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A Study on Complex t-Neutrosophic Graph with Intention to Preserve Biodiversity

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Abstract: This study introduces the notion of complex t-neutrosophic graphs (CTNGs) as a powerful tool for understanding and displaying complex interactions that are sometimes difficult to understand. It demonstrates that CTNGs may accurately reflect complicated interactions involving several components or dimensions within a particular scenario. It also instructs the basic set operations of CTNGs and analyzes notions like homomorphism and isomorphism within this framework. Furthermore, the research describes a practical application of CTNGs. It illustrates their value in addressing biodiversity conservation by taking into account a variety of relevant factors. The paper uses this application to highlight the flexibility and effectiveness of CTNGs as a tool for decision-makers to visualize and prioritize activities targeted at improving biodiversity conservation.

Keywords: graph theory; t-neutrosophic graph; decision-making and sustainability

MSC: 05C72; 03B52; 68R10; 05C60; 05C90

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1. Introduction

Decision-making is an essential part of both personal lives and corporate administration. Its relevance stems from its potential to influence the success or failure of an organization. Managers make decisions at all stages of the management cycle, from planning to control, and the quality of those decisions has a substantial impact on their performance. Managers who lack excellent decision-making abilities are unable to efficiently perform other managerial functions such as planning, organizing, supervising, regulating, and staffing. The decision-making process should be cumulative and participatory, promoting organizational progress. In contexts characterized by ambiguity and vagueness, uncertainty management solutions become critical. Fuzzy decision-making settings implement such tactics by utilizing fuzzy set theory, which is effective at expressing circumstances when data lack clarity or accuracy. This theory gives membership degrees to items in a set, taking into account the uncertainty inherent in decision-making.

1.1. Fuzzy Set

Zadeh [1] fuzzy set theory provides a mathematical framework for addressing ambiguity, vagueness, and imprecision in computational perception. Since its origin, fuzzy set theory has been successfully applied in a variety of technological and scientific disciplines, including consumer electronics, control systems, image processing, robotics, artificial intelligence, and industrial automation. Furthermore, it has proved useful in operations research areas such as project management, decision theory, supply chain management, queuing theory, and quality control. Authors such as Kandel [2], Klir, and Yuan [3], as well as Mendel [4] and Zimmermann [5], write introductory works that explain the concepts and principles of fuzzy set theory to facilitate further comprehension and application.

1.2. Intuitionistic Fuzzy Set

The intuitionistic fuzzy set (IFS) is an extension of the fuzzy set that includes both membership and non-membership functions, expanding Zadeh's original notion. Atanassov [6] proposed IFS as an extension of Zadeh's fuzzy set, which in turn extends on the standard concept of a set. These sets provide a flexible framework for dealing with the uncertainty and ambiguity that are inherent in decision-making processes. De et al. [7] presented operations on IFS and investigated their varied properties, shedding light on key components of these procedures. IFS is important in fuzzy mathematics because it has a wide range of real-world applications, such as pattern identification, machine learning, decision-making, and market forecasting. Ejegwa et al. [8] offered a thorough analysis of numerous IFS models in actual circumstances. Smarandache [9] established Neutrosophy in 1998, which investigates the genesis, nature, and scope of neutralities, as well as their interconnections across many ideational spectrums. A neutrosophic set is defined by three membership functions: truth, indeterminacy, and falsity. This framework is an effective mathematical tool for extending the ideas of classical sets, fuzzy sets [1], IFSs [5], interval-valued fuzzy sets [10], paraconsistent sets, dialetheist sets, paradoxist sets, and tautological sets [9]. Neutrosophic sets are effective at managing the uncertain and inconsistent information that is omnipresent in daily life. In recent years, scholars throughout the world have conducted substantial studies on neutrosophic sets. Wang et al. [11] researched single-valued neutrosophic sets, making them more useful in scientific and technical sectors where reflecting uncertainty, incompleteness, imprecision, and inconsistency is critical. Hanafy et al. [12,13] investigated the correlation coefficient of neutrosophic sets, whereas Ye [14] looked into the correlation coefficient of single-valued neutrosophic sets. Broumi and Smaradache [15] defined the correlation coefficient for interval neutrosophic sets, whereas Salama et al. [16] investigated neutrosophic sets and topological spaces.

1.3. Fuzzy, Intuitionistic Fuzzy, and Neutrosophic Graph Theory

Graphs are a useful method to illustrate relationships between items, with edges indicating relationships and vertices representing objects. Graph theory is a strong tool for understanding and simplifying complicated systems. In mathematical chemistry, molecular descriptors are important for investigating the structure of molecules using mathematical approaches. Chemical graph theory investigates the intersection of chemistry, graph theory, and mathematics, using molecular graphs to represent atoms and bonds in compounds, allowing for the depiction of connections. Creating a Fuzzy Graph Model is necessary when expressing item connections when there is uncertainty. Fuzzy graphs are excellent mathematical tools for dealing with unknown elements. Rosenfeld [17] pioneered research in fuzzy graph theory, which was followed by Mordeson and Chang-Shyh's [18] discussion of fuzzy graph operations and Bhattacharya's [19] proof of graph theoretic findings. Bhutani [20] investigated the automorphisms of fuzzy graphs. Fuzzy graph theory has several applications in science and engineering, including broadcast communications, production, social networks, artificial intelligence, data processing, and neural systems. Intuitionistic fuzzy graphs (IFGs) provide a finer model of human cognition and decision-making processes, allowing for the exact determination of acceptance or rejection within a set. Shannon and Atanassov [21] introduced IFGs and intuitionistic fuzzy relations, which have been studied for their properties. Karunambigai and Atanassov [22] focused on IFG operations, whereas Gani and Begum [23] investigated IFG size, order, and degree. Sundas and Akram [24] explained how intuitionistic fuzzy soft graphs may be used to solve decision-making difficulties. Yaqoob et al. [25] contributed to the advancement of complicated intuitionistic fuzzy graph theory. Abida and Faryal [26] classified essential operations for complex IFGs into direct, semi-strong, strong, and modular products. Quek et al. [27] develop pentapartitioned neutrosophic graphs and show how they may be used to determine the safest pathways and towns in response to COVID-19. They investigate mathematical models that take neutrosophic components into account, resulting in a complete framework for assessing complex pandemic responses. AL Al-Omeri and Kaviyarasu [28] present a method for detecting online streaming services based on the Max product of complement in neutrosophic graphs. Their method uses neutrosophic graphs to model and evaluate internet streaming networks, assisting in the discovery and optimization of streaming services. Broumi et al. [29] introduce the notion of single-valued neutrosophic graphs, which expands graph theory to include neutrosophic components. Their work strengthens the theoretical underpinning of neutrosophic graph theory by providing insights into the representation and analysis of ambiguous and indeterminate data and later provides complex fermatean neutrosophic graphs and illustrates their use in decision-making processes. By incorporating fermatean neutrosophic aspects into graph theory, they present a unique paradigm for dealing with choice issues in uncertain contexts [30]. Akram and Shahzadi [31] investigate basic mathematical characteristics and procedures related to neutrosophic graph theory. Their work advances the creation of mathematical methods for assessing uncertain and indeterminate data represented by neutrosophic graphs. Yaqoob and Akram [32] define complex neutrosophic graphs, which expand the idea of neutrosophic graphs to include complex-valued neutrosophic constituents. Their work broadens the scope of neutrosophic graph theory by introducing a flexible framework for modeling and evaluating complicated systems with uncertain knowledge. Şahin's [33] approach to neutrosophic graph theory with applications provides insights into the theoretical underpinnings and practical applications of neutrosophic graph models. Their work advances neutrosophic graph theory, making it easier to analyze complex systems in uncertain environments. Kaviyarasu's [34] research focuses on the mathematical and structural features of neutrosophic graphs with regular edge patterns. Their study advances our understanding of neutrosophic graph topologies, providing insights into the representation and processing of ambiguous data. Mohammed Alqahtani et al. [35] investigate the use of complicated neutrosophic graphs in hospital infrastructure design. Their research shows how complicated neutrosophic graph models may be used to enhance hospital infrastructure layouts, hence increasing efficiency and resource allocation in healthcare institutions. Asima Razzaque et al. [36] explain a comprehensive analysis to reduce poverty by using t-intuitionistic fuzzy graphs. In [37–41] discussed some diction making by using FG, IFG, NFG and CNFGs. When compared to traditional complex neutrosophic fuzzy sets, the use of complex t-neutrosophic fuzzy sets (CTNFSs) offers significant benefits in negotiating ambiguity and uncertainty. This strategic approach provides a flexible technique for dealing with the uncertainties and ambiguities that are inherent in decision-making processes. Furthermore, complex t-neutrosophic fuzzy models are gaining popularity as a way to bridge the gap between traditional numerical models used in engineering and research and the symbolic models found in expert systems.

The theoretical framework of complex neutrosophic graphs (CNGs) is a useful tool for illuminating and clarifying complicated and ambiguous difficulties faced in actual situations. This efficacy stems from its capacity to successfully communicate the intrinsic features of unpredictability, complexity, imprecision, and uncertainty connected with the things contained inside these sets. However, to address practical problems about truth membership, indeterminacy membership, and falsity membership functions, it is necessary to reformulate these techniques using precise numerical values. To address this issue, the idea of CTNGs was proposed, which employs linear t-norm and t-conorm operators. The implementation of CTNGs originates from the need for a systematic and adaptive technique to successfully manage ambiguity and enable decision-making based on predefined criteria. In this case, the parameter 't' simplifies the procedure by defining specific criteria for determining the degree of truth membership, indeterminacy membership and falsity membership. In many real circumstances, decision-making must account for varied levels of confidence. Introducing the parameter 't' in CTNGs seeks to overcome standard

NFG limits by providing precise control over stringency, improving customization, allowing for independent decision thresholds, increasing flexibility, and decreasing ambiguity. These advantages establish CTNGs as an effective strategy for representing uncertainty and promoting well-informed decision-making in situations demanding a personalized and controlled approach to uncertainty management.

The benefit of CTNGs extends to comprehending and negotiating complicated choice settings, where standard NFG fails. These graphs offer decision-makers significant tools for studying and evaluating various options by completely illustrating the complex interplay between input and outcome factors. Complex fuzzy connections enable decision-makers to analyze numerous criteria and their interdependence methodically, allowing for a more holistic approach to tackling complex decision-making difficulties. The complex approach of CTNGs provides a considerable improvement in decision-making, especially in scenarios defined by truth membership, indeterminacy membership, falsity membership, and the parameter 't'. It represents a break from the constraints imposed by binary logic, providing the path for greater precision in decision-making processes. Finally given some exiting work in Table 1.

Table 1. Some relevant studies.

Years	Reference	Technique Used	Decision-Making
2022	37	Degree and distance of fuzzy graph	Urban public transportation problem for finding the best place for a bus stop
2023	38	Spherical Fuzzy Zagreb Energy	Selecting location
2018	39	Neutrosophic Cubic Graphs	Real-life applications in industries
2020	40	Neutrosophic graph	Application in wireless network
2020	41	t-fuzzy graphs	Find minimum distance
2023	36	t-intuitionistic fuzzy graphs	Application in poverty reduction
2024	35	Complex neutrosophic graphs	Hospital infrastructure design
2024	Present	Complex t-neutrosophic graph	Biodiversity conservation

1.4. Motivation

Understanding complex interactions in many contexts, such as biodiversity conservation, is critical for making good decisions. However, standard approaches may fail to capture the complexities of such multiple events. As a result, there is a need for a sophisticated tool capable of correctly representing and analyzing complicated relationships.

1.5. Novelty

The use of CTNGs as a unique tool for expressing and evaluating complex relationships is what distinguishes this work. While there are other graph-based models, including complex t-neutrosophic sets (CTNS) brings a new depth to the topic. This integration allows for the depiction of uncertainty, indeterminacy, and inconsistency in complex systems, making the suggested technique ideal for modeling real-world events with various interacting elements. Furthermore, the paper investigates basic set operations, homomorphism, and isomorphism within the context CTNGs, adding to its originality.

1.6. Goal

- To introduce CTNGs and its associated properties.
- To give the new framework of CTNGs with the intention of preserving biodiversity.
- To develop theoretical foundations for CTNGs.
- To illustrate that CTNGs can be used in real applications to address biodiversity conservation.
- To show the impact of isomorphism and homomorphism in CTNGs for making better decisions preserving biodiversity.

1.7. Objective

- Complex objective: A mathematical tool for modeling ambiguous or imprecise information inside a graph structure is the T-neutrosophic graph. T-neutrosophic sets, a generalization of fuzzy sets, intuitionistic fuzzy sets, and classical sets, are included into the idea of traditional graphs to expand upon it.
- Expand the scope of standard graph theory principles and techniques to intricate t-neutrosophic graphs, enabling more thorough examination and resolution of issues in intricate systems.
- Utilize complex t-neutrosophic graphs in decision-making processes where multiple conflicting criteria or uncertain information need to be considered simultaneously.
- Utilize intricate t-neutrosophic graphs to solve practical issues in a variety of fields, including biological, transportation, social, and communication networks.

1.8. Key Contribution

The main contribution of this study is the development of CTNGs, an effective tool for visualizing and understanding complex relationships, especially in the context of biodiversity protection. The work illustrates how CTNGs effectively capture complex interactions between several conservation variables, supporting well-informed decision-making. By means of real-world implementations, it demonstrates the adaptability of CTNGs in tackling issues related to biodiversity conservation by taking into account several pertinent aspects including species preservation, habitat preservation, and climate change adaption. The incorporation of indeterminacy, truth membership, and falsity membership functions into CTNGs gives decision-makers an understanding of conservation endeavors, empowering them to efficiently prioritize actions. Furthermore, the parameter 't' in CTNGs enables modification to fit various sensitivities and situations, enabling decision-makers to manage risk and uncertainty and navigate through complex decision-making environments. All things considered, this study provides a thorough framework and analytical tools that improve biodiversity conservation decision-making processes, greatly advancing ecosystem preservation and human well-being.

The paper progresses as follows: The 'Basics of CTNGs' section clarifies key terminology to highlight the originality of the offered work. Various set-theoretical operations are investigated with graphical representations in the next section, 'Symmetric Operations on CTNGs'. Following that, the section 'Isomorphism CTNGs' defines homomorphisms and isomorphisms inside CTNGs. Moving on, the 'Application of CTNG in biodiversity conservation' section employs the newly developed strategy to promote ecosystem preservation. Finally, the research finishes with sensitivity analysis, comparative analysis, and definite conclusions summarizing the findings.

2. Basics of CTNGs

Definition 1. *Given a universal set U, let G be its neutrosophic set (NS) and t∈ [0,1]. Known as a t-neutrosophic set (TNS), the* $NS_{\mathcal{G}_t}$ *of U is defined as* $T_{\mathcal{G}_t}(\upsilon_1) = min\{T_{\mathcal{G}_t}(\upsilon_1), t\}$, $I_{\mathcal{G}_t}(\upsilon_1) = max\{I_{\mathcal{G}_t}(\upsilon_1), 1-t\}$, *and* $F_{\mathcal{G}_t}(\upsilon_1) = max\{F_{\mathcal{G}_t}(\upsilon_1), 1-t\}$, $\forall\ \upsilon_1 \in U$. *The form of TNS is* $\mathcal{G}_t = \{\upsilon_1, T_{\mathcal{G}_t}(\upsilon_1), I_{\mathcal{G}_t}(\upsilon_1), F_{\mathcal{G}_t}(\upsilon_1) : \upsilon_1 \in U\}$ *where* $T_{\mathcal{G}_t}, I_{\mathcal{G}_t,}$ *and* $F_{\mathcal{G}_t}$ *are functions that assign a degree of truth membership, indeterminacy membership, and falsity membership, respectively. Moreover, the functions* $T_{\mathcal{G}_t}, I_{\mathcal{G}_t}$ *and* $F_{\mathcal{G}_t}$ *satisfy the condition* $0 \leq T_{\mathcal{G}}(\upsilon_1) + I_{\mathcal{G}}(\upsilon_1) + F_{\mathcal{G}}(\upsilon_1) \leq 3$.

Definition 2. *A complex neutrosophic set (CNS) A, defined on a universe of discourse X is an objective of the form* $A = \left\{\upsilon_1,\ T_{\mathcal{G}_t}(\upsilon_1)e^{iT\tau_{\mathcal{G}_t}(\upsilon_1)}, I_{\mathcal{G}_t}(\upsilon_1)e^{iI\sigma_{\mathcal{G}_t}(\upsilon_1)},\ F_{\mathcal{G}_t}(\upsilon_1)e^{iF\rho_{\mathcal{G}_t}(\upsilon_1)}\right\}$, *where* $i = \sqrt{-1},\ (T_{\mathcal{G}_t}(\upsilon_1),\ I_{\mathcal{G}_t}(\upsilon_1),\ F_{\mathcal{G}_t}(\upsilon_1)) \in [0,1], 0 \leq T\tau_{\mathcal{G}_t}(\upsilon_1),\ \ I\sigma_{\mathcal{G}_t}(\upsilon_1),\ F\rho_{\mathcal{G}_t}(\upsilon_1) \leq 2\pi$.

Definition 3. *For a given simple graph* $\mathcal{G}$ *= (V,E), let* $\mathcal{G}_t = (A, B)$ *be a neutrosophic graph (NG). The notation* $\mathcal{G}_t = (A_t,\ B_t)$ *denotes a CTNG, where* $A_t = \left\{\left(\upsilon_i, T\vartheta_{\mathcal{G}t}(\upsilon_i)e^{iT\tau_{\mathcal{G}t}(\upsilon_i)},\right.\right.$

$I\vartheta_{\mathcal{G}t}(\upsilon_i)e^{iI\sigma_{\mathcal{G}t}(\upsilon_i)},\ F\vartheta_{\mathcal{G}t}(\upsilon_i)e^{iF\rho_{\mathcal{G}t}(\upsilon_i)}\Big) : \upsilon_i \in V\Big\}$ *is a CTNS on V and* $B_t = \Big\{\Big((\upsilon_i, \upsilon_j),$ $T\vartheta_{\mathcal{G}t}(\upsilon_i, \upsilon_j)e^{iT\tau_{\mathcal{G}t}(\upsilon_i,\ \upsilon_j)},\ I\vartheta_{\mathcal{G}t}(\upsilon_i, \upsilon_j)e^{iI\sigma_{\mathcal{G}t}(\upsilon_i,\ \upsilon_j)}, F\vartheta_{\mathcal{G}t}(\upsilon_i, \upsilon_j)e^{iF\rho_{\mathcal{G}t}(\upsilon_i,\ \upsilon_j)}\Big) : (\upsilon_i,\ \upsilon_j) \in E\Big\}$ *is a CTNS on* $E \subseteq V \times V$, *such that* $\forall(\upsilon_i, \upsilon_j) \in E$.

$$T\vartheta_{B_t}(\upsilon_i, \upsilon_j)e^{iT\tau_{B_t}(\upsilon_i,\ \upsilon_j)} \le min\{T\vartheta_{A_t}(\upsilon_i),\ T\vartheta_{A_t}(\upsilon_j)\}e^{i\ min\{T\tau_{A_t}(\upsilon_i),\ T\tau_{A_t}(\upsilon_j)\}}$$

$$I\vartheta_{B_t}(\upsilon_i,\ \upsilon_j)e^{I\sigma_{B_t}(\upsilon_i,\ \upsilon_j)} \le max\{I\vartheta_{A_t}(\upsilon_i),\ I\vartheta_{A_t}(\upsilon_j)\}e^{i\ max\{I\sigma_{A_t}(\upsilon_i),\ I\sigma_{A_t}(\upsilon_j)\}}$$

$$F\vartheta_{B_t}(\upsilon_i,\ \upsilon_j)e^{iF\rho_{B_t}(\upsilon_i,\ \upsilon_j)} \le max\{F\vartheta_{A_t}(\upsilon_i),\ F\vartheta_{A_t}(\upsilon_j)\}e^{imax\{F\rho_{A_t}(\upsilon_i),\ F\rho_{A_t}(\upsilon_j)\}}$$

Satisfy the condition

$$0 \le T\vartheta_{A_t}(\upsilon_i) + I\vartheta_{A_t}(\upsilon_i) + F\vartheta_{A_t}(\upsilon_i) \le 3 \text{ and } 0 \le T\vartheta_{B_t}(\upsilon_i,\ \upsilon_j) + I\vartheta_{B_t}(\upsilon_i,\ \upsilon_j) + F\vartheta_{B_t}(\upsilon_i,\ \upsilon_j) \le 3.$$

Example 1. *Examine the* $G' = (V, E)$ *in which* $V = \{\upsilon_1, \upsilon_2, \upsilon_3\}$ *and* $E = \{\upsilon_1\upsilon_2,\ \upsilon_1\upsilon_3,\ \upsilon_1\upsilon_4,\ \upsilon_2\upsilon_3,\ \upsilon_3\upsilon_4\}$. *Let A be a complex t-neutrosophic subset (CTNSs) of V and B be a CTNS of* $E \subseteq V \times V$, *as given at* $t = 0.60e^{i0.9\pi}$ *in Figure 1*

$$A_{0.60e^{i0.6\pi}} = \begin{Bmatrix} (\upsilon_1, 0.2e^{i0.3\pi}, 0.5e^{i0.6\pi}, 0.7e^{i0.8\pi}), \\ (\upsilon_2, 0.4e^{i0.5\pi}, 0.5e^{i0.6\pi}, 0.65e^{i0.6\pi}), \\ (\upsilon_3, 0.3e^{i0.4\pi}, 0.6e^{i0.7\pi}, 0.8e^{i0.9\pi}), \\ (\upsilon_4, 0.3e^{i0.4\pi}, 0.6e^{i0.7\pi}, 0.8e^{i0.9\pi}) \end{Bmatrix}$$

and

$$B_{0.60e^{i0.6\pi}} = \begin{Bmatrix} (\upsilon_1\upsilon_2, 0.2e^{i0.1\pi}, 0.6e^{i0.7\pi}, 0.7e^{i0.9\pi}), \\ (\upsilon_1\upsilon_3, 0.1e^{i0.3\pi}, 0.9e^{i0.8\pi}, 0.7e^{i0.8\pi}), \\ (\upsilon_1\upsilon_4, 0.2e^{i0.3\pi}, 0.6e^{i0.7\pi}, 0.8e^{i0.9\pi}), \\ (\upsilon_2\upsilon_3, 0.1e^{i0.2\pi}, 0.4e^{i0.3\pi}, 0.6e^{i0.4\pi}), \\ (\upsilon_3\upsilon_4, 0.1e^{i0.3\pi}, 0.9e^{i0.7\pi}, 0.6e^{i0.7\pi}) \end{Bmatrix}.$$

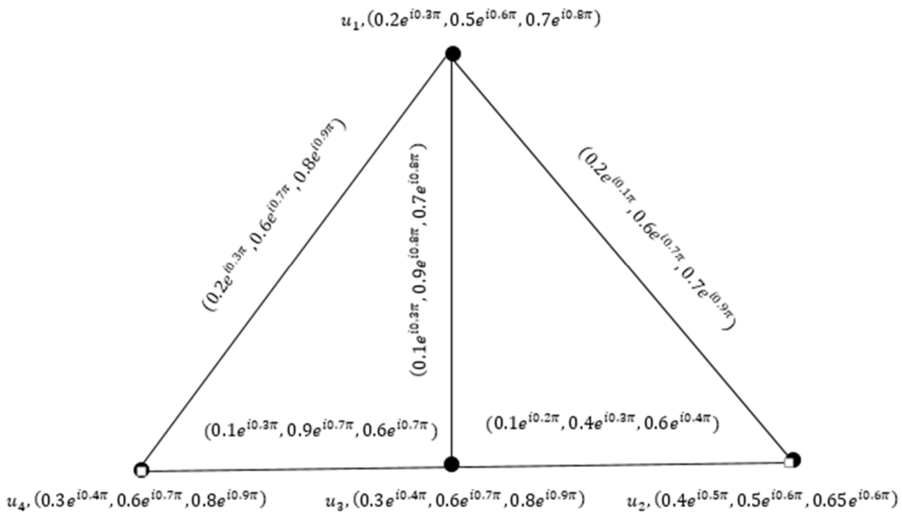

Figure 1. The graphical representation of a CTNG, where t = $0.6e^{i0.9\pi}$; $\mathcal{G}_{0.6} = (A_{0.60e^{i0.6\pi}}, B_{0.60e^{i0.6\pi}})$.

Definition 4. *Let* $\mathcal{G}_t = (A_t, B_t)$ *be a CTNG; then,* $\mathcal{H}_t = (A'_t, B'_t)$ *is considered a complex neutrosophic subgraph (CTNSG) if* $A'_t \subseteq A_t$ *and* $B'_t \subseteq B_t$.

Definition 5. *A CTNG $\mathcal{G}_t = (A_t, B_t)$ is termed a complete CTNG if it satisfies the following condition:*

$$T\vartheta_{B_t}(\upsilon_1, \upsilon_2)e^{iT\tau_{B_t}(\upsilon_1,\ \upsilon_2)} = min\{T\vartheta_{A_t}(\upsilon_1), T\vartheta_{A_t}(\upsilon_2)\}e^{i\ min\{T\tau_{A_t}(\upsilon_1),\ T\tau_{A_t}(\upsilon_2)\}}$$

$$I\vartheta_{B_t}(\upsilon_1, \upsilon_2)e^{iI\sigma_{B_t}(\upsilon_1,\ \upsilon_2)} = max\{I\vartheta_{A_t}(\upsilon_1), I\vartheta_{A_t}(\upsilon_2)\}e^{i\ max\{I\sigma_{A_t}(\upsilon_1),\ I\sigma_{A_t}(\upsilon_2)\}}$$

$$F\vartheta_{B_t}(\upsilon_1, \upsilon_2)e^{iF\rho_{B_t}(\upsilon_1,\ \upsilon_2)} = max\{F\vartheta_{A_t}(\upsilon_1), F\vartheta_{A_t}(\upsilon_2)\}\ e^{i\ max\ \{F\rho_{A_t}(\upsilon_1),\ F\rho_{A_t}(\upsilon_2)\}}, \forall\ (\upsilon_1, \upsilon_2) \in E.$$

Example 2. *Figure 2 illustrates the entire $0.6e^{i0.5\pi}$-NG $\mathcal{G}_t$.*

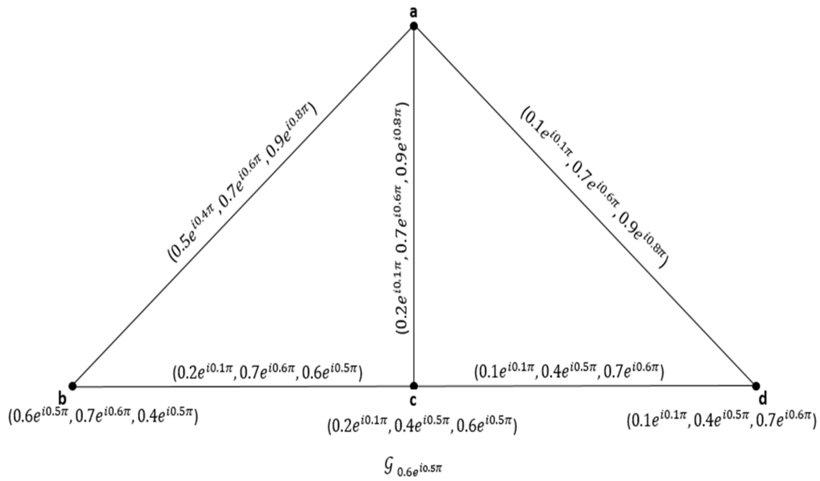

Figure 2. $\mathcal{G}_{0.6e^{i0.5\pi}}$.

Definition 6. *In the CTNG, the order is defined as follows:*

$$O(\mathcal{G}_t) = \begin{pmatrix} \sum\limits_{\upsilon_1 \in V} T\vartheta_{A_t}(\upsilon_1)\ e^{\sum_{\upsilon_1 \in V} T\tau_{A_t}(\upsilon_1)}, \\ \sum\limits_{\upsilon_1 \in V} I\vartheta_{A_t}(\upsilon_1)e^{\sum_{\upsilon_1 \in V} I\sigma_{A_t}(\upsilon_1)}, \\ \sum\limits_{\upsilon_1 \in V} F\vartheta_{A_t}(\upsilon_1)e^{\sum_{\upsilon_1 \in V} F\rho_{A_t}(\upsilon_1)} \end{pmatrix}$$

Example 3. *The order of CTNG $\mathcal{G}_t$ is $(1.0e^{i1.5\pi}, 2.5e^{i2.7\pi}, 2.75e^{i3.0\pi})$ from Example 1.*

Definition 7. *The CTNG has a size defined by*

$$S(\mathcal{G}_t) = \begin{pmatrix} \sum\limits_{(\upsilon_1,\upsilon_2) \in E} T\vartheta_{B_t}(\upsilon_1, \upsilon_2)e^{i\ T\tau_{B_t}(\upsilon_1,\upsilon_2)}, \\ \sum\limits_{(\upsilon_1,\upsilon_2) \in E} I\vartheta_{B_t}(\upsilon_1, \upsilon_2)e^{i\ I\sigma_{B_t}(\upsilon_1,\upsilon_2)}, \\ \sum\limits_{(\upsilon_1,\upsilon_2) \in E} F\vartheta_{B_t}(\upsilon_1, \upsilon_2)e^{i\ F\rho_{B_t}(\upsilon_1,\upsilon_2)} \end{pmatrix}$$

Definition 8. *In the CTNG, the degree of vertex υ_1 in $\mathcal{G}_t$ is defined as follows:*

1. $deg_{\mathcal{G}_t}(\upsilon_1) = \left(deg_{T_{B_t}}(\upsilon_1), deg_{I_{B_t}}(\upsilon_1), deg_{F_{B_t}}(\upsilon_1)\right)$

$$deg_{\mathcal{G}_t}(\upsilon_1) = \begin{pmatrix} \sum_{(\upsilon_1,\upsilon_2)\in E} T\vartheta_{B_t}(\upsilon_1,\upsilon_2)e^{i\ T\tau_{B_t}(\upsilon_1,\upsilon_2)}, \\ \sum_{(\upsilon_1,\upsilon_2)\in E} I\vartheta_{B_t}(\upsilon_1,\upsilon_2)e^{i\ I\sigma_{B_t}(\upsilon_1,\upsilon_2)}, \\ \sum_{(\upsilon_1,\upsilon_2)\in E} F\vartheta_{B_t}(\upsilon_1,\upsilon_2)e^{i\ F\rho_{B_t}(\upsilon_1,\upsilon_2)} \end{pmatrix}$$

2. *The minimum degree $\delta(\mathcal{G}_t)$ of the CTNG is given by*

$$\delta(\mathcal{G}_t) = \left(\delta_{T\vartheta_{B_t}}(\mathcal{G}_t)e^{i\delta_{T\tau_{B_t}}(\mathcal{G}_t)}, \delta_{I\vartheta_{B_t}}(\mathcal{G}_t)e^{i\delta_{I\sigma_{B_t}}(\mathcal{G}_t)}.\delta_{F_{\vartheta B_t}}(\mathcal{G}_t)e^{i\delta_{F\rho_{B_t}}(\mathcal{G}_t)}\right)$$

$$\delta(\mathcal{G}_t) = \begin{pmatrix} \min\left\{deg_{T\vartheta_{B_t}}(\upsilon_1)\right\}e^{i\ min\{deg_{T\tau_{B_t}}(\upsilon_1)\}}, \\ \min\left\{deg_{I\vartheta_{B_t}}(\upsilon_1)\right\}e^{i\ min\{deg_{I\sigma_{B_t}}(\upsilon_1)\}}, \\ \min\left\{deg_{F\vartheta_{B_t}}(\upsilon_1)\right\}e^{i\ \min\{deg_{F\rho_{B_t}}(\upsilon_1)\}} \end{pmatrix}\upsilon_1 \in V$$

3. *The maximum degree $\Delta(\mathcal{G}_t)$of the CTNG is given by*

$$\Delta(\mathcal{G}_t) = \left(\Delta_{\mathrm{T}\vartheta_{\mathrm{B}_t}}(\mathcal{G}_t)\mathrm{e}^{\mathrm{i}\Delta_{\mathrm{T}\tau_{\mathrm{B}_t}}(\mathcal{G}_t)}, \Delta\vartheta_{\mathrm{I}_{\mathrm{B}_t}}(\mathcal{G}_t)\mathrm{e}^{\mathrm{i}\Delta_{\mathrm{I}\sigma_{\mathrm{B}_t}}(\mathcal{G}_t)}, \Delta\vartheta_{\mathrm{F}_{\mathrm{B}_t}}(\mathcal{G}_t)\mathrm{e}^{\mathrm{i}\Delta_{\mathrm{F}\rho_{\mathrm{B}_t}}(\mathcal{G}_t)}\right)$$

$$\Delta(\mathcal{G}_t) = \begin{pmatrix} \max\left\{\mathrm{deg}_{\mathrm{T}\vartheta_{\mathrm{B}_t}}(\upsilon_1)\right\}\mathrm{e}^{\mathrm{imax}\{\mathrm{deg}_{\mathrm{T}\tau_{\mathrm{B}_t}}(\upsilon_1)\}}, \\ \max\left\{\mathrm{deg}_{\mathrm{I}\vartheta_{\mathrm{B}_t}}(\upsilon_1)\right\}\mathrm{e}^{\mathrm{imax}\{\mathrm{degz}_{\mathrm{I}\sigma_{\mathrm{B}_t}}(\upsilon_1)\}}, \\ \max\left\{\mathrm{deg}_{\mathrm{F}\vartheta_{\mathrm{B}_t}}(\upsilon_1)\right\}\mathrm{e}^{\mathrm{imax}\{\mathrm{deg}_{\mathrm{F}\rho_{\mathrm{B}_t}}(\upsilon_1)\}} \end{pmatrix}\upsilon_1 \in V$$

Example 4. *From Example 1, the degree of vertex in $\mathcal{G}_t$ is*

$$deg_{\mathcal{G}_t}(a) = \left(0.5e^{i0.7\pi}, 2.1e^{i2.2\pi}, 2.2e^{i2.6\pi}\right);\ deg_{\mathcal{G}_t}(b) = \left(0.3e^{i0.3\pi}, 1e^{i1.0\pi}, 1.3e^{i1.3\pi}\right);$$

$$deg_{\mathcal{G}_t}(c) = \left(0.3e^{i0.8\pi}, 2.2e^{i1.8\pi}, 1.9e^{i1.9\pi}\right);\ deg_{\mathcal{G}_t}(d) = \left(0.3e^{i0.6\pi}, 1.5e^{i1.4\pi}, 1.4e^{i1.6\pi}\right)$$

3. Operation on CTNG

3.1. Cartesian Product of CTNG

Definition 9. *Let $\mathcal{G}_t = (A_t, B_t)$ and $\mathcal{G}_t' = (A_t', B_t')$ be any two CTNGs of G = (V, E) and G′= (V′, E′), respectively. In the Cartesian product $\mathcal{G}_t \times \mathcal{G}_t'$ of two CTNGs, $\mathcal{G}_t$ and $\mathcal{G}_t'$ are defined by $(A_t \times A_t',\ B_t \times B_t')$ where $A_t \times A_t'$ and $B_t \times B_t'$ are CTNSs on $V \times V' = \{(\upsilon_1,\omega_1),(\upsilon_2,\omega_2) : \upsilon_1\ \&\ \upsilon_2 \in V; \omega_1\ \&\ \omega_2 \in V'\}$ and $E \times E' = \{(\upsilon_1,\omega_1),(\upsilon_2,\omega_2) : \upsilon_1 = \upsilon_2, \upsilon_1\ \&\ \upsilon_2 \in V,$ $(\omega_1,\omega_2) \in E'\}\ U\ \{(\upsilon_1,\omega_1),(\upsilon_2,\omega_2) : \omega_1 = \omega_2, \omega_1\ \&\ \omega_2 \in V', (\upsilon_1,\upsilon_2) \in E\}\ U\ \{(\upsilon_1,\omega_1),$ $(\upsilon_2,\omega_2) : \omega_1 \neq \omega_2, \upsilon_1 \neq \upsilon_2, (\omega_1,\omega_2) \in E', (\upsilon_1,\upsilon_2) \in E\}$, respectively, which satisfies the following condition:*

1. $\forall(\upsilon_1,\omega_1) \in V \times V'$
   (a) $T\vartheta_{A_t\times A_t'}(\upsilon_1,\omega_1)e^{iT\tau_{A_t\times A_t'}(\upsilon_1,\omega_1)} = min\left\{T\vartheta_{A_t}(\upsilon_1), T\vartheta_{A_{t'}}(\omega_1)\right\}e^{imin\{T\tau_{A_t}(\upsilon_1),T\tau_{A_{t'}}(\omega_1)\}}$
   (b) $I\vartheta_{A_t\times A_t'}(\upsilon_1,\omega_1)e^{iI\sigma_{A_t\times A_t'}(\upsilon_1,\omega_1)} = max\left\{I\vartheta_{A_t}(\upsilon_1), I\vartheta_{A_{t'}}(\omega_1)\right\}e^{imax\{I\sigma_{A_t}(\upsilon_1),I\sigma_{A_{t'}}(\omega_1)\}}$
   (c) $F\vartheta_{A_t\times A_t'}(\upsilon_1,\ \omega_1)e^{iF\rho_{A_t\times A_t'}(\upsilon_1,\ \omega_1)} = max\left\{F\vartheta_{A_t}(\upsilon_1),\ F\vartheta_{A_{t'}}(\omega_1)\right\}e^{i\ max\{F\rho_{A_t}(\upsilon_1),\ F\rho_{A_{t'}}(\omega_1)\}}$

2. *If* $\upsilon_1 = \upsilon_2$ *and* $\forall\ (\omega_1, \omega_2) \in E'$

(a) $$T\vartheta_{B_t \times B'_t}((\upsilon_1, \omega_1), (\upsilon_2, \omega_2))e^{iT\tau_{B_t \times B'_t}((\upsilon_1,\omega_1),(\upsilon_2,\omega_2))} = min\left\{T\vartheta_{A_t}(\upsilon_1),\ T\vartheta_{B_{t'}}(\omega_1, \omega_2)\right\}e^{i\ min\{T\tau_{A_t}(\upsilon_1),\ T\tau_{B_{t'}}(\omega_1,\ \omega_2)\}}$$

(b) $$I\vartheta_{B_t \times B'_t}((\upsilon_1, \omega_1), (\upsilon_2, \omega_2))e^{iI\sigma_{B_t \times B'_t}((\upsilon_1,\omega_1),(\upsilon_2,\omega_2))} = max\left\{I\vartheta_{A_t}(\upsilon_1),\ I\vartheta_{B_{t'}}(\omega_1, \omega_2)\right\}e^{imax\{I\sigma_{A_t}(\upsilon_1),\ I\sigma_{B_{t'}}(\omega_1,\ \omega_2)\}}$$

(c) $$F\vartheta_{B_t \times B'_t}((\upsilon_1, \omega_1), (\upsilon_2, \omega_2))e^{iF\rho_{B_t \times B'_t}((\upsilon_1,\ \omega_1),\ (\upsilon_2,\ \omega_2))} = max\left\{F\vartheta_{A_t}(\upsilon_1),\ F\vartheta_{B_{t'}}(\omega_1, \omega_2)\right\}e^{imax\{F\rho_{A_t}(\upsilon_1),\ F\rho_{B_{t'}}(\omega_1,\ \omega_2)\}}$$

3. *If* $\omega_1 = \omega_2$ *and* $\forall\ (\upsilon_1, \upsilon_2) \in E$

(a) $$T\vartheta_{B_t \times B'_t}((\upsilon_1, \omega_1), (\upsilon_2, \omega_2))e^{iT\tau_{B_t \times B'_t}((\upsilon_1,\omega_1),(\upsilon_2,\omega_2))} = min\left\{T\vartheta_{B_t}(\upsilon_1,\ \upsilon_2),\ T_{\vartheta A_{t'}}(\omega_1)\right\}e^{imin\{T\tau_{B_t}(\upsilon_1,\ \upsilon_2),\ T\tau_{A_{t'}}(\omega_1)\}}$$

(b) $$I\vartheta_{B_t \times B'_t}((\upsilon_1, \omega_1), (\upsilon_2, \omega_2))e^{i\ I\sigma_{B_t \times B'_t}((\upsilon_1,\ \omega_1),\ (\upsilon_2,\ \omega_2))} = max\left\{I\vartheta_{B_t}(\upsilon_1, \upsilon_2),\ I\vartheta_{A_{t'}}(\omega_1)\right\}e^{i\ max\{I\sigma_{B_t}(\upsilon_1,\upsilon_2),\ I\sigma_{A_{t'}}(\omega_1)\}}$$

(c) $$F\vartheta_{B_t \times B'_t}((\upsilon_1, \omega_1), (\upsilon_2, \omega_2))e^{i\ F\rho_{B_t \times B'_t}((\upsilon_1,\ \omega_1),\ (\upsilon_2,\ \omega_2))} = max\left\{F\vartheta_{B_t}(\upsilon_1, \upsilon_2), F\vartheta_{A_{t'}}(\omega_1)\right\}e^{i\ max\{F\rho_{B_t}(\upsilon_1,\ \upsilon_2),\ F\rho_{A_{t'}}(\omega_1)\}}$$

Example 5. *Figure 3a,b illustrate two* $0.4e^{i0.7\pi}$*-NG* G_t *and* G'_t*, which are the elements of consideration. The Cartesian product* $G_{0.4}e^{i0.5\pi} \times G'_{0.4}e^{i0.5\pi}$*, which corresponds to them, is seen in Figure 4.*

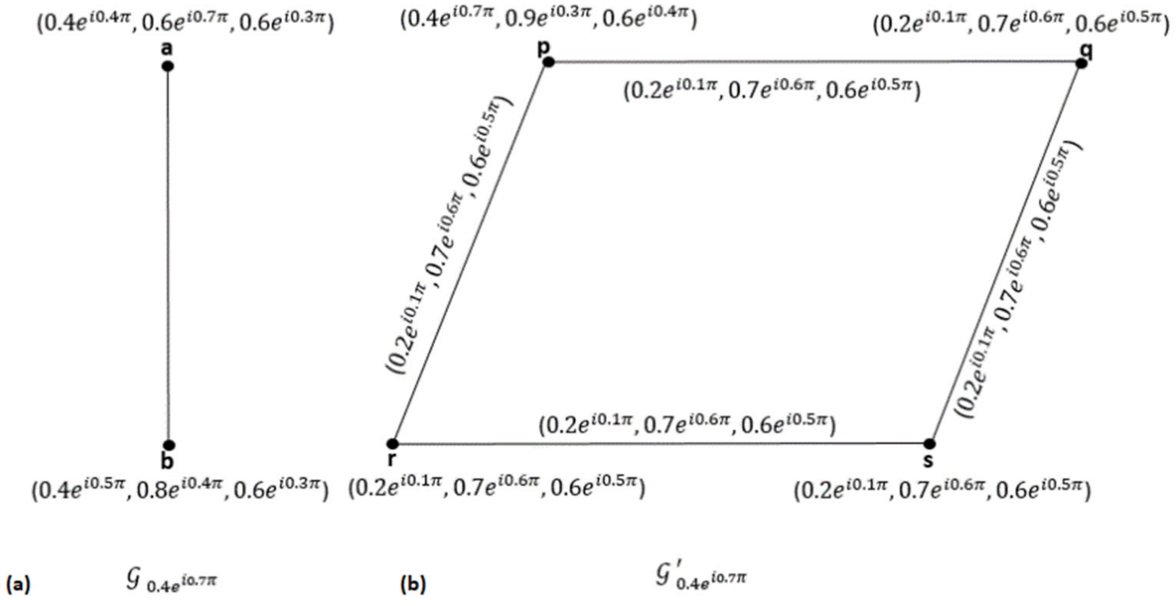

Figure 3. (**a**). $0.4e^{i0.7\pi}$-NG. (**b**). $0.4e^{i0.7\pi}$-NG.

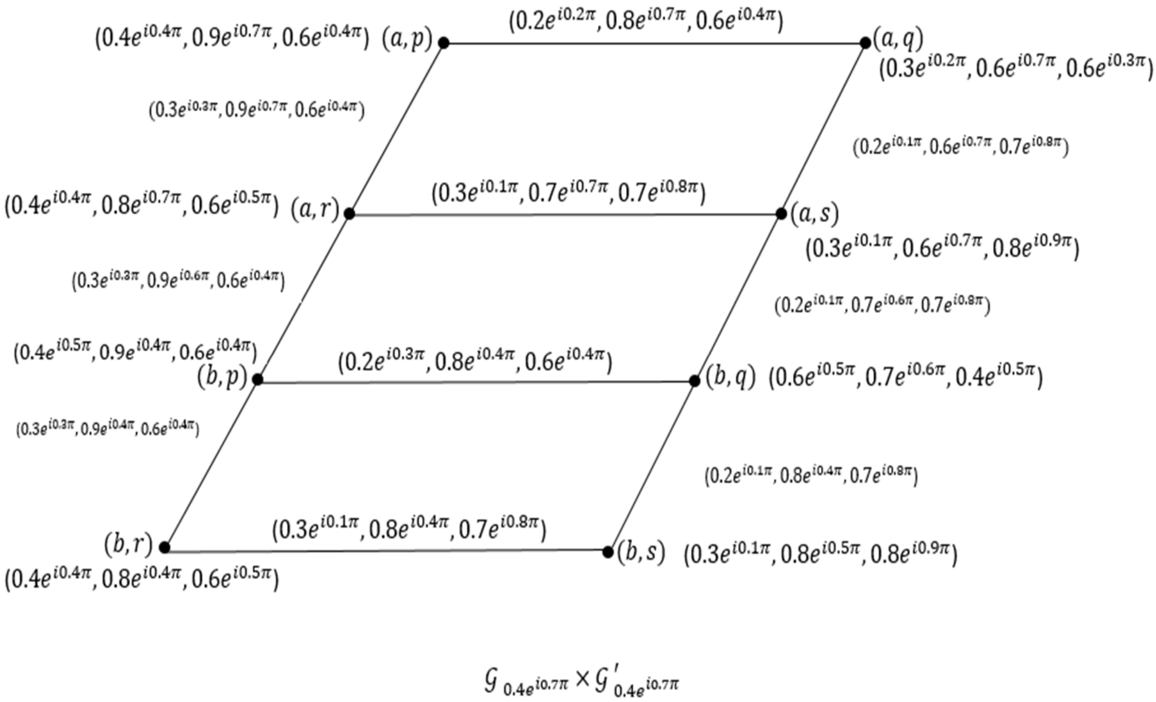

Figure 4. The corresponding Cartesian product $\mathcal{G}_{0.4e^{i0.7\pi}} \times \mathcal{G}'_{0.4e^{i0.7\pi}}$.

Definition 10. *The degree of a vertex in $\mathcal{G}_t \times \mathcal{G}'_t$ is defined as followes: for any $(\upsilon_1, \omega_1) \in V \times V'$.*

$$deg_{\mathcal{G}_t \times \mathcal{G}'_t}(\upsilon_1, \omega_1) = \begin{pmatrix} deg\left\{T\vartheta_{B_t \times B'_t}((\upsilon_1,\omega_1),(\upsilon_2,\omega_2))\right\} e^{iT\tau_{B_t \times B'_t}((\upsilon_1,\omega_1),(\upsilon_2,\omega_2))}, \\ deg\left\{I\vartheta_{B_t \times B'_t}((\upsilon_1,\omega_1),(\upsilon_2,\omega_2))\right\} e^{iI\sigma_{B_t \times B'_t}((\upsilon_1,\omega_1),(\upsilon_2,\omega_2))}, \\ deg\left\{F\vartheta_{B_t \times B'_t}((\upsilon_1,\omega_1),(\upsilon_2,\omega_2))\right\} e^{iF\rho_{B_t \times B'_t}((\upsilon_1,\omega_1),(\upsilon_2,\omega_2))} \end{pmatrix}$$

where

$$deg\left\{T\vartheta_{B_t \times B'_t}((\upsilon_1,\omega_1),(\upsilon_2,\omega_2))\right\} e^{iT\tau_{B_t \times B'_t}((\upsilon_1,\omega_1),\ (\upsilon_2,\omega_2))}$$

$$= \sum_{\upsilon_1=\upsilon_2,(\omega_1,\omega_2)\in E'} min\left\{T\vartheta_{A_t}(\upsilon_1), T\vartheta_{B_{t'}}(\omega_1,\omega_2)\right\} e^{i\sum_{\upsilon_1=\upsilon_2,(\omega_1,\omega_2)\in E'} min\{T\tau_{A_t}(\upsilon_1),T\tau_{B_{t'}}(\omega_1,\omega_2)\}}$$
$$+\sum_{\omega_1=\omega_2,(\upsilon_1,\upsilon_2)\in E} min\left\{T\vartheta_{B_t}(\upsilon_1,\upsilon_2), T\vartheta_{A_{t'}}(\omega_1)\right\} e^{i\sum_{\omega_1=\omega_2,(\upsilon_1,\upsilon_2)\in E} min\{T_{\tau B_t}(\upsilon_1,\upsilon_2),T_{\tau A_{t'}}(\omega_1)\}}$$
$$+\sum_{\omega_1\neq\omega_2,\upsilon_1\neq\upsilon_2} min\left\{T\vartheta_{B_t}(\upsilon_1,\upsilon_2), T\vartheta_{B_{t'}}(\omega_1,\omega_2)\right\} e^{i\sum_{\omega_1\neq\omega_2,\upsilon_1\neq\upsilon_2} min\{T\tau_{B_t}(\upsilon_1,\upsilon_2),T\tau_{B_{t'}}(\omega_1,\omega_2)\}},$$

$$deg\left\{I\vartheta_{B_t \times B'_t}((\upsilon_1,\omega_1),(\upsilon_2,\omega_2))\right\} e^{iI\sigma_{B_t \times B'_t}((\upsilon_1,\omega_1),(\upsilon_2,\omega_2))}$$

$$= \sum_{\upsilon_1=\upsilon_2,(\omega_1,\omega_2)\in E'} max\left\{I\vartheta_{A_t}(\upsilon_1), I\vartheta_{B_{t'}}(\omega_1,\omega_2)\right\} e^{i\sum_{\upsilon_1=\upsilon_2,(\omega_1,\omega_2)\in E'} max\{I\sigma_{A_t}(\upsilon_1),I\sigma_{B_{t'}}(\omega_1,\omega_2)\}}$$
$$+\sum_{\omega_1=\omega_2,(\upsilon_1,\upsilon_2)\in E} max\left\{I\vartheta_{B_t}(\upsilon_1,\upsilon_2), I\vartheta_{A_{t'}}(\omega_1)\right\} e^{i\sum_{\omega_1=\omega_2,(\upsilon_1,\upsilon_2)\in E} max\{I\sigma_{B_t}(\upsilon_1,\upsilon_2),\ I\sigma_{A_{t'}}(\omega_1)\}}$$
$$+\sum_{\omega_1\neq\omega_2,\upsilon_1\neq\upsilon_2} max\left\{I\vartheta_{B_t}(\upsilon_1,\upsilon_2), I\vartheta_{B_{t'}}(\omega_1,\omega_2)\right\} e^{i\sum_{\omega_1\neq\omega_2,\upsilon_1\neq\upsilon_2} max\{I\sigma_{B_t}(\upsilon_1,\upsilon_2),\ I\sigma_{B_{t'}}(\omega_1,\omega_2)\}}$$

and

$$deg\left\{F\vartheta_{B_t\times B_t'}((\upsilon_1,\omega_1),(\upsilon_2,\omega_2))\right\}e^{i\ F\rho_{B_t\times B_t'}((\upsilon_1,\omega_1),(\upsilon_2,\omega_2))}$$

$$=\sum_{\upsilon_1=\upsilon_2,(\omega_1,\omega_2)\in E'} max\left\{F\vartheta_{A_t}(\upsilon_1),F\vartheta_{B_{t'}}(\omega_1,\omega_2)\right\}e^{i\sum_{\upsilon_1=\upsilon_2,(\omega_1,\omega_2)\in E'} max\{F\rho_{A_t}(\upsilon_1),F\rho_{B_{t'}}(\omega_1,\omega_2)\}}$$

$$+\sum_{\omega_1=\omega_2,(\upsilon_1,\upsilon_2)\in E} max\left\{F\vartheta_{B_t}(\upsilon_1,\upsilon_2),F\vartheta_{A_{t'}}(\omega_1)\right\}e^{i\sum_{\omega_1=\omega_2,(\upsilon_1,\upsilon_2)\in E} max\{F\rho_{B_t}(\upsilon_1,\upsilon_2),\ F\rho_{A_{t'}}(\omega_1)\}}$$

$$+\sum_{\omega_1\neq\omega_2,\upsilon_1\neq\upsilon_2} max\left\{F\vartheta_{B_t}(\upsilon_1,\upsilon_2),F\vartheta_{B_{t'}}(\omega_1,\omega_2)\right\}e^{i\sum_{\omega_1=\omega_2,(\upsilon_1,\upsilon_2)\in E} max\{F\rho_{B_t}(\upsilon_1,\upsilon_2),F\rho_{A_{t'}}(\omega_1)\}}.$$

Example 6. *From example 5, each vertex in $\mathcal{G}_t\times\mathcal{G}_t'$ has the following degree:*

$$deg_{\mathcal{G}_t\times\mathcal{G}_t'}(a,p)=\left(0.5e^{i0.5\pi},1.7e^{i1.4\pi},1.2e^{i0.8\pi}\right),\ deg_{\mathcal{G}_t\times\mathcal{G}_t'}(a,q)=\left(0.4e^{i0.3\pi},1.4e^{i1.4\pi},1.3e^{i1.2\pi}\right)$$

$$deg_{\mathcal{G}_t\times\mathcal{G}_t'}(a,r)=\left(0.9e^{i0.7\pi},\ 2.5e^{i2.0\pi},1.9e^{i1.6\pi}\right),\ deg_{\mathcal{G}_t\times\mathcal{G}_t'}(a,s)=\left(0.7e^{i0.3\pi},2.0e^{i2.0\pi},2.1e^{i2.4\pi}\right),$$

$$deg_{\mathcal{G}_t\times\mathcal{G}_t'}(b,p)=\left(0.8e^{i0.9\pi},2.6e^{i1.4\pi},1.8e^{i1.2\pi}\right),\ deg_{\mathcal{G}_t\times\mathcal{G}_t'}(b,q)=\left(0.6e^{i0.5\pi},2.3e^{i1.4\pi},2e^{i2.0\pi}\right)$$

$$deg_{\mathcal{G}_t\times\mathcal{G}_t'}(b,r)=\left(0.5e^{i0.2\pi},\ 1.6e^{i0.8\pi},1.3e^{i1.2\pi}\right),\ deg_{\mathcal{G}_t\times\mathcal{G}_t'}(b,s)=\left(0.5e^{i0.2\pi},1.6e^{i0.8\pi},1.4e^{i1.6\pi}\right)$$

Theorem 1. *Two CTNGs are Cartesian products, and the result is another CTNG.*

Proof. For $\mathrm{A_t}\times\mathrm{A_t'}$, the condition is obvious. Assuming that $\mathrm{u_1}\in\mathrm{V}$ and $(\omega_1,\omega_2)\in\mathrm{E'}$,

$$\mathrm{T}\vartheta_{\mathrm{B_t\times B_t'}}((\upsilon_1,\omega_1),(\upsilon_1,\omega_2))\mathrm{e}^{\mathrm{iT}\tau_{\mathrm{B_t\times B_t'}}((\upsilon_1,\omega_1),(\upsilon_1,\omega_2))}$$

$$=\min\left\{\mathrm{T}\vartheta_{\mathrm{A_t}}(\upsilon_1),\mathrm{T}\vartheta_{\mathrm{B_{t'}}}(\omega_1,\omega_2)\right\}\mathrm{e}^{\mathrm{imin}\{\mathrm{T}\tau_{\mathrm{A_t}}(\mathrm{u_1}),\mathrm{T}\tau_{\mathrm{B_{t'}}}(\omega_1,\omega_2)\}}$$

$$\leq\min\left\{\mathrm{T}\vartheta_{\mathrm{A_t}}(\mathrm{u_1}),\min\left\{\mathrm{T}\vartheta_{\mathrm{A_{t'}}}(\omega_1),\mathrm{T}\vartheta_{\mathrm{A_{t'}}}(\omega_2)\right\}\right\}\mathrm{e}^{\mathrm{i}\min\{\mathrm{T}\tau_{\mathrm{A_t}}(\upsilon_1),\min\{\mathrm{T}\tau_{\mathrm{A_{t'}}}(\omega_1),\mathrm{T_{A_{t'}}}(\omega_2)\}\}}$$

$$\leq\min\left\{\min\left\{\mathrm{T}\vartheta_{\mathrm{A_t}}(\upsilon_1),\mathrm{T}\vartheta_{\mathrm{A_{t'}}}(\omega_1)\right\},\min\left\{\mathrm{T}\vartheta_{\mathrm{A_t}}(\upsilon_1),\mathrm{T}\vartheta_{\mathrm{A_{t'}}}(\omega_2)\right\}\right\}$$

$$\mathrm{e}^{\mathrm{i}\min\{\min\{\mathrm{T}\tau_{\mathrm{A_t}}(\upsilon_1),\mathrm{T}\tau_{\mathrm{A_{t'}}}(\omega_1)\},\min\{\mathrm{T}\tau_{\mathrm{A_t}}(\upsilon_1),\mathrm{T}\tau_{\mathrm{A_{t'}}}(\omega_2)\}\}}$$

$$\mathrm{T}\vartheta_{\mathrm{B_t\times B_t'}}((\upsilon_1,\omega_1),(\upsilon_2,\omega_2))\mathrm{e}^{\mathrm{iT}\tau_{\mathrm{B_t\times B_t'}}((\upsilon_1,\omega_1),(\upsilon_2,\omega_2))}$$

$$=\min\left\{\mathrm{T}\vartheta_{\mathrm{A_t\times A_t'}}(\upsilon_1,\omega_1),\mathrm{T}\vartheta_{\mathrm{A_t\times A_t'}}(\omega_1,\omega_2)\right\}\mathrm{e}^{\mathrm{imin}\{\mathrm{T}\tau_{\mathrm{A_t\times A_t'}}(\upsilon_1,\omega_1),\mathrm{T}\tau_{\mathrm{A_t\times A_t'}}(\omega_1,\omega_2)\}}$$

Consequently,

$$\mathrm{T}\vartheta_{\mathrm{B_t\times B_t'}}((\upsilon_1,\omega_1),(\upsilon_2,\omega_2))\mathrm{e}^{\mathrm{iT}\tau_{\mathrm{B_t\times B_t'}}((\upsilon_1,\omega_1),(\upsilon_2,\omega_2))}$$

$$\leq\min\left\{\mathrm{T}\vartheta_{\mathrm{A_t\times A_t'}}(\upsilon_1,\omega_1),\mathrm{T}\vartheta_{\mathrm{A_t\times A_t'}}(\omega_1,\omega_2)\right\}\mathrm{e}^{\mathrm{imin}\{\mathrm{T}\tau_{\mathrm{A_t\times A_t'}}(\upsilon_1,\omega_1),\mathrm{T}\tau_{\mathrm{A_t\times A_t'}}(\omega_1,\omega_2)\}}$$

if $\upsilon_1\in\mathrm{V}$, $(\omega_1,\omega_2)\in\mathrm{E'}$. Similarly for

$$\mathrm{I}\vartheta_{\mathrm{B_t\times B_t'}}((\upsilon_1,\omega_1),(\upsilon_2,\omega_2))\mathrm{e}^{\mathrm{iI}\sigma_{\mathrm{B_t\times B_t'}}((\upsilon_1,\omega_1),(\upsilon_2,\omega_2))}$$

$$\leq\max\left\{\mathrm{I}\vartheta_{\mathrm{A_t\times A_t'}}(\upsilon_1,\omega_1),\mathrm{I}\vartheta_{\mathrm{A_t\times A_t'}}(\omega_1,\omega_2)\right\}\mathrm{e}^{\mathrm{imax}\{\mathrm{I}\sigma_{\mathrm{A_t\times A_t'}}(\upsilon_1,\omega_1),\mathrm{I}\sigma_{\mathrm{A_t\times A_t'}}(\omega_1,\omega_2)\}}$$

if $\upsilon_1 \in V$, $(\omega_1, \omega_2) \in E'$ and

$$F\vartheta_{B_t \times B'_t}((\upsilon_1, \omega_1), (\upsilon_2, \omega_2))e^{iF\rho_{B_t \times B'_t}((\upsilon_1,\omega_1),(\upsilon_2,\omega_2))}$$
$$\leq \max\left\{F\vartheta_{A_t \times A'_t}(\upsilon_1, \omega_1), F\vartheta_{A_t \times A'_t}(\omega_1, \omega_2)\right\}e^{i\max\{F\rho_{A_t \times A'_t}(\upsilon_1,\omega_1), F\rho_{A_t \times A'_t}(\omega_1,\omega_2)\}}$$

if $\upsilon_1 \in V$, $(\omega_1, \omega_2) \in E'$. Likewise, we can demonstrate it for $w_1 \in V'$,$(\upsilon_1, \upsilon_2) \in E$. □

3.2. Composistion of CTNG

Definition 11. *In the composition $\mathcal{G}_t \circ \mathcal{G}'_t$ of two CTNGs, $\mathcal{G}_t$ and $\mathcal{G}'_t$ is a CTNG and defined as a pair $(A_t \circ A'_t,\ B_t \circ B'_t)$ where $(A_t \circ A'_t)$ and $(B_t \circ B'_t)$ are CTNSs on $V \times V' = \{(\upsilon_1, \omega_1), (\upsilon_2, \omega_2) : \upsilon_1 \ \&\ \upsilon_2 \in V; \omega_1 \ \&\ \omega_2 \in V'\}$ and $E \times E' = \{(\upsilon_1, \omega_1), (\upsilon_2, \omega_2) : \upsilon_1 = \upsilon_2, \upsilon_1 \ \&\ \upsilon_2 \in V, (\omega_1, \omega_2) \in E'\}$ U $\{(\upsilon_1, \omega_1), (\upsilon_2, \omega_2) : \omega_1 = \omega_2, \omega_1 \ \&\ \omega_2 \in V', (\upsilon_1, \upsilon_2) \in E\}$ U $\{(\upsilon_1, \omega_1), (\upsilon_2, \omega_2) : \omega_1 \neq \omega_2, \upsilon_1 \neq \upsilon_2, (\omega_1, \omega_2) \in E', (\upsilon_1, \upsilon_2) \in E\}$, respectively, which satisfies the following condition:*

1. $\forall((\upsilon_1, \omega_1) \in V \circ V'$
   (a) $T\vartheta_{A_t \circ A'_t}(\upsilon_1, \omega_1)e^{iT\tau_{A_t \circ A'_t}(\upsilon_1,\omega_1)} = min\left\{T\vartheta_{A_t}(\upsilon_1), T\vartheta_{A_{t'}}(\omega_1)\right\}e^{imin\{T\tau_{A_t}(\upsilon_1), T\tau_{A_{t'}}(\omega_1)\}}$
   (b) $I\vartheta_{A_t \circ A'_t}(\upsilon_1, \omega_1)e^{iI\sigma_{A_t \circ A'_t}(\upsilon_1,\omega_1)} = max\left\{I\vartheta_{A_t}(\upsilon_1), I\vartheta_{A_{t'}}(\omega_1)\right\}e^{imax\{I\sigma_{A_t}(\upsilon_1), I\sigma_{A_{t'}}(\omega_1)\}}$
   (c) $F\vartheta_{A_t \circ A'_t}(\upsilon_1, \omega_1)e^{iF\rho_{A_t \circ A'_t}(\upsilon_1,\omega_1)} = max\left\{F\vartheta_{A_t}(\upsilon_1), F\vartheta_{A_{t'}}(\omega_1)\right\}e^{imax\{F\rho_{A_t}(\upsilon_1), F\rho_{A_{t'}}(\omega_1)\}}$
2. *If* $\upsilon_1 = \upsilon_2$ *and* $\forall\ (\omega_1, \omega_2) \in E'$
   (i) $T\vartheta_{B_t \circ B'_t}((\upsilon_1, \omega_1), (\upsilon_2, \omega_2))e^{iT\tau_{B_t \circ B'_t}((\upsilon_1,\omega_1),(\upsilon_2,\omega_2))} = min\left\{T\vartheta_{A_t}(\upsilon_1), T\vartheta_{B_{t'}}(\omega_1, \omega_2)\right\}e^{imin\{T\tau_{A_t}(\upsilon_1), T\tau_{B_{t'}}(\omega_1,\omega_2)\}}$
   (ii) $I\vartheta_{B_t \circ B'_t}((\upsilon_1, \omega_1), (\upsilon_2, \omega_2))e^{i\ I\sigma_{B_t \circ B'_t}((\upsilon_1,\ \omega_1),\ (\upsilon_2,\ \omega_2))} = max\left\{I\vartheta_{A_t}(\upsilon_1), I\vartheta_{B_{t'}}(\omega_1, \omega_2)\right\}e^{imax\{I\sigma_{A_t}(\upsilon_1),\ I\rho_{B_{t'}}(\omega_1,\ \omega_2)\}}$
   (iii) $F\vartheta_{B_t \circ B'_t}((\upsilon_1, \omega_1), (\upsilon_2, \omega_2))e^{i\ F\rho_{B_t \circ B'_t}((\upsilon_1,\ \omega_1),\ (\upsilon_2,\ \omega_2))} = max\left\{F\vartheta_{A_t}(\upsilon_1), F\vartheta_{B_{t'}}(\omega_1, \omega_2)\right\}e^{imax\{F\rho_{A_t}(\upsilon_1),\ F\rho_{B_{t'}}(\omega_1,\ \omega_2)\}}$
3. *If* $\omega_1 = \omega_2$ *and* $\forall\ (\upsilon_1, \upsilon_2) \in E$
   (a) $T\vartheta_{B_t \circ B'_t}((\upsilon_1, \omega_1), (\upsilon_2, \omega_2))eiT\tau Bt \circ Bt'\upsilon1, \omega1, \upsilon2, \omega2 = min\left\{T\vartheta_{B_t}(\upsilon_1, \upsilon_2), T\vartheta_{A_{t'}}(\omega_1)\right\}e^{imin\{T\tau_{B_t}(\upsilon_1,\upsilon_2), T\tau_{A_{t'}}(\omega_1)\}}$
   (b) $I\vartheta_{B_t \circ B'_t}((\upsilon_1, \omega_1), (\upsilon_2, \omega_2))e^{iI\sigma_{B_t \circ B'_t}((\upsilon_1,\ \omega_1),\ (\upsilon_2,\ \omega_2))} = max\left\{I\vartheta_{B_t}(\upsilon_1, \upsilon_2), I\vartheta_{A_{t'}}(\omega_1)\right\}e^{i\ max\{I\sigma_{B_t}(\upsilon_1,\ \upsilon_2),\ I\sigma_{A_{t'}}(\omega_1)\}}$
   (c) $F\vartheta_{B_t \circ B'_t}((\upsilon_1, \omega_1), (\upsilon_2, \omega_2))e^{iF\rho_{B_t \circ B'_t}((\upsilon_1,\ \omega_1),\ (\upsilon_2,\ \omega_2))} = max\left\{F\vartheta_{B_t}(\upsilon_1, \upsilon_2), F\vartheta_{A_{t'}}(\omega_1)\right\}e^{i\ max\{F\rho_{B_t}(\upsilon_1,\upsilon_2),\ F\rho_{A_{t'}}(\omega_1)\}}$
4. *If* $\omega_1 \neq \omega_2$ *and* $\forall\ (\upsilon_1, \upsilon_2) \in E$
   (a) $T\vartheta_{B_t \circ B'_t}((\upsilon_1, \omega_1), (\upsilon_2, \omega_2))e^{iT\tau_{B_t \circ B'_t}((\upsilon_1,\omega_1),(\upsilon_2,\omega_2))} = min\left\{T\vartheta_{B_t}(\upsilon_1, \upsilon_2), T\vartheta_{A_{t'}}(\omega_1), T\vartheta_{A_{t'}}(\omega_2)\right\}e^{min\{T\tau_{B_t}(\upsilon_1,\ \upsilon_2),\ T\tau_{A_{t'}}(\omega_1),\ T\tau_{A_{t'}}(\omega_2)\}}$
   (b) $I\vartheta_{B_t \circ B'_t}((\upsilon_1, \omega_1), (\upsilon_2, \omega_2))e^{i\ I\sigma_{B_t \circ B'_t}((\upsilon_1,\ \omega_1),\ (\upsilon_2,\ \omega_2))} = max\left\{I\vartheta_{B_t}(\upsilon_1, \upsilon_2), I\vartheta_{A_{t'}}(\omega_1), I\vartheta_{A_{t'}}(\omega_2)\right\}e^{i\ max\{I\sigma_{B_t}(\upsilon_1,\upsilon_2),\ I\sigma_{A_{t'}}(\omega_1),\ I\sigma_{A_{t'}}(\omega_2)\}}$
   (c) $F\vartheta_{B_t \circ B'_t}((\upsilon_1, \omega_1), (\upsilon_2, \omega_2))e^{i\ F\rho_{B_t \circ B'_t}((\upsilon_1,\ \omega_1),\ (\upsilon_2,\ \omega_2))} = max\left\{F\vartheta_{B_t}(\upsilon_1, \upsilon_2), F\vartheta_{A_{t'}}(\omega_1), F\vartheta_{A_{t'}}(\omega_2)\right\}e^{i\ max\{F\rho_{B_t}(\upsilon_1,\upsilon_2),\ F\rho_{A_{t'}}(\omega_1),\ F\rho_{A_{t'}}(\omega_2)\}}$

Example 7. *Consider the two* $0.6e^{i0.5\pi}$ *-CTNGs* $\mathcal{G}_t$ *and* $\mathcal{G}'_t$ *illustrated in Figure 5a,b and then their corresponding composition* $\mathcal{G}_t \circ \mathcal{G}'_t$ *see in Figure 6.*

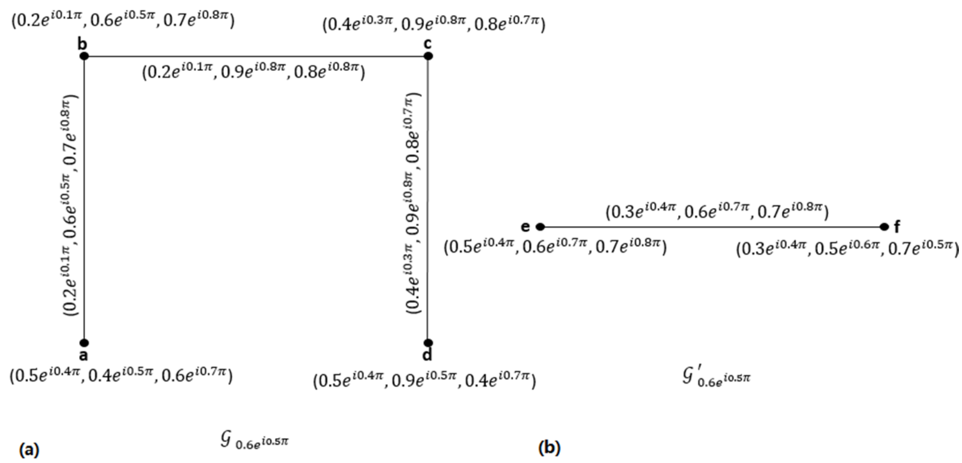

Figure 5. (**a**). $0.6e^{i0.5\pi}$-$\mathcal{G}_t$. (**b**). $0.6e^{i0.5\pi}$-$\mathcal{G}'_t$.

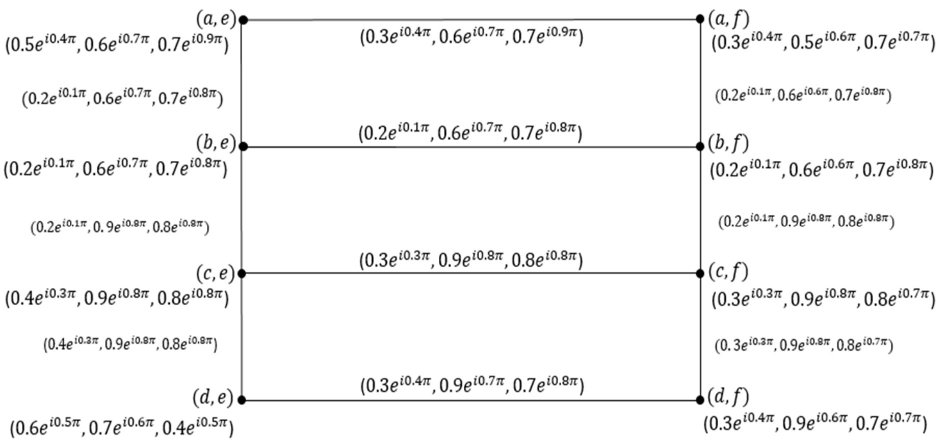

Figure 6. $\mathcal{G}_{0.6e^{i0.5\pi}} \circ \mathcal{G}'_{0.6e^{i0.5\pi}}$.

Definition 12. *The following defines the degree of vertex in* $\mathcal{G}_t \circ \mathcal{G}'_t$, *for any*

$$(\upsilon_1, \omega_1) \in V \times V'; deg_{\mathcal{G}_t \circ \mathcal{G}'_t}(\upsilon_1, \omega_1) = \begin{pmatrix} deg\left\{T\vartheta_{B_t \circ B'_t}((\upsilon_1, \omega_1), (\upsilon_2, \omega_2))\right\} e^{i\{T\tau_{B_t \circ B'_t}((\upsilon_1,\omega_1),(\upsilon_2,\omega_2))\}}, \\ deg\left\{I\vartheta_{B_t \circ B'_t}((\upsilon_1, \omega_1), (\upsilon_2, \omega_2))\right\} e^{I\sigma_{B_t \circ B'_t}((\upsilon_1,\omega_1),(\upsilon_2,\omega_2))}, \\ deg\left\{F\vartheta_{B_t \circ B'_t}((\upsilon_1, \omega_1), (\upsilon_2, \omega_2))\right\} e^{i\, F\rho_{B_t \circ B'_t}((\upsilon_1,\omega_1),(\upsilon_2,\omega_2))}. \end{pmatrix}$$

where

$$deg\left\{T\vartheta_{B_t \circ B'_t}((\upsilon_1, \omega_1), (\upsilon_2, \omega_2))\right\} e^{i\{T\tau_{B_t \circ B'_t}((\upsilon_1,\omega_1),(\upsilon_2,\omega_2))\}}$$

$$= \sum\nolimits_{\upsilon_1 = \upsilon_2, (\omega_1, \omega_2) \in E'} min\left\{T\vartheta_{A_t}(\upsilon_1), T\vartheta_{B_{t'}}(\omega_1, \omega_2)\right\} e^{i\sum_{\upsilon_1=\upsilon_2,(\omega_1,\omega_2)\in E'} min\{T\tau_{A_t}(\upsilon_1), T\tau_{B_{t'}}(\omega_1,\omega_2)\}} +$$

$$\sum\nolimits_{\omega_1 = \omega_2, (\upsilon_1, \upsilon_2) \in E} min\left\{T\vartheta_{B_t}(\upsilon_1, \upsilon_2), T\vartheta_{A_{t'}}(\omega_1)\right\} e^{i\sum_{\omega_1=\omega_2,(\upsilon_1,\upsilon_2)\in E} min\{T\tau_{B_t}(\upsilon_1,\upsilon_2),\ T\tau_{A_{t'}}(\omega_1)\}} +$$

$$\sum\nolimits_{\omega_1 \neq \omega_2, (\upsilon_1, \upsilon_2) \in E} min\left\{T\vartheta_{B_t}(\upsilon_1, \upsilon_2), T\vartheta_{A_{t'}}(\omega_1), T\vartheta_{A_{t'}}(\omega_2)\right\} e^{i\sum_{\omega_1\neq\omega_2,(\upsilon_1,\upsilon_2)\in E} min\{T\tau_{B_t}(\upsilon_1,\upsilon_2),\ T\tau_{A_{t'}}(\omega_1), T\tau_{A_{t'}}(\omega_2)\}}$$

$$deg\left\{I\vartheta_{B_t\circ B_t'}((\upsilon_1,\omega_1),(\upsilon_2,\omega_2))\right\}e^{i\{I\sigma_{B_t\circ B_t'}((\upsilon_1,\omega_1),(\upsilon_2,\omega_2))\}} =$$

$$\sum\nolimits_{\upsilon_1=\upsilon_2,(\omega_1,\omega_2)\in E'} max\left\{I\vartheta_{A_t}(\upsilon_1), I\vartheta_{B_{t'}}(\omega_1,\omega_2)\right\}e^{i\sum_{\upsilon_1=\upsilon_2,(\omega_1,\omega_2)\in E'} max\{I\sigma(\upsilon_1),\ I\sigma_{B_{t'}}(\omega_1,\omega_2)\}} +$$

$$\sum\nolimits_{\omega_1=\omega_2,(\upsilon_1,\upsilon_2)\in E} max\left\{I\vartheta_{B_t}(\upsilon_1,\upsilon_2), I\vartheta_{A_{t'}}(\omega_1)\right\}e^{i\sum_{\omega_1=\omega_2,(\upsilon_1,\upsilon_2)\in E} max\{I\sigma_{B_t}(\upsilon_1,\upsilon_2),I\sigma_{A_{t'}}(\omega_1)\}} +$$

$$\sum\nolimits_{\omega_1\neq\omega_2,(\upsilon_1,\upsilon_2)\in E} max\left\{I\vartheta_{B_t}(\upsilon_1,\upsilon_2), I\vartheta_{A_{t'}}(\omega_1), I\vartheta_{A_{t'}}(\omega_2)\right\}e^{i\sum_{\omega_1\neq\omega_2,(\upsilon_1,\upsilon_2)\in E} max\{I\sigma_{B_t}(\upsilon_1,\upsilon_2),\ I\sigma_{A_{t'}}(\omega_1),\ I\sigma_{A_{t'}}(\omega_2)\}}$$

and

$$deg\left\{F\vartheta_{B_t\circ B_t'}((\upsilon_1,\omega_1),(\upsilon_2,\omega_2))\right\}e^{i\{I\rho_{B_t\circ B_t'}((\upsilon_1,\omega_1),(\upsilon_2,\omega_2))\}}$$

$$= \sum_{\upsilon_1=\upsilon_2,(\omega_1,\omega_2)\in E'} max\left\{F\vartheta_{A_t}(\upsilon_1), F\vartheta_{B_{t'}}(\omega_1,\omega_2)\right\}e^{i\sum_{\upsilon_1=\upsilon_2,(\omega_1,\omega_2)\in E'} max\{F\rho(\upsilon_1),\ F\rho_{B_{t'}}(\omega_1,\omega_2)\}}$$

$$+ \sum_{\omega_1=\omega_2,(\upsilon_1,\upsilon_2)\in E} max\left\{F\vartheta_{B_t}(\upsilon_1,\upsilon_2), F\vartheta_{A_{t'}}(\omega_1)\right\}e^{i\sum_{\omega_1=\omega_2,(\upsilon_1,\upsilon_2)\in E} max\{F\rho_{B_t}(\upsilon_1,\upsilon_2),\ F\rho_{A_{t'}}(\omega_1)\}}$$

$$+ \sum_{\omega_1\neq\omega_2,(\upsilon_1,\upsilon_2)\in E} max\left\{F\vartheta_{B_t}(\upsilon_1,\upsilon_2), F\vartheta_{A_{t'}}(\omega_1), F\vartheta_{A_{t'}}(\omega_2)\right\}e^{i\sum_{\omega_1\neq\omega_2,(\upsilon_1,\upsilon_2)\in E} max\{F\rho_{B_t}(\upsilon_1,\upsilon_2),F\rho_{A_{t'}}(\omega_1),\ F\rho_{A_{t'}}(\omega_2)\}}$$

Example 8. *From Example 7 the degree of each vertices in $\mathcal{G}_t\circ\mathcal{G}_t'$ are*

$$deg_{\mathcal{G}_t\circ\mathcal{G}_t'}(a,e) = \left(0.5e^{i0.5\pi}, 1.2e^{i1.4\pi}, 1.4e^{i1.7\pi}\right), deg_{\mathcal{G}_t\circ\mathcal{G}_t'}(a,f) = \left(0.5e^{i0.5\pi}, 1.2e^{i1.3\pi}, 1.4e^{i1.7\pi}\right),$$

$$deg_{\mathcal{G}_t\circ\mathcal{G}_t'}(b,e) = \left(0.6e^{i0.3\pi},\ 2.1e^{i2.2\pi}, 2.2e^{i2.4\pi}\right),\ deg_{\mathcal{G}_t\circ\mathcal{G}_t'}(b,f) = \left(0.6e^{i0.3\pi}, 2.1e^{i2.1\pi}, 2.2e^{i2.4\pi}\right),$$

$$deg_{\mathcal{G}_t\circ\mathcal{G}_t'}(c,e) = \left(0.9e^{i0.7\pi}, 2.7e^{i2.4\pi}, 1.8e^{i1.2\pi}\right),\ deg_{\mathcal{G}_t\circ\mathcal{G}_t'}(c,f) = \left(0.8e^{i0.7\pi}, 2.7e^{i2.4\pi}, 2.4e^{i2.3\pi}\right)$$

$$deg_{\mathcal{G}_t\circ\mathcal{G}_t'}(d,e) = \left(0.9e^{i0.9\pi},\ 1.6e^{i1.8\pi}, 1.1e^{i1.3\pi}\right),\ deg_{\mathcal{G}_t\circ\mathcal{G}_t'}(d,f) = \left(0.6e^{i0.8\pi}, 1.8e^{i1.3\pi}, 1.4e^{i1.5\pi}\right)$$

3.3. Union of CTNG

Definition 13. *Let $G = (V, E)$ and $G' = (V', E')$ be any two CTNGs, such that $\mathcal{G}_t = (A_t, B_t)$, and $\mathcal{G}_t' = (A_t', B_t')$. The union $\mathcal{G}_t \cup \mathcal{G}_t'$ of these two CTNGs is defined, under certain assumptions, as $(A_t \cup A_t', B_t \cup B_t')$, where $A_t \cup A_t'$ and $B_t \cup B_t'$, respectively, represent CTNSs on $V \cup V'$ and $E \cup E'$, which satisfies the following condition:*

(1) *If $\upsilon_1 \in V$ and $\upsilon_1 \notin V'$*
   (a) $T\vartheta_{A_t\cup A_t'}(\upsilon_1)e^{iT\tau_{A_t\cup A_t'}(\upsilon_1)} = T\vartheta_{A_t}(\upsilon_1)e^{iT\tau_{A_t}(\upsilon_1)}$
   (b) $I\vartheta_{A_t\cup A_t'}(\upsilon_1)e^{iI\sigma_{A_t\cup A_t'}(\upsilon_1)} = I\vartheta_{A_t}(\upsilon_1)e^{iI\sigma_{A_t}(\upsilon_1)}$
   (c) $F\vartheta_{A_t\cup A_t'}(\upsilon_1)e^{iF\rho_{A_t\cup A_t'}(\upsilon_1)} = F\vartheta_{A_t}(\upsilon_1)e^{iF\rho_{A_t}(\upsilon_1)}$

(2) *If $\upsilon_1 \notin V$ and $\upsilon_1 \in V'$*
   (a) $T\vartheta_{A_t\cup A_t'}(\upsilon_1)e^{iT\tau_{A_t\cup A_t'}(\upsilon_1)} = T\vartheta_{A_t'}(\upsilon_1)e^{iT\tau_{A_t'}(\upsilon_1)}$
   (b) $I\vartheta_{A_t\cup A_t'}(\upsilon_1)e^{iI\sigma_{A_t\cup A_t'}(\upsilon_1)} = I\vartheta_{A_t'}(\upsilon_1)e^{iI\sigma_{A_t'}(\upsilon_1)}$
   (c) $F\vartheta_{A_t\cup A_t'}(\upsilon_1)e^{iF\rho_{A_t\cup A_t'}(\upsilon_1)} = F\vartheta_{A_t'}(\upsilon_1)e^{iF\rho_{A_t'}(\upsilon_1)}$

(3) *If $\upsilon_1 \in V \cap V'$*
   (a) $T\vartheta_{A_t\cup A_t'}(\upsilon_1)e^{iT\tau_{A_t\cup A_t'}(\upsilon_1)} = max\left\{T\vartheta_{A_t}(\upsilon_1), T\vartheta_{A_t'}(\upsilon_1)\right\}e^{imax\{T\tau_{A_t}(\upsilon_1),T\tau_{A_t'}(\upsilon_1)\}}$
   (b) $I\vartheta_{A_t\cup A_t'}(\upsilon_1)e^{ie^{iI\sigma_{A_t\cup A_t'}(\upsilon_1)}} = min\left\{I\vartheta_{A_t}(\upsilon_1), I\vartheta_{A_t'}(\upsilon_1)\right\}e^{imin\{I\sigma_{A_t}(\upsilon_1),I\sigma_{A_t'}(\upsilon_1)\}}$

(c) $F\vartheta_{A_t \cup A'_t}(\upsilon_1) e^{iF\rho_{A_t \cup A'_t}(\upsilon_1)} = min\left\{F\vartheta_{A_t}(\upsilon_1), F\vartheta_{A'_t}(\upsilon_1)\right\} e^{imin\{F\rho_{A_t}(\upsilon_1), F\rho_{A'_t}(\upsilon_1)\}}$

(4) *If* $(\upsilon_1, \omega_1) \in E$ *and* $(\upsilon_1, \omega_1) \notin E'$

(a) $T\vartheta_{B_t \cup B'_t}(\upsilon_1, \omega_1) e^{iT\tau_{B_t \cup B'_t}(\upsilon_1, \omega_1)} = T\vartheta_{B_t}(\upsilon_1, \omega_1) e^{iT\tau_{B_t}(\upsilon_1, \omega_1)}$

(b) $I\vartheta_{B_t \cup B'_t}(\upsilon_1, \omega_1) e^{iI\sigma_{B_t \cup B'_t}(\upsilon_1, \omega_1)} = I\vartheta_{B_t}(\upsilon_1, \omega_1) e^{iI\sigma_{B_t}(\upsilon_1, \omega_1)}$

(c) $F\vartheta_{B_t \cup B'_t}(\upsilon_1, \omega_1) e^{iF\rho_{B_t \cup B'_t}(\upsilon_1, \omega_1)} = F\vartheta_{B_t}(\upsilon_1, \omega_1) e^{iF\rho_{B_t}(\upsilon_1, \omega_1)}$

(5) *If* $(\upsilon_1, \omega_1) \notin E$ *and* $(\upsilon_1, \omega_1) \in E'$

(a) $T\vartheta_{B_t \cup B'_t}(\upsilon_1, \omega_1) e^{iT\tau_{B_t \cup B'_t}(\upsilon_1, \omega_1)} = T\vartheta_{B'_t}(\upsilon_1, \omega_1) e^{iT\tau_{B'_t}(\upsilon_1, \omega_1)}$

(b) $I\vartheta_{B_t \cup B'_t}(\upsilon_1, \omega_1) e^{iI\sigma_{B_t \cup B'_t}(\upsilon_1, \omega_1)} = I\vartheta_{B'_t}(\upsilon_1, \omega_1) e^{iI\sigma_{B'_t}(\upsilon_1, \omega_1)}$

(c) $F\vartheta_{B_t \cup B'_t}(\upsilon_1, \omega_1) e^{iF\rho_{B_t \cup B'_t}(\upsilon_1, \omega_1)} = F\vartheta_{B'_t}(\upsilon_1, \omega_1) e^{iF\rho_{B'_t}(\upsilon_1, \omega_1)}$

(6) *If* $(\upsilon_1, \omega_1) \in E \cap E'$

(a) $T\vartheta_{B_t \cup B'_t}(\upsilon_1, \omega_1) e^{iT\tau_{B_t \cup B'_t}(\upsilon_1, \omega_1)}$
$= max\left\{T\vartheta_{B_t}(\upsilon_1, \omega_1), T\vartheta_{B'_t}(\upsilon_1, \omega_1)\right\} e^{max\{T\tau_{B_t}(\upsilon_1, \omega_1), T\tau_{B'_t}(\upsilon_1, \omega_1)\}}$

(b) $I\vartheta_{B_t \cup B'_t}(\upsilon_1, \omega_1) e^{iI\sigma_{B_t \cup B'_t}(\upsilon_1, \omega_1)}$
$= min\left\{I\vartheta_{B_t}(\upsilon_1, \omega_1), I\vartheta_{B'_t}(\upsilon_1, \omega_1)\right\} e^{min\{I\sigma_{B_t}(\upsilon_1, \omega_1), I\sigma_{B'_t}(\upsilon_1, \omega_1)\}}$

(c) $F\vartheta_{B_t \cup B'_t}(\upsilon_1, \omega_1) e^{iF\rho_{B_t \cup B'_t}(\upsilon_1, \omega_1)}$
$= min\left\{F\vartheta_{B_t}(\upsilon_1, \omega_1), F\vartheta_{B'_t}(\upsilon_1, \omega_1)\right\} e^{min\{F\rho_{B_t}(\upsilon_1, \omega_1), F\rho_{B'_t}(\upsilon_1, \omega_1)\}}$

Example 9. *Consider the two* $0.7e^{i0.6\pi}$*-CTNGs* $\mathcal{G}_t$ *and* $\mathcal{G}'_t$ *shown in Figure 7a,b.*

Figure 8 *depicts the graphical representation of the union:* $\mathcal{G}_{0.7} \cup \mathcal{G}'_{0.7}$ *of two 0.7-CTNGs* $\mathcal{G}_{0.7}$ *and* $\mathcal{G}'_{0.7}$.

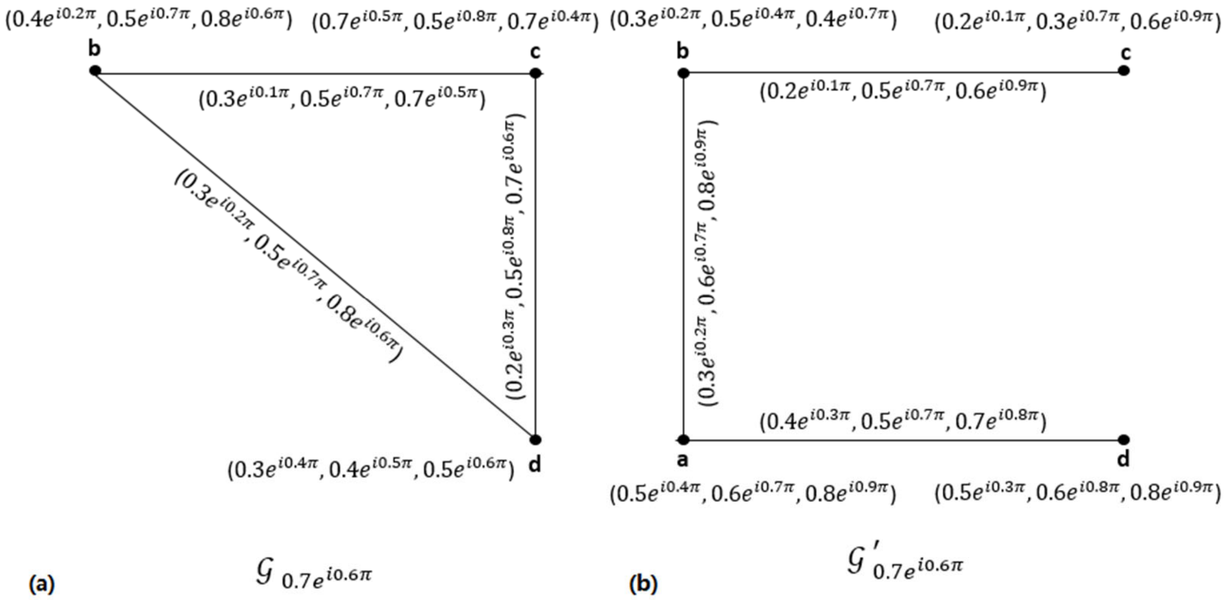

Figure 7. (**a**). $0.7e^{i0.6\pi} - NG\mathcal{G}_{0.7e^{i0.6\pi}}$. (**b**). $0.7e^{i0.6\pi} - NG\mathcal{G}'_{0.7e^{i0.6\pi}}$.

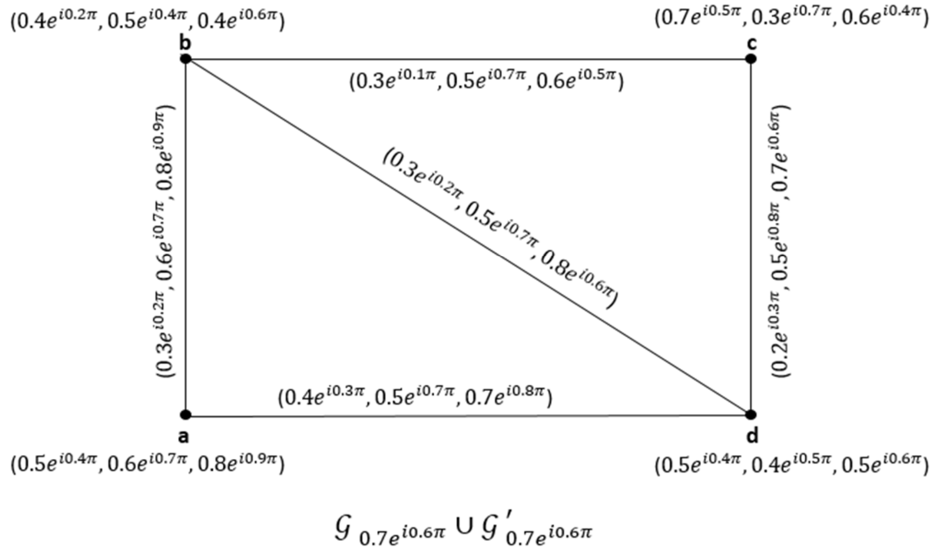

Figure 8. $\mathcal{G}_{0.7} \cup \mathcal{G}'_{0.7}$.

Definition 14. *The degree of vertex* (υ_1, ω_1) *at a CTNG for any* $(\upsilon_1, \omega_1) \in V \times V'$

$$deg_{\mathcal{G}_t \cup \mathcal{G}'_t}(\upsilon_1, \omega_1) = \begin{pmatrix} deg\left\{T\vartheta_{B_t \cup B'_t}(\upsilon_1, \omega_1)\right\}e^{i\ \{T\tau_{B_t \cup B'_t}(\upsilon_1, \omega_1)\}}, \\ deg\left\{I\vartheta_{B_t \cup B'_t}(\upsilon_1, \omega_1)\right\}e^{i\ \{I\sigma_{B_t \cup B'_t}(\upsilon_1, \omega_1)\}}, \\ deg\left\{F\vartheta_{B_t \cup B'_t}(\upsilon_1, \omega_1)\right\}e^{i\ \{F\rho_{B_t \cup B'_t}(\upsilon_1,\ \omega_1)\}} \end{pmatrix}$$

where

$$deg\left\{T\vartheta_{B_t \cup B'_t}(\upsilon_1, \omega_1)e^{i\ T\tau_{B_t \cup B'_t}(\upsilon_1, \omega_1)}\right\}$$

$$= \sum_{(\upsilon_1,\ \omega_1) \in E\ ,(\upsilon_1,\ \omega_1) \notin E'} T\vartheta_{B_t}(\upsilon_1, \omega_1)e^{i\sum_{(\upsilon_1,\ \ \omega_1) \in E\ ,(\upsilon_1,\ \ \omega_1) \notin E'} T\tau_{B_t}(\upsilon_1, \omega_1)}$$

$$+ \sum_{(\upsilon_1,\ \omega_1) \notin E, (\upsilon_1, \omega_1) \in E'} T\vartheta_{B'_t}(\upsilon_1, \omega_1)e^{i\sum_{(\upsilon_1, \omega_1) \notin E, (\upsilon_1, \omega_1) \in E'} T\tau_{B'_t}(\upsilon_1, \omega_1)}$$

$$+\sum_{(\upsilon_1,\ \omega_1) \in E \cap E'} max\left\{T\vartheta_{B_t}(\upsilon_1, \omega_1), T\vartheta_{B'_t}(\upsilon_1, \omega_1)\right\}e^{\sum_{(\upsilon_1, \omega_1) \in E \cap E'} max\{T\tau_{B_t}(\upsilon_1, \omega_1),\ T\tau_{B'_t}(\upsilon_1, \omega_1)\}},$$

$$deg\left\{I\vartheta_{B_t \cup B'_t}(\upsilon_1, \omega_1)e^{i\ I\sigma_{B_t \cup B'_t}(\upsilon_1, \omega_1)}\right\}$$

$$= \sum_{(\upsilon_1,\ \omega_1) \in E\ ,(\upsilon_1,\ \omega_1) \notin E'} I\vartheta_{B_t}(\upsilon_1, \omega_1)e^{i\sum_{(\upsilon_1, \omega_1) \in E\ ,(\upsilon_1, \omega_1) \notin E'} I\sigma_{B_t}(\upsilon_1,\ \omega_1)}$$

$$+\sum_{(\upsilon_1, \omega_1) \notin E, (\upsilon_1,\ \omega_1) \in E'} I\vartheta_{B'_t}(\upsilon_1, \omega_1)e^{i\sum_{(\upsilon_1, \omega_1) \notin E, (\upsilon_1, \omega_1) \in E'} I\sigma_{B'_t}(\upsilon_1,\ \omega_1)}$$

$$+\sum_{(\upsilon_1, \omega_1) \in E \cap E'} min\left\{I\vartheta_{B_t}(\upsilon_1, \omega_1), I\vartheta_{B'_t}(\upsilon_1, \omega_1)\right\}e^{\sum_{(\upsilon_1, \omega_1) \in E \cap E'} min\{I\sigma_{B_t}(\upsilon_1, \omega_1),\ I\sigma_{B'_t}(\upsilon_1,\ \omega_1)\}}$$

and

$$deg\left\{F\vartheta_{B_t\cup B_t'}(\upsilon_1,\omega_1)e^{i\ F\rho_{B_t\cup B_t'}(\upsilon_1,\omega_1)}\right\}$$

$$=\sum\nolimits_{(\upsilon_1,\omega_1)\in E\ ,(\upsilon_1,\omega_1)\notin E'}\ F\vartheta_{B_t}(\upsilon_1,\omega_1)e^{i\sum_{(\upsilon_1,\ \ \omega_1)\in E\ ,(\upsilon_1,\ \ \omega_1)\notin E'}\ F\rho_{B_t}(\upsilon_1,\ \omega_1)}$$

$$+\sum\nolimits_{\ (\upsilon_1,\omega_1)\notin E,(\upsilon_1,\omega_1)\in E'}\ F\vartheta_{B_t'}(\upsilon_1,\omega_1)e^{i\sum_{\ (\upsilon_1,\omega_1)\notin E,(\upsilon_1,\ \ \omega_1)\in E'}\ F\rho_{B_t'}(\upsilon_1,\ \omega_1)}$$

$$+\sum\nolimits_{\ (\upsilon_1,\omega_1)\in E\cap E'}\ min\left\{F\vartheta_{B_t}(\upsilon_1,\omega_1),F\vartheta_{B_t'}(\upsilon_1,\omega_1)\right\}e^{\sum_{\ (\upsilon_1,\omega_1)\in E\cap E'}\ \ min\{F\rho_{B_t}(\upsilon_1,\omega_1),\ F\rho_{B_t'}(\upsilon_1,\omega_1)\}}$$

3.4. Joining of CTNGs

Definition 15. *Consider two CTNGs $\mathcal{G}_t=(A_t,B_t)$ and $\mathcal{G}_t'=(A_t',B_t')$. These CTNGs' joining operation $\mathcal{G}_t+\mathcal{G}_t'$ is described as (A_t+A_t',B_t+B_t'), where A_t+A_t' produces a CTNG on $V\cup V'$ and B_t+B_t' forms a CTNG on $E\cup E'\cup E''$, subject to certain requirements.*

(1) *If $\upsilon_1\in V$ and $\upsilon_1\notin V'$*

(a) $T\vartheta_{A_t+A_t'}(\upsilon_1)e^{iT\tau_{A_t+A_t'}(\upsilon_1)}=T\vartheta_{A_t}(\upsilon_1)e^{iT\tau_{A_t}(\upsilon_1)}$

(b) $I\vartheta_{A_t+A_t'}(\upsilon_1)e^{iI\sigma_{A_t+A_t'}(\upsilon_1)}=I\vartheta_{A_t}(\upsilon_1)e^{iI\sigma_{A_t}(\upsilon_1)}$

(c) $F\vartheta_{A_t+A_t'}(\upsilon_1)e^{iF\rho_{A_t+A_t'}(\upsilon_1)}=F\vartheta_{A_t}(\upsilon_1)e^{iF\rho(\upsilon_1)}$

(2) *If $\upsilon_1\notin V$ and $\upsilon_1\in V'$*

(a) $T\vartheta_{A_t+A_t'}(\upsilon_1)e^{iT\tau_{A_t+A_t'}(\upsilon_1)}=T\vartheta_{A_t'}(\upsilon_1)e^{iT\tau_{A_t'}(\upsilon_1)}$

(b) $I\vartheta_{A_t+A_t'}(\upsilon_1)e^{iI\sigma_{A_t+A_t'}(\upsilon_1)}=I\vartheta_{A_t'}(\upsilon_1)e^{iI\sigma_{A_t'}(\upsilon_1)}$

(c) $F\vartheta_{A_t+A_t'}(\upsilon_1)e^{iF\rho_{A_t+A_t'}(\upsilon_1)}=F\vartheta_{A_t'}(\upsilon_1)e^{iF\rho_{A_t'}(\upsilon_1)}$

(3) *If $\upsilon_1\in V\cap V'$*

(a) $T\vartheta_{A_t+A_t'}(\upsilon_1)e^{iT\tau_{A_t+A_t'}(\upsilon_1)}=max\left\{T\vartheta_{A_t}(\upsilon_1),T\vartheta_{A_t'}(\upsilon_1)\right\}e^{imax\{T\tau_{A_t}(\upsilon_1),T\tau_{A_t'}(\upsilon_1)\}}$

(b) $I\vartheta_{A_t+A_t'}(\upsilon_1)e^{iI\sigma_{A_t+A_t'}(\upsilon_1)}=min\left\{I\vartheta_{A_t}(\upsilon_1),I\vartheta_{A_t'}(\upsilon_1)\right\}e^{imin\{T\sigma_{A_t}(\upsilon_1),T\sigma_{A_t'}(\upsilon_1)\}}$

(c) $F\vartheta_{A_t+A_t'}(\upsilon_1)e^{iF\rho_{A_t+A_t'}(\upsilon_1)}=min\left\{F\vartheta_{A_t}(\upsilon_1),F\vartheta_{A_t'}(\upsilon_1)\right\}e^{imin\{F\rho_{A_t}(\upsilon_1),F\rho_{A_t'}(\upsilon_1)\}}$

(4) *If $(\upsilon_1,\omega_1)\in E$ and $(\upsilon_1,\omega_1)\notin E'$*

(a) $T\vartheta_{B_t+B_t'}(\upsilon_1,\omega_1)e^{iT\tau_{B_t+B_t'}(\upsilon_1,\omega_1)}=T\vartheta_{B_t}(\upsilon_1,\omega_1)e^{iT\tau_{B_t}(\upsilon_1,\omega_1)}$

(b) $I\vartheta_{B_t+B_t'}(\upsilon_1,\omega_1)e^{iI\sigma_{B_t+B_t'}(\upsilon_1,\omega_1)}=I\vartheta_{B_t}(\upsilon_1,\omega_1)e^{iI\sigma_{B_t}(\upsilon_1,\omega_1)}$

(c) $F\vartheta_{B_t+B_t'}(\upsilon_1,\omega_1)e^{iF\rho_{B_t+B_t'}(\upsilon_1,\omega_1)}=F\vartheta_{B_t}(\upsilon_1,\omega_1)e^{iF\rho_{B_t}(\upsilon_1,\omega_1)}$

(5) *If $(\upsilon_1,\omega_1)\notin E$ and $(\upsilon_1,\omega_1)\in E'$*

(a) $T\vartheta_{B_t+B_t'}(\upsilon_1,\omega_1)e^{iT\tau_{B_t+B_t'}(\upsilon_1,\omega_1)}=T\vartheta_{B_t'}(\upsilon_1,\omega_1)e^{iT\tau_{B_t'}(\upsilon_1,\omega_1)}$

(b) $I\vartheta_{B_t+B_t'}(\upsilon_1,\omega_1)e^{iI\sigma_{B_t+B_t'}(\upsilon_1,\omega_1)}=I\vartheta_{B_t'}(\upsilon_1,\omega_1)e^{iI\sigma_{B_t'}(\upsilon_1,\omega_1)}$

(c) $F\vartheta_{B_t+B_t'}(\upsilon_1,\omega_1)e^{iF\rho_{B_t+B_t'}(\upsilon_1,\omega_1)}=F\vartheta_{B_t'}(\upsilon_1,\omega_1)e^{iF\rho_{B_t'}(\upsilon_1,\omega_1)}$

(6) *If $(\upsilon_1,\omega_1)\in E\cap E'$*

(a) $T\vartheta_{B_t+B_t'}(\upsilon_1,\omega_1)e^{i\ T\tau_{B_t+B_t'}(\upsilon_1,\omega_1)}$
$=max\left\{T\vartheta_{B_t}(\upsilon_1,\omega_1),T\vartheta_{B_t'}(\upsilon_1,\omega_1)\right\}e^{imax\{T\tau_{B_t}(\upsilon_1,\omega_1),T\tau_{B_t'}(\upsilon_1,\omega_1)\}}$

(b) $I\vartheta_{B_t+B_t'}(\upsilon_1,\omega_1)e^{i\ I\sigma_{B_t+B_t'}(\upsilon_1,\omega_1)}$
$=\ min\left\{I\vartheta_{B_t}(\upsilon_1,\omega_1),I\vartheta_{B_t'}(\upsilon_1,\omega_1)\right\}e^{imin\{I\sigma_{B_t}(\upsilon_1,\omega_1),\ I\sigma_{B_t'}(\upsilon_1,\omega_1)\}}$

(c) $F\vartheta_{B_t+B_t'}(\upsilon_1,\omega_1)e^{i\,F\rho_{B_t+B_t'}(\upsilon_1,\omega_1)}$
$= min\left\{F\vartheta_{B_t}(\upsilon_1,\omega_1), F\vartheta_{B_t'}(\upsilon_1,\omega_1)\right\}e^{imin\{F\rho_{B_t}(\upsilon_1,\omega_1),\, F\rho_{B_t'}(\upsilon_1,\omega_1)\}}$

(7) *If* $(\upsilon_1,\omega_1) \in E''$

(a) $T\vartheta_{B_t+B_t'}(\upsilon_1,\omega_1)\, e^{iT\tau_{B_t+B_t'}(\upsilon_1,\omega_1)} = max\left\{T\vartheta_{B_t}(\upsilon_1), T\vartheta_{B_t'}(\omega_1)\right\}e^{imax\{T\tau_{B_t}(\upsilon_1),T\tau_{B_t'}(\omega_1)\}}$

(b) $I\vartheta_{B_t+B_t'}(\upsilon_1,\omega_1)e^{iI\sigma_{B_t+B_t'}(\upsilon_1,\omega_1)} = min\left\{I\vartheta_{B_t}(\upsilon_1), I\vartheta_{B_t'}(\omega_1)\right\}e^{imin\{I\sigma_{B_t}(\upsilon_1),I\sigma_{B_t'}(\omega_1)\}}$

(c) $F\vartheta_{B_t+B_t'}(\upsilon_1,\omega_1)e^{iF\rho_{B_t+B_t'}(\upsilon_1,\omega_1)} = min\left\{F\vartheta_{B_t}(\upsilon_1), F\vartheta_{B_t'}(\omega_1)\right\}e^{imin\{F\rho_{B_t}(\upsilon_1),F\rho_{B_t'}(\omega_1)\}}$

Example 10. *From Example 9, the graphical representation of* $0.4e^{i0.7\pi}$*-CTNG* $\mathcal{G}_t + \mathcal{G}_t'$ *is shown in Figure 9.*

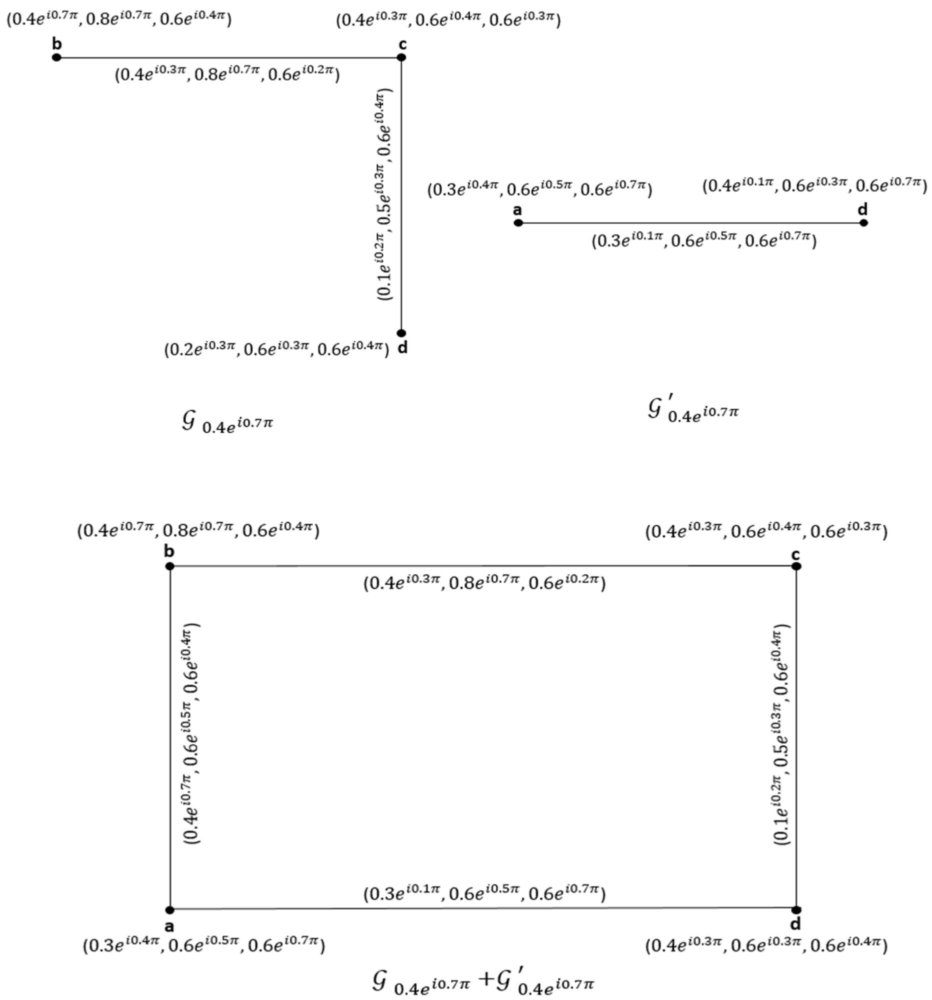

Figure 9. $\mathcal{G}_{0.4e^{i0.7\pi}} + \mathcal{G}'_{0.4e^{i0.7\pi}}$.

Definition 16. *Consider the following two CTNGs* $\mathcal{G}_t$ *and* $\mathcal{G}_t'$*. The vertex degree in the CTNG* $\mathcal{G}_t + \mathcal{G}_t'$ *is described below. For any* $(\upsilon_1,\omega_1) \in V \times V'$*.*

$$deg_{\mathcal{G}_t+\mathcal{G}_t'}(\upsilon_1,\omega_1) = \begin{pmatrix} deg\left\{T\vartheta_{B_t+B_t'}(\upsilon_1,\omega_1)\right\}e^{i\{T\tau_{B_t+B_t'}(\upsilon_1,\omega_1)\}}, \\ deg\left\{I\vartheta_{B_t+B_t'}(\upsilon_1,\omega_1)\right\}e^{i\{I\sigma_{B_t+B_t'}(\upsilon_1,\omega_1)\}}, \\ deg\left\{F\vartheta_{B_t+B_t'}(\upsilon_1,\omega_1)\right\}e^{i\{F\rho_{B_t+B_t'}(\upsilon_1,\omega_1)\}} \end{pmatrix}$$

where

$$deg\left\{T_{B_t+B_t'}(\upsilon_1,\omega_1)e^{iT\tau_{B_t+B_t'}(\upsilon_1,\omega_1)}\right\} =$$
$$\Big(\sum_{(\upsilon_1,\omega_1)\in E\ ,(\upsilon_1,\omega_1)\notin E'} T\vartheta_{B_t}(\upsilon_1,\omega_1)e^{i\sum_{(\upsilon_1,\omega_1)\in E\ ,(\upsilon_1,\omega_1)\notin E'} T\tau_{B_t}(\upsilon_1,\omega_1)}+$$
$$\sum_{(\upsilon_1,\omega_1)\notin E,(\upsilon_1,\omega_1)\in E'} T\vartheta_{B_t'}(\upsilon_1,\omega_1)e^{i\sum_{(\upsilon_1,\omega_1)\notin E,(\upsilon_1,\omega_1)\in E'} T\tau_{B_t'}(\upsilon_1,\omega_1)}+$$
$$\sum_{(\upsilon_1,\omega_1)\in E+E'} max\left\{T\vartheta_{B_t}(\upsilon_1,\omega_1),T\vartheta_{B_t'}(\upsilon_1,\omega_1)\right\}e^{i\sum_{(\upsilon_1,\omega_1)\in E\cap E'} max\{T\tau_{B_t}(\upsilon_1,\omega_1),\ T\tau_{B_t'}(\upsilon_1,\omega_1)\}}+$$
$$\sum_{(\upsilon_1,\omega_1)\in E''} max\left\{T\vartheta_{B_t}(\upsilon_1),T\vartheta_{B_t'}(\omega_1)\right\}e^{i\sum_{(\upsilon_1,\omega_1)\in E''} max\{T\tau_{B_t}(\upsilon_1),T\tau_{B_t'}(\omega_1)\}}\Big),$$

$$deg\left\{I\vartheta_{B_tB_t'}(\upsilon_1,\omega_1)e^{iI\sigma_{B_t+B_t'}(\upsilon_1,\omega_1)}\right\}$$
$$=\Bigg(\sum_{(\upsilon_1,\omega_1)\in E\ ,(\upsilon_1,\omega_1)\notin E'} I\vartheta_{B_t}(\upsilon_1,\omega_1)e^{i\sum_{(\upsilon_1,\omega_1)\in E\ ,(\upsilon_1,\omega_1)\notin E'} I\sigma_{B_t}(\upsilon_1,\omega_1)}$$
$$+\sum_{(\upsilon_1,\omega_1)\notin E,(\upsilon_1,\omega_1)\in E'} I\vartheta_{B_t'}(\upsilon_1,\omega_1)e^{i\sum_{(\upsilon_1,\omega_1)\notin E,(\upsilon_1,\omega_1)\in E'} I\sigma_{B_t'}(\upsilon_1,\omega_1)}+$$
$$\sum_{(\upsilon_1,\omega_1)\in E\cap E'} max\left\{I\vartheta_{B_t}(\upsilon_1,\omega_1),I\vartheta_{B_t'}(\upsilon_1,\omega_1)\right\}e^{i\sum_{(\upsilon_1,\omega_1)\in E\cap E'} max\{I\sigma_{B_t}(\upsilon_1,\omega_1),\ I\sigma_{B_t'}(\upsilon_1,\omega_1)\}}$$
$$+\sum_{(\upsilon_1,\omega_1)\in E''} max\left\{I\vartheta_{B_t}(\upsilon_1),I\vartheta_{B_t'}(\omega_1)\right\}e^{i\sum_{(\upsilon_1,\omega_1)\in E''} max\{I\sigma_{B_t}(\upsilon_1),\ I\sigma_{B_t'}(\omega_1)\}}\Bigg)$$

and

$$deg\left\{F\vartheta_{B_t+B_t'}(\upsilon_1,\omega_1)e^{iF\rho_{B_t+B_t'}(\upsilon_1,\omega_1)}\right\}$$
$$=\Bigg(\sum_{(\upsilon_1,\omega_1)\in E\ ,(\upsilon_1,\omega_1)\notin E'} F\vartheta_{B_t}(\upsilon_1,\omega_1)\,e^{i\sum_{(\upsilon_1,\omega_1)\in E\ ,(\upsilon_1,\omega_1)\notin E'} F\rho_{B_t}\upsilon_1,\omega_1}$$
$$+\sum_{(\upsilon_1,\omega_1)\notin E,(\upsilon_1,\omega_1)\in E'} F\vartheta_{B_t'}(\upsilon_1,\omega_1)e^{i\sum_{(\upsilon_1,\omega_1)\notin E,(\upsilon_1,\omega_1)\in E'} F\rho_{B_t'}(\upsilon_1,\omega_1)}$$
$$+\sum_{(\upsilon_1,\omega_1)\in E+E'} max\left\{F\vartheta_{B_t}(\upsilon_1,\omega_1),F\vartheta_{B_t'}(\upsilon_1,\omega_1)\right\}e^{i\sum_{(\upsilon_1,\omega_1)\in E\cap E'} max\{F\rho_{B_t}(\upsilon_1,\omega_1),\ F\rho_{B_t'}(\upsilon_1,\omega_1)\}}$$
$$+\sum_{(\upsilon_1,\omega_1)\in E''} max\left\{F\vartheta_{B_t}(\upsilon_1),F\vartheta_{B_t'}(\omega_1)\right\}e^{i\sum_{(\upsilon_1,\omega_1)\in E''} max\{F\rho_{B_t}(\upsilon_1),\ F\rho_{B_t'}(\omega_1)\}}\Bigg)$$

Theorem 2. *For any two CTNGs $\mathcal{G}_t = (A_t, B_t)$ and $\mathcal{G}_t' = (A_t', B_t')$ of G = (V,E) and G′ = (V′,E′), respectively, where $V \cap V' \neq \varnothing$, their union $\mathcal{G}_t \cup \mathcal{G}_t' = (A_t \cup A_t', B_t \cup B_t')$ is a CTNG of G = G∪ G′ if $\mathcal{G}_t$ and $\mathcal{G}_t'$ are the CTNGs of G and G′, respectively.*

Proof. Let us assume a CTNG, $\mathcal{G}_t \cup \mathcal{G}_t'$. Let $(\upsilon_1,\omega_1) \in E, (\upsilon_1,\omega_1) \notin E'$, and

$$(\upsilon_1,\omega_1) \in V - V'.$$

Consider

$$\begin{aligned}\mathrm{T\vartheta_{B_t}(\upsilon_1,\omega_1)e^{iT\tau_{B_t}(\upsilon_1,\omega_1)}} &= \mathrm{T\vartheta_{B_tUB_t'}(\upsilon_1,\omega_1)e^{iT\tau_{B_tUB_t'}(\upsilon_1,\omega_1)}} \\ &\leq \min\left\{\mathrm{T\vartheta_{A_t\cup A_t'}(\upsilon_1),T\vartheta_{A_t\cup A_t'}(\omega_1)}\right\}\mathrm{e^{imin\{T\tau_{A_t\cup A_t'}(\upsilon_1),T\tau_{A_t\cup A_t'}(\omega_1)\}}} \\ &= \min\{\mathrm{T\vartheta_{A_t}(\upsilon_1),T\vartheta_{A_t}(\omega_1)}\}\mathrm{e^{imin\{T\tau_{A_t}(\upsilon_1),T\tau_{A_t}(\omega_1)\}}}\end{aligned}$$

Consequently,

$$\mathrm{I\vartheta_{B_t}(\upsilon_1,\omega_1)e^{iI\sigma_{B_t}(\upsilon_1,\omega_1)}} \leq \max\{\mathrm{I\vartheta_{A_t}(\upsilon_1),I\vartheta_{A_t}(\omega_1)}\}\mathrm{e^{imax\{I\sigma_{A_t}(\upsilon_1),I\sigma_{A_t}(\omega_1)\}}}$$

$$\begin{aligned}\text{Also } \mathrm{I\vartheta_{B_t}(\upsilon_1,\omega_1)e^{iI\sigma_{B_t}(\upsilon_1,\omega_1)}} &= \mathrm{I\vartheta_{B_tUB_t'}(\upsilon_1,\omega_1)e^{iI\sigma_{B_tUB_t'}(\upsilon_1,\omega_1)}} \\ &\leq \max\left\{\mathrm{I\vartheta_{A_t\cup A_t'}(\upsilon_1),I\vartheta_{A_t\cup A_t'}(\omega_1)}\right\}\mathrm{e^{imax\{I\sigma_{A_t\cup A_t'}(\upsilon_1),I\sigma_{A_t\cup A_t'}(\omega_1)\}}} \\ &= \max\{\mathrm{I\vartheta_{A_t}(\upsilon_1),I\vartheta_{A_t}(\omega_1)}\}\mathrm{e^{imax\{I\sigma_{A_t}(\upsilon_1),I\sigma_{A_t}(\omega_1)\}}}\end{aligned}$$

Consequently,

$$\mathrm{I\vartheta_{B_t}(\upsilon_1,\omega_1)e^{iI\sigma_{B_t}(\upsilon_1,\omega_1)}} \leq \max\{\mathrm{I\vartheta_{A_t}(\upsilon_1),I\vartheta_{A_t}(\omega_1)}\}\mathrm{e^{imax\{I\sigma_{A_t}(\upsilon_1),I\sigma_{A_t}(\omega_1)\}}}$$

Consequently,

$$\mathrm{F\vartheta_{B_t}(\upsilon_1,\omega_1)e^{iF\rho_{B_t}(\upsilon_1,\omega_1)}} \leq \max\{\mathrm{F\vartheta_{A_t}(\upsilon_1),F\vartheta_{A_t}(\omega_1)}\}\mathrm{e^{imax\{F\rho_{A_t}(\upsilon_1),F\rho_{A_t}(\omega_1)\}}}$$

Also,

$$\begin{aligned}\mathrm{F\vartheta_{B_t}(\upsilon_1,\omega_1)e^{iF\rho_{B_t}(\upsilon_1,\omega_1)}} &= \mathrm{F\vartheta_{B_tUB_t'}(\upsilon_1,\omega_1)e^{iF\rho_{B_tUB_t'}(\upsilon_1,\omega_1)}} \\ &\leq \max\left\{\mathrm{F\vartheta_{A_t\cup A_t'}(\upsilon_1),F\vartheta_{A_t\cup A_t'}(\omega_1)}\right\}\mathrm{e^{imax\{F\rho_{A_t\cup A_t'}(\upsilon_1),F\rho_{A_t\cup A_t'}(\omega_1)\}}} \\ &= \max\{\mathrm{F\vartheta_{A_t}(\upsilon_1),F\vartheta_{A_t}(\omega_1)}\}\mathrm{e^{imax\{F\rho_{A_t}(\upsilon_1),F\rho_{A_t}(\omega_1)\}}}\end{aligned}$$

Consequently,

$$\mathrm{F\vartheta_{B_t}(\upsilon_1,\omega_1)e^{iF\rho_{B_t}(\upsilon_1,\omega_1)}} \leq \max\{\mathrm{F\vartheta_{A_t}(\upsilon_1),F\vartheta_{A_t}(\omega_1)}\}\mathrm{e^{imax\{F\sigma_{A_t}(\upsilon_1),F\sigma_{A_t}(\omega_1)\}}}$$

This establishes $\mathcal{G}_t = (A_t, B_t)$ as a CTNG. Similarly, we conclude $\mathcal{G}_t' = (A_t', B_t')$ as a CTNG of G''. Assuming $\mathcal{G}_t$ and $\mathcal{G}_t'$ and understanding that the merging of two CTNGs generates a CTNG, it follows that $\mathcal{G}_t \cup \mathcal{G}_t'$. □

4. Isomorphism of CTNGs

Definition 17. *Let $\mathcal{G}_t = (A_t, B_t)$ and $\mathcal{G}_t' = (A_t', B_t')$ be any two CTNGs of G = (V, E) and G′ = (V′, E′), respectively. A homomorphism θ from CTNG $\mathcal{G}_t$ and $\mathcal{G}_t'$ is a mapping $\theta : V \to V'$, satisfying the following conditions:*

1. $T\vartheta_{A_t}(\upsilon_1)e^{i\,T\vartheta_{A_t}(\upsilon_1)} \leq T\vartheta_{A_t'}(\theta(\upsilon_1))e^{i\,T\tau_{A_t'}(\theta(\upsilon_1))}$,
$I\vartheta_{A_t}(\upsilon_1)e^{i\,I\sigma_{A_t}(\upsilon_1)} \leq I\vartheta_{A_t'}(\theta(\upsilon_1))e^{i\,I\sigma_{A_t'}(\theta(\upsilon_1))}$, *and*
$F\vartheta_{A_t}(\upsilon_1)e^{i\,F\rho_{A_t}(\upsilon_1)} \leq F\vartheta_{A_t'}(\theta(\upsilon_1))e^{i\,F\rho_{A_t'}(\theta(\upsilon_1))}$; $\forall \upsilon_1 \in V$.
2. $T\vartheta_{B_t}(\upsilon_1,\omega_1)e^{i\,T\tau_{B_t}(\upsilon_1,\,\omega_1)} \leq T\vartheta_{B_t'}(\theta(\upsilon_1),\theta(\omega_1))\,e^{i\,T\tau_{B_t'}(\theta(\upsilon_1),\theta(\omega_1))}$,
$I\vartheta_{B_t}(\upsilon_1,\omega_1)e^{i\,I\sigma_{B_t}(\upsilon_1,\omega_1)} \leq I\vartheta_{B_t'}(\theta(\upsilon_1),\theta(\omega_1))e^{i\,I\sigma_{B_t'}(\theta(\upsilon_1),\theta(\omega_1))}$, *and*
$F\vartheta_{B_t}(\upsilon_1,\omega_1)e^{i\,F\rho_{B_t}(\upsilon_1,\omega_1)} \leq F\vartheta_{B_t'}(\theta(\upsilon_1),\theta(\omega_1))e^{i\,F\rho_{B_t'}(\theta(\upsilon_1),\theta(\omega_1))}$; $\forall(\upsilon_1,\omega_1) \in E$.

Definition 18. *A weak isomorphism $\theta : V \to V'$, from CTNG $\mathcal{G}_t$ to $\mathcal{G}_t'$, must meet the following conditions:*

$$T\vartheta_{A_t}(\upsilon_1)e^{I\ T\tau_{A_t}(\upsilon_1)} \le T\vartheta_{A'_t}(\theta(\upsilon_1))e^{I\ T\tau_{A'_t}(\theta(\upsilon_1))},$$

$$I\vartheta_{A_t}(\upsilon_1)e^{i\ I\sigma_{A_t}(\upsilon_1)} \le I\vartheta_{A'_t}(\theta(\upsilon_1))e^{i\ I\sigma_{A'_t}(\theta(\upsilon_1))}, and$$

$$F\vartheta_{A_t}(\upsilon_1)e^{iF\rho_{A_t}(\upsilon_1)} \le F\vartheta_{A'_t}(\theta(\upsilon_1))e^{i\ F\rho_{A'_t}(\theta(\upsilon_1))}; \forall \upsilon_1 \in V.$$

Definition 19. *A strong co-isomorphism is defined as a bijective mapping $\theta : V \to V'$ between any two CTNGs, $\mathcal{G}_t = (A_t, B_t)$ and $\mathcal{G}'_t = (A'_t, B'_t)$ of G = (V,E) and G′ = (V′,E′), respectively, which meets the following conditions:*

1. $T\vartheta_{A_t}(\upsilon_1)e^{iT\tau_{A_t}(\upsilon_1)} \le T\vartheta_{A'_t}(\theta(\upsilon_1))e^{T\tau_{A'_t}(\theta(\upsilon_1))}$,
   $I\vartheta_{A_t}(\upsilon_1)e^{iI\sigma_{A_t}(\upsilon_1)} \le I\vartheta_{A'_t}(\theta(\upsilon_1))e^{iI\sigma_{A'_t}(\theta(\upsilon_1))}$, *and*
   $F\vartheta_{A_t}(\upsilon_1)e^{iF\rho_{A_t}(\upsilon_1)} \le F\vartheta_{A'_t}(\theta(\upsilon_1))e^{i\ F\rho_{A'_t}(\theta(\upsilon_1))}; \forall \upsilon_1 \in V.$
2. $T\vartheta_{B_t}(\upsilon_1,\omega_1)e^{iT\tau_{B_t}(\upsilon_1,\omega_1)} \le T\vartheta_{B'_t}(\theta(\upsilon_1),\theta(\omega_1))e^{iT\tau_{B'_t}(\theta(\upsilon_1),\theta(\omega_1))}$,
   $I\vartheta_{B_t}(\upsilon_1,\omega_1)e^{iI\sigma_{B_t}(\upsilon_1,\omega_1)} \le I\vartheta_{B'_t}(\theta(\upsilon_1),\theta(\omega_1))e^{iI\sigma_{B'_t}(\theta(\upsilon_1),\ \theta(\omega_1))}$, *and*
   $F\vartheta_{B_t}(\upsilon_1,\omega_1)e^{iF\rho_{B_t}(\upsilon_1,\ \omega_1)} \le F\vartheta_{B'_t}(\theta(\upsilon_1),\theta(\omega_1))e^{iF\rho_{B'_t}(\theta(\upsilon_1),\ \theta(\omega_1))}; \forall(\upsilon_1,\omega_1) \in E.$
3. $T\vartheta_{B_t}(\upsilon_1,\omega_1)e^{iT\tau_{B_t}(\upsilon_1,\ \omega_1)} = T\vartheta_{B'_t}(\theta(\upsilon_1),\theta(\omega_1))e^{iT\tau_{B'_t}(\theta(\upsilon_1),\ \theta(\omega_1))}$,
   $I\vartheta_{B_t}(\upsilon_1,\omega_1)e^{iI\sigma_{B_t}(\upsilon_1,\ \omega_1)} = I\vartheta_{B'_t}(\theta(\upsilon_1),\theta(\omega_1))e^{iI\sigma_{B'_t}(\theta(\upsilon_1),\ \theta(\omega_1))}$, *and*
   $F\vartheta_{B_t}(\upsilon_1,\omega_1)e^{iF\rho_{B_t}(\upsilon_1,\ \omega_1)} = F\vartheta_{B'_t}(\theta(\upsilon_1),\theta(\omega_1))e^{iF\rho_{B'_t}(\theta(\upsilon_1),\ \theta(\omega_1))}; \forall(\upsilon_1,\omega_1) \in E.$

Definition 20. *An isomorphism between CTNGs $\mathcal{G}_t = (A_t, B_t)$ and $\mathcal{G}'_t = (A'_t, B'_t)$ is a bijective homomorphism mapping $\theta : V \to V'$ (written as $\mathcal{G}_t \approx \mathcal{G}'_t$) which satisfies the following conditions:*

1. $T\vartheta_{A_t}(\upsilon_1)e^{iT\tau_{A_t}(\upsilon_1)} \le T\vartheta_{A'_t}(\theta(\upsilon_1))e^{T\tau_{A'_t}(\theta(\upsilon_1))}$,
   $I\vartheta_{A_t}(\upsilon_1)e^{iI\sigma_{A_t}(\upsilon_1)} \le I\vartheta_{A'_t}(\theta(\upsilon_1))e^{iI\sigma_{A'_t}(\theta(\upsilon_1))}$, *and*
   $F\vartheta_{A_t}(\upsilon_1)e^{iF\rho_{A_t}(\upsilon_1)} \le F\vartheta_{A'_t}(\theta(\upsilon_1))e^{iF\rho_{A'_t}(\theta(\upsilon_1))}; \forall \upsilon_1 \in V.$
2. $T\vartheta_{B_t}(\upsilon_1,\omega_1)e^{i\ T\tau_{B_t}(\upsilon_1,\omega_1)} = T\vartheta_{B'_t}(\theta(\upsilon_1),\theta(\omega_1))e^{iT\tau_{B'_t}(\theta(\upsilon_1),\theta(\omega_1))}$,
   $I\vartheta_{B_t}(\upsilon_1,\omega_1)e^{i\ I\sigma_{B_t}(\upsilon_1,\omega_1)} = I\vartheta_{B'_t}(\theta(\upsilon_1),\theta(\omega_1))\ e^{iI\sigma_{B'_t}(\theta(\upsilon_1),\theta(\omega_1))}$, *and*
   $F\vartheta_{B_t}(\upsilon_1,\omega_1)e^{iF\rho_{B_t}(\upsilon_1,\omega_1)} = F\vartheta_{B'_t}(\theta(\upsilon_1),\theta(\omega_1))e^{iF\rho_{B'_t}(\theta(\upsilon_1),\theta(\omega_1))}; \forall(\upsilon_1,\omega_1) \in E.$

Example 11. *According to the following figures, take the two $0.8e^{i0.7\pi}$-$\mathcal{G}_t$ and $\mathcal{G}'_t$ as shown in Figure 10a,b.*

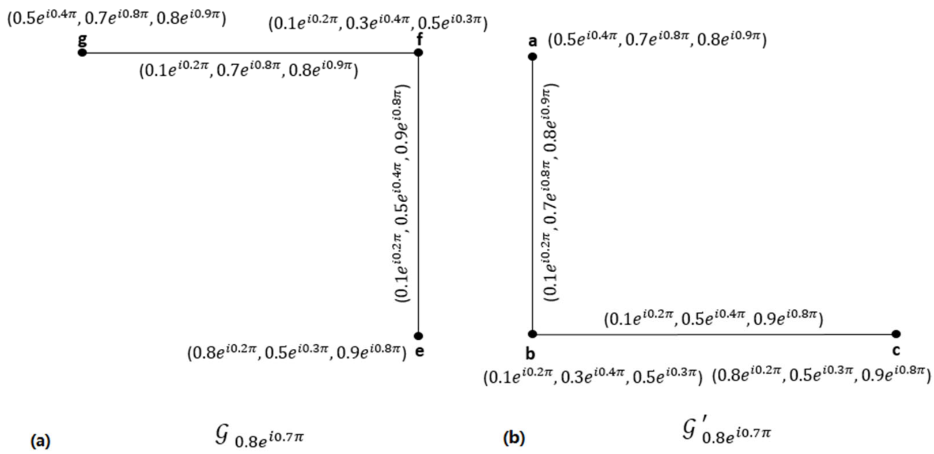

Figure 10. (**a**) $0.8e^{i0.7\pi} - NG\mathcal{G}_{0.8e^{i0.7\pi}}$. (**b**). $0.8e^{i0.7\pi} - NG\mathcal{G}'_{0.8e^{i0.7\pi}}$.

According to Definition (20), the mapping ζ(a) = g, ζ(b) = f, and ζ(c) = e gives us

$$\mathcal{G}_{\ 0.8e^{i0.7\pi}} \approx \mathcal{G}'_{0.8e^{i0.7\pi}}$$

Theorem 3. *The characteristics of an equivalence relation are satisfied by the connection of an isomorphism between CTNGs.*

Proof. Both symmetry and reflexivity are clear. The isomorphism of $\mathcal{G}_t$ onto $\mathcal{G}'_t$ and $\mathcal{G}'_t$ onto $\mathcal{G}''_t$, respectively, are denoted by the notations $\varphi : V \to V'$ and $\theta : V' \to V''$. Accordingly, $\theta \circ \varphi: V \to V''$ is a bijective map from V' to V'', and it is defined as follows:

$$(\theta \circ \varphi)(\mho_1) = \theta(\varphi(\mho_1)),\ \forall \mho_1 \in V$$

For a map $\varphi : V \to V'$ defined by $\varphi(\mho_1) = \omega_1, \forall \mho_1 \in V$, it is an isomorphism. Considering Definition (20), we have

$$T\vartheta_{A_t}(\mho_1)e^{iT\tau_{A_t}(\mho_1)} = T\vartheta_{A'_t}(\varphi(\mho_1))e^{iT\tau_{A'_t}(\varphi(\mho_1))} = T\vartheta_{A'_t}(\omega_1)e^{iT\tau_{A'_t}(\varphi(\mho_1))}, \forall \mho_1 \in V \quad (1)$$

$$I\vartheta_{A_t}(\mho_1)e^{i\ I\sigma_{A_t}(\mho_1)} = I\vartheta_{A'_t}(\varphi(\mho_1))e^{iI\sigma_{A'_t}(\varphi(\mho_1))} = I\vartheta_{A'_t}(\omega_1)\ e^{iI\sigma_{A'_t}(\omega_1)}, \forall \mho_1 \in V \quad (2)$$

$$F\vartheta_{A_t}(\mho_1)e^{iF\rho_{A_t}(\mho_1)} = F\vartheta_{A'_t}(\varphi(\mho_1))e^{iF\rho_{A'_t}(\varphi(\mho_1))} = F\vartheta_{A'_t}(\omega_1)e^{iF\rho_{A'_t}(\omega_1)}, \forall \mho_1 \in V \quad (3)$$

and

$$\begin{aligned} T\vartheta_{B_t}(\mho_1, \mho_2)e^{iT\tau_{B_t}(\mho_1,\mho_2)} &= T\vartheta_{B'_t}(\varphi(\mho_1), \varphi(\mho_2))e^{iT\tau_{B'_t}(\varphi(\mho_1),\varphi(\mho_2))} \\ &= T\vartheta_{B'_t}(\omega_1, \omega_2)e^{iT\tau_{B'_t}(\omega_1,\omega_2)}, \forall(\mho_1, \mho_2) \in E \end{aligned} \quad (4)$$

$$\begin{aligned} I\vartheta_{B_t}(\mho_1, \mho_2)e^{iI\sigma_{B_t}(\mho_1,\mho_2)} &= I\vartheta_{B'_t}(\varphi(\mho_1), \varphi(\mho_2))e^{iI\sigma_{B'_t}(\varphi(\mho_1),\varphi(\mho_2))} \\ &= I\vartheta_{B'_t}(\omega_1, \omega_2)e^{iI\sigma_{B'_t}(\omega_1,\omega_2)}, \forall(\mho_1, \mho_2) \in E \end{aligned} \quad (5)$$

$$\begin{aligned} F\vartheta_{B_t}(\mho_1, \mho_2)e^{iF\rho_{B_t}(\mho_1,\mho_2)} &= F\vartheta_{B'_t}(\varphi(\mho_1), \varphi(\mho_2))e^{iF\rho_{B'_t}(\varphi(\mho_1),\varphi(\mho_2))} \\ &= F\vartheta_{B'_t}(\omega_1, \omega_2)e^{iF\rho_{B'_t}(\omega_1,\omega_2)}\forall(\mho_1, \mho_2) \in E \end{aligned} \quad (6)$$

In the same way, we obtain that

$$T\vartheta_{A'_t}(\omega_1)e^{iT\tau_{A'_t}(\omega_1)} = T\vartheta_{A''_t}(v_1)e^{iT\tau_{A''_t}(v_1)}, \forall\ \omega_1 \in V' \quad (7)$$

$$I\vartheta_{A'_t}(\omega_1)e^{iI\sigma_{A'_t}(\omega_1)} = I\vartheta_{A''_t}(v_1)e^{iI\sigma_{A''_t}(v_1)}, \forall\ \omega_1 \in V' \quad (8)$$

$$F\vartheta_{A'_t}(\omega_1)e^{iF\rho_{A'_t}(\omega_1)} = F\vartheta_{A''_t}(v_1)e^{iF\rho_{A''_t}(v_1)}, \forall\ \omega_1 \in V' \quad (9)$$

and

$$T\vartheta_{B'_t}(\omega_1, \omega_2)e^{iT\tau_{B'_t}(\omega_1,\omega_2)} = T\vartheta_{B''_t}(v_1, v_2)e^{iT\tau_{B''_t}(v_1,v_2)}, \forall(\omega_1, \omega_2) \in E' \quad (10)$$

$$I\vartheta_{B'_t}(\omega_1, \omega_2)e^{iI\sigma_{B'_t}(\omega_1,\omega_2)} = I\vartheta_{B''_t}(v_1, v_2)e^{iI\sigma_{B''_t}(v_1,v_2)}, \forall(\omega_1, \omega_2) \in E' \quad (11)$$

$$F\vartheta_{B'_t}(\omega_1, \omega_2)e^{iF\rho_{B'_t}(\omega_1,\omega_2)} = F\vartheta_{B''_t}(v_1, v_2)e^{iF\rho_{B''_t}(v_1,v_2)}, \forall(\omega_1, \omega_2) \in E' \quad (12)$$

By using the relations (1) and (7) and $\varphi(\upsilon_1) = \omega_1$, $\forall\ \upsilon_1 \in V$, we have

$$\begin{aligned} T\vartheta_{A_t}(\upsilon_1)e^{T\tau_{A_t}(\upsilon_1)} &= T\vartheta_{A_t'}(\varphi(\upsilon_1))e^{iT\tau_{A_t'}(\varphi(\upsilon_1))} \\ &= T\vartheta_{A_t'}(\omega_1)e^{iT\tau_{A_t'}(\omega_1)} \\ &= T\vartheta_{A_t''}(\theta(\omega_1))e^{iT\tau_{A_t''}(\theta(\omega_1))} \\ &= T\vartheta_{A_t''}(\theta(\varphi(\upsilon_1)))e^{i\,T\tau_{A_t''}(\theta(\varphi(\upsilon_1)))} \end{aligned}$$

By using the relations (2) and (8) and $\varphi(\upsilon_1) = \omega_1$, $\forall\ \upsilon_1 \in V$, we have

$$\begin{aligned} I\vartheta_{A_t}(\upsilon_1)e^{I\sigma_{A_t}(\upsilon_1)} &= I\vartheta_{A_t'}(\varphi(\upsilon_1))e^{iI\sigma_{A_t'}(\varphi(\upsilon_1))} \\ &= I\vartheta_{A_t'}(\omega_1)e^{iI\sigma_{A_t'}(\omega_1)} \\ &= I\vartheta_{A_t''}(\theta(\omega_1))e^{iI\sigma_{A_t''}(\theta(\omega_1))} \\ &= I\vartheta_{A_t''}(\theta(\varphi(\upsilon_1)))e^{i\,I\sigma_{A_t''}(\theta(\varphi(\upsilon_1)))} \end{aligned}$$

By using the relations (3) and (9) and $\varphi(\upsilon_1) = \omega_1$, $\forall\ \upsilon_1 \in V$, we have

$$\begin{aligned} F\vartheta_{A_t}(\upsilon_1)e^{F\rho_{A_t}(\upsilon_1)} &= F\vartheta_{A_t'}(\varphi(\upsilon_1))e^{iF\rho_{A_t'}(\varphi(\upsilon_1))} \\ &= F\vartheta_{A_t'}(\omega_1)e^{iF\rho_{A_t'}(\omega_1)} \\ &= F\vartheta_{A_t''}(\theta(\omega_1))e^{iF\rho_{A_t''}(\theta(\omega_1))} \\ &= F\vartheta_{A_t''}(\theta(\varphi(\upsilon_1)))e^{i\,F\rho_{A_t''}(\theta(\varphi(\upsilon_1)))} \end{aligned}$$

When using the relations (4) and (10), the outcome is

$$\begin{aligned} T\vartheta_{B_t}(\upsilon_1,\upsilon_2)e^{iT\tau_{B_t}(\upsilon_1,\upsilon_2)} &= T\vartheta_{B_t'}(\omega_1,\omega_2)e^{iT\tau_{B_t'}(\omega_1,\ \ \omega_2)} \\ &= T\vartheta_{B_t''}(\theta(\omega_1),\theta(\omega_2))e^{T\tau_{B_t''}(\theta(\omega_1)\ ,\ \ \theta(\omega_2))} \\ &= T\vartheta_{B_t''}(\theta(\varphi(\upsilon_1)),\theta(\varphi(\upsilon_2)))e^{iT\tau_{B_t''}(\theta(\varphi(\upsilon_1)),\ \ \theta(\varphi(\upsilon_2)))} \end{aligned}$$

When using the relations (5) and (11), the outcome is

$$\begin{aligned} I\vartheta_{B_t}(\upsilon_1,\upsilon_2)e^{iI\sigma_{B_t}(\upsilon_1,\upsilon_2)} &= I\vartheta_{B_t'}(\omega_1,\omega_2)e^{iI\sigma_{B_t'}(\omega_1,\omega_2)} \\ &= I\vartheta_{B_t''}(\theta(\omega_1),\theta(\omega_2))e^{I\sigma_{B_t''}(\theta(\omega_1),\ \ \theta(\omega_2))} \\ &= I\vartheta_{B_t''}(\theta(\varphi(\upsilon_1)),\theta(\varphi(\upsilon_2)))e^{iI\sigma_{B_t''}(\theta(\varphi(\upsilon_1)),\ \ \theta(\varphi(\upsilon_2)))} \end{aligned}$$

When using the relations (6) and (12), the outcome is

$$\begin{aligned} F\vartheta_{B_t}(\upsilon_1,\upsilon_2)e^{iF\rho_{B_t}(\upsilon_1,\upsilon_2)} &= F\vartheta_{B_t'}(\omega_1,\omega_2)e^{iF\rho_{B_t'}(\omega_1,\ \omega_2)} \\ &= F\vartheta_{B_t''}(\theta(\omega_1),\theta(\omega_2))e^{F\rho_{B_t''}(\theta(\omega_1),\ \ \theta(\omega_2))} \\ &= F\vartheta_{B_t''}(\theta(\varphi(\upsilon_1)),\theta(\varphi(\upsilon_2)))e^{iF\rho_{B_t''}(\theta(\varphi(\upsilon_1)),\ \ \theta(\varphi(\upsilon_2)))} \end{aligned}$$

Hence, $\mathcal{G}_t$ and $\mathcal{G}_t''$ are isomorphic to each other via $\theta \circ \varphi$. As a result, the proof is finished. □

5. Real-World Applications in Biodiversity Conservation

In this section, we consider applications for biodiversity conservation based on CTNGs and TNGs.

5.1. Experiment Description

The experiments used CTNGs to investigate and address biodiversity conservation issues. CTNGs enabled decision-makers to evaluate confusing data and estimate many aspects of biodiversity conservation, such as habitat protection, species conservation, sustainable land-use practices, ecosystem restoration, climate change adaption, and public awareness and education. The CTNG framework used indeterminacy, truth membership, and falsity membership functions to characterize interactions between conservation factors, reflecting the degree of alignment or divergence from specified factors.

The findings are as follows:

1. CTNGs made it easier to identify critical aspects that contribute to biodiversity conservation, giving decision-makers insight into the complicated relationships between conservation elements.
2. Using CTNGs to examine the links between conservation factors and prioritize interventions allowed decision-makers to make more informed conservation program decisions.
3. The parameter 't' in CTNGs allows decision-makers to tailor the graphs to their domain knowledge and problem, resulting in better targeted interventions and informed biodiversity conservation decisions.
4. Visual representations of CTNGs shed light on the intricate relationships between conservation factors, contributing to the formulation of effective conservation strategies and biodiversity preservation.

5.2. Application of CTNGs in Biodiversity Conservation

The loss of biodiversity presents huge worldwide issues, affecting ecosystems, economies, and human well-being. Factors such as habitat degradation, pollution, overexploitation, and climate change all contribute to biodiversity loss, endangering species survival and ecosystem health. Addressing biodiversity protection demands a comprehensive approach that takes into account the intricate interplay of numerous elements. Using CTNGs, decision-makers may examine ambiguous data and estimate biodiversity conservation components. This method allows for the identification of crucial factors and improves decision-making in conservation programs.

In Figure 11, Key aspects contributing to biodiversity conservation include habitat protection (B_1) , species conservation (B_2), sustainable land-use practices (B_3), ecosystem restoration (B_4), climate change adaptation (B_5), and public awareness and education (B_6). Let $\mathcal{M} = \{B_1, B_2, B_3, B_4, B_5, B_6\}$ be the vertex of factors that strongly contribute to biodiversity conservation. The edges represent the degree of connection or link between components using t-neutrosophic values. Within the CTNG paradigm, indeterminacy, truth membership, and falsity membership functions represent the relationship between diverse factors, capturing the degree to which an element aligns with or diverges from specified factors. The integration of these functions provides a thorough knowledge of relationships between elements and variables, which can help decision-makers evaluate conservation efforts. The parameter't' enables decision-makers to modify the complicated TNG to their domain knowledge and problem, allowing for more targeted interventions and informed biodiversity conservation decision-making. Visual representations employing CTNGs shed light on the complex connections between conservation elements, assisting in the creation of successful conservation strategies and the preservation of biodiversity for future generations. Furthermore, various parameter values for 't' in neutrosophic graphs indicate different attitudes toward risk and uncertainty. The direction of threshold values for indeterminacy, truth membership, and falsity membership allows decision-makers to stress or downplay certain features depending on the desired level of uncertainty and confidence.

The parameter 't' in neutrosophic graphs allows for customization to varied circumstances and sensitivities. Adjusting the 't' value allows decision-makers to experiment with different scenarios by varying the balance of element acceptance and rejection. This versatility is essential for making decisions in unclear contexts or changing conditions.

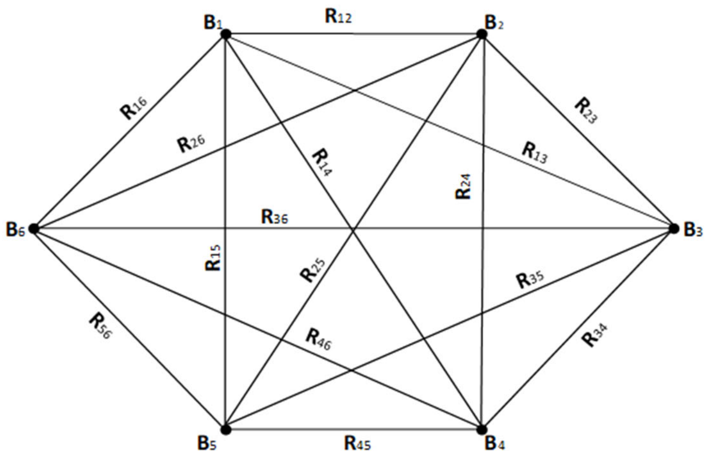

Figure 11. Graphical representation of biodiversity conservation.

Additionally, varying parameter values of 't' signify distinct perspectives on risk and uncertainty. The orientation of threshold values for membership and non-membership empowers decision-makers to accentuate or downplay specific facts, aligning with their preferred degrees of acceptance and rejection. The parameter 't' facilitates the customization of CTNGs to suit diverse contexts and sensitivities. By adjusting the variable 't', decision-makers can investigate numerous possibilities by modulating the balance between favorable and opposing viewpoints. This capability proves crucial when navigating decisions in uncertain environments or amid ongoing changes.

The parameter 't' allows decision-makers to customize CTNGs to match their subject expertise and unique issue requirements. Furthermore, differing parameter values for 't' represent varied attitudes toward risk and uncertainty. The direction of threshold values for truth membership, indeterminacy membership, and falsity membership allows decision-makers to emphasize or downplay certain facts based on the desired levels of truth membership, indeterminacy membership, and falsity membership. This parameter, designated as 't', allows CTNGs to be customized to fit a variety of settings and sensitivity levels. Changing the variable 't' allows decision-makers to investigate a range of options by altering the balance between positive and opposing opinions. This capacity is critical while making judgments in unclear circumstances or amid continual change. Table 2 shows the complex neutrosophic fuzzy sets (CNFSs) and 0.7-CNFS defined at the vertices.

Table 2. Vertices of NS and $0.7e^{0.9\pi}$-NS.

Vertices	CNS	Complex 0.7-NS
B_1	$\left(0.8e^{i0.7\pi}, 0.4e^{i0.3\pi}, 0.4e^{i\ 0.3\pi}\right)$	$\left(0.7e^{i0.7\pi}, 0.4e^{i0.3\pi}, 0.4e^{i\ 0.3\pi}\right)$
B_2	$\left(0.6e^{i0.5\pi}, 0.3e^{i0.2\pi}, 0.1e^{i\ 0.1\pi}\right)$	$\left(0.6e^{i0.5\pi}, 0.3e^{i0.3\pi}, 0.3e^{i\ 0.3\pi}\right)$
B_3	$\left(0.5e^{i0.4\pi}, 0.7e^{i0.3\pi}, 0.6e^{i\ 0.4\pi}\right)$	$\left(0.5e^{i0.4\pi}, 0.7e^{i0.3\pi}, 0.6e^{i\ 0.4\pi}\right)$
B_4	$\left(0.8e^{i0.7\pi}, 0.9e^{i0.4\pi}, 0.7e^{i\ 0.6\pi}\right)$	$\left(0.7e^{i0.7\pi}, 0.9e^{i0.4\pi}, 0.7e^{i\ 0.6\pi}\right)$
B_5	$\left(0.9e^{i0.8\pi}, 0.6e^{i0.7\pi}, 0.5e^{i\ 0.4\pi}\right)$	$\left(0.7e^{i0.7\pi}, 0.6e^{i0.7\pi}, 0.5e^{i\ 0.4\pi}\right)$
B_6	$\left(0.4e^{i0.3\pi}, 0.5e^{i0.4\pi}, 0.2e^{i\ 0.1\pi}\right)$	$\left(0.4e^{i0.3\pi}, 0.5e^{i0.4\pi}, 0.3e^{i\ 0.3\pi}\right)$

As seen in Table 3, edge R_{12}, which connects habitat protection to species conservation, represents a link between encouraging habitat protection and species conservation. In the context of edge $R_{12} = \left(0.5e^{i0.4\pi}, 0.4e^{i0.3\pi}, 0.4e^{i\,0.3\pi}\right)$, a truth membership degree of $0.5e^{i0.4\pi}$ implies a relationship between these variables, $0.4e^{i0.3\pi}$ implies the indeterminacy membership value between these variables, whereas a falsity membership of $0.4e^{i\,0.3\pi}$ indicates a lesser correlation efforts. In this context, the parameter 't' represents the component that can lead to a 70% reduction in biodiversity conservation. Moving on to Table 4, the application of Part (1) of Definition (8) produces the following outcomes. The edges' score function is defined below. By using this formula, the score value of each edge can be defined.

$$SV = \left| \left(\text{deg}T\vartheta(l_j) - \text{deg}I\vartheta(l_j) - \text{deg}F\vartheta(l_j)\right) + \frac{1}{2\pi}\left(\text{degT}\tau(l_j) - \text{degI}\sigma(l_j) - \text{degF}\rho\right)(l_j) \right|,$$
$$1 \leq j \leq 6.$$

Table 3. Edges of CNS and complex $0.7e^{0.9\pi}$-NS.

Edges	Complex 0.7-NS
$R_{12} = (l_1, l_2)$	$\left(0.5e^{i0.4\pi}, 0.4e^{i0.3\pi}, 0.4e^{i\,0.3\pi}\right)$
$R_{13} = (l_1, l_3)$	$\left(0.4e^{i0.3\pi}, 0.6e^{i0.3\pi}, 0.6e^{i\,0.4\pi}\right)$
$R_{14} = (l_1, l_4)$	$\left(0.6e^{i0.6\pi}, 0.8e^{i0.4\pi}, 0.7e^{i\,0.6\pi}\right)$
$R_{15} = (l_1, l_5)$	$\left(0.6e^{i0.6\pi}, 0.6e^{i0.7\pi}, 0.5e^{i\,0.4\pi}\right)$
$R_{16} = (l_1, l_6)$	$\left(0.3e^{i0.2\pi}, 0.5e^{i0.4\pi}, 0.4e^{i\,0.3\pi}\right)$
$R_{23} = (l_2, l_3)$	$\left(0.4e^{i0.3\pi}, 0.6e^{i0.3\pi}, 0.6e^{i\,0.4\pi}\right)$
$R_{24} = (l_2, l_4)$	$\left(0.4e^{i0.3\pi}, 0.8e^{i0.3\pi}, 0.7e^{i\,0.6\pi}\right)$
$R_{25} = (l_2, l_5)$	$\left(0.4e^{i0.3\pi}, 0.5e^{i0.6\pi}, 0.5e^{i\,0.4\pi}\right)$
$R_{26} = (l_2, l_6)$	$\left(0.3e^{i0.2\pi}, 0.5e^{i0.4\pi}, 0.3e^{i\,0.3\pi}\right)$
$R_{34} = (l_3, l_4)$	$\left(0.2e^{i0.3\pi}, 0.8e^{i0.4\pi}, 0.7e^{i\,0.5\pi}\right)$
$R_{35} = (l_3, l_5)$	$\left(0.4e^{i0.3\pi}, 0.7e^{i0.7\pi}, 0.6e^{i\,0.4\pi}\right)$
$R_{36} = (l_3, l_6)$	$\left(0.3e^{i0.2\pi}, 0.7e^{i0.4\pi}, 0.6e^{i\,0.4\pi}\right)$
$R_{45} = (l_4, l_5)$	$\left(0.6e^{i0.6\pi}, 0.8e^{i0.6\pi}, 0.7e^{i\,0.6\pi}\right)$
$R_{46} = (l_4, l_6)$	$\left(0.4e^{i0.3\pi}, 0.8e^{i0.3\pi}, 0.7e^{i\,0.6\pi}\right)$
$R_{56} = (l_5, l_6)$	$\left(0.4e^{i0.3\pi}, 0.6e^{i0.7\pi}, 0.5e^{i\,0.4\pi}\right)$

Table 4. Table of truth, indeterminate, and falsity membership degree of each factor.

Factor	Degree of Each Factor
B_1	$\deg(l_1) = \left(2.4e^{i\,2.1\pi},\ 2.9e^{i\,2.1\pi}, 2.6e^{i\,2.0\pi}\right)$
B_2	$\deg(l_2) = \left(2.0e^{i\,1.5\pi},\ 2.8e^{i\,1.9\pi}, 2.5e^{i\,2.0\pi}\right)$
B_3	$\deg(l_3) = \left(2.9e^{i\,1.4\pi},\ 3.4\,e^{i\,2.1\pi}, 3.1e^{i\,2.1\pi}\right)$
B_4	$\deg(l_4) = \left(2.4e^{i\,2.1\pi},\ 4.0e^{i\,2.0\pi}, 3.5e^{i\,2.9\pi}\right)$
B_5	$\deg(l_5) = \left(2.4e^{i\,2.1\pi},\ 3.2e^{i\,3.3\pi}, 2.8e^{i\,2.2\pi}\right)$
B_6	$\deg(l_6) = \left(1.8e^{i\,1.25\pi},\ 3.1e^{i\,2.2\pi}, 2.4e^{i\,2.0\pi}\right)$

Table 5 shows the results obtained by using the scoring function formula stated in Table 4. Figure 12 shows a visual depiction of the score function for the parameters listed in Table 5. $B_4 = 9.49822971$ has the greatest value, showing that B_4 is the most influential factor in biodiversity conservation (parameter 't'). From the above Table 5, it is clear that ecosystem restoration will have an impact in biodiversity conversation.

Table 5. Score value of CTNGs.

Factor	Score Value $SV(l_j)$ of CTNG
Habitat protection (B_1)	6.24159265
Species conservation (B_2)	5.06991118
Sustainable land-use practice (B_3)	7.99822971
Ecosystem restoration (B_4)	9.49822971
Climate change adaptation (B_5)	8.94070751
Public awareness and education (B_6)	8.51238898

5.3. Performance Comparative Analysis

The CTNGs improve upon neutrosophic graphs by incorporating the parameter 't' for fine-tuning uncertainty modeling to individual requirements and domain features. Adjusting 't' allows decision-makers to more precisely describe diverse decision-making circumstances, resulting in a more nuanced depiction of ambiguity and vagueness. With applications covering a wide range of scenarios and decision-making processes, the changeable parameter 't' allows modification to capture variable degrees of caution or optimism, responding to unique requirements. This adaptability is crucial in problem-solving contexts where several degrees of ambiguity, reluctance, and decision preferences must be handled concurrently. CTNGs develop in complex decision-making scenarios such as medical diagnosis, pattern identification, and decision support systems because of their capacity to tolerate varying degrees of ambiguity and reluctance. The graph's uniformity implies that all elements have an equal link to the component under study, independent of truth, indeterminacy, and falsity membership levels. Furthermore, assigning 't' a truth membership value of 0.6 implies a strong link, whilst an indeterminacy membership value of 0.3 suggests either a high, weak, or medium-strength relationship, and a falsity membership value of 0.4 indicates a weak connection between elements. Using the supplied 't' values, decision-makers may adjust the CTNS to their background and scenario. Furthermore, varied 't' values correlate to different perspectives on risk and uncertainty. The continuous uniformity implies that all features are regarded to be equally related to the component under discussion, with no visible differentiation depending on their level of truth, indeterminacy, and falsity membership. The persistence of uncertainty in linking these aspects with the factor is consistent across all dimensions. Choosing a parameter value of 't' around zero implies a need for greater accuracy in determining the consequences for biodiversity conservation.

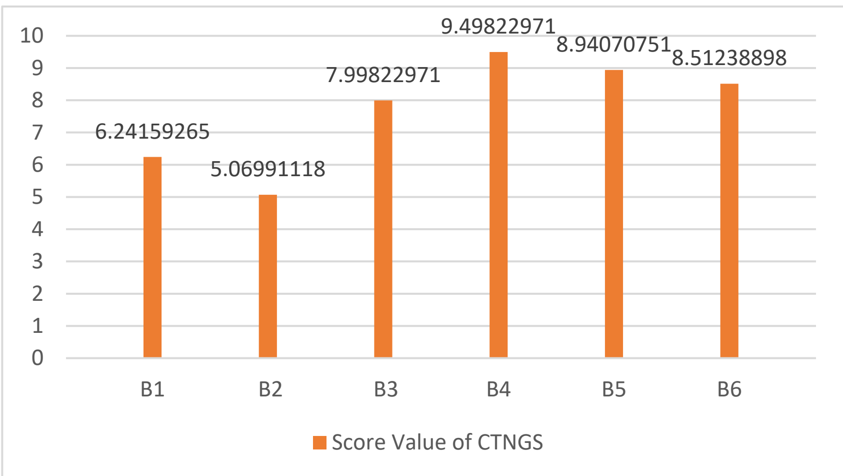

Figure 12. Graphical representation of score values of CTNGs.

5.4. Sensitivity Analysis

The present investigation's sensitivity analysis focuses on the parameter 't' in CTNGs, highlighting its critical importance in biodiversity conservation decision-making processes. 't' enables decision-makers to change the balance of element acceptance and rejection, reflecting different attitudes toward risk and uncertainty. Decision-makers can experiment with alternative situations by changing the value of 't', highlighting or downplaying key aspects based on desired confidence levels. This flexibility promotes efficient decision-making in unknown contexts and changing situations, which is critical for handling the challenges of biodiversity protection. Furthermore, the capacity to adapt 't' helps conservation plans match with different viewpoints on risk and uncertainty, allowing decision-makers to appropriately prioritize operations.

6. Conclusions

The notion of complex t-neutrosophic graphs (CTNGs) was introduced in this study, and some basic aspects of this phenomena were investigated. There have been many studies and demonstrations of graphical representations for numerous set-theoretical operations of CTNGs. Furthermore, a definition and an examination of some of the fundamental components of a complement of CTNGs were provided. The concepts of CTNG homomorphisms and isomorphisms were presented. Additionally, an example of how the recently developed technique can be used in biodiversity conservation was shown.

The consequences of biodiversity preservation lack discernible specificity when a parameter value 't' near 0 is used. Conversely, when the value of the parameter 't' gets closer to 1, it indicates a significant and apparent association with accomplishing goals associated with biodiversity preservation. The parameter 't' in CTNGs expresses the degree of confidence or doubt on the success of biodiversity preservation initiatives. The two extremes of this metric represent a strong association and/or negligible impact with the desired outcome. By employing this calibrated parameter, decision-makers can accurately modify how uncertainty is portrayed and how it impacts analytical outcomes, resulting in a more complex and adaptable framework for addressing the complex challenges associated with poverty alleviation.

Future work: Future studies might focus on creating complex optimization algorithms that are customized to the properties of CTNGs, allowing for more efficient and effective decision-making in biodiversity conservation and other complex system analysis fields. These algorithms might include multi-objective optimization strategies to find optimum solutions while taking into account competing objectives, as well as uncertainty quantification methods to effectively manage uncertainty in decision-making processes. Furthermore, additional validation studies and case studies in a variety of ecological situations would give empirical proof of CTNGs' relevance and efficacy, making them more widely adopted by decision-makers and conservation practitioners. Then, we will extend this work to various symmetric differences in complex t-neutrosophic graphs.

Our main objective for the next research is to tackle MCDM problems—supplier selection, risk management, and renewable energy selection—by using the suggested approach. Neural networks, clustering, feature selection, and risk management will all be impacted by the suggested strategies. Furthermore, a few sophisticated methods for generating decisions involving complicated spherical fuzzy Aczel Alsina aggregation operators will also be examined in relation to the tactics discussed in this paper.

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