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Interval-Valued Linguistic q-Rung Orthopair Fuzzy TODIM with Unknown Attribute Weight Information

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Abstract: It is widely known that symmetry does exist in management systems, such as economics, management, and even daily life. In addition, effective and qualified decision-making methods can enhance the performance and symmetry of management systems. Hence, this paper focuses on a decision-making method. Linguistic interval-valued q-rung orthopair fuzzy sets (LIVq-ROFSs) have recently been proposed as being effective in describing decision-makers' evaluation values in complex situations. This paper proposes a novel multi-attribute group decision-making (MAGDM) method with LIVq-ROFSs to handle realistic decision-making problems. The main contributions of this study are three-fold. First, a new method for determining the weight information of attributes based on decision makers' evaluation values is proposed. Second, the classical TODIM is extended into LIVq-ROFSs and a new decision-making method is proposed. Third, our proposed MAGDM method is applied to a real decision-making problem to reveal its effectiveness.

Keywords: linguistic interval-valued q-rung orthopair fuzzy sets; attributes' weights determination methods; TODIM; multi-attribute group decision making

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1. Introduction

Decision making is one of the most important activities in economics, management, and even daily life. Recently, more and more researchers have paid attention to multi-attribute group decision making (MAGDM). MAGDM refers to a collection of decision-making problems in which a group of experts are invited to express their opinions over alternatives under a set of attributes. MAGDM method collects different opinions from different experts, making the final decision-making results fairer and more reliable. The recently proposed q-rung orthopair fuzzy set (q-ROFS) [1] characterized by a membership degree and a non-membership degree is a powerful tool to handle decision makers' complex evaluation information. In addition, the advantages and superiorities of q-ROFSs over intuitionistic fuzzy sets (IFSs) [2] and Pythagorean fuzzy sets (PFSs) [3] are widely known in the field of MAGDM. Hence, q-ROFSs have been widely applied in real MAGDM problems and many novel achievements have been reported. For instance, Jana et al. [4] introduced a new series of new operations for q-rung orthopair fuzzy numbers (q-ROFNs) and, based on which, a novel collection of q-rung orthopair fuzzy aggregation operators is developed. Yu et al. [5] extended the classical q-ROFSs into q-rung orthopair cubic fuzzy sets, introduced their Maclaurin symmetric mean operators, and applied them to an emerging technology enterprises evaluation problem. Wang and Yang [6] considered MAGDM situations in which a partitioned relationship exists among q-ROFNs, and they introduced a series of q-rung orthopair fuzzy partitioned Bonferroni means. Liu et al. [7] combined the power average and Maclaurin symmetric mean operators under q-ROFSs and introduced some hybrid aggregation operators. Comparative analysis illustrates the advantages and superiorities of the proposed operators. Based on continuous Archimedean T-Norms and

T-Conorms, Ai et al. [8] introduced a series of q-rung orthopair fuzzy integrals for aggregating continuous q-ROFNs, and applied them in MAGDM. Wei et al. [9] introduced a series of aggregations operators for fusing q-rung orthopair fuzzy information, studied their properties and applied them in decision-making. For more recent developments of q-ROFS-based MAGDM methods, readers are suggested to refer to [10–14].

The above references reveal that the q-ROFSs have good performance in describing decision makers' fuzzy and uncertain decision-making information in realistic MAGDM problems. However, sometimes q-ROFSs still have defects when describing decision-makers' assessments. As a matter of fact, sometimes decision makers prefer to use linguistic terms to present membership degrees and non-membership degrees. For the sake of convenience, we call them linguistic membership degrees (LMDs) and linguistic non-membership degrees (LNMDs). For example, Zhang [15] generalized the traditional IFSs into linguistic IFSs (LIFSs), where the intuitionistic fuzzy membership and non-membership degrees are denoted by linguistic terms. Due to this characteristic, LIFSs are more powerful and suitable than IFSs, and they have received much attention in the field of MAGDM [16–18]. Afterward, motived by LIFSs, Garg [19] introduced the concept of linguistic Pythagorean fuzzy sets (LPFSs), where LMDs and LNMDs are employed in the traditional way to PFSs, and linguistic Pythagorean fuzzy sets (LPFSs) are developed. As PFSs are more powerful than IFSs, LPFSs are also more flexible and robust than LIFSs. Similarly, LPFSs have also received much attention and readers can find LPFS-based decision-making methods in [20–22]. However, LIFSs and LPFSs still have shortcomings when handling realistic MAGDM problems. We provide the following instance to illustrate their drawbacks. Let $S = \{s_t | 0 \leq t \leq l\}$ be a continuous linguistic term set with odd cardinality and $\alpha = (s_\delta, s_\eta)$ be a linguistic fuzzy number defined on S. If α is a linguistic intuitionistic fuzzy number, then $\delta + \eta \leq l$; and if α is a linguistic Pythagorean fuzzy number, then $\delta^2 + \eta^2 \leq l^2$. Now a group of professors are invited to evaluate the quality of discipline. Let S = {s_0 = '*very poor*', s_1 = '*poor*', s_2 = '*slightly poor*', s_3 = '*fair*', s_4 = '*slightly good*', s_5 = '*good*', s_6 = '*very good*'} be a linguistic term set, then the group of professors would like to use s_4 and s_5 to be the LMD and LNMD, respectively. Obviously, the ordered pair cannot be denoted by LIFSs and LPFSs. This example indicates the drawback of LIFSs and LPFSs.

To overcome the defect of LIFSs and LPFSs, Liu and Liu [23] proposed the concept of linguistic q-rung orthopair fuzzy sets (Lq-ROFSs), which use LMD and LNMD in the classical q-ROFSs. Lq-ROFSs take the advantages of q-ROFSs, and, hence, they are more powerful and flexible than LIFs and LPFSs. Since its appearance, the Lq-ROFS-based MAGDM method has become a new research direction. Liu et al. [24] investigated their point aggregation operators, discussed their properties, and applied them to MAGDM problems. Based on the linguistic scale function, Liu et al. [25] extended the reference deal TOPSIS method into Lq-ROFSs, proposed a novel MAGDM method, and applied it to postgraduate entrance qualification assessment. Bao and Shi [26] introduced the ELECTRE-based MAGDM method under Lq-ROFSs and used robot evaluation and selection. Liu and Liu [27] developed a series of linguistic q-rung orthopair fuzzy power Muirhead mean operators, which not only reduce the bad effect of unduly high or low aggregated values, but also consider the interrelationship among aggregated values. More recent developments of Lq-ROFS-based MAGDM methods can be found in [28–32]. Recently, the idea of linguistic interval-valued q-rung orthopair fuzzy sets (LIVq-ROFSs) [33] was developed for handling realistic MAGDM problems. In a LIVq-ROFS, two uncertain linguistic variables are used to denote the LMD and LNMD. Hence, LIVq-ROFS can be regarded as a general form of Lq-ROFS and a linguistic term. Afterward, more and more scholars started to investigate LIVq-ROFSs as well as their applications in decision-making. For example, Gong [34] proposed a series of entropy measures and used them to determine the weight vector of attributes in decision making. Gurmani et al. [35] proposed basic operational rules as well as a new score function of LIVq-ROFSs, and finally, a VIKOR-based MAGDM method was developed.

The above-mentioned references reveal the good performance of LIVq-ROFSs in handling MAGDM problems. However, recent research still has limitations. In most research on LIVq-ROFSs, the weight vector of attributes is usually assumed to be known, which is somewhat inconsistent with reality. Due to the complexity of real MAGDM problems, the weight information of attributes is usually unknown. However, most LIVq-ROFS-based MAGDM methods only consider known attributes. Hence, sometimes they are powerless to handle realistic MAGDM problems. Although Khan et al.'s [33] method considers unknown attribute weight information, they only consider incomplete weight information. In other words, Khan et al.'s [33] method can only deal with MAGDM problems where weight information is partially known. However, we often encounter completely unknown attribute weight situations. Hence, it is necessary to propose a new MAGDM method for handling completely unknown weight situations. In addition, the TODIM (an acronym in Portuguese for Iterative Multi-criteria Decision Making) method introduced by Gomes and Lima [36] is a powerful decision-making approach. The primary advantage of TODIM is that it takes decision-makers' psychological behaviors into consideration when determining the ranking outcome of alternatives. Hence, the ranking results produced by TODIM are usually more reliable than some other methods. Due to these advantages, TODIM has been widely used in picture fuzzy sets [37], neutrosophic sets [38], Pythagorean fuzzy sets [39], complex interval-valued intuitionistic fuzzy sets [40], etc. Nevertheless, nothing has been performed with TODIM under LIVq-ROFSs. Considering the advantage of TODIM, it is of high necessity to study TODIM under LIVq-ROFSs.

The main works of this study are three-fold. First, we study TODIM under LIVq-ROFSs and propose a new MAGDM method. Second, a method for determining the weight information of attributes is proposed. Third, our proposed MAGDM is applied to a realistic problem to illustrate its rightness and advantages. The rest of this paper is organized as follows. Section 2 reviews some basic notions that will be used in the following sections. Section 3 proposes a new MAGDM method under LIVq-ROFSs. Section 4 applies the proposed method to a realistic MAGDM problem to demonstrate its effectiveness. Summarization is presented in Section 5.

2. Some Basic Notions

In this Section, some basic notions are reviewed.

2.1. Linguistic Interval-Valued q-Rung Orthopair Fuzzy Sets

Definition 1 [33]. *Let $X = \{x_1, x_2, \ldots, x_n\}$ be a finite set, and $S = \{s_t | 0 \leq t \leq l\}$ is a continuous linguistic term set. Then, a linguistic interval-valued q-rung orthopair fuzzy set (LIV-q-ROFS) A defined on X can be expressed as*

$$X = \{x_i, \widetilde{s}_M(x_i), \widetilde{s}_N(x_i) | x_i \in X\} \quad (1)$$

where $\widetilde{s}_M = [\widetilde{s}_{m_1}(x_i), \widetilde{s}_{m_2}(x_i)]$ and $\widetilde{s}_N = [\widetilde{s}_{n_1}(x_i), \widetilde{s}_{n_2}(x_i)]$ are all subsets of $[s_0, s_L]$ in which s_0 and s_L represent the minimum and maximum linguistic variable of the linguistic term set, and are said to be the linguistic membership degree and linguistic non-membership degree of the element $x_i \in X$ to S. For any $x_i \in X$, $(\widetilde{s}_{m_2})^q + (\widetilde{s}_{n_2})^q \leq \widetilde{s}_l^q$ (i.e., $m_2^q + n_2^q \leq l^q$) always satisfies for $x_i \in X$. For the sake of convenience, we call the order pair $\left([\widetilde{s}_{m_1}(x_i), \widetilde{s}_{m_2}(x_i)], [\widetilde{s}_{n_1}(x_i), \widetilde{s}_{n_2}(x_i)]\right)$ a linguistic interval-valued q-rung orthopair fuzzy value (LIVq-ROFV), which can be denoted by $\xi = ([\widetilde{s}_a, \widetilde{s}_b], [\widetilde{s}_c, \widetilde{s}_d])$ for simplicity.

Some basic operational rules of LIVq-ROFVs are presented by Khan et al. [33].

Definition 2 [33]. *Let $\xi_1 = \left([\widetilde{s}_{a_1}, \widetilde{s}_{b_1}], [\widetilde{s}_{c_1}, \widetilde{s}_{d_1}]\right)$, $\xi_2 = \left([\widetilde{s}_{a_2}, \widetilde{s}_{b_2}], [\widetilde{s}_{c_2}, \widetilde{s}_{d_2}]\right)$ and $\xi = ([\widetilde{s}_a, \widetilde{s}_b], [\widetilde{s}_c, \widetilde{s}_d])$ by any three LIVq-ROFVs defined on a continuous linguistic term set $S = \{s_t | 0 \leq t \leq l\}$, and λ be a positive real number, then*

(1) $\xi_1 \oplus \xi_2 = \left(\left[\widetilde{s}_{(a_1^q+a_2^q-a_1^q a_2^q/l^q)^{1/q}}, \widetilde{s}_{(b_1^q+b_2^q-b_1^q b_2^q/l^q)^{1/q}}\right], \left[\widetilde{s}_{c_1c_2/l}, \widetilde{s}_{d_1d_2/l}\right]\right);$

(2) $\xi_1 \otimes \xi_2 = \left(\left[\widetilde{s}_{a_1a_2/l}, \widetilde{s}_{b_1b_2/l}\right], \left[\widetilde{s}_{(c_1^q+c_2^q-c_1^q c_2^q/l^q)^{1/q}}, \widetilde{s}_{(d_1^q+d_2^q-d_1^q d_2^q/l^q)^{1/q}}\right]\right);$

(3) $\lambda\xi = \left(\left[\widetilde{s}_{(l^q-l^q(1-a^q/l^q)^\lambda)^{1/q}}, \widetilde{s}_{(l^q-l^q(1-b^q/l^q)^\lambda)^{1/q}}\right], \left[\widetilde{s}_{l(c/l)^\lambda}, \widetilde{s}_{l(d/l)^\lambda}\right]\right);$

(4) $\xi^\lambda = \left(\left[\widetilde{s}_{l(a/l)^\lambda}, \widetilde{s}_{l(b/l)^\lambda}\right], \left[\widetilde{s}_{(l^q-l^q(1-c^q/l^q)^\lambda)^{1/q}}, \widetilde{s}_{(l^q-l^q(1-d^q/l^q)^\lambda)^{1/q}}\right]\right).$

Khan et al. [33] also defined the score and accuracy values and comparison rules to compare any two LIVq-ROFVs.

Definition 3 [33]. *Let* $\xi = ([\widetilde{s}_a, \widetilde{s}_b], [\widetilde{s}_c, \widetilde{s}_d])$ *be a LIVq-ROFV defined on a continuous linguistic term set* $S = \{s_t | 0 \le t \le l\}$. *The score and accuracy values of* ξ *can be defined by*

$$S(\xi) = \widetilde{s}_{\left(\frac{l^q+a^q+b^q-c^q-d^q}{4}\right)^{1/q}} \tag{2}$$

and

$$H(\xi) = \widetilde{s}_{\left(\frac{l^q+a^q+b^q+c^q+d^q}{4}\right)^{1/q}} \tag{3}$$

Based on the score and accuracy values of LIVq-ROFVs, the comparison method for any two LIVq-ROFVs is presented as follows.

Definition 4 [33]. *Let* $\xi_1 = \left(\left[\widetilde{s}_{a_1}, \widetilde{s}_{b_1}\right], \left[\widetilde{s}_{c_1}, \widetilde{s}_{d_1}\right]\right)$ *and* $\xi_2 = \left(\left[\widetilde{s}_{a_2}, \widetilde{s}_{b_2}\right], \left[\widetilde{s}_{c_2}, \widetilde{s}_{d_2}\right]\right)$ *be any two LIVq-ROFVs defined on a continuous linguistic term set* $S = \{s_t | 0 \le t \le l\}$, *then we have*

(1) *If* $S(\xi_1) < S(\xi_2)$, *then* $\xi_1 < \xi_2$;
(2) *If* $S(\xi_1) = S(\xi_2)$, *then*
(3) *If* $H(\xi_1) < H(\xi_2)$, *then* $\xi_1 < \xi_2$;
(4) *If* $H(\xi_1) = H(\xi_2)$, *then* $\xi_1 = \xi_2$.

The distance measure between any two LIVq-ROFVs is presented as follows.

Definition 5 [33]. *Let* $\xi_1 = \left(\left[\widetilde{s}_{a_1}, \widetilde{s}_{b_1}\right], \left[\widetilde{s}_{c_1}, \widetilde{s}_{d_1}\right]\right)$ *and* $\xi_2 = \left(\left[\widetilde{s}_{a_2}, \widetilde{s}_{b_2}\right], \left[\widetilde{s}_{c_2}, \widetilde{s}_{d_2}\right]\right)$ *be any two LIVq-ROFVs defined on a continuous linguistic term set* $S = \{s_t | 0 \le t \le l\}$. *The generalized distance measure between* ξ_1 *and* ξ_2 *is defined as follows*

$$d(\xi_1, \xi_2) = \left(\frac{1}{4(l^q)^\delta}\left(\left|a_1^q - a_2^q\right|^\delta + \left|b_1^q - b_2^q\right|^\delta + \left|c_1^q - c_2^q\right|^\delta + \left|d_1^q - d_2^q\right|^\delta\right)\right)^{1/\delta} \tag{4}$$

In the formula, the above-generalized distance of LIVq-ROFVs can be reduced to Euclidean distance when $\delta = 2$, and reduce to Hamming distance when $\delta = 1$.

To aggregate a series of LIVq-ROFVs, Khan et al. [33] introduced the linguistic interval-valued q-rung orthopair fuzzy weighted average (LIVq-ROFWA) operator.

Definition 6 [33]. *Let* $\xi_j = \left(\left[\widetilde{s}_{a_j}, \widetilde{s}_{b_j}\right], \left[\widetilde{s}_{c_j}, \widetilde{s}_{d_j}\right]\right) (j = 1, 2, \ldots, n)$ *be a collection of LIVq-ROFVs, and* $w = (w_1, w_2, \ldots, w_n)^T$ *be the corresponding weight vector, satisfying* $\sum_{j=1}^n w_j = 1$ *and* $0 \le w_j \le 1$. *The LIVq-ROFWA operator is defined as follows*

$$LIVq - ROFWA(\xi_1, \xi_2, \ldots, \xi_n) = \sum_{j=1}^n w_j \xi_j \tag{5}$$

2.2. TODIM

The TODIM is a powerful method as it can consider decision-makers' psychological behaviors. Hence, decision results produced by TODIM are more reliable. The main steps of TODIM are briefly introduced as follows. Let $A = \{A_1, A_2, \ldots, A_m\}$ be a collection of alternatives that are to be evaluated and $G = \{G_1, G_2, \ldots, G_n\}$ be a collection of attributes. A group of decision makers uses crisp numbers to describe their evaluation information and finally, a decision matrix is obtained, which can be denoted as $X = \left(x_{ij}\right)_{m \times n}$. The weight vector of attributes is $w = (w_1, w_2, \ldots, w_n)^T$, such that $\sum_{j=1}^{n} w_j = 1$ and $0 \leq w_j \leq 1$. For a convenient description, we use $Z = \left(z_{ij}\right)_{m \times n}$ to denote the normalized decision matrix. The main steps of TODIM are presented as follows.

Step 1. Calculate the relative weight of attribute G_k to a reference attribute G_r, which is

$$w_{kr} = w_k / w_r, \tag{6}$$

where w_k is the weight of attribute G_k and $w_r = \max\{w_j | j = 1, 2, \ldots, n\}$.

Step 2. Calculate the dominance degree of alternative A_i over A_j with respect to attribute G_k by

$$\Phi_k(A_i, A_j) = \begin{cases} \sqrt{w_{kr}\left(z_{ik} - z_{jk}\right) / \sum_{l=1}^{n} w_{lr}}, \; if \; z_{ik} \geq z_{jk} \\ -\frac{1}{\theta}\sqrt{\frac{\sum_{l=1}^{n} w_{lr}\left(z_{jk} - z_{ik}\right)}{w_{kr}}}, \; if \; z_{ik} < z_{jk} \end{cases}, \tag{7}$$

where $\theta > 0$, representing the attenuation factor of the losses.

Step 3. Calculate the overall dominance degree of alternative A_i over A_j with the following

$$\Phi(A_i, A_j) = \sum_{k=1}^{n} \Phi_k(A_i, A_j). \tag{8}$$

Step 4. Calculate the overall performance of alternative A_i with the following

$$\phi(A_i) = \sum_{j=1}^{m} \Phi(A_i, A_j). \tag{9}$$

Step 5. Calculate the normalized overall performance of alternative A_i with the following

$$\omega(A_i) = \frac{\phi(A_i) - min\phi(A_j)}{max\phi(A_j) - min\phi(A_j)}, \tag{10}$$

Step 6. Rank the alternative according to $\xi(A_i)$.

3. A Novel MAGDM Method as Well as Its Detailed Steps

This study introduced the TODIM under a linguistic interval-valued q-rung orthopair fuzzy environment. In other words, this paper proposed a novel MAGDM method based on TODIM for dealing with decision-making problems wherein decision-makers' evaluations are expressed by LIVq-ROFVs. This Section introduces the detailed steps of our proposed method. In order to do this, we first introduce the description of a typical MAGDM problem under LIVq-ROFSs. Afterward, a method for determining the weight information of attributes is proposed. Finally, the steps of our proposed MAGDM method are presented.

3.1. Description of a Typical MAGDM Problem Based on LIVq-ROFSs

Let us consider a MAGDM problem based on LIVq-ROFSs. We assume there is a set of alternatives $A = \{A_1, A_2, \ldots, A_m\}$ and a collection of attributes, denoted by $G = \{G_1, G_2, \ldots, G_n\}$. The weight information of attributes is completely unknown. To evaluate the m alternatives under the n attributes, a group of decision makers are invited to express their opinions and the group of decision makers can be denoted by $D = \{D_1, D_2, \ldots, D_e\}$. The weight of decision makers is $\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_e)^t$, such that $\sum_{f=1}^{e} \lambda_f = 1$ and $0 \leq$

$\lambda_f \leq 1$. Let $S = \{s_t | 0 \leq t \leq l\}$ be a pre-defined continuous linguistic term set, and for the decision maker $D_f(f = 1, 2, \ldots, e)$ utilizes a LIVq-ROFV $\xi_{ij}^f = \left(\left[\widetilde{s}_{a_{ij}^f}, \widetilde{s}_{b_{ij}^f}\right], \left[\widetilde{s}_{c_{ij}^f}, \widetilde{s}_{d_{ij}^f}\right]\right)$ to express his/her opinion for the attribute $G_j(j = 1, 2, \ldots, n)$ of alternative $A_i(i = 1, 2, \ldots, n)$. Finally, we can obtain a collection of linguistic interval-valued q-rung orthopair fuzzy decision matrices, denoted by $M^f = \left(\xi_{ij}^f\right)_{m \times n}$.

3.2. The Process of Determining the Weight Vector of Attributes

It is known that the weight information of attributes plays an important role. However, it is not easy to determine the weight vector of attributes. Generally speaking, decision makers can provide the weight information subjectively after discussion and negotiation. However, it is not a sufficient method to determine the attribute vector, because of the complexity of decision-making problems. Therefore, it is necessary to determine the weights of attributes by some objective methods. As a matter of fact, many scholars have realized the importance of attribute weight determination and some methods to objectively determine weight information have been proposed. For example, Ye [41] proposed a method to determine weight information based on the assumption that reasonable weight values of attributes should make the overall score value of all alternatives as large as possible. By maximizing group consensus, Zhang [42] introduced a nonlinear optimization model to determine the attributes' weights under hesitant fuzzy sets. Biswas and Sarkar [43] proposed an entropy measure for PFSs and based on which, they introduced a method to determine the attributes' weights for handling Pythagorean fuzzy MAGDM. In [44], a q-rung orthopair fuzzy criteria importance through an inter-criteria correlation method based on derived weights of decision makers, and standard deviation and correlation coefficient were developed to determine weights of attributes. Recently, Zhang et al. [45] proposed a method to calculate the weights of attributes under probabilistic linguistic term sets. In their opinions, the weight vector of attributes should not only make the total deviation between all alternatives and the positive ideal solution to be a minimum, but also make the Shannon information entropy be a maximum. Later on, Tang et al. [46] incorporated the above ideas under probabilistic dual Pythagorean hesitant fuzzy sets and introduced an optimization model to calculate the weights of attributes. In this paper, we also use the main ideas of Zhang et al. [45] to determine the weight information of attributes under LIVq-ROFSs. The above-mentioned references reveal that the idea is effective for determining the attributes' weight information. Hence, this paper incorporates this idea when calculating the vector of attributes. According to Zhang et al. [45] and Tang et al.'s [46] studies, when determining the weights of attributes, the following two rules should be taken into consideration. In the following, for convenient expression, we assume that the comprehensive decision matrix is denoted by $A = (a_{ij})_{m \times n}$, where a_{ij} is a crisp number. Let $w = (w_1, w_2, \ldots, w_n)^T$, such that $\sum_{j=1}^{n} w_j = 1$ and $0 \leq w_j \leq 1$. Let $a_j^* = max_i a_{ij}$.

Rule I. The weight vector of attributes should make the total deviation between all the alternatives and the positive ideal solution (PIS) to be a minimum.

Rule II. The weight vector should make the Shannon information entropy be a maximum.

When considering Rule I, we can use

$$\Delta_j(w) = w_j \sqrt{\sum_{i=1}^{m} \left(a_j^* - a_{ij}\right)^2}, \tag{11}$$

to denote the deviation of alternative A_i and a_j^* with respect to attribute G_j. Therefore, the sum of all deviations can be denoted as follows

$$\Delta(w) = \sum_{j=1}^{n} \Delta_j(w) = \sum_{j=1}^{n} w_j \sqrt{\sum_{i=1}^{m} \left(a_j^* - a_{ij}\right)^2}, \tag{12}$$

When considering Rule II, we can use

$$H = -k\sum_{j=1}^{n} w_j ln w_j, \quad (13)$$

to denote the Shannon information entropy measure. Hence, by considering both Rule I and Rule II, we can establish the following optimization model, i.e.,

$$min\left\{u\sum_{j=1}^{n} w_j\sqrt{\sum_{i=1}^{m}\left(a_j^* - a_{ij}\right)^2} + (1-u)\sum_{j=1}^{n} w_j ln w_j\right\} \text{ s. t. } w_j \geq 0 \text{ and } \sum_{j}^{m} w_j = 1$$

where $1 > u > 0$ denotes the balance coefficient between the two mentioned objectives. Generally, we usually assume that the two objectives have the same importance, and we can set $u = 0.5$, and the following result by solving this optimization model can be obtained, which is shown as follows:

$$w_j = \frac{exp\left(-\frac{u}{1-u}\sqrt{\sum_{i=1}^{n}\left(b_j^* - b_{ij}\right)^2} - 1\right)}{\sum_{j=1}^{n} exp\left(-\frac{u}{1-u}\sqrt{\sum_{i=1}^{n}\left(b_j^* - b_{ij}\right)^2} - 1\right)}, \quad (14)$$

Based on the above analysis, and the attributes' weights determination method, in the following, we present our proposed MAGDM method based on TODIM.

Step 1. Normalize the original decision matrices. As there are benefit and cost types of attributes, the original decision matrices should be transformed into

$$\xi_{ij}^f = \begin{cases} \left(\left[\widetilde{s}_{a_{ij}^f}, \widetilde{s}_{b_{ij}^f}\right], \left[\widetilde{s}_{c_{ij}^f}, \widetilde{s}_{d_{ij}^f}\right]\right), \ G_j \in I_1 \\ \left(\left[\widetilde{s}_{c_{ij}^f}, \widetilde{s}_{d_{ij}^f}\right], \left[\widetilde{s}_{a_{ij}^f}, \widetilde{s}_{b_{ij}^f}\right]\right), G_j \in I_2 \end{cases}, \quad (15)$$

where I_1 and I_2 denote benefit and cost types of attributes, respectively.

Step 2. For attribute $G_j(j = 1, 2, \ldots, n)$ of alternative $A_i(i = 1, 2, \ldots, m)$, use the LIVq-ROFWA operator to calculate the overall evaluation value, i.e.,

$$\xi_{ij} = LIVq - ROFWA\left(\xi_{ij}^1, \xi_{ij}^2, \ldots, \xi_{ij}^f\right), \quad (16)$$

Step 3. Calculate the weights of attributes according to Equation (13).

Step 4. Calculate the relative weight of attribute G_k to a reference attribute G_r, that is

$$w_{kr} = w_k / w_r, \quad (17)$$

where w_k is the weight of attribute G_k and $w_r = max\{w_j | j = 1, 2, \ldots, n\}$.

Step 5. Calculate the dominance degree of alternative A_i over A_j with respect to attribute G_k by

$$\Phi(A_i, A_j) = \begin{cases} \sqrt{d\left(\xi_{ik}, \xi_{jk}\right)w_{kr} / \sum_{l=1}^{n} w_{lr}}, \ if \ \xi_{ik} \geq \xi_{jk} \\ -\frac{1}{\theta}\sqrt{\frac{\sum_{l=1}^{n} w_{lr} d\left(\xi_{ik}, \xi_{jk}\right)}{w_{kr}}}, \ if \ \xi_{ik} < \xi_{jk} \end{cases}, \quad (18)$$

where $d\left(\xi_{ik}, \xi_{jk}\right)$ denotes the distance between ξ_{ik} and ξ_{jk}, $\theta > 0$, representing the attenuation factor of the losses.

Step 6. Calculate the overall dominance degree of alternative A_i over A_j with the following

$$\Phi(A_i, A_j) = \sum_{k=1}^{n} \Phi_k(A_i, A_j), \quad (19)$$

Step 7. Calculate the overall performance of alternative A_i with the following

$$\phi(A_i) = \sum_{j=1}^{m} \phi(A_i, A_j), \tag{20}$$

Step 8. Calculate the normalized overall performance of alternative A_i with the following

$$\xi(A_i) = \frac{\phi(A_i) - min\phi(A_j)}{max\phi(A_j) - min\phi(A_j)}, \tag{21}$$

Step 9. Rank the alternative according to $\omega(A_i)$.

4. Numerical Example

This Section presents a numerical example to demonstrate the calculation process of our proposed MAGDM method. Operating in an innovation ecosystem—the synthesis of your new offerings and that of other firms creates a coherent customer solution—carries risk [47]. Because most breakthrough innovations cannot succeed in isolation. They need complementary innovation to attract customers. For example, in the early 1990s, high-quality HDTV plans seized the mass market. However, the key supporting elements—signal compression technology and broadcasting standards—have not yet been introduced. However, as high-definition TV manufacturers wait for complementary innovations to catch up, a new competitive landscape and competitors have already emerged. Therefore, the success of the final delivery of a breakthrough product depends on the development and deployment of all other solution components [48]. In many aspects of breakthrough innovation delivery, team motivation issues, supply issues, financial difficulties, and leadership crises can all disrupt the projects of partners [49]. Enterprises need to coordinate all stakeholders involved in the innovation ecosystem, and evaluate the delay risks caused by the adoption of each component of innovation in this stage (such as supply risks caused by rising raw material prices). And based on the level of all delay risks, make hierarchical decisions on the classification and optimization methods of each risk, specifically allocating which risks should be handled internally and which risks should be shared with third parties [50].

4.1. The Description of the Problem

Assuming that a technology enterprise uses MAGDM analysis to assess the delivery risks of four key complementary components that need to be submitted to the innovation ecosystem in order to determine the delivery risk of a breakthrough invention, namely, the delivery of battery components (A_1), microchip components (A_2), screen components (A_3), and system programs (A_4). Once the causes of risks are identified, optimization solutions typically emerge. For example, if a company relies excessively on a single partner leading to high adoption segment prices, the enterprise might design a product with a flexible interface, supporting the integration of more collaborators. This type of risk will be "Handled Internally by the Enterprise" (G_1). If complementary companies lack the motivation to develop their products, they might reach an exclusive licensing agreement, thus alleviating concerns about competition from complementary merchants. This type of risk will be "Jointly Handled by the Enterprise and Complementary Businesses" (G_2). If the complementary product launches late or is priced too high, the company might seek new partners. This type of risk will be "Handled by a Third Party" (G_3). Consequently, six types of businesses are ultimately categorized into three types of optimization methods.

The project decision-making team is composed of three groups of executives with technical backgrounds, elected from the enterprise and complementary merchants, including a group of experts in the field of electromechanical engineering (D_1), a group of experts in product design (D_2), and a group of information technology experts (D_3).

They express their preferences about the four types of businesses using multiplicative preference relations, additive preference relations, and linguistic distribution preference relations, respectively. Among them, the decision makers use a set of linguistic terms: very poor, poor, fair, good, and very good.

We have delineated four pivotal risk indicators: initiative risks, interdependence risks, integration risks [47], and structural inertia risks [51]. Initiative risks embody the familiar uncertainties of managing a project. Interdependence risks involve the uncertainties of coordinating with complementary innovators. Integration risks are the uncertainties presented by the adoption process across the value chain. Structural inertia risks are the uncertainty of the structural inertia of the innovation ecosystem.

Initiative risks (G_1): This attribute is utilized to evaluate the inherent uncertainties involved in managing innovation projects by leading enterprises within an innovation ecosystem [47]. The assessment of initiative risks involves the leading enterprise evaluating the internal factors of the project that could lead to delays, with a focus on the project's own internal elements such as the feasibility of the technology, the accurate assessment of market demands, the appropriateness of resource allocation, and the team's execution capability [52]. Regardless of whether the breakthrough innovation project is a high-tech chip, such as fast-moving consumer goods such as breakfast cereal or an intangible financial service product, it faces the risk of market loss due to the inability to deliver on time and according to specifications. In addressing such risks, the emphasis is on accurately assessing the challenges and potential obstacles faced by the project and formulating corresponding strategies to mitigate these risks, ensuring the smooth progression of the project.

Interdependence risks (G_2): This attribute is employed to evaluate the uncertainties encountered during the coordination process between leading enterprises and complementors, a common risk in innovation ecosystems [47]. In a large-scale technology project, the success of the project may not only depend on the efforts of the leading enterprise but also on the timely delivery of critical components or services by multiple partners [53]. Failure to deliver as promised by any party leads to overall delays, potentially lasting weeks, months, or even years. Not only the party falling behind but also all parties that complement it bear the consequences of the delay. Assessing interdependence risks entails evaluating each complementor of the product. For example, partners may cause delays due to internal development challenges, regulatory gaps, motivational issues, financial difficulties, leadership crises, or even their own interdependencies with other parties [54]. If a project exhibits high interdependence risk, managers can specifically optimize response measures and set reasonable delivery timelines.

Integration risks (G_3): This attribute is used to assess the uncertainties associated with the adoption process of a product or service throughout the entire value chain [47]. Especially in complex ecosystems, the success of an innovation depends not only on the acceptance by end-users but also on the adoption by intermediaries or distributors [55]. The management of integration risks involves ensuring that the innovative project can be smoothly integrated into the existing market structures and value chains, which includes understanding and influencing the decision-making factors at various intermediary stages. For instance, an innovative medical device might require the approval and support of hospitals, insurance companies, and government agencies for successful market entry. Each adopter along the value chain requires time to understand the product, agree to test it, and accept the trial results. The higher the innovation's position on the value chain, the greater the number of intermediaries that must adopt the innovation before it reaches the delivery and sales stage. Once the assessment of integration risks is completed, a series of moderate improvements along the downstream chain (such as coordinated design, pre-marketing, and channel incentive management) may expedite the product's reach to the consumer and potentially require fewer resources.

Structural Inertia Risks (G_4): This attribute is used to assess the uncertainty of innovation being affected by the structural inertia of the ecosystem. According to Organizational Ecology Theory, the innovation of ecosystems is also influenced by structural inertia [56]. Generally speaking, the older the complementors and intermediaries involved in an innovation, the more likely it is to be influenced by the structural inertia of the innovation ecosystem [51,57]. The presence of structural inertia implies that organizations tend to maintain the status quo, making it difficult to respond quickly to dynamic market environments [58]. Specifically, structural inertia can lead to members of the innovation ecosystem being subject to legitimacy constraints, and forming unique behavioral norms and a fixed information processing structure. This affects the operational efficiency of certain parts of the innovation ecosystem, thereby increasing the probability of delivery delays; especially in complex ecosystems such as healthcare, where businesses often turn to the government to help overcome this inertia. For instance, many IT suppliers are currently lobbying their governments to mandate the implementation of digital medical records. These efforts replace one mode of delay (legislative and administrative lags) for another (the monumental collective inertia of the health care system).

To determine the best alternative, a group of decision experts (D_1, D_2 and D_3) are invited to evaluate the performance of the four alternatives under the attributes. We assume the weight vector of decision makers is $\lambda = (0.3, 0.2, 0.5)^T$. Let S = {s_0 = '*Extremely poor*', s_1 = '*very poor*', s_2 = '*poor*', s_3 = '*slightly poor*', s_4 = '*fair*', s_5 = '*slightly good*', s_6 = '*good*', and s_7 = '*very good*'} be a linguistic term set. The group of decision makers use LIVq-ROFVs to express their evaluation values and the original decision matrixes are presented in Tables 1–3. In the following, we use the proposed MAGDM method that presents the following.

Table 1. The original decision matrix provided by D_1.

	G_1	G_2	G_3	G_4
A_1	$([s_2, s_3][s_3, s_7])$	$([s_3, s_4][s_3, s_8])$	$([s_1, s_6][s_2, s_4])$	$([s_3, s_6][s_4, s_5])$
A_2	$([s_2, s_2][s_2, s_5])$	$([s_4, s_6][s_1, s_5])$	$([s_3, s_7][s_1, s_2])$	$([s_2, s_3][s_4, s_6])$
A_3	$([s_3, s_5][s_1, s_4])$	$([s_5, s_6][s_1, s_3])$	$([s_1, s_4][s_3, s_3])$	$([s_1, s_2][s_4, s_7])$
A_4	$([s_1, s_4][s_2, s_5])$	$([s_2, s_3][s_3, s_5])$	$([s_6, s_7][s_1, s_3])$	$([s_1, s_5][s_2, s_4])$

Table 2. The original decision matrix provided by D_2.

	G_1	G_2	G_3	G_4
A_1	$([s_2, s_6][s_3, s_4])$	$([s_1, s_3][s_3, s_5])$	$([s_2, s_3][s_5, s_6])$	$([s_2, s_3][s_2, s_7])$
A_2	$([s_3, s_4][s_6, s_6])$	$([s_3, s_4][s_6, s_7])$	$([s_1, s_6][s_2, s_5])$	$([s_3, s_3][s_7, s_7])$
A_3	$([s_5, s_6][s_2, s_6])$	$([s_5, s_6][s_1, s_2])$	$([s_4, s_5][s_2, s_6])$	$([s_2, s_6][s_4, s_6])$
A_4	$([S_3, S_6][S_3, S_5])$	$([S_4, S_4][S_4, S_4])$	$([S_3, S_7][S_2, S_4])$	$([S_2, S_3][S_1, S_3])$

Table 3. The original decision matrix provided by D_3.

	G_1	G_2	G_3	G_4
A_1	$([s_3, s_4][s_1, s_6])$	$([s_3, s_4][s_4, s_7])$	$([s_3, s_5][s_3, s_4])$	$([s_2, s_7][s_5, s_6])$
A_2	$([s_4, s_7][s_6, s_6])$	$([s_4, s_5][s_3, s_7])$	$([s_6, s_7][s_2, s_3])$	$([s_4, s_5][s_3, s_7])$
A_3	$([s_2, s_6][s_1, s_6])$	$([s_2, s_7][s_2, s_6])$	$([s_2, s_4][s_1, s_2])$	$([s_3, s_7][s_1, s_2])$
A_4	$([s_1, s_2][s_1, s_6])$	$([s_4, s_6][s_1, s_7])$	$([s_5, s_6][s_2, s_3])$	$([s_3, s_6][s_5, s_7])$

4.2. The Decision-Making Process

Step 1. Standardize the original decision matric (seen in Table 1), which is the same as the step 1 in Approach I.

Step 2. Compute the relative weight of attribute G_j to the reference attribute G_r. According to Equation (13)

$$w_{jr} = (0.9857, 0.7538, 1, 0.8719)^{\mathrm{T}}$$

Step 3. For attribute $G_j(j = 1, 2, \ldots, n)$ of alternative $A_i(i = 1, 2, \ldots, m)$, use the LIVq-ROFWA operator to calculate the overall evaluation value, which is shown in Table 4.

Table 4. The overall evaluation values of the alternatives.

	G_1	G_2
A_1	$([s_{2.6008}, s_{4.7921}], [s_{1.7321}, s_{5.4792}])$	$([s_{2.6843}, s_{3.7612}], [s_{3.4641}, s_{6.4992}])$
A_2	$([s_{3.4835}, s_{6.1294}], [s_{6.1294}, s_{5.7852}])$	$([s_{5.7852}, s_{5.0493}], [s_{2.9649}, s_{5.4444}])$
A_3	$([s_{3.6831}, s_{5.8451}], [s_{1.2311}, s_{5.5326}])$	$([s_{4.1283}, s_{6.6064}], [s_{1.4142}, s_{3.7567}])$
A_4	$([s_{2.0757}, s_{4.5205}], [s_{1.5971}, s_{5.4772}])$	$([s_{3.7639}, s_{5.2174}], [s_{1.8882}, s_{5.5330}])$
	G_1	G_2
A_1	$([s_{2.5316}, s_{4.9256}], [s_{4.9256}, s_{4.5174}])$	$([s_{2.2804}, s_{6.3216}], [s_{3.6325}, s_{6.0590}])$
A_2	$([s_{5.0282}, s_{6.7828}], [s_{1.7411}, s_{3.2245}])$	$([s_{3.4835}, s_{4.2865}], [s_{4.0973}, s_{6.7875}])$
A_3	$([s_{2.8907}, s_{4.3664}], [s_{1.5337}, s_{3.0157}])$	$([s_{2.5316}, s_{6.4087}], [s_{2.0000}, s_{3.5726}])$
A_4	$([s_{4.9256}, s_{6.6064}], [s_{1.7411}, s_{3.2704}])$	$([s_{2.5316}, s_{5.3113}], [s_{2.5686}, s_{4.8540}])$

Step 4. Compute the dominance of each alternative A_i over each alternative A_k.

$$\Phi_k(A_i, A_k) = \begin{bmatrix} 0 & -0.5064 & -1.9294 & -1.0706 \\ -0.9094 & 0 & -2.2284 & -0.6610 \\ -0.3007 & -0.2484 & 0 & -0.0913 \\ -0.5177 & -0.3671 & -2.5473 & 0 \end{bmatrix}, \ i,k = 1,2,3,4$$

Step 5. Compute the overall prospect value of alternative A_i.

$$\phi(A_1) = -0.8173, \ \phi(A_2) = -0.7047$$
$$\phi(A_3) = -1.1297, \ \phi(A_4) = -0.3474$$

Step 6. Rank the alternatives and we can obtain $A_4 \succ A_2 \succ A_1 \succ A_3$, and A_4 is the best alternative.

5. Conclusions

It is known that Lq-ROFS is a powerful information representation tool that can effectively describe DMs' evaluation information in complicated MAGDM problems. More and more researchers have noticed the flexibility and power of Lq-ROFSs in solving decision-making problems. This paper proposes a novel MAGDM method wherein decision-makers' evaluation values are in the form of Lq-ROFSs. In the newly developed MAGDM method, we propose a way to determine the weights of attributes. Afterward, we introduced a TODIM method for determining the ranking of alternatives. The advantages of our proposed method are obvious. First of all, our proposed method is based on the information representation tool Lq-ROFSs, and therefore it is more suitable and sufficient to handle realistic decision-making problems in economics and management. Second, our method can solve MAGDM problems where the weight vector of attributes is completely unknown. We established a model to determine the weight vector of attributes. The model is based on two assumptions; i.e., the weight vector of attributes should make total deviation between all the alternatives and the PIS to be a minimum (Rule I) and should make the Shannon information entropy be a maximum (Rule II). Third, our proposed MAGDM method is based on TODIM, which is an interactive decision-making approach that can take DMs' psychological behaviors into consideration. Hence, the above-mentioned three advantages make our proposed method more suitable and powerful for handling realistic MAGDM problems. Afterward, we used a numerical example to show the effectiveness of our method. The numerical example proves that our developed MAGDM method can effectively solve real decision-making problems. In the future, we will continue our research from two aspects. First of all, in this paper, the DMs are independent. In other words, DMs express their evaluation values independently. However, in real decision-making problems, DMs are usually related, which means that there is relationship among DMs. Recently, more and more researchers have noticed this phenomenon and some of them have attempted to investigate MAGDM problems from the perspective of social networks [59]. Therefore, we plan to investigate MAGDM methods based on social networks in the future. Second, in this paper, we assume there are only several DMs. However, in some real MAGDM problems, there are dozens even thousands of DMs. Recently, some researchers have conducted MAGDM research from the perspective of large-scale group decision makers [60]. Hence, in the future, we will investigate the Lq-ROFS-based decision-making method in large-scale group decision-making situations.

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