of Denmark and Sweden were determined by differences in the third decimal place (the difference between $D(\alpha)$ for Denmark and Sweden was less than 0.01 for all α). Similarly, for Ireland, the indicator values for both Denmark and Sweden were close to the ideal points and far from the anti-ideal points.

Table 18. The values of the $D(\alpha)$ measure for EU countries in 2022 with different coefficients α.

Countries	$D(\alpha)$ Values for Different Coefficients α											Max-Min
	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	
Belgium	0.683	0.675	0.668	0.660	0.652	0.645	0.637	0.630	0.622	0.615	0.607	0.075
Bulgaria	0.217	0.210	0.204	0.197	0.190	0.183	0.177	0.170	0.163	0.157	0.150	0.067
Czechia	0.624	0.616	0.607	0.598	0.590	0.581	0.573	0.564	0.555	0.547	0.538	0.086
Denmark	0.770	0.764	0.759	0.754	0.749	0.744	0.739	0.734	0.729	0.723	0.718	0.051
Germany	0.550	0.540	0.531	0.521	0.512	0.502	0.493	0.483	0.474	0.465	0.455	0.095
Estonia	0.757	0.745	0.733	0.721	0.709	0.696	0.684	0.672	0.660	0.648	0.636	0.120
Ireland	0.847	0.835	0.822	0.809	0.797	0.784	0.772	0.759	0.747	0.734	0.721	0.126
Greece	0.446	0.433	0.419	0.406	0.392	0.378	0.365	0.351	0.338	0.324	0.310	0.136
Spain	0.662	0.652	0.642	0.633	0.623	0.613	0.603	0.593	0.583	0.573	0.564	0.098
France	0.680	0.674	0.668	0.662	0.656	0.651	0.645	0.639	0.633	0.627	0.621	0.060
Croatia	0.634	0.620	0.607	0.593	0.579	0.565	0.552	0.538	0.524	0.510	0.496	0.138
Italy	0.553	0.542	0.531	0.520	0.508	0.497	0.486	0.474	0.463	0.452	0.440	0.113
Cyprus	0.433	0.415	0.397	0.379	0.361	0.343	0.325	0.306	0.288	0.270	0.252	0.182
Latvia	0.683	0.674	0.665	0.656	0.647	0.638	0.629	0.620	0.611	0.602	0.592	0.091
Lithuania	0.695	0.684	0.674	0.663	0.653	0.642	0.632	0.621	0.611	0.600	0.590	0.105
Luxembourg	0.668	0.665	0.661	0.658	0.655	0.651	0.648	0.645	0.641	0.638	0.635	0.033
Hungary	0.540	0.530	0.520	0.510	0.499	0.489	0.479	0.469	0.459	0.449	0.438	0.102
Malta	0.510	0.508	0.506	0.505	0.503	0.501	0.499	0.497	0.495	0.493	0.491	0.019
Netherlands	0.736	0.731	0.726	0.722	0.717	0.713	0.708	0.703	0.699	0.694	0.689	0.046
Austria	0.638	0.635	0.632	0.630	0.627	0.624	0.621	0.618	0.615	0.612	0.609	0.029
Poland	0.657	0.645	0.633	0.622	0.610	0.598	0.586	0.574	0.563	0.551	0.539	0.118
Portugal	0.652	0.646	0.641	0.635	0.629	0.624	0.618	0.613	0.607	0.602	0.596	0.056
Romania	0.187	0.181	0.175	0.169	0.162	0.156	0.150	0.144	0.138	0.132	0.126	0.061
Slovenia	0.702	0.698	0.694	0.690	0.685	0.681	0.677	0.673	0.669	0.665	0.661	0.041
Slovakia	0.476	0.474	0.472	0.470	0.468	0.466	0.464	0.462	0.460	0.457	0.455	0.021
Finland	0.734	0.728	0.722	0.716	0.709	0.703	0.697	0.691	0.685	0.679	0.672	0.062
Sweden	0.760	0.756	0.751	0.747	0.742	0.738	0.733	0.729	0.724	0.720	0.715	0.045

Table 19. The rankings of EU countries obtained by the $D(\alpha)$ measure for different coefficients α.

Country	Position of Countries for $D(\alpha)$ Measure and Different Coefficients α										
	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
Belgium	10	9	10	10	11	10	10	10	10	10	11
Bulgaria	26	26	26	26	26	26	26	26	26	26	26
Czechia	18	18	17	17	17	17	17	17	17	17	17
Denmark	2	2	2	2	2	2	2	2	2	2	2
Germany	20	20	20	19	19	19	20	20	20	20	21
Estonia	4	4	4	5	6	6	6	7	7	7	7
Ireland	1	1	1	1	1	1	1	1	1	1	1
Greece	24	24	24	24	24	24	24	24	24	24	24
Spain	13	13	13	14	15	15	15	15	15	15	15
France	11	10	9	9	8	9	9	9	9	9	9
Croatia	17	17	18	18	18	18	18	18	18	18	18
Italy	19	19	19	20	20	21	21	21	21	22	22
Cyprus	25	25	25	25	25	25	25	25	25	25	25
Latvia	9	11	11	12	12	12	12	12	13	13	13
Lithuania	8	8	8	8	10	11	11	11	12	14	14
Luxembourg	12	12	12	11	9	8	8	8	8	8	8
Hungary	21	21	21	21	22	22	22	22	23	23	23
Malta	22	22	22	22	21	20	19	19	19	19	19
Netherlands	5	5	5	4	4	4	4	4	4	4	4
Austria	16	16	16	15	14	14	13	13	11	11	10
Poland	14	15	15	16	16	16	16	16	16	16	16
Portugal	15	14	14	13	13	13	14	14	14	12	12
Romania	27	27	27	27	27	27	27	27	27	27	27
Slovenia	7	7	7	7	7	7	7	6	6	6	6
Slovakia	23	23	23	23	23	23	23	23	22	21	20
Finland	6	6	6	6	5	5	5	5	5	5	5
Sweden	3	3	3	3	3	3	3	3	3	3	3

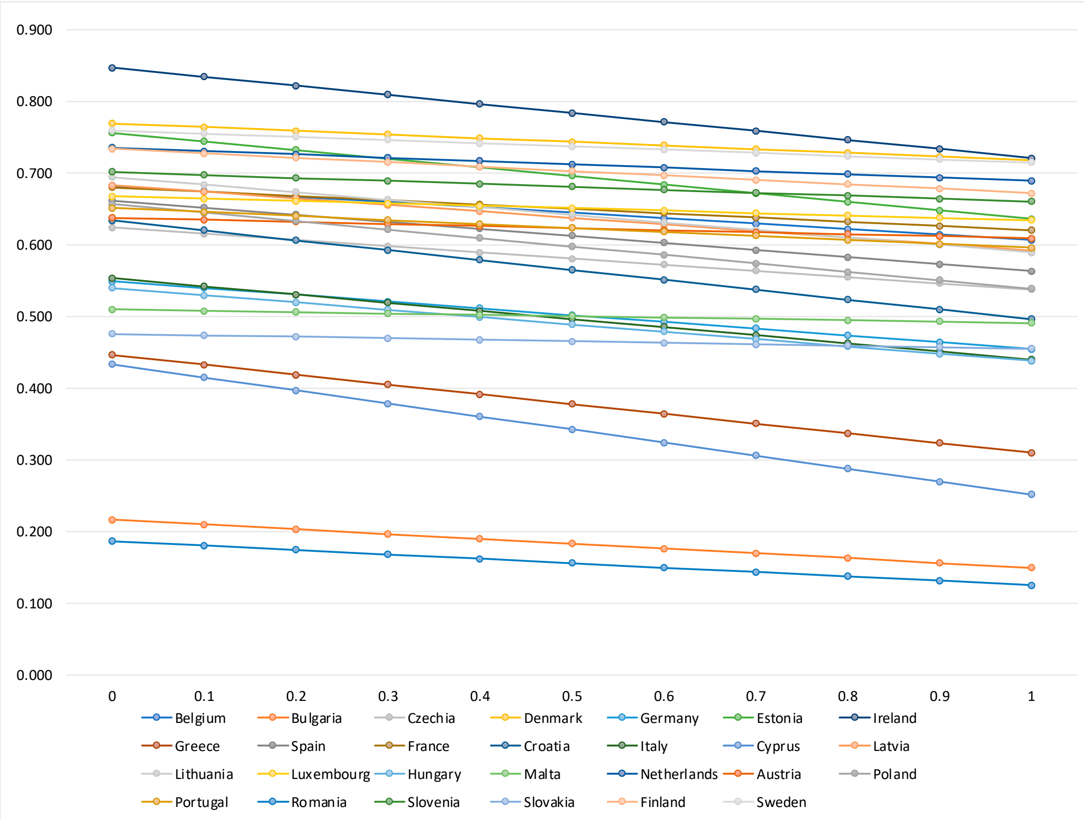

Figure 4. Graphical representation of $D(\alpha)$ measure and ranking of EU countries in the area of education in 2022 obtained for different coefficients α for MIDIA method.

From Figure 4, it is easy to observe that for the countries occupying the last four positions in the rankings, their $D(\alpha)$ values for each chosen α significantly deviated from the values of the other member states. For Romania (27th position), the values ranged from 0.126 to 0.187, for Bulgaria (26th position) from 0.150 to 0.217, for Cyprus (25th position) from 0.252 to 0.433, and for Greece (24th position) from 0.310 to 0.446. The $D(\alpha)$ values for the remaining countries ranged from 0.438 to 0.757, and their positions changed depending on the chosen α value. The greatest changes in position depending on the chosen alpha coefficient were observed for two countries: Lithuania and Austria, which changed their ranking positions by as much as six places. Lithuania, with $\alpha = 0$ occupied the eighth position, while for $\alpha = 1$, it was ranked 14th. With $\alpha = 0.5$, Lithuania held the 12th position. Austria achieved the 16th, 14th, and 10th positions for α equal to 0, 0.5, and 1, respectively. From Figure 5, we can observe that for seven other countries, their ranking position for $\alpha = 0.5$ differed from the positions based on $D(0)$ and $D(1)$. These countries were: Germany in the 19th position (20th for $D(0)$ and 21st for $D(1)$), Estonia in the 6th position (4th for $D(0)$ and 7th for $D(1)$), Italy in the 21st position (19th for $D(0)$ and 22nd for $D(1)$), Latvia in the 12th position (9th for $D(0)$ and 13th for $D(1)$), Hungary in the 22nd position (21st for $D(0)$ and 23rd for $D(1)$), Malta in the 20th position (22nd for $D(0)$ and 19th for $D(1)$), and Portugal in the 13th position (15th for $D(0)$ and 12th for $D(1)$).

Table 17 and Figure 4 also show that for Germany, Estonia, Spain, Italy, and Luxembourg, the rankings for the $D(0)$, $D(0.1)$, and $D(0.2)$ measures are identical; for Lithuania, Hungary, Malta, and Finland, this holds additionally for $D(0.3)$; and for Slovenia and Slovakia, the rankings are the same from $D(0)$ to $D(0.6)$. On the other hand, Czechia, France, and Croatia maintain the same positions from $D(0.2)$ to $D(1)$, the Netherlands and Poland from $D(0.3)$ to $D(1)$, and Finland from $D(0.4)$ to $D(1)$. As α increases, the positions of Estonia, Spain, Italy, Croatia, Latvia, Lithuania, Hungary, and Poland systematically decrease, while the positions of Czechia, Luxembourg, Malta, the Netherlands, Austria, and Finland systematically increase. The positions of Belgium, Germany, Spain, France, and Portugal change in various directions.

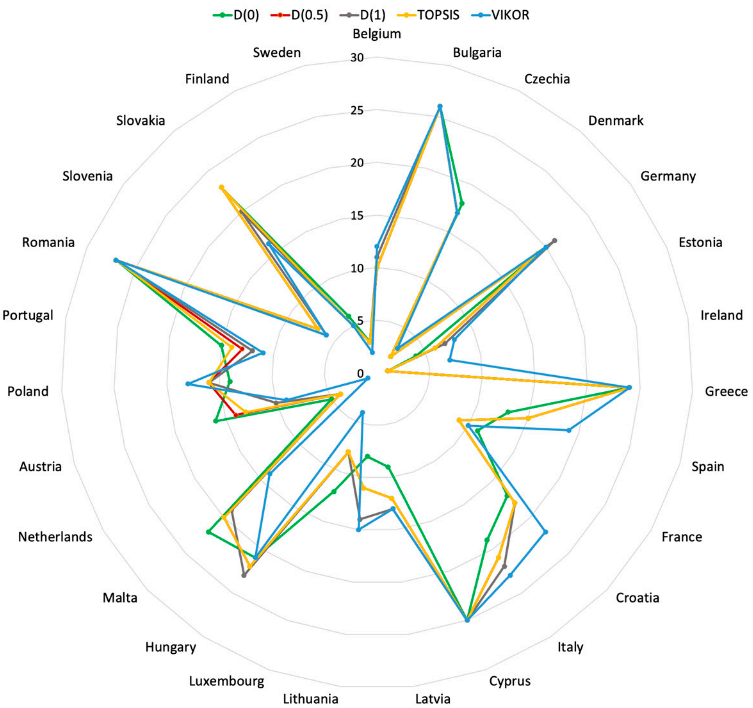

Figure 5. Graphical representation of ranking of EU countries in the area of education in 2022 for TOPSIS, VIKOR, and for $\alpha = 0$, $\alpha = 0.5$, $\alpha = 1$ using the MIDIA method.

Spearman's rank correlation coefficients between the values of the $D(\alpha)$ measure for different α values are presented in Table 20. These values indicate a strong correlation between the rankings. The lowest values are observed when comparing the rankings obtained using $D(1)$ with $D(0)$ (0.947), $D(0)$ with $D(0.9)$ (0.953), and $D(1)$ with $D(0.2)$ (0.954).

Table 20. The Spearman coefficient for $D(\alpha)$ and different coefficients α.

Spearman Coefficient	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.0	1.000										
0.1	0.998	1.000									
0.2	0.996	0.999	1.000								
0.3	0.991	0.996	0.997	1.000							
0.4	0.982	0.987	0.989	0.995	1.000						
0.5	0.978	0.983	0.984	0.991	0.998	1.000					
0.6	0.976	0.980	0.982	0.988	0.996	0.999	1.000				
0.7	0.974	0.979	0.980	0.987	0.996	0.998	0.999	1.000			
0.8	0.964	0.969	0.971	0.980	0.991	0.995	0.997	0.998	1.000		
0.9	0.953	0.960	0.961	0.972	0.986	0.991	0.992	0.993	0.997	1.000	
1.0	0.947	0.954	0.955	0.966	0.982	0.986	0.988	0.989	0.995	0.999	1.000

In summary, the sensitivity analysis conducted in measure $D(\alpha)$ by varying the different coefficients α shows partial consistency of ranking for $D(\alpha)$, as in Example 2 (see Section 4).

Finally, we compared the results obtained using the MIDIA method with those from other well-known methods based on reference points, namely TOPSIS and VIKOR. To clarify the relationships revealed by these approaches, we compared and discussed the results obtained using MIDIA with parameters $\alpha = 0$, $\alpha = 0.5$, $\alpha = 1$, alongside the results from TOPSIS and VIKOR with $v = 0.5$. In Table 21, the values of T_i for TOPSIS and Q_i for VIKOR, along with the rankings of EU countries in 2022 based on these values, are presented.

Table 21. The values of T_i for TOPSIS and Q_i for VIKOR, and rankings based on these values for 27 EU countries in 2022.

EU Country	T_i	Ranking TOPSIS	Q_i	Ranking VIKOR
Belgium	0.635	10	0.379	12
Bulgaria	0.203	26	0.971	26
Czechia	0.575	17	0.461	17
Denmark	0.732	2	0.153	3
Germany	0.502	19	0.558	20
Estonia	0.675	6	0.239	8
Ireland	0.753	1	0.221	7
Greece	0.393	24	0.828	24
Spain	0.603	15	0.526	19
France	0.642	9	0.296	10
Croatia	0.557	18	0.606	22
Italy	0.497	21	0.632	23
Cyprus	0.367	25	0.850	25
Latvia	0.626	12	0.402	13
Lithuania	0.629	11	0.425	15
Luxembourg	0.647	8	0.163	4
Hungary	0.490	22	0.575	21
Malta	0.501	20	0.419	14
Netherlands	0.703	4	0.066	1
Austria	0.620	13	0.254	9
Poland	0.588	16	0.484	18
Portugal	0.617	14	0.320	11
Romania	0.176	27	1.000	27
Slovenia	0.674	7	0.192	6
Slovakia	0.466	23	0.459	16
Finland	0.691	5	0.188	5
Sweden	0.727	3	0.071	2

In Figure 5, the rankings of EU countries based on the MIDIA method for three values $\alpha = 0$, $\alpha = 0.5$, $\alpha = 1$ are presented, along with the rankings obtained using the TOPSIS and VIKOR for $v = 0.5$.

The results presented in Tables 19 and 21, as well as Figure 5, provide the opportunity to compare the rankings obtained using the MIDIA method with those obtained using the TOPSIS and VIKOR methods. It is worth noting that the ranking obtained using the TOPSIS method was almost identical to that obtained using the MIDIA method, with $\alpha = 0.5$. The changes of one position concerned only two EU countries. In the case of Austria, there was a change from the 14th position for the MIDIA method to the 13th position for TOPSIS, while, for Portugal, the position changed from 13th (for MIDIA) to 14th (for TOPSIS). The Spearman correlation coefficient for the rankings obtained using the MIDIA method ($\alpha = 0.5$) and TOPSIS was very strong, equal to 0.999. Comparing the rankings obtained by the MIDIA method with $\alpha = 0$ and $\alpha = 1$ with the rankings obtained by the TOPSIS method, the differences were slightly larger. The Spearman correlation coefficients were 0.977 and 0.987, respectively. The largest difference of position in the comparison of TOPSIS with $D(0)$ was for Luxembourg, which moved from 12th position for $D(0)$ to 8th position for TOPSIS. However, when comparing the rankings obtained by the MIDIA method with $\alpha = 1$ and TOPSIS, the largest positional change (by three positions) was observed for three countries: Lithuania (moving from 14th position for $D(1)$ to 11th position for TOPSIS), Austria (moving from 10th position for $D(1)$ to 13th position for TOPSIS), and Slovakia (moving from 20th position for $D(1)$ to 23rd position for TOPSIS). It is worth noting that

15 countries maintained their positions in the rankings obtained by the $D(1)$ and TOPSIS methods, while, in the comparison of TOPSIS with $D(0)$, 10 EU countries did not change their positions.

Comparing the rankings obtained using the MIDIA method for $\alpha = 0.5$ and the VIKOR method for $v = 0.5$, we can state that the differences are more significant. Only in six cases did the rankings remain unchanged (for Bulgaria, the Czech Republic, Greece, Cyprus, Romania, and Finland). The biggest change in the ranking, by seven positions (from 23rd place for MIDIA to 16th place for VIKOR), concerned Slovakia. A surprising change also occurred for Ireland, which ranked first using the MIDIA method, but only seventh using the VIKOR method. A difference of six positions can also be observed for Malta. In this case, however, it was an improvement from 20th place using MIDIA to 14th using VIKOR. The Spearman correlation coefficient for the MIDIA method ($\alpha = 0.5$) and VIKOR ($v = 0.5$) was 0.925. The greatest consistency in rankings can be observed when comparing the results obtained by the MIDIA method with $\alpha = 1$ and the VIKOR method. In this case, the Spearman correlation coefficient was 0.954. The largest positional difference (by six positions) in the comparison of VIKOR with $D(1)$ was, similar to $D(0.5)$, for Ireland, which moved from 1st place for $D(1)$ to 7th for VIKOR. It is worth noting that eight countries did not change their positions. However, the Spearman correlation coefficient between the rankings obtained by the $D(0)$ method and VIKOR was the lowest, at 0.854. In this case, six countries did not change their positions. The largest changes (by eight positions) were for Luxembourg (from 12th for $D(0)$ to 4th for VIKOR) and Malta (from 22nd for $D(0)$ to 14th for VIKOR). Additionally, three countries changed their ranking by seven positions: Lithuania (from 8th for $D(0)$ to 15th for VIKOR), Austria (from 16th for $D(0)$ to 9th for VIKOR), and Slovakia (from 23rd for $D(0)$ to 16th for VIKOR).

6. Discussion

The proposed $D(\alpha)$ method, by incorporating the importance coefficient, allows for a balanced evaluation of alternatives by considering both distances to the ideal solution (α) and the distance from the anti-ideal solution ($1 - \alpha$). This balance prevents any single criterion from dominating the decision-making process. This hybrid approach aims to select alternatives that are close to the ideal solution while being far from the anti-ideal solution.

The coefficient of importance α can be determined by consulting with experts about the goals. The ability to adjust alpha allows decision-makers to customize the evaluation criteria based on the specific needs and objectives of the decision context. This flexibility ensures that the method can adapt to different priorities and preferences. The integrating coefficient provides a clearer and more structured approach to decision-making, helping DM understand the trade-offs between prioritizing distances to ideal solutions and avoiding distances to anti-ideal solutions.

It is worth noting that the MIDIA method integrates elements from both the TOPSIS and VIKOR methods, offering a balanced approach to multi-criteria decision-making by focusing on the distances from ideal and anti-ideal points.

TOPSIS emphasizes the distance between the ideal solution and the anti-ideal solution. In TOPSIS, the best alternative is the one that is closest to the ideal solution and farthest from the anti-ideal solution. This method involves normalizing the decision matrix, calculating a weighted normalized decision matrix, and then determining the Euclidean distances to the ideal and anti-ideal points. The final ranking is based on the relative closeness of each alternative to the ideal solution.

VIKOR, on the other hand, is designed to introduce a compromise solution and also considers the distance to the ideal and anti-ideal solutions, but introduces a compromise ranking based on preference and satisfaction measures. VIKOR also normalizes the decision matrix and uses weighted scores, but focuses on aggregating the distances through a utility-based ranking method, allowing for a more nuanced reflection of DMs preferences. VIKOR uses a linear combination of the best and worst alternatives to compute a ranking index that reflects a compromise between proximity to the ideal and anti-ideal solutions.

The MIDIA method synthesizes these approaches by incorporating the structure of TOPSIS and the compromise perspective of VIKOR. MIDIA uses a weighted system to account for both the ideal and anti-ideal distances, allowing for the incorporation of asymmetric preferences. It steps beyond simple proximity measures by integrating coefficients that reflect the importance of each distance, thus enabling a more flexible and tailored evaluation process. This method follows a comprehensive nine-step process, starting from determining the decision matrix to ranking the alternatives based on their relative closeness to an aggregated measure of ideal and anti-ideal distances. By combining elements from both TOPSIS and VIKOR, MIDIA provides a more adaptable framework that can reflect varied decision-making contexts. The use of a dual-reference point system (ideal and anti-ideal) and the ability to weigh the importance of distances enable MIDIA to address asymmetrical preferences more effectively. This makes MIDIA particularly valuable in scenarios where decision-makers prioritize different aspects of the ideal and anti-ideal solutions, thus providing a more nuanced and behaviorally consistent decision-making tool.

A brief comparison of the three methods, TOPSIS, VIKOR, and MIDIA, based on ideal and anti-ideal points is presented below

- TOPSIS is a method based on the concept of the distance to the ideal solution and the anti-ideal solution. VIKOR is a compromise ranking method that also considers the distance to ideal and anti-ideal solutions, but introduces compromise ranking based on preference and satisfaction measures. MIDIA is a method based on the concept of weighted distance to the ideal solution and the anti-ideal solution.
- The TOPSIS method focuses on the Euclidean distance to the ideal and anti-ideal solutions. At the same time, VIKOR uses compromise measures that consider both the sum of weighted deviations and the maximum weighted deviation. The MIDIA method, similarly to TOPSIS, focuses on the Euclidean distance to the ideal and anti-ideal solutions, but by implementing coefficients, it allows for balancing and considering the asymmetry in the importance of these distances.
- TOPSIS offers a simple ranking based on the closeness coefficient. In the case of VIKOR, the ranking process is more complex, considering various compromise measures, which can lead to different solutions depending on the model parameters. Similarly, in MIDIA, the final ranking depends on the coefficients, reflecting the relative importance of the distances to the ideal and anti-ideal solutions.
- TOPSIS is ideal for situations where decision-maker preferences are clear and the goal is to find the solution closest to the ideal. On the other hand, VIKOR is better suited for cases where a compromise solution is needed, considering different aspects of decision-maker satisfaction. By introducing coefficients, MIDIA can tailor the decision-making process to reflect varied preferences regarding the importance of the distances to the ideal and anti-ideal points. This method can introduce more flexibility and balance in the evaluation.

In summary, all methods have unique features and are used depending on the specific requirements of multi-criteria analysis. The choice between TOPSIS, VIKOR, and MIDIA depends on the decision-making context, expectations regarding compromises, the importance of ideal and anti-ideal solutions in evaluating alternatives, and the nature of the available data.

7. Conclusions and Further Research

Most multi-criteria methods try to incorporate human thinking into their procedures [78]. In line with this approach, the proposed multi-criteria method MIDIA captures data using the concepts of ideal and anti-ideal points and inserts the concept of weights into aggregation procedures. The multi-criteria algorithm MIDIA proposed in this paper is user-friendly because it is based on fundamental concepts—the ideal solution and the anti-ideal solution—that are familiar from everyday experiences. Moreover, considering combinations in a multi-criteria method as a mix of distances with different weights from the ideal and anti-ideal points results in different aggregation value measures and provides valuable insights from a behavioral perspective. The decision-making structure of this method is flexible and readily adaptable to decision-makers preferences.

The proposed multi-criteria distance measure, MIDIA, offers several key advantages that enhance its effectiveness in decision-making processes.

Balanced Consideration: By incorporating coefficients of distance importance, the formula can balance the importance of proximity to the ideal solution (α) and the anti-ideal solution ($1 - \alpha$). This balance ensures that some aspects dominate disproportionately, leading to a more comprehensive evaluation of alternatives.

Flexibility in Decision-Making: The ability to adjust coefficients of distance importance allows decision-makers to tailor the evaluation process according to the specific priorities and objectives of the decision context. This flexibility ensures that the method can adapt to varying degrees of emphasis on achieving ideal outcomes versus avoiding worst-case scenarios.

Handling Asymmetry: Many decision scenarios involve asymmetrical preferences associated with achieving goals. The MIDIA method addresses this by explicitly accounting for asymmetry through the parameter α, which ranges from 0 to 1.

Special Case with $\alpha = 1$ (Hellwig Measure): When α equals 1, the formula simplifies to the Hellwig measure [5]. This measure is particularly insightful as it focuses solely on the distance from the ideal solution, providing a clear and direct assessment of how well alternatives perform relative to the best possible outcome.

Enhanced Decision Clarity: By integrating both aspects—distance to the ideal and distance from the anti-ideal—the combined formula enhances decision clarity. It offers a more nuanced understanding of each alternative's performance across multiple criteria, facilitating informed decision-making processes.

Application in Various Fields: The utility of such a combined approach extends across diverse fields, including engineering, finance, and public policy, where decisions often hinge on balancing optimal outcomes with risk mitigation strategies.

In summary, the MIDIA, with its coefficients of distance importance approach, provides decision-makers with a robust toolset to effectively evaluate alternatives, consider multiple dimensions of performance, and make well-informed decisions that align with strategic objectives and decision-maker preferences.

This work acknowledges certain limitations that will serve as subjects for further research. In the paper, the focus was limited to a few examples that served as illustrations of the usability of the MIDIA approach and the impact coefficient on the final ranking. These limited examples did not fully capture the approach's versatility and effectiveness across a broader range of scenarios. The study did not encompass a wide variety of datasets, which raises questions about the generalizability of the findings. Future research should aim to explore and quantify the relationships between the properties of the decision matrix and the rankings obtained by different coefficients. This could provide deeper insights into how the properties of the dataset influence ranking outcomes. Expanding the study to include different datasets or simulation studies would help in assessing the generalizability of the results. This would involve testing the MIDIA method across various contexts to determine its robustness and applicability. In the practical example, the results obtained by the MIDIA method were compared with the results obtained by TOPSIS and VIKOR for $v = 0.5$. The obtained rankings of EU countries in the area of education in 2022 indicate a large similarity between the rankings obtained by MIDIA for $\alpha = 0.5$ and TOPSIS and significantly larger differences in the positions of countries obtained by the MIDIA and VIKOR methods. Considering comparisons with alternative methods beyond the TOPSIS and VIKOR approaches, especially for $\alpha = 0.5$, could provide a broader perspective on the performance of the proposed approach. This would help in understanding how the MIDIA approach stands relative to other decision-making methodologies. Investigating the methods of determining coefficients from a behavioral perspective could be valuable. For instance, in the context of negotiation problems, developing a negotiation scoring system where ideal and anti-ideal points are treated as aspiration and reservation levels could offer practical insights. By addressing these limitations and pursuing the suggested future studies, researchers can enhance the understanding and applicability of the MIDIA approach in various decision-making contexts.

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Abbreviations

The following abbreviations are used in this manuscript:

MCDM	Multi-criteria decision-making
DM	Decision maker
VIKOR	VlseKriterijuska Optimizacija I Komoromisno Resenje
TOPSIS	Technique for Ordering Preferences by Similarity to Ideal Solution
MIDIA	A Multi-Criteria Method Integrating Distances to Ideal and Anti-ideal Points

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