symmetry

MDPI

Article

The Research on Strategic Choices of Food Supply Chain Considering Information Symmetry and Cost Sharing

Jianhua Wang * and Ji Xu

School of Business, Jiangnan University, Wuxi 214000, China; jnuxuji@163.com
* Correspondence: jianhua.w@jiangnan.edu.cn

Abstract: In the digital economy era, information symmetry, transparency, and traceability in food supply chains have increasingly garnered consumer attention. To motivate supply chain members to engage in product traceability, this paper examines the competitive and cooperative dynamics among participants in the food supply chain over continuous time. By developing a differential game model involving manufacturers and retailers with three decision-making modes, we solve the model using the Hamilton–Jacobi–Bellman (HJB) equation and perform a simulation analysis to assess the impact of different modes on overall supply chain profits. Additionally, we analyze how various parameters affect the profits of manufacturers and retailers. The key findings of this study indicate that centralized decision-making enhances the overall benefits of the food supply chain. Among the three decision-making models, the cost-sharing model proves to be the optimal approach, as it leads to a Pareto improvement in the profits of both manufacturers and retailers. These conclusions provide valuable insights for supply chain members seeking to optimize product traceability and enhance supply chain efficiency, as well as for government authorities involved in traceable supply chain governance.

Keywords: digital economy; food supply chain; food traceability; information symmetry

Academic Editors: Jian-Qiang Wang and Hsien-Chung Wu

Received: 5 December 2024
Revised: 10 January 2025
Accepted: 16 January 2025
Published: 18 January 2025

Citation: Wang, J.; Xu, J. The Research on Strategic Choices of Food Supply Chain Considering Information Symmetry and Cost Sharing. *Symmetry* **2025**, *17*, 142. https://doi.org/10.3390/sym17010142

1. Introduction

Food is a fundamental component of people's livelihoods and has long garnered significant consumer attention. Food traceability plays a crucial role in protecting consumer rights. Unlike other products, food directly impacts consumer health, making its supply chain subject to more stringent regulatory standards. Many countries and regions mandate a high level of traceability within the food supply chain to ensure the rapid identification of sources and the timely implementation of corrective actions in the event of food safety concerns. Furthermore, the short lifecycle and limited shelf life of food products necessitate the close monitoring of each stage of production, distribution, and consumption through traceability systems.

1.1. Digital Traceability

With the rise of the digital economy, food traceability has undergone significant technological innovations and transformations. Digital traceability technologies, such as cloud computing, big data with "one product, one code", and blockchain-based solutions, have, to an extent, provided critical support for the digital governance of food safety risks [1]. Existing traceability frameworks are primarily designed for food and agricultural products. Chen, J. et al. [2] addressed issues related to the redundancy filtering of large data volumes and on-chain storage for traceability. They proposed a trusted storage and traceability system based on blockchain and internet of things (IoT) technologies. Due to its numerous advantages, blockchain-based traceability methods have been extensively researched and applied across various production sectors and in daily life. Leveraging blockchain's decentralization, Zhu, J. et al. [3] developed a dynamic multi-center collaborative certification and traceability model for supply chain participants, utilizing blockchain characteristics and hashing principles. Zheng, K. et al. [4] designed a multi-party traceability platform for agricultural product quality and safety, based on distributed storage, computing technologies, and spatiotemporal data integration.

In the context of enhancing the depth of product traceability, Opara, L. [5] identified six key components of traceability systems in agriculture and food supply chains: product traceability, process traceability, genetic traceability, input traceability, disease/pest traceability, and measurement traceability. These elements guide the design and implementation of agricultural and food traceability systems. Concerning modular traceability design, Olsen, P. et al. [6] outlined the general components of traceability systems in food, focusing on identifying supply chain units, recording unit attributes, and tracking their connections and transformations throughout the supply chain. Purwandoko, P. et al. [7] developed a traceability framework for organic rice certification, intended to guide the design of traceability systems for organic rice products. Regarding regulatory aspects, Regattieri, A. et al. [8] analyzed legal provisions and regulatory schemes for food traceability, providing a general framework for achieving the essential functions of an effective traceability system.

1.2. Information Sharing

In the development of a digital traceability system, information sharing plays a pivotal role. Research by Song, H. et al. [9] has found that the adoption of traceability systems is primarily influenced by the willingness of companies to share traceability data. Encouraging supply chain participants to share this information enhances the operational efficiency of traceability systems. However, private information can provide competitive advantages to certain stakeholders. For instance, Zhang, L. and Yang, J. [10] have argued that manufacturers who access private demand data can adjust wholesale prices that maximize their profits. This practice, however, may increase wholesale costs for retailers and intensify competition in the retail market. With regard to retailers, Jin, W. et al. [11] observed that they act as intermediaries, transmitting consumer needs, habits, and preferences to upstream supply chain participants. Information sharing between retailers and consumers supports the sustainable development of the supply chain. Finally, from the consumer's perspective, Jiang, N. et al. [12] emphasized that the construction of an information traceability system in the supply chain should meet the informational needs of consumers so as to enhance the market competitiveness of the supply chain. Furthermore, through visualized traceability data, consumers can independently assess and choose safe food products. If an issue arises with purchased food, consumers can quickly trace the problem's source and identify the responsible party [13].

1.3. Cost Sharing

Cost sharing is an effective tool for supply chain coordination and incentive alignment. Scholars have widely applied cost-sharing strategies to enhance supply chain performance in areas such as quality management, product development, and carbon reduction. Song, H. et al. [9] have suggested that improving the willingness of companies to share traceability information could be achieved through the reasonable allocation of traceability costs. Zhen, Y. et al. [14] have proposed a cost-sharing contract for blockchain investment in a digital publishing supply chain, consisting of publishers, sales platforms, and consumers. Their work addresses optimal pricing and copyright protection decisions under both traditional and blockchain copyright protection models. Additionally, some domestic and international scholars have conducted in-depth research on cost-sharing contracts in other areas. For example, Cai, J. et al. [15] introduced cost-sharing contracts in scenarios where both manufacturers and retailers are risk averse, only the retailer is risk averse, or only the manufacturer is risk averse, optimizing the performance of green supply chains and exploring the optimal decisions of supply chain members. Yang, H. et al. [16] proposed a manufacturer-led two-level supply chain model, considering retailers' fairness concerns in both pre-sale and non-pre-sale scenarios, and verified the effectiveness of cost-sharing contracts and their role in supply chain games.

1.4. Differential Games

In recent years, an increasing number of scholars, both domestic and international, have employed differential game models to examine supply chain management issues. This approach is particularly relevant as interactions among supply chain members often represent a continuous and dynamic process, reflecting long-term decision-making challenges. Hong, J. et al. [17] used a differential game model to investigate quality control coordination in a supply chain comprising a supplier and a manufacturer. Their findings indicate that cooperative quality control leads to a more favorable Nash equilibrium compared with non-cooperative solutions. Ma, D. and Hu, J. [18] introduced fairness concerns among members in a closed-loop supply chain and applied differential games to derive dynamic equilibrium solutions. Luisito, B. [19] applied differential game theory to study cooperation and non-cooperation models between two countries managing cross-border CO_2 pollution. His analysis showed that feedback strategies could reduce CO_2 emissions and enhance environmental performance. Zhou, M. et al. [20] examined the dynamic effects of alliances between supply chain members on advertising investments using differential games. Their study found that such alliances incentivize manufacturers and retailers to increase advertising investment. However, there remains limited research on the specific application of differential game strategies to food supply chains.

1.5. Paper Contributions

A review of the literature reveals that most studies on cost-sharing in supply chains focus on static game models. Similarly, in existing traceability research, the level of product traceability is often treated as a static parameter. However, in practice, factors such as aging production equipment and delayed traceability innovations can lead to a gradual decline in product traceability over time.

This paper distinguishes itself from most existing literature, which predominantly addresses static problems, by employing a differential game approach to capture the time-varying nature of product traceability. It considers decision-making under varying risk preferences of supply chain members and analyzes the optimal dynamic decisions for both individual members and the entire system within cooperative, non-cooperative Nash, and Stackelberg game scenarios. The paper compares these decision-making models to identify the optimal dynamic strategy for traceable food supply chains. By addressing the competitive and cooperative relationships among game participants over continuous time, it provides valuable decision-making insights for supply chain members to enhance supply chain efficiency. Moreover, it offers a robust foundation for traceable supply chain governance by government authorities. Additionally, the paper contributes to the academic development of differential game theory within the field of supply chain management.

2. Materials and Methods

2.1. Problem Description

Given the perishable, seasonal, and loss-prone nature of food products, this paper considers the short lifecycle of food, which encompasses the key stages of production, sales, and consumption. The supply chain under investigation consists of an upstream manufacturer that implements a digital traceability system and a downstream retailer that promotes traceability to consumers. The focus of this research is on the shared burden of traceability costs within this supply chain. The operational model of the supply chain system studied in this paper is shown in Figure 1.

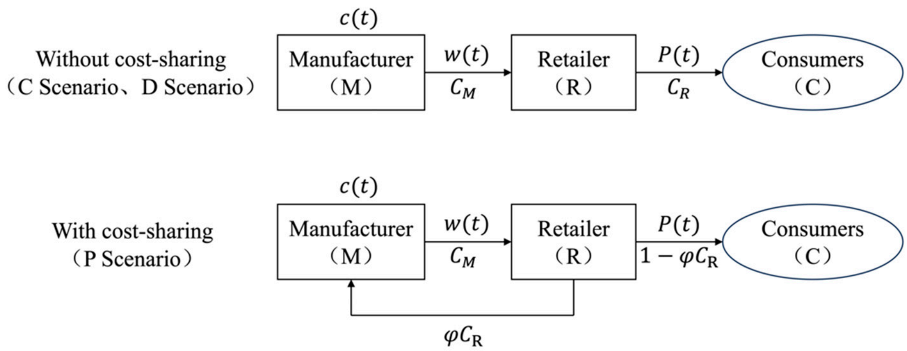

Figure 1. Supply chain structure diagram.

In this supply chain system, the manufacturer directly impacts the product's traceability level by implementing the digital traceability system. Conversely, the retailer indirectly affects market demand by promoting the product's traceability to consumers. To encourage the retailer's active promotion of traceability, the upstream manufacturer is willing to share part of the traceability promotion costs with the retailer.

2.2. Model Assumptions and Notation Explanation

To study the relevant issues, this paper makes the following assumptions about the supply chain model.

H1. *This paper considers a two-level supply chain consisting of a single manufacturer (M) and a single retailer (R) after the establishment of a digital traceability system. Given that the manufacturer faces greater pressure than the retailer during the implementation of the traceability system, the paper adopts a Stackelberg differential game model in which the manufacturer leads, and the downstream retailer follows.*

H2. *The initial investment in the traceability system determines the level of traceability of the product. The higher the investment, the higher the traceability level of the product, though it also involves greater resource consumption. Let the effort level of the manufacturer's traceability investment be denoted as $I(t)$, and the associated cost of traceability investment be represented as Equation (1):*

$$C_M(I(t))\frac{1}{2}\kappa I^2 \quad (1)$$

where κ is the cost coefficient for traceability investment. Similarly, let the retailer's promotional effort level for traceable food be denoted as $E(t)$, with the associated promotional cost given by Equation (2):

$$C_R(E(t))\frac{1}{2}\lambda E^2 \quad (2)$$

where λ is the cost coefficient for promotional efforts.

H3. *The level of product traceability is related to the manufacturer's investment effort, but over time, the traceability level will undergo dynamic changes. This dynamic process is described by Equation (3):*

$$\dot{\tau}(t) = \gamma I(t) - \delta\tau(t), \tau(0) = \tau_0 \tag{3}$$

where γ represents the impact coefficient indicating the change in traceability per unit effort, and δ reflects the degradation of traceability over time due to factors such as aging production equipment or the lag in traceability innovation. τ_0 represents the initial traceability level of the product.

H4. *The demand for traceable food is linearly related to the product's traceability level, the retailer's promotional effort, and the price. The basic demand function for traceable food is given by Equation (4):*

$$D(t) = \alpha - p(t) + \mu\tau(t) + \nu E(t) \tag{4}$$

where α is the potential market demand, which is common knowledge between both parties. μ represents the impact of traceability on the demand for traceable food, and ν represents the impact of promotional efforts on the demand for traceable food.

H5. *The manufacturer determines the wholesale price $w(t)$ based on the traceability level, cost-sharing contract, and manufacturing costs $c(t)$, among other factors. The retailer, in turn, sets the retail price $p(t)$ based on the wholesale price and other relevant factors.*

H6. *Both the manufacturer and the retailer have a discount rate of $\rho(\rho > 0)$.*

All parameters stated in the assumptions and their definitions are summarized in Table 1.

Table 1. Parameter Summary.

Parameter	Definition
$I(t)$	Manufacturer's traceability investment effort
$E(t)$	Retailer's traceability promotion effort
κ	Traceability investment cost coefficient
λ	Traceability promotion cost coefficient
γ	Traceability impact coefficient on product
δ	Traceability degradation coefficient
α	Potential market demand
μ	Impact coefficient of traceability on demand
ν	Impact coefficient of promotion on demand
$c(t)$	Manufacturing cost
$w(t)$	Wholesale price
$p(t)$	Retail price

2.3. Cooperative Game Analysis Under Centralized Decision-Making (C Scenario)

In the context of centralized decision-making, it is assumed that a central decision-maker exists whose objective is to maximize the overall profit of the supply chain. Although it is difficult to identify such a central decision-maker in practical scenarios, this optimal decision serves as a benchmark for evaluating the effectiveness of contracts. This approach aims to investigate the impact of centralized decision-making on supply chain efficiency under ideal conditions and provide a reference for subsequent decentralized decision-making. Therefore, when analyzing the coordination effects of contracts, the optimal outcome under centralized decision-making is used as a benchmark for comparison with ideal scenarios.

Under centralized decision-making, the long-term objective function model of the supply chain is given by Equation (5):

$$R_s^c(\tau, t) = \max_{p(t),I(t),E(t)} \int_0^\infty e^{-\rho t}\{(p(t) - c(t))D(t) - C_M(I(t)) - C_R(E(t))\}dt \tag{5}$$

To obtain the equilibrium strategy for the problem, the Hamilton–Jacobi–Bellman (HJB) equation from the differential game approach is employed to solve Equation (5). Given the complexity of solving the long-term objective function model and following the approach of Xia, Z. et al. [21], the time variable t is omitted for notational simplicity. The solution is as follows:

Proposition 1. *The equilibrium solution under the centralized decision-making scenario is as follows:*

The optimal decision equilibrium for the supply chain system is given by Equation (6):

$$p_s^{c^*} = \frac{(\alpha + c + \mu\tau)\lambda - c\nu^2}{2\lambda - \nu^2}, I_s^{c^*} = \frac{\gamma(2z_1^*\tau + z_2^*)}{\kappa}, E_s^{c^*} = \frac{(\alpha - c + \mu\tau)\nu}{2\lambda - \nu^2} \tag{6}$$

The optimal profit value function for the supply chain system is given by Equation (7):

$$R_s^{c^*} = e^{-\rho t}(z_1^*\tau^2 + z_2^*\tau + z_3^*) \tag{7}$$

The trajectory of the product traceability level over time is given by Equation (8):

$$\tau^c = (\tau_0 + \frac{z_2^*\gamma^2}{2z_1^*\gamma^2\tau - \delta\kappa})e^{\frac{2z_1^*\gamma^2\tau - \delta\kappa}{\kappa}t} - \frac{z_2^*\gamma^2}{2z_1^*\gamma^2 - \delta\kappa} \tag{8}$$

where

$$\begin{cases} z_1^* = 2\delta + \rho + \sqrt{\frac{\kappa(2\delta+\rho)^2}{4\gamma^2} - \frac{\mu^2\lambda}{2\lambda-\nu^2}} \\ z_2^* = \frac{\mu\lambda\kappa(\alpha-c)}{(2\lambda-\nu^2)(\kappa\delta+\kappa\rho-2\gamma^2 z_1)} \\ z_3^* = \frac{\lambda(\alpha-c)^2}{2\rho(2\lambda-\nu^2)} + \frac{\gamma^2 z_2^2}{2\kappa\rho} \end{cases}$$

The detailed derivation of Proposition 1 can be found in the Proof in Appendix A.

Under centralized decision-making, this analysis examines the effects of various parameters on the decision variables of supply chain members, including market potential demand (α), traceability input cost coefficient (κ), traceability promotional cost coefficient (λ), product traceability level impact coefficient (γ), traceability level decay coefficient (δ), and the influence of traceability promotion on demand (ν). As stated in Lemma 1:

Lemma 1. *Sensitivity analysis of relevant parameters under centralized decision-making.*

$$\frac{\partial\tau^c}{\partial\alpha} > 0, \frac{\partial\tau^c}{\partial\kappa} < 0, \frac{\partial\tau^c}{\partial\lambda} < 0, \frac{\partial\tau^c}{\partial\gamma} > 0, \frac{\partial\tau^c}{\partial\delta} < 0, \frac{\partial\tau^c}{\partial\nu} > 0;$$

$$\frac{\partial p_s^{c^*}}{\partial\alpha} > 0, \frac{\partial p_s^{c^*}}{\partial\kappa} < 0, \frac{\partial p_s^{c^*}}{\partial\lambda} > 0, \frac{\partial p_s^{c^*}}{\partial\gamma} > 0, \frac{\partial p_s^{c^*}}{\partial\delta} < 0, \frac{\partial p_s^{c^*}}{\partial\nu} > 0;$$

$$\frac{\partial E_s^{c^*}}{\partial\alpha} > 0, \frac{\partial E_s^{c^*}}{\partial\kappa} < 0, \frac{\partial E_s^{c^*}}{\partial\lambda} < 0, \frac{\partial E_s^{c^*}}{\partial\gamma} > 0, \frac{\partial E_s^{c^*}}{\partial\delta} < 0, \frac{\partial E_s^{c^*}}{\partial\nu} > 0;$$

$$\frac{\partial I_s^{c^*}}{\partial\alpha} > 0, \frac{\partial I_s^{c^*}}{\partial\kappa} < 0, \frac{\partial I_s^{c^*}}{\partial\lambda} < 0, \frac{\partial I_s^{c^*}}{\partial\gamma} > 0, \frac{\partial I_s^{c^*}}{\partial\delta} < 0, \frac{\partial I_s^{c^*}}{\partial\nu} > 0.$$

From Lemma 1, we conclude that, under centralized decision-making, the following apply:

1. The product traceability level, retail price, manufacturer's traceability effort, and retailer's traceability promotion effort are all positively correlated with market potential demand. Specifically, as market potential demand increases, these factors also increase. This indicates that market demand is a critical determinant of product development, and that conducting pre-market surveys is essential for making optimal decisions in the game.
2. The product traceability level, retail price, manufacturer's traceability effort, and retailer's traceability promotional effort are all negatively correlated with the traceability input cost coefficient. As traceability input costs rise, these factors decline. This suggests that, as the difficulty of enhancing product traceability increases, manufacturers' incentives to invest in traceability decrease, resulting in lower product traceability levels. This, in turn, affects product pricing and the traceability efforts of both manufacturers and retailers. Therefore, controlling traceability costs is essential, and government subsidies or incentives could reduce the costs of building traceability systems, thereby encouraging greater investment in traceability efforts and promoting the sustainable development of the traceable food industry.
3. The product traceability level, manufacturer's traceability effort, and retailer's traceability promotional efforts are all negatively correlated with the traceability promotional cost coefficient. In contrast, the retail price is positively correlated with this coefficient. This suggests that higher promotional costs directly impact the manufacturer's traceability input, which in turn reduces the product's traceability level. Furthermore, as traceability promotion costs increase, the retail price of the product also increases.
4. The product traceability level, retail price, manufacturer's traceability efforts, and retailer's traceability promotional efforts are all positively correlated with the product traceability level impact coefficient. This indicates that, as the product's traceability level improves, manufacturers are more likely to invest in traceability efforts, and retailers are also more inclined to promote the product's traceability. Additionally, as the traceability attributes of the product improve, its price tends to increase accordingly.
5. The product traceability level, retail price, manufacturer's traceability effort, and retailer's traceability promotion effort are all negatively correlated with the traceability level decay coefficient. This implies that the faster the product's traceability level naturally deteriorates, the more difficult it becomes for manufacturers and retailers to intensify their investments in traceability efforts.
6. The product traceability level, retail price, manufacturer's traceability efforts, and retailer's traceability promotional efforts are all positively correlated with the impact of traceability promotion on demand. This indicates that, as market demand increases, manufacturers place greater emphasis on improving product traceability and are willing to invest more in traceability efforts. Similarly, retailers will enhance their promotional efforts, resulting in a corresponding rise in the retail price.

2.4. Non-Cooperative Game Analysis Under Decentralized Decision-Making Scenario (D Scenario)

In this scenario, the manufacturer and the retailer make independent decisions to maximize their individual profits. Their respective long-term objective functions are represented by Equations (9) and (10):

$$\max_{I} R_M^d = \int_0^\infty e^{-\rho t}\{(w-c)D - C_M(I)\}dt \tag{9}$$

$$\max_{p,E} R_R^d = \int_0^{\infty} e^{-\rho t}\{(p-w)D - C_R(E)\}dt \tag{10}$$

The solution method is analogous to Proposition 1, yielding the following results:

Proposition 2. *The equilibrium solution under the decentralized decision-making scenario is as follows:*

7. *The equilibrium solution for the manufacturer and retailer is given by Equation (11):*

$$p_R^{d*} = \frac{(\alpha + w + \mu\tau)\lambda - w\nu^2}{2\lambda - \nu^2}, I_M^{d*} = \frac{a_1^*\gamma}{\kappa}, E_R^{d*} = \frac{(\alpha - w + \mu\tau)\nu}{2\lambda - \nu^2} \tag{11}$$

8. *The optimal profit functions for the manufacturer and the retailer are given by Equations (12) and (13):*

$$R_R^{d*} = e^{-\rho t}\left(b_1^*\tau^2 + b_2^*\tau + b_3^*\right) \tag{12}$$

$$R_R^{d*} = e^{-\rho t}\left(b_1^*\tau^2 + b_2^*\tau + b_3^*\right) \tag{13}$$

9. *The trajectory of the product traceability level over time is given by Equation (14):*

$$\tau^d = (\tau_0 - \frac{a_1^*\gamma^2}{\delta\kappa})e^{-\delta\tau t} + \frac{a_1^*\gamma^2}{\delta\kappa} \tag{14}$$

where

$$\begin{cases} a_1^* = \frac{\mu\lambda(w-c)}{(2\lambda-\nu^2)(\delta+\rho)} \\ a_2^* = \frac{\lambda(\alpha-w)(w-c)}{\rho(2\lambda-\nu^2)} + \frac{\gamma^2 a_1^2}{2\kappa\rho} \end{cases}; \begin{cases} b_1^* = \frac{\mu^2\lambda}{2(2\lambda-\nu^2)(2\delta+\rho)} \\ b_2^* = \frac{\mu\lambda(\alpha-w)}{(2\lambda-\nu^2)(\delta+\rho)} + \frac{2\gamma^2 a_1 b_1}{\kappa(\delta+\rho)} \\ b_3^* = \frac{\lambda(\alpha-w)^2}{2\rho(2\lambda-\nu^2)} + \frac{\gamma^2 a_1 b_2}{\kappa\rho} \end{cases}$$

The detailed derivation of Proposition 2 can be found in the Proof in Appendix A.

Under decentralized decision-making, this analysis examines the impact of various parameters on the decision variables of supply chain members, including market potential demand (α), traceability input cost coefficient (κ), traceability promotional cost coefficient (λ), product traceability level impact coefficient (γ), traceability level decay coefficient (δ), and the influence of traceability promotion on demand (ν). As stated in Lemma 2:

Lemma 2. *Sensitivity analysis of relevant parameters under decentralized decision-making.*

$$\frac{\partial\tau^d}{\partial\alpha} > 0,\ \frac{\partial\tau^d}{\partial\kappa} < 0,\ \frac{\partial\tau^d}{\partial\lambda} < 0,\ \frac{\partial\tau^d}{\partial\gamma} > 0,\ \frac{\partial\tau^d}{\partial\delta} < 0,\ \frac{\partial\tau^d}{\partial\nu} > 0;$$

$$\frac{\partial p_R^{d*}}{\partial\alpha} > 0,\ \frac{\partial p_R^{d*}}{\partial\kappa} < 0,\ \frac{\partial p_R^{d*}}{\partial\lambda} > 0,\ \frac{\partial p_R^{d*}}{\partial\gamma} > 0,\ \frac{\partial p_R^{d*}}{\partial\delta} < 0,\ \frac{\partial p_R^{d*}}{\partial\nu} > 0;$$

$$\frac{\partial E_R^{d*}}{\partial\alpha} > 0,\ \frac{\partial E_R^{d*}}{\partial\kappa} < 0,\ \frac{\partial E_R^{d*}}{\partial\lambda} < 0,\ \frac{\partial E_R^{d*}}{\partial\gamma} > 0,\ \frac{\partial E_R^{d*}}{\partial\delta} < 0,\frac{\partial E_R^{d*}}{\partial\nu} > 0;$$

$$\frac{\partial I_M^{d*}}{\partial\alpha} > 0,\ \frac{\partial I_M^{d*}}{\partial\kappa} < 0,\ \frac{\partial I_M^{d*}}{\partial\lambda} < 0,\ \frac{\partial I_M^{d*}}{\partial\gamma} > 0,\ \frac{\partial I_M^{d*}}{\partial\delta} < 0,\ \frac{\partial I_M^{d*}}{\partial\nu} > 0.$$

From Lemma 2, it can be concluded that, under decentralized decision-making, the relationships between product traceability level, retail price, manufacturer's traceability effort, retailer's traceability promotional effort, and the parameters of market potential demand, traceability input cost coefficient, traceability promotional cost coefficient, product traceability level impact coefficient, traceability level decay coefficient, and demand impact coefficient are consistent with those in Lemma 1. That is, conclusions 1–6 in Lemma 2 align with those in Lemma 1.

Comparison of decision variables: Furthermore, by comparing $p_s^{c^*}$ with $p_R^{d^*}$, $I_s^{c^*}$ with $I_M^{d^*}$, and $E_s^{c^*}$ with $E_R^{d^*}$, we find that $p_s^{c^*} > p_R^{d^*}$; $I_s^{c^*} > I_M^{d^*}$; $E_s^{c^*} > E_R^{d^*}$. This indicates that, under centralized decision-making, the optimal decisions of both the manufacturer and retailer are superior to those made under decentralized decision-making. Given that the overall profit function comparison is relatively complex, the next section will provide a comparative analysis of the supply chain's total profit and the profit variations of each player in both scenarios, using numerical examples.

2.5. Principal–Agent Game Under the Cost-Sharing Contract (P Scenario)

This section analyzes the impact of the cost-sharing contract on the supply chain system under the cost-sharing contract scenario. To incentivize the retailer to actively engage in traceability promotion, the manufacturer is willing to bear a portion of the cost of traceability promotion, with the sharing ratio denoted as φ.

In this scenario, both the manufacturer and the retailer independently make decisions with the goal of maximizing their respective profits. In the first step, the manufacturer determines the effort level $I(t)$ for traceability investment and the cost-sharing ratio φ for the contract. In the second step, the retailer, based on the manufacturer's decisions, determines the retail price p and the effort level $E(t)$ for traceability promotion.

The manufacturer's long-term objective function is given by Equation (15):

$$\max_{\varphi,I} R_M^p = \int_0^\infty e^{-\rho t}\{(w-c)D - C_M(I) - \varphi C_R(E)\}dt \tag{15}$$

The retailer's long-term objective function is given by Equation (16):

$$\max_{p,E} R_R^p = \int_0^\infty e^{-\rho t}\{(p-w)D - (1-\varphi)C_R(E)\}dt \tag{16}$$

Proposition 3. *The equilibrium solution under the cost-sharing scenario is as follows:*

The equilibrium solutions for the manufacturer and the retailer are given by Equation (17):

$$\begin{aligned} I_M^{p^*} &= \frac{\gamma\left(m_1^*\tau + m_2^*\right)}{\kappa}, E_R^{p^*} = \frac{(\alpha - w + \mu\tau)\nu}{2(1-\varphi^*)\lambda - \nu^2}, \\ \varphi^* &= \frac{4(w-c)\left(\alpha\nu^2 + \mu\tau\nu^2 - \alpha\lambda - w\lambda - \mu\tau\lambda\right) - 1}{4\lambda(\alpha - w + \mu\tau)^2\nu^2}, \\ p_R^{p^*} &= \frac{(\alpha + w + \mu\tau)\lambda - w\nu^2}{2(1-\varphi^*)\lambda - \nu^2} \end{aligned} \tag{17}$$

The optimal profit functions for the manufacturer and the retailer are given by Equation (18) and Equation (19):

$$R_M^{p^*} = e^{-\rho t}\left(m_1^*\tau^2 + m_2^* + m_3^*\right) \tag{18}$$

$$R_R^{p^*} = e^{-\rho t}\left(n_1^*\tau^2 + n_2^*\tau + n_3^*\right) \tag{19}$$

The trajectory of the product's traceability level over time is given by Equation (20):

$$\tau^p = (\tau_0 - \frac{m_2^*\gamma^2}{m_1^*\gamma^2 - \delta\kappa})e^{\frac{m_1^*\gamma^2 - \delta\kappa}{\kappa}t} - \frac{m_2^*\gamma^2}{m_1^*\gamma^2 - \delta\kappa} \tag{20}$$

where

$$\begin{cases} m_1^* = 2\delta + \rho + \sqrt{\frac{\kappa(2\delta+\rho)^2}{\gamma^2} + \frac{\varphi\nu^2\mu\lambda}{2\lambda-\nu^2}} \\ m_2^* = \frac{2\mu\lambda[(w-c)-\varphi(\alpha-w)]}{2[\kappa(\delta+\rho)-\gamma^2 m_1][2(1-\varphi)\lambda-\nu^2]^2} \\ m_3^* = \frac{2(1-\lambda)(\alpha-w)(w-c)\left[2(1-\varphi)\lambda-\nu^2\right]-\varphi\lambda(\alpha-w)^2\nu^2}{2[2(1-\varphi)\lambda-\nu^2]^2} + \frac{\gamma^2 m_2^2}{2\kappa\rho} \end{cases}$$

$$\begin{cases} n_1^* = \frac{(1-\varphi)\mu^2\lambda\kappa}{2[2(1-\varphi)\lambda-\nu^2][(2\delta+\rho)\kappa-4\gamma^2 m_1]} \\ n_2^* = \frac{(1-\varphi)(\alpha-w)\mu\lambda\kappa}{[2(1-\varphi)\lambda-\nu^2][(\delta+\rho)\kappa-2\gamma^2 m_1]} + \frac{2\gamma^2 n_1 m_2}{(\delta+\rho)\kappa-2\gamma^2 m_1} \\ n_3^* = \frac{\lambda(1-\varphi)(\alpha-w)^2}{2\rho[2(1-\varphi)\lambda-\nu^2]} + \frac{\gamma^2 n_2 m_2}{\kappa\rho} \end{cases}$$

The detailed derivation of Proposition 3 can be found in the Proof in Appendix A.

Under cost-sharing arrangements, this analysis examines the impact of various parameters on the decision variables of supply chain members, including market potential demand (α), traceability input cost coefficient (κ), traceability promotional cost coefficient (λ), product traceability level impact coefficient (γ), traceability level decay coefficient (δ), and the influence of traceability promotion on demand (ν). As stated in Lemma 3:

Lemma 3. *Sensitivity analysis of relevant parameters under cost-sharing arrangements.*

$$\frac{\partial\tau^p}{\partial\alpha} > 0,\ \frac{\partial\tau^p}{\partial\kappa} < 0,\ \frac{\partial\tau^p}{\partial\lambda} < 0,\ \frac{\partial\tau^p}{\partial\gamma} > 0,\ \frac{\partial\tau^p}{\partial\delta} < 0,\ \frac{\partial\tau^p}{\partial\nu} > 0;$$

$$\frac{\partial p_R^{p^*}}{\partial\alpha} > 0,\ \frac{\partial p_R^{p^*}}{\partial\kappa} < 0,\ \frac{\partial p_R^{p^*}}{\partial\lambda} > 0,\ \frac{\partial p_R^{p^*}}{\partial\gamma} > 0,\ \frac{\partial p_R^{p^*}}{\partial\delta} < 0,\ \frac{\partial p_R^{p^*}}{\partial\nu} > 0;$$

$$\frac{\partial E_R^{p^*}}{\partial\alpha} > 0,\ \frac{\partial E_R^{p^*}}{\partial\kappa} < 0,\ \frac{\partial E_R^{p^*}}{\partial\lambda} < 0,\ \frac{\partial E_R^{p^*}}{\partial\gamma} > 0,\ \frac{\partial E_R^{p^*}}{\partial\delta} < 0,\ \frac{\partial E_R^{p^*}}{\partial\nu} > 0;$$

$$\frac{\partial I_M^{p^*}}{\partial\alpha} > 0,\ \frac{\partial I_M^{p^*}}{\partial\kappa} < 0,\ \frac{\partial I_M^{p^*}}{\partial\lambda} < 0,\ \frac{\partial I_M^{p^*}}{\partial\gamma} > 0,\ \frac{\partial I_M^{p^*}}{\partial\delta} < 0,\ \frac{\partial I_M^{p^*}}{\partial\nu} > 0.$$

From Lemma 3, it can be concluded that, under decentralized decision-making, the relationships between product traceability level, retail price, manufacturer's traceability input effort, retailer's traceability promotional effort, and the parameters of market potential demand, traceability input cost coefficient, traceability promotional cost coefficient, product traceability level impact coefficient, traceability level decay coefficient, and demand impact coefficient are consistent with those in Lemma 1. That is, conclusions 1–7 in Lemma 3 align with those in Lemma 2.

Comparison of decision variables: Furthermore, by comparing $p_s^{c^*}, p_R^{d^*}$ with $p_R^{p^*}$; $I_s^{c^*}, I_M^{d^*}$ with $I_M^{p^*}$; and $E_s^{c^*}, E_R^{d^*}$ with $E_R^{p^*}$, we observe that $p_s^{c^*} > p_R^{p^*} > p_R^{d^*}$; $I_s^{c^*} > I_M^{p^*} > I_M^{d^*}$; $E_s^{c^*} > E_R^{p^*} > E_R^{d^*}$. This suggests that cost-sharing contracts can, to some extent, achieve a Pareto improvement in the decision-making of supply chain members.

2.6. Comparative Analysis

The equilibrium solutions and optimal profits for each member of the supply chain under three different scenarios are summarized in Table 2.

Table 2. Equilibrium solutions and optimal profits.

	Manufacturer's Equilibrium Solution	Retailer's Equilibrium Solution	Manufacturer's Optimal Profit	Retailer's Optimal Profit
Centralized decision	$\frac{\gamma(2z_1^*\tau+z_2^*)}{\kappa}$	$\frac{(\alpha-c+\mu\tau)\nu}{2\lambda-\nu^2}$	$\frac{1}{2}e^{-\rho t}(z_1^*\tau^2+z_2^*\tau+z_3^*)$	$\frac{1}{2}e^{-\rho t}(z_1^*\tau^2+z_2^*\tau+z_3^*)$
Decentralized decision	$\frac{a_1^*\gamma}{\kappa}$	$\frac{(\alpha-w+\mu\tau)\nu}{2\lambda-\nu^2}$	$e^{-\rho t}(a_1^*\tau+a_2^*)$	$e^{-\rho t}(b_1^*\tau^2+b_2^*\tau+b_3^*)$
Cost sharing	$\frac{\gamma(m_1^*\tau+m_2^*)}{\kappa}$	$\frac{(\alpha-w+\mu\tau)\nu}{2(1-\varphi^*)\lambda-\nu^2}$	$e^{-\rho t}(m_1^*\tau^2+m_2^*+m_3^*)$	$e^{-\rho t}(n_1^*\tau^2+n_2^*\tau+n_3^*)$

By comparing the equilibrium solutions of each parameter, the following conclusions can be drawn:

1. since $2z_1^*-m_1^*=2\delta+\rho+2\sqrt{\frac{\kappa(2\delta+\rho)^2}{4\gamma^2}-\frac{\mu^2\lambda}{2\lambda-\nu^2}}-\sqrt{\frac{\kappa(2\delta+\rho)^2}{\gamma^2}+\frac{\varphi\nu^2\mu\lambda}{2\lambda-\nu^2}}>0$; $z_2^*-m_2^*=\frac{\mu\lambda\kappa(\alpha-c)}{[\kappa(\delta+\rho)-2\gamma^2z_1](2\lambda-\nu^2)}-\frac{2\mu\lambda[(w-c)-\varphi(\alpha-w)]}{2[\kappa(\delta+\rho)-\gamma^2m_1][2(1-\varphi)\lambda-\nu^2]^2}>0$; $\tau_\infty^c-\tau_\infty^p=\frac{m_2^*\gamma^2}{m_1^*\gamma^2-\delta\kappa}-\frac{z_2^*\gamma^2}{2z_1^*\gamma^2-\delta\kappa}>0$, it follows that $I_s^{c^*}-I_M^{p^*}>0$. Similarly, $I_M^{p^*}-I_M^{d^*}>0$, and thus $I_M^{d^*}<I_M^{p^*}<I_s^{c^*}$.
2. $E_s^{c^*}=\frac{(\alpha-c+\mu\tau)\nu}{2\lambda-\nu^2}$, $E_R^{d^*}=\frac{(\alpha-w+\mu\tau)\nu}{2\lambda-\nu^2}$, $E_R^{p^*}=\frac{(\alpha-w+\mu\tau)\nu}{2(1-\varphi^*)\lambda-\nu^2}$. Generally, $w>c$, so $E_R^{d^*}<E_R^{p^*}<E_s^{c^*}$.

From the above conclusions, we can derive the following:

1. If the manufacturer shares the traceability promotion costs with the retailer, both the manufacturer's effort in traceability input and the retailer's efforts in traceability promotion are increased compared with the decentralized decision-making scenario without cost-sharing. This suggests that a cost-sharing contract can optimize the decisions of both the manufacturer and the retailer, thereby improving the overall decision-making quality of the supply chain.
2. In centralized decision-making, both the manufacturer's effort in traceability input and the retailer's efforts in traceability promotion exceed those in the other two scenarios. This further demonstrates that centralized decision-making yields the optimal outcome for the supply chain under ideal conditions, serving as a benchmark for comparing the effects of contractual collaboration.

The comparison of the optimal profits of supply chain system and its members will be presented in the next section using specific numerical examples, to assess the impact of the cost-sharing contract on the overall benefit of the supply chain system.

3. Results

Based on the derivation of the differential game, it was found that the optimal profits for both the supply chain system and its members in the three differential game models involving the manufacturer and retailer depend on the selection of parameters. Therefore, a numerical example is used to analyze and compare the profits under the above-mentioned scenarios.

Referring to the study by Xu, C. et al. [22], the relevant parameters for the numerical example are assumed as follows: $\kappa=0.8$, $\lambda=0.6$, $\gamma=2$, $\delta=1$, $\alpha=20$, $c=3$, $w=8$, $\rho=0.9$. Additionally, based on the findings of Hong, J. and Huang, P. [17], who surveyed an actual manufacturer's supply chain, the parameters $\mu=0.8$, $\nu=0.6$ are assumed.

By substituting these parameters into the analytical expressions and using MATLAB, the following results can be obtained: graphs showing the changes in the overall profit of the supply chain system under three different scenarios, the profit variations for both the manufacturer and retailer before and after cost-sharing, and the effects of the relevant parameters on the Pareto improvement of the overall supply chain profit.

3.1. Overall Profit Changes in the Supply Chain System

As shown in Figure 2, in a centralized decision-making scenario, the total profit of the supply chain system reaches its maximum, consistent with the theoretical derivations. When the manufacturer and retailer make decentralized decisions individually, the overall profit of the supply chain is minimized. However, if the manufacturer and retailer reach a cost-sharing agreement, the system's overall profit improves to some extent.

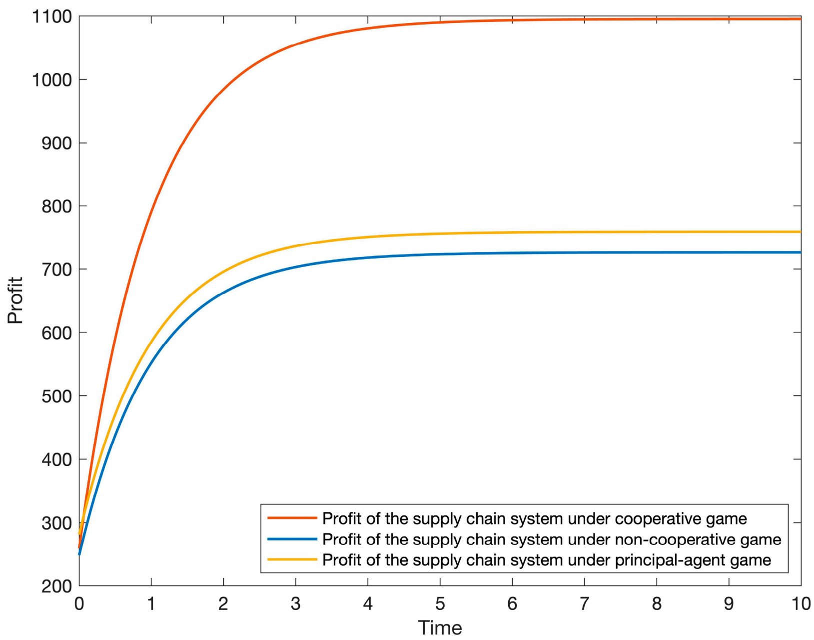

Figure 2. Profits of the supply chain system under three scenarios.

Furthermore, the figure clearly indicates that the total profit under centralized decision-making is consistently much higher than in the other two scenarios. Additionally, the profit growth rate under centralized decision-making significantly exceeds that of the other two cases. This indicates that centralized decision-making is significantly superior to decentralized decision-making and may offer valuable insights for addressing traceability issues in food supply chains.

3.2. Profit Changes of the Manufacturer and Retailer

As shown in Figure 3, transitioning from decentralized decision-making to a decentralized decision-making scenario with a cost-sharing contract results in increased profits for both the manufacturer and the retailer. This occurs because the manufacturer is willing to absorb part of the costs, which encourages the retailer to promote traceability efforts more actively, thereby boosting market demand. As a result, profits for both the manufacturer and the retailer are enhanced. This demonstrates that the cost-sharing contract improves the optimal profits for both parties.

Moreover, by comparing the vertical distance between the profit changes of the manufacturer and retailer in the figure, it is evident that the profit difference for the retailer is larger when a cost-sharing agreement is in place. This indicates that the cost-sharing contract has a more pronounced positive effect on the retailer. This is because the manufacturer, to incentivize the retailer's traceability promotion, shares part of the traceability promotion costs, which leads to an increase in the manufacturer's costs. On the other hand, for the retailer, the traceability promotion not only boosts market demand but also benefits from cost-sharing. Therefore, the impact of the cost-sharing contract on the retailer's profits is greater than its effect on the manufacturer's profits.

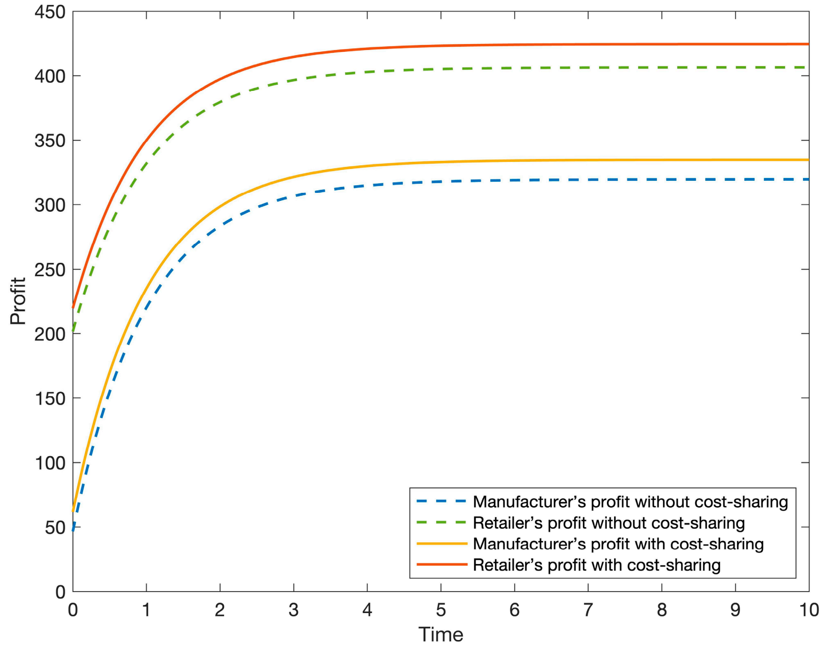

Figure 3. Comparison of manufacturer and retailer profits before and after cost sharing.

3.3. Dynamic Evolution of Product Traceability Level

Using MATLAB to illustrate the dynamic evolution of the product traceability level over time, as well as the relationship between the total system profit and the product's green level trajectory.

As shown in Figure 4, in all three scenarios, the product's traceability level consistently increases over time, eventually stabilizing. Moreover, the traceability level under centralized decision-making is consistently higher than that under decentralized decision-making, and cost-sharing contracts can lead to a Pareto improvement in traceability levels to some extent.

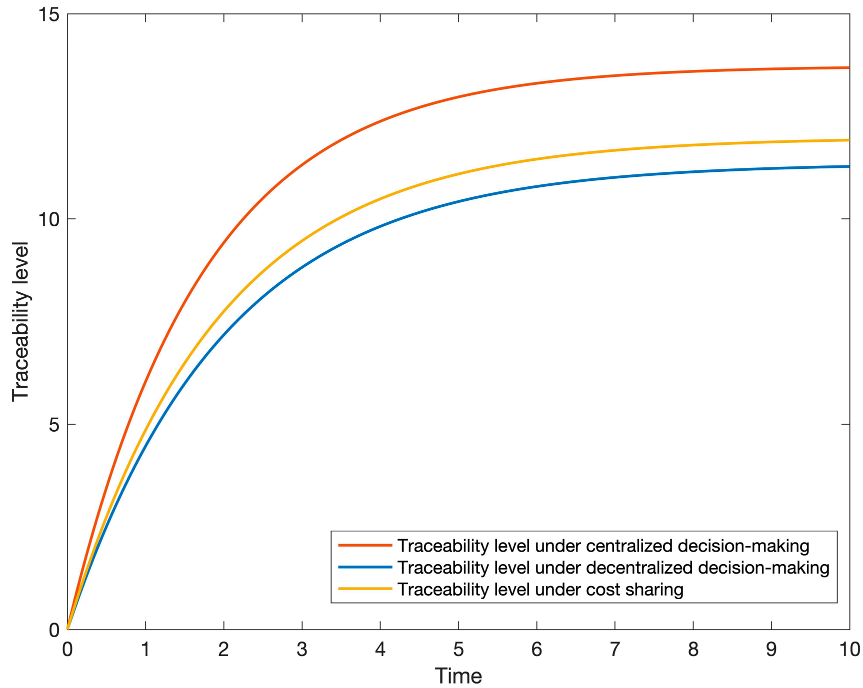

Figure 4. Dynamic evolution of product traceability level.

3.4. The Impact of Relevant Parameters on the Overall Supply Chain Profit

As shown in Figure 5, as the impact coefficient ν of traceability promotion efforts on traceable food demand increases, the overall profit of the supply chain rises across all three scenarios. Moreover, the Pareto improvement effect also enhances as ν increases.

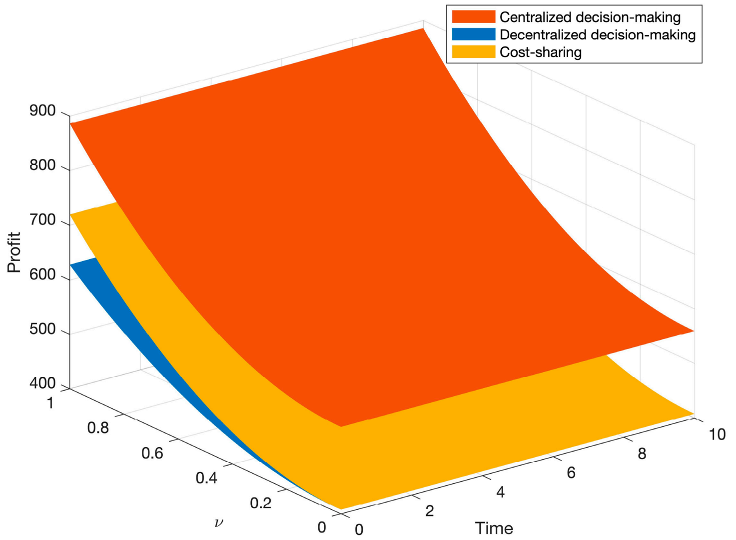

Figure 5. The impact of parameter ν on the overall profit of the supply chain.

The impact coefficient ν represents the sensitivity of consumers to the retailer's traceability promotion efforts. This suggests that the greater the consumer sensitivity to the retailer's traceability efforts, the more effectively the retailer's promotion can stimulate consumer interest in purchasing traceable food, thereby boosting market demand and increasing profits. Furthermore, the incentive effect of the cost-sharing contract becomes more pronounced, encouraging the retailer to invest more in traceability promotion efforts.

As shown in Figure 6, with the increase in the traceable promotion cost coefficient λ, the overall profit of the supply chain under all three scenarios gradually decreases. Additionally, the Pareto improvement effect weakens as λ increases.

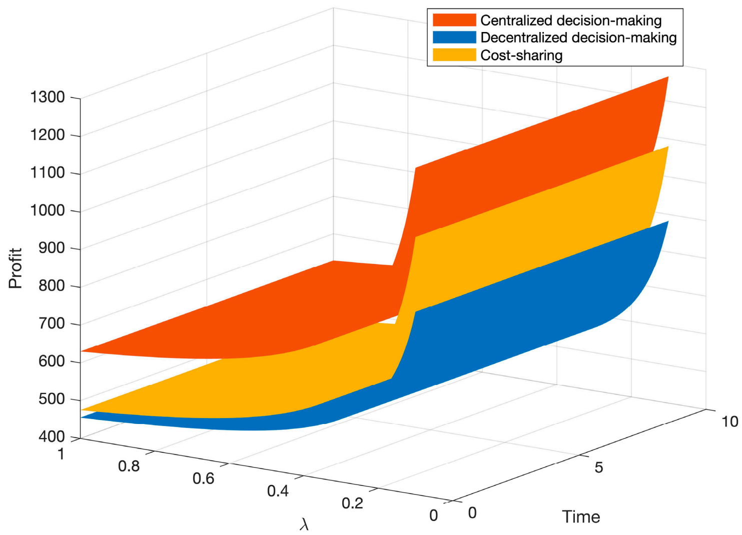

Figure 6. The Impact of parameter λ on overall supply chain profit.

The larger the coefficient λ, the higher the traceable promotion cost for the retailer. In the case of an increasing traceable promotion cost coefficient, even if the manufacturer shares a certain proportion of the retailer's promotion cost, the incentive effect of this cost-sharing behavior becomes less pronounced as λ increases. Consequently, this leads to a decrease in the retailer's motivation for traceable promotional activities, resulting in a decline in the overall profit.

4. Discussion

This study investigates the competitive and cooperative relationships among various players in the food supply chain over continuous time. A differential game model is developed to represent three decision-making modes: manufacturer, retailer, and the supply chain. By solving and analyzing the model, we perform simulation analysis to examine overall changes in supply chain profits under different modes, as well as the profit variations for both the manufacturer and the retailer. The study finds that, under different decision-making modes, the profits of the manufacturer, retailer, and the entire supply chain system fluctuate. Centralized decision-making maximizes the overall profit of the food supply chain system; however, this is often difficult to implement in practice. In contrast, decentralized decision-making allows the manufacturer and retailer to each achieve their optimal profits within a competitive relationship. Cost-sharing contracts, however, can lead to a Pareto improvement in the optimal strategies of both the manufacturer and the retailer, thereby enhancing the overall efficiency of the supply chain.

Unlike existing studies on traceable food supply chains, which typically adopt static models, this research uses a differential game approach to examine the dynamic changes in product traceability levels and their impact on the decision-making of supply chain members. This approach offers new perspectives and methods for studying traceable food supply chains. The findings also provide practical guidance for supply chain decision-makers and offer valuable insights for business managers.

However, there are still gaps and limitations in the research, particularly in terms of the content and methods employed, which warrant further refinement and expansion, as follows:

1. This study analyzes supply chain decisions by focusing solely on the positions of primary supply chain players, excluding the influence of third parties, such as government subsidy policies, on overall decision-making. Future research could incorporate government policy subsidies into the model to explore their impact on the decision-making of supply chain participants in greater detail.
2. The study centers on a two-tier supply chain consisting of a single manufacturer and a single retailer. However, real-world supply chains typically involve multiple manufacturers and retailers, and may even follow a three-tier structure. Additionally, information asymmetry exists among these entities. Future research could develop decision models for two-tier or three-tier supply chains with multiple manufacturers and retailers to address these complexities.
3. This research employs theoretical models to coordinate the supply chain through the establishment of various contract models, with feasibility discussed using numerical examples and sensitivity analysis. However, it lacks case studies based on real-world data. Future studies could integrate actual case studies to refine and enhance the models.

5. Conclusions

This study applies differential game theory to examine dynamic strategic issues within traceable food supply chains. It considers the product traceability level, the manufacturer's effort in traceability, and the retailer's promotional efforts as decision variables, with the product traceability level serving as the state variable. The optimal decisions of supply chain members and the maximum profit of the entire supply chain are analyzed under decentralized decision-making, centralized decision-making, and cost-sharing strategies. The study yields the following conclusions:

1. Under different decision-making modes, the profits of the manufacturer, retailer, and supply chain system vary. Centralized decision-making maximizes the overall profit of the food supply chain system, but it is often difficult to achieve in practice. Decentralized decision-making allows the manufacturer and retailer to reach their optimal profits within a competitive relationship. Cost-sharing contracts can significantly improve the optimal profits of the supply chain.
2. The greater the sensitivity of consumers to the retailer's traceability efforts (denoted as the coefficient ν), the more significant the Pareto improvement. Conversely, the higher the traceability cost coefficient λ, the smaller the Pareto improvement effect.

The findings of this study not only provide guidance for food supply chain decision-makers in making optimal supply chain decisions, but also offer practical insights for improving the management efficiency of traceable supply chains. Additionally, these results can serve as a reference for government policy formulation:

1. Reducing the operational costs of traceability systems is crucial. The traceability costs borne by manufacturers affect the decision-making behaviors of all participants. Currently, the high entry barriers and initial investment required for food digital traceability systems in China create numerous challenges for food sourcing and safety supervision. Therefore, the government can lower the construction costs of traceability systems, through subsidies, rewards, and other methods, to encourage enterprises to participate more actively in building traceability systems.
2. Establishment of a unified food traceability information platform. While centralized decision-making and cost-sharing contracts can significantly increase the overall supply chain profits, they are often difficult to implement in practice. Establishing a unified information platform can link upstream and downstream stakeholders in the supply chain, creating a cooperative mechanism for information sharing and risk-sharing, thus maximizing the overall supply chain benefits.
3. Accelerating the improvement of food traceability laws and regulations. Food traceability is closely linked to food safety. Formulating comprehensive food traceability regulations is essential for ensuring food safety. By clarifying the responsibilities and obligations of each participant and stage in the food supply chain, standardizing information-sharing protocols and procedures, and implementing specific requirements for traceability system development, the regulatory framework for food traceability can be gradually enhanced.

Author Contributions: Conceptualization, J.W.; methodology, J.X.; formal analysis, J.W.; writing—original draft preparation, J.X. All authors have read and agreed to the published version of the manuscript.

Funding: This research was funded by the Major Projects of the National Social Science Fund of China under grant number 20&ZD117.

Data Availability Statement: Data are contained within the article.

Conflicts of Interest: The authors declare no conflicts of interest.

Appendix A

Proof of Proposition 1. Referring to the HJB equation method, it can be derived from Equation (5) that the optimal profit value function for the supply chain system satisfies the following HJB Equation (A1):

$$\rho V_s^c(\tau) = \max_{p,I,E}\left\{(p-c)(\alpha - p + \mu\tau + \nu E) - \frac{1}{2}\kappa I^2 - \frac{1}{2}\lambda E^2 + V_s^{c\prime}(\tau)(\gamma I - \delta\tau)\right\} \quad \text{(A1)}$$

The function $V_s^c(\tau)$ represents the profit function of the supply chain system, while $V_s^c(\tau)$ denotes the partial derivative of the profit function with respect to the product traceability level τ.

By taking the partial derivatives of Equation (A1) with respect to p, I and E, we obtain the following system of equations:

$$\begin{cases} \alpha - 2p + \mu\tau + \nu E + c = 0 \\ -\kappa I + \gamma V_s^{c\prime}(\tau) = 0 \\ (p-c)\nu - \lambda E = 0 \end{cases}$$

Solving this system of equations yields the following:

$$\begin{cases} p_s^{c^*} = \frac{(\alpha + c + \mu\tau)\lambda - c\nu^2}{2\lambda - \nu^2} \\ I = \frac{\gamma V_s^{c\prime}(\tau)}{\kappa} \\ E_s^{c^*} = \frac{(\alpha - c + \mu\tau)\nu}{2\lambda - \nu^2} \end{cases}$$

Substituting p, I and E into Equation (A1), we obtain Equation (A2):

$$\rho V_s^c(\tau) = \&\frac{\lambda(\alpha - c + \mu\tau)^2}{2(2\lambda - \nu^2)} - \delta\tau V_s^{c\prime}(\tau) + \frac{\gamma^2\left[V_S^{c\prime}(\tau)\right]^2}{2\kappa} \quad \text{(A2)}$$

From Equation (A2), we observe that the total profit $V_s^c(\tau)$ of the supply chain system is a quadratic function of the product traceability level τ.

Assume the specific form of the function $V_s^c(\tau)$ is given by Equation (A3), as follows:

$$V_s^c(\tau) = z_1\tau^2 + z_2\tau + z_3 \quad \text{(A3)}$$

where z_1, z_2 and z_3 are undetermined coefficients. Substituting Equation (A3) into Equation (A2) and simplifying, the undetermined coefficients can be calculated as Equation (A4):

$$\begin{cases} z_1^* = 2\delta + \rho + \sqrt{\frac{\kappa(2\delta+\rho)^2}{4\gamma^2} - \frac{\mu^2\lambda}{2\lambda - \nu^2}} \\ z_2^* = \frac{\mu\lambda\kappa(\alpha - c)}{(2\lambda - \nu^2)(\kappa\delta + \kappa\rho - 2\gamma^2 z_1)} \\ z_3^* = \frac{\lambda(\alpha - c)^2}{2\rho(2\lambda - \nu^2)} + \frac{\gamma^2 z_2^2}{2\kappa\rho} \end{cases} \quad \text{(A4)}$$

By substituting z_1^*, z_2^* and z_3^* into Equation (A3), the expression for the total profit function of the supply chain system $V_s^c(\tau)$ can be obtained as Equation (A5):

$$V_s^c(\tau) = z_1^*\tau^2 + z_2^*\tau + z_3^* \quad \text{(A5)}$$

Next, by substituting z_1^*, z_2^* and z_3^* into the equation $I = \frac{\gamma V_s^{c\prime}(\tau)}{\kappa}$, we obtain the expression for $I_s^{c^*}$ as: $I_s^{c^*} = \frac{\gamma\left(2z_1^*\tau + z_2^*\right)}{\kappa}$.

Substituting $I_s^{c^*}$ into the differential equation governing the product traceability level evolution, we obtain Equation (A6):

$$\dot{\tau}(t) = \frac{2z_1^*\gamma^2\tau - \delta\kappa}{\kappa}\tau + \frac{z_2^*\gamma^2}{\kappa},\ \tau(0) = \tau_0 \tag{A6}$$

From Equation (A6), the trajectory of the product traceability level over time can be calculated as Equation (A7):

$$\tau^c = (\tau_0 + \frac{z_2^*\gamma^2}{2z_1^*\gamma^2\tau - \delta\kappa})e^{\frac{2z_1^*\gamma^2\tau - \delta\kappa}{\kappa}t} - \frac{z_2^*\gamma^2}{2z_1^*\gamma^2 - \delta\kappa} \tag{A7}$$

□

Proof of Proposition 2. Following the method of the HJB equation, as demonstrated in Proposition 1, the retailer's optimal profit value function, based on Equation (10), satisfies the following HJB equation:

$$\rho V_R^d(\tau) = \max_{p,E}\left\{(p-w)(\alpha - p + \mu\tau + \nu E) - \frac{1}{2}\lambda E^2 + V_R^{d'}(\tau)(\gamma I - \delta\tau)\right\} \tag{A8}$$

By taking the partial derivatives of Equation (A8) with respect to $p,\ E$, the following system of equations is obtained:

$$\begin{cases} \alpha - 2p + \mu\tau + \nu E + w = 0 \\ (p-w)\nu - \lambda E = 0 \end{cases}$$

Solving this system yields the following:

$$\begin{cases} p_R^{d^*} = \frac{(\alpha + w + \mu\tau)\lambda - w\nu^2}{2\lambda - \nu^2} \\ E_R^{d^*} = \frac{(\alpha - w + \mu\tau)\nu}{2\lambda - \nu^2} \end{cases}$$

Based on the equilibrium solution, it can be observed that the retail price p set by the retailer is positively correlated with market demand α, wholesale price w, the demand impact coefficient of traceability μ, and the product traceability level τ. The effort in traceability promotion E is positively correlated with market demand α, the traceability impact coefficient μ, the product traceability level τ, and the traceability promotion impact coefficient ν, while it is negatively correlated with the traceability promotion cost coefficient λ. This indicates that, as the product traceability level and related market demand increase, the retailer is willing to invest more resources in traceability promotion. Conversely, if the traceability promotion cost increases, the retailer's enthusiasm for promotion will decrease accordingly.

Similarly, following the proof process for the retailer, the manufacturer's optimal profit value function satisfies the following HJB Equation (A9):

$$\rho V_M^d(\tau) = \max_{I}\left\{(w-c)(\alpha - p + \mu\tau + \nu E) - \frac{1}{2}\kappa I^2 + {V_M^d}'(\tau)(\gamma I - \delta\tau)\right\} \tag{A9}$$

Substituting $p_R^{d^*}$ and $E_R^{d^*}$ into Equation (A9), and taking the partial derivative with respect to I, we obtain the following:

$$I = \frac{\gamma {V_M^d}'(\tau)}{\kappa}$$

Substituting $p_R^{d^*}$, $I_R^{d^*}$ and $E_R^{d^*}$ into Equations (A8) and (A9), and simplifying, we obtain Equations (A10) and (A11):

$$\rho V_M^d(\tau) = \frac{\lambda(\alpha - w + \mu\tau)(w - c)}{2\lambda - \nu^2} - \delta\tau V_M^{d\,\prime}(\tau) + \frac{\gamma^2 [V_M^{d\,\prime}(\tau)]^2}{2\kappa} \tag{A10}$$

$$\rho V_R^d(\tau) = \&\frac{\lambda(\alpha - w + \mu\tau)^2}{2(2\lambda - \nu^2)} - \delta\tau V_R^{d\prime}(\tau) + \frac{\gamma^2 V_R^{d\prime}(\tau) V_M^{d\,\prime}(\tau)}{\kappa} \tag{A11}$$

From the above expressions, it can be observed that the manufacturer's profit $V_M^d(\tau)$ has a linear relationship with the product traceability level τ, while the retailer's profit $V_R^d(\tau)$ has a quadratic relationship with τ.

Let the specific expressions for the functions $V_M^d(\tau)$ and $V_R^d(\tau)$ be as Equations (A12) and (A13):

$$V_M^d(\tau) = a_1\tau + a_2 \tag{A12}$$

$$V_R^d(\tau) = b_1\tau^2 + b_2\tau + b_3 \tag{A13}$$

where $a_1, a_2; b_1, b_2, b_3$ are undetermined coefficients. Substituting Equation (A12) into Equation (A10) and Equation (A13) into Equation (A11), and simplifying, the undetermined coefficients can be calculated as follows in Equations (A14) and (A15):

$$\begin{cases} a_1^* = \frac{\mu\lambda(w-c)}{(2\lambda-\nu^2)(\delta+\rho)} \\ a_2^* = \frac{\lambda(\alpha-w)(w-c)}{\rho(2\lambda-\nu^2)} + \frac{\gamma^2 a_1^2}{2\kappa\rho} \end{cases} \tag{A14}$$

$$\begin{cases} b_1^* = \frac{\mu^2\lambda}{2(2\lambda-\nu^2)(2\delta+\rho)} \\ b_2^* = \frac{\mu\lambda(\alpha-w)}{(2\lambda-\nu^2)(\delta+\rho)} + \frac{2\gamma^2 a_1 b_1}{\kappa(\delta+\rho)} \\ b_3^* = \frac{\lambda(\alpha-w)^2}{2\rho(2\lambda-\nu^2)} + \frac{\gamma^2 a_1 b_2}{\kappa\rho} \end{cases} \tag{A15}$$

Substituting a_1^* and a_2^* into Equation (A12), and b_1^*, b_2^* and b_3^* into Equation (A13), the expressions for the manufacturer's profit function $V_M^d(\tau)$ and the retailer's profit function $V_R^d(\tau)$ are obtained as Equations (A16) and (A17):

$$V_M^d(\tau) = a_1^*\tau + a_2^* \tag{A16}$$

$$V_R^d(\tau) = b_1^*\tau^2 + b_2^*\tau + b_3^* \tag{A17}$$

Substituting a_1^* and a_2^* into $I = \frac{\gamma V_M^{d\,\prime}(\tau)}{\kappa}$, we obtain $I_M^{d^*} = \frac{a_1^*\gamma}{\kappa}$.

Substituting $I_M^{d^*}$ into the differential equation governing the product traceability level dynamics, we obtain Equation (A18):

$$\dot{\tau}(t) = -\delta\tau + \frac{a_1^*\gamma^2}{\kappa}, \tau(0) = \tau_0 \tag{A18}$$

From Equation (A18), the trajectory of the product traceability level over time is derived as Equation (A19):

$$\tau^d = (\tau_0 - \frac{a_1^*\gamma^2}{\delta\kappa})e^{-\delta\tau t} + \frac{a_1^*\gamma^2}{\delta\kappa} \tag{A19}$$

□

Proof of Proposition 3. Following the method of the HJB equation, as demonstrated in Proposition 2, the retailer's optimal profit value function, based on Equation (16), satisfies the following HJB equation:

$$\rho V_R^p(\tau) = \max_{p,E}\left\{(p-w)(\alpha - p + \mu\tau + \nu E) - (1-\varphi)\frac{1}{2}\lambda E^2 + V_R^{p\prime}(\tau)(\gamma I - \delta\tau)\right\} \quad \text{(A20)}$$

By taking the partial derivatives of Equation (A20) with respect to p and E, the following system of equations is obtained:

$$\begin{cases} \alpha - 2p + \mu\tau + \nu E + w = 0 \\ (p-w)\nu - (1-\varphi)\lambda E = 0 \end{cases}$$

Solving this system yields the following:

$$\begin{cases} p_R^{p^*} = \frac{(\alpha + w + \mu\tau)\lambda - w\nu^2}{2(1-\varphi)\lambda - \nu^2} \\ E_R^{p^*} = \frac{(\alpha - w + \mu\tau)\nu}{2(1-\varphi)\lambda - \nu^2} \end{cases}$$

Following the same approach for the manufacturer, the optimal profit value function satisfies the following HJB Equation (A21):

$$\rho V_M^p(\tau) = \max_{\varphi,I}\left\{(w-c)(\alpha - p + \mu\tau + \nu E) - \frac{1}{2}\kappa I^2 - \varphi\frac{1}{2}\lambda E^2 + V_M^{p\ \prime}(\tau)(\gamma I - \delta\tau)\right\} \quad \text{(A21)}$$

Substituting $p_R^{p^*}$ and $E_R^{p^*}$ into Equation (A21), and taking the partial derivatives with respect to φ and I, the following system of equations is obtained:

$$\begin{cases} \frac{4\lambda(w-c)\left(\alpha\nu^2 + \mu\tau\nu^2 - \alpha\lambda - w\lambda - \mu\tau\lambda\right) - (\lambda^2 + 4\varphi\lambda^2)(\alpha - w + \mu\tau)^2\nu^2}{2(2(1-\varphi)\lambda - \nu^2)^2} = 0 \\ -\kappa I + \gamma V_M^{p\ \prime}(\tau) = 0 \end{cases}$$

Solving this system yields the following:

$$\begin{cases} \varphi^* = \frac{4(w-c)\left(\alpha\nu^2 + \mu\tau\nu^2 - \alpha\lambda - w\lambda - \mu\tau\lambda\right) - 1}{4\lambda(\alpha - w + \mu\tau)^2\nu^2} \\ I = \frac{\gamma V_M^{p\ \prime}(\tau)}{\kappa} \end{cases}$$

Substituting $p_R^{p^*}$, φ^*, I and $E_R^{p^*}$ into Equations (A20) and (A21), and simplifying, we obtain Equations (A22) and (A23):

$$\rho V_M^p(\tau) = \frac{(1-\varphi)\lambda(\alpha - w + \mu\tau)(w-c)}{2(1-\varphi)\lambda - \nu^2} - \frac{\varphi\lambda(\alpha - w + \mu\tau)^2\nu^2}{2[2(1-\varphi)\lambda - \nu^2]^2} - \delta\tau V_M^{p\ \prime}(\tau) + \frac{\gamma^2\left[V_M^{p\ \prime}(\tau)\right]^2}{2\kappa} \quad \text{(A22)}$$

$$\rho V_R^p(\tau) = \frac{(1-\varphi)\lambda(\alpha - w + \mu\tau)^2}{2[2(1-\varphi)\lambda - \nu^2]} - \delta\tau V_R^{p\prime}(\tau) + \frac{\gamma^2 V_R^{p\prime}(\tau)V_M^{p\ \prime}(\tau)}{\kappa} \quad \text{(A23)}$$

From the above equations, it can be observed that the manufacturer's profit $V_M^p(\tau)$ and the retailer's profit $V_R^p(\tau)$ are both quadratic functions of the traceability level τ.

Let the specific forms of the functions $V_M^p(\tau)$ and $V_R^p(\tau)$ be Equations (A24) and (A25), as follows:

$$V_M^p(\tau) = m_1\tau^2 + m_2\tau + m_3 \quad \text{(A24)}$$

$$V_R^p(\tau) = n_1\tau^2 + n_2\tau + n_3 \quad \text{(A25)}$$

where $m_1, m_2, m_3; n_1, n_2, n_3$ are undetermined coefficients. Substituting Equations (A24) into Equation (A22), and Equation (A25) into Equation (A23), and simplifying, the undetermined coefficients can be calculated as Equations (A26) and (A27):

$$\begin{cases} m_1^* = 2\delta + \rho + \sqrt{\frac{\kappa(2\delta+\rho)^2}{\gamma^2} + \frac{\varphi\nu^2\mu\lambda}{2\lambda-\nu^2}} \\ m_2^* = \frac{2\mu\lambda[(w-c)-\varphi(\alpha-w)]}{2[\kappa(\delta+\rho)-\gamma^2 m_1][2(1-\varphi)\lambda-\nu^2]^2} \\ m_3^* = \frac{2(1-\lambda)(\alpha-w)(w-c)[2(1-\varphi)\lambda-\nu^2]-\varphi\lambda(\alpha-w)^2\nu^2}{2[2(1-\varphi)\lambda-\nu^2]^2} + \frac{\gamma^2 m_2^2}{2\kappa\rho} \end{cases} \tag{A26}$$

$$\begin{cases} n_1^* = \frac{(1-\varphi)\mu^2\lambda\kappa}{2[2(1-\varphi)\lambda-\nu^2][(2\delta+\rho)\kappa-4\gamma^2 m_1]} \\ n_2^* = \frac{(1-\varphi)(\alpha-w)\mu\lambda\kappa}{[2(1-\varphi)\lambda-\nu^2][(\delta+\rho)\kappa-2\gamma^2 m_1]} + \frac{2\gamma^2 n_1 m_2}{(\delta+\rho)\kappa-2\gamma^2 m_1} \\ n_3^* = \frac{\lambda(1-\varphi)(\alpha-w)^2}{2\rho[2(1-\varphi)\lambda-\nu^2]} + \frac{\gamma^2 n_2 m_2}{\kappa\rho} \end{cases} \tag{A27}$$

Substituting m_1^*, m_2^* and m_3^* into Equation (A24), and n_1^*, n_2^* and n_3^* into Equation (A25), the expressions for the manufacturer's profit function $V_M^p(\tau)$ and the retailer's profit function $V_R^p(\tau)$ are obtained as Equations (A28) and (A29):

$$V_M^p(\tau) = m_1^*\tau^2 + m_2^* + m_3^* \tag{A28}$$

$$V_R^p(\tau) = n_1^*\tau^2 + n_2^*\tau + n_3^* \tag{A29}$$

Substituting m_1^*, m_2^* and m_3^* into $I = \frac{\gamma V_M^{p\,\prime}(\tau)}{\kappa}$, we obtain $I_M^{p^*} = \frac{\gamma\left(m_1^*\tau + m_2^*\right)}{\kappa}$.

Substituting $I_M^{p^*}$ into the differential equation for the traceability level change process, we obtain Equation (A30):

$$\dot{\tau}(t) = \frac{m_1^*\gamma^2 - \delta\kappa}{\kappa}\tau + \frac{m_2^*\gamma^2}{\kappa},\ \tau(0) = \tau_0 \tag{A30}$$

From Equation (A30), the trajectory of the product traceability level over time is derived as Equation (A31):

$$\tau^p = (\tau_0 - \frac{m_2^*\gamma^2}{m_1^*\gamma^2 - \delta\kappa})e^{\frac{m_1^*\gamma^2-\delta\kappa}{\kappa}t} - \frac{m_2^*\gamma^2}{m_1^*\gamma^2 - \delta\kappa} \tag{A31}$$

References

1. Zhang, B.; Ma, R. Digital Governance Logic and Optimization Path of Food Safety Risks: Based on the Information Ecosystem Perspective. *Dongyue Acad. J.* **2023**, *44*, 128–136+192. [CrossRef]
2. Chen, J.; Luo, D.; Tang, C. Trustworthy Traceability System for Agricultural Internet of Things Based on Blockchain. *J. Inf. Secur.* **2022**, *7*, 139–149. [CrossRef]
3. Zhu, J.; Fu, Y. Dynamic Multi-Center Collaborative Certification Model for Supply Chain Based on Blockchain. *J. Netw. Inf. Secur.* **2016**, *2*, 27–33.
4. Zheng, K.; Liu, S. Research and Design of a Multilateral Platform for Agricultural Product Quality Safety Traceability. *China Agric. Sci. Technol. Bull.* **2017**, *19*, 52–58. [CrossRef]
5. Opara, L. Traceability in agriculture and food supply chain: A review of basic concepts, technological implications, and future prospects. *Eur. J. Oper. Res.* **2003**, *1*, 101–106.
6. Olsen, P.; Borit, M. The components of a food traceability system. *Trends Food Sci. Technol.* **2018**, *77*, 143–149. [CrossRef]
7. Purwandoko, P.; Seminar, K.; Sutrisno; Sugiyanta. Framework for design of traceability system on organic rice certification. *IOP Conf. Ser. Earth Environ. Sci.* **2018**, *147*, 012044. [CrossRef]
8. Regattieri, A.; Gamberi, M.; Manzini, R. Traceability of food products: General framework and experimental evidence. *J. Food Eng.* **2007**, *81*, 347–356. [CrossRef]

9. Song, H.; Wang, R.; Ma, W. Research on Information Sharing and Coordination Mechanism of Food Supply Chain Traceability Based on Differential Game. *J. Huazhong Agric. Univ. (Soc. Sci. Ed.)* **2018**, *03*, 144–151+160. [CrossRef]
10. Zhang, L.; Yang, J. Information Sharing and Financing Decisions in Dual-Channel Supply Chains Considering Retailer Financial Constraints. *Manag. Rev.* **2023**, *35*, 280–291. [CrossRef]
11. Jin, W.; Wang, S.; Deng, S. Research on the Impact of Internet Technology Applications on the Spatial Organization of Retail Industry. *Hum. Geogr.* **2018**, *33*, 1–10. [CrossRef]
12. Jiang, N.; Zhang, L.; Chen, Y. Supply Chain Information Traceability Based on Consumer Experience Demands and Its System Construction. *Inf. Theory Pract.* **2018**, *41*, 123–128. [CrossRef]
13. Xia, H.; Liu, J. How Blockchain Technology Can Integrate with Agriculture? Insights from Japan's Experience. *Rural Econ.* **2022**, *01*, 20–29.
14. Zhen, Y.; Tian, J.; Wang, W. Blockchain Investment and Copyright Protection Strategies in Digital Publishing Supply Chain. *Syst. Manag. J.* **2024**, *33*, 589–600.
15. Cai, J.; Jiang, L.; Yang, M. Risk-Informed Green Supply Chain Decisions under Asymmetric Information. *Syst. Eng. Theory Pract.* **2024**, *44*, 1615–1632.
16. Yang, H.; Chen, X.; Shi, W. Green Product Supply Chain Decisions under Presale Model Considering Retailer Fairness Concerns. *China Manag. Sci.* **2023**, *9*, 1–14. [CrossRef]
17. Hong, J.; Huang, P. Dynamic Coordination Mechanism for Quality Control in Two-Level Supply Chain. *J. Manag. Eng.* **2011**, *25*, 62–65+94. [CrossRef]
18. Ma, D.; Hu, J. Dynamic Equilibrium Strategy for Retailer Recovery in Closed-Loop Supply Chain with Fairness Concerns. *China Manag. Sci.* **2019**, *27*, 70–78. [CrossRef]
19. Bertinalli, L.; Camacho, C.; Zou, B. Carbon capture and storage and transboundary pollution: A differential game approach. *Eur. J. Oper. Res.* **2014**, *237*, 721–728. [CrossRef]
20. Zhou, M.; Lin, J. Cooperative advertising and pricing models in a dynamic marketing channel. *J. Syst. Sci. Syst. Eng.* **2014**, *23*, 94–110. [CrossRef]
21. Xia, Z.; Sun, S. Research on Quality Behavior Coordination Mechanism in Secondary Pork Supply Chain Based on Differential Game. *Oper. Res. Manag.* **2014**, *23*, 198–205.
22. Xu, C.; Zhao, D.; Yuan, B. Differential Game Model of Joint Emission Reduction and Low-Carbon Promotion in Upstream and Downstream Supply Chains. *J. Manag. Sci. China* **2016**, *19*, 53–65.