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Magnetic Transport in Spin Antiferromagnets for Spintronics Applications

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Abstract: Had magnetic monopoles been ubiquitous as electrons are, we would probably have had a different form of matter, and power plants based on currents of these magnetic charges would have been a familiar scene of modern technology. Magnetic dipoles do exist, however, and in principle one could wonder if we can use them to generate magnetic currents. In the present work, we address the issue of generating magnetic currents and magnetic thermal currents in electrically-insulating low-dimensional Heisenberg antiferromagnets by invoking the (broken) electricity-magnetism duality symmetry. The ground state of these materials is a spin-liquid state that can be described well via the Jordan–Wigner fermions, which permit an easy definition of the magnetic particle and thermal currents. The magnetic and magnetic thermal conductivities are calculated in the present work using the bond–mean field theory. The spin-liquid states in these antiferromagnets are either gapless or gapped liquids of spinless fermions whose flow defines a current just as the one defined for electrons in a Fermi liquid. The driving force for the magnetic current is a magnetic field with a gradient along the magnetic conductor. We predict the generation of a magneto-motive force and realization of magnetic circuits using low-dimensional Heisenberg antiferromagnets. The present work is also about claiming that what the experiments in spintronics attempt to do is trying to treat the magnetic degrees of freedoms on the same footing as the electronic ones.

Keywords: spin liquids; spintronics; magnetic transport; duality symmetry

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1. Introduction

The issue of the adequate definition of the spin current had attracted significant interest because of its importance in spintronics' applications [1–3]. An et al. [4] used the relativistic Dirac equation in order to define such a current. In addition, many other authors argued that the spin transport includes both linear displacement of spins as well as angular motion due to the rotation of the spins. One of the earliest problems encountered in the definition of the spin current is the satisfaction of the continuity equation [5,6]. It is interesting that spintronics experiments attempt to marry in practice between spin currents and electric currents, and create one current from the other and vice versa. It is as if these experiments try to prove in a practical manner some sort of symmetry between electricity and magnetism. We propose that, at a more fundamental level, these experiments attempt to prove the duality electricity–magnetism symmetry, which is missing from the Maxwell equations in the presence of matter (Maxwell equations are symmetric under the duality symmetry transformation in vacuum). While this symmetry is broken at the monopole level, it could approximately hold at the dipole level in materials where the charge degrees of freedom are practically frozen due to a large energy gap in their excitations. In such materials, the magnetic degrees of freedom carried by magnetic dipoles are responsible for the low-lying energy excitations. The low-dimensional (chains and ladders) Heisenberg antiferromagnets constitute a good example of these materials.

For these low-dimensional antiferromagnets, a natural way to deal with the difficulties associated with the definition of the magnetic particle and heat currents is the usage of the duality symmetry of electromagnetism. In the remainder of this manuscript, we will refer to the spin current and spin thermal current in these materials as magnetic current and magnetic thermal current, respectively. This relabeling is necessary in order to best reflect this symmetry between electricity and magnetism. It is well known that the Maxwell equations would have been fully symmetric under the duality transformation if magnetic monopoles existed. If they did, magnetic currents would have been defined in the same way as electric currents. In the present real situation where the duality symmetry between electricity and magnetism is broken in the presence of sources (matter), the magnetic dipoles resulting from the spins' degrees of freedom of electrons do exist, however. In the Heisenberg antiferromagnets, these magnetic dipoles interact and form the so-called spin liquids that bear interesting similarities with the Fermi liquid states of electrons in conventional metals.

We define the magnetic current and magnetic thermal current after transforming the spin degrees of freedom using the Jordan–Wigner (JW) transformation in one-dimension (1D), or its generalized sisters in the case of ladders [7,8]. This approach is well suited for insulating antiferromagnets, like the linear-chain compound Sr_2CuO_3. Such materials are electrically insulating because the electric charge degrees of freedom are suppressed by large excitation energy gaps, and are characterized by spin-1/2 moments that are arranged on chains or ladders. Due to their strong spatial anisotropic magnetic exchange interactions and large quantum fluctuations, they do not magnetically order even at very low temperatures. One of the interesting consequences of using the JW transformation is the definition of a magnetic current (rather than a spin current) because such a transformation puts the treatment of the spin degrees of freedom on the same footing as the electronic charge degrees of freedom in metals. We claim that this one-to-one correspondence between the magnetic moments (spins) in the Heisenberg antiferromagnets and electrons (charges) in metals is reminiscent of the duality symmetry of electricity and magnetism in vacuum, or even in matter had the magnetic monopoles [9] been ubiquitous as electrons do–that is to say, that the original 1D JW and its higher dimension generalized sisters transform the spins into spinless fermions that behave exactly like electrons as far as Fermi statistics and transport properties are concerned.

The Heisenberg quantum antiferromagnets are modeled with the Heisenberg Hamiltonian that consists of exchange interactions between spins on adjacent sites. The 1D case relevant for Sr_2CuO_3, for example, is simpler to analyze, and will be used throughout this paper. Note, however, that the results of this work can be generalized to three-leg ladder systems, which behave as effective single Heisenberg chains especially when the interchain interaction is much greater than the intrachain one [10]. The two-leg Heisenberg ladder is, however, gapped [11], and an approach will be developed in the near future by taking into account this gap. Upon using the JW transformation, the 1D Hamiltonian maps into that of spinless fermions with a tight-binding kinetic energy term corresponding to the XY part of the spin Hamiltonian, and a repulsive interaction between JW fermions on adjacent sites resulting from the Ising term of the Hamiltonian. Afterwards, we define particle and thermal currents for these spinless fermions in the same way as for electrons in a metal, and use the techniques of transport theory including the Kubo formula for calculating the conductivity and the Green–Kubo formula for the magnetic thermal conductivity.

The driving force for the magnetic current of the spinless fermions can be provided by an external magnetic field with a gradient along the chain of spins. The reason for this is that the JW transformation maps the magnetic field in the Zeeman-coupling term onto the chemical potential for the spinless JW fermions, as is well known. Thus, a gradient in the magnetic field forces the spinless fermions to flow along the chain in order to lower their energy, just as electrons do in order to lower their energy when a gradient in the chemical potential is applied to them. Note that a magnetic field with a gradient, rather than a uniform magnetic field alone, is needed for the present case of a magnetic current because this magnetic current is not that of magnetic monopoles, but that of magnetic dipoles. This is similar to the fact that a gradient in the electric field can be the driving force for an electric dipole. The experimental work by Hirobe et al. [12] reported the observation of spin current in Sr_2CuO_3, which resulted from a temperature gradient. Indeed a temperature gradient ∇T can generate a flow of the JW fermions just as it does for electrons in metals. We, however, argue for and support the more convenient utilization of a magnetic field gradient. It is worth mentioning that we think that the Heisenberg antiferromagnets can be incorporated into spintronics devices without using electric contacts. The magnetic fields generated by circulating electric currents in the regular electric circuits of a given device can be taken advantage of to induce a magnetic current in the Heisenberg antiferromagnet part of the device.

The present paper is organized as follows. In Section 2, the nature of the JW fermions is discussed in connection with the (broken) electricity-magnetism duality symmetry. In Section 3, a review of the bond–mean-field theory (BMFT) applied to the Heisenberg chain in a magnetic field is presented. In Section 4, the particle current density, the Green's and spectral functions are calculated for the JW fermions. In Section 5, the current–current correlation function is calculated and used to derive the conductivity of the JW fermions. Section 6 deals with the calculation of the magnetic thermal conductivity. In Section 7, the main result of the present work is discussed, and predictions for potential applications are outlined. Conclusions are drawn in Section 8.

2. The JW Transformation and Duality Symmetry

In addition to the fact that the JW transformation preserves the spin commutation relation as required, we think that a more profound aspect of this transformation is related to the electricity–magnetism duality (broken) symmetry as explained in the introduction. This transformation maps magnetic dipoles (magnetic degrees of freedom) onto spinless fermions whose statistics and physics are similar to the ones of electrons, except for the electric charge. The 1D JW transformation [13] reads as:

$$\begin{aligned} S_i^- &= c_i e^{i\sum_{j=1}^{i-1} c_j^\dagger c_j}, \\ S_i^z &= c_i^\dagger c_i - 1/2, \end{aligned} \tag{1}$$

where i or j label the chain sites, S_i^- is the spin ladder operator, and S_i^z is the z-component of the spin operator. c_i ($c_i^\dagger$) is the JW annihilation (creation) operator. We believe that there is a profound reason behind the fact that the JW fermions satisfy the same statistics as the original electrons that carry the spin (thus magnetic) degrees of freedom, and that this is not a mere accident. The JW transformation bears in it the footprint of the electricity-magnetism duality symmetry of the Maxwell equations in the vacuum. The rational for this proposal is that if the Maxwell equations included magnetic monopoles, one could have had a transformation between electrons and magnetic monopoles in any given study of the electronic and magnetic properties of any material, and that the magnetic and electronic properties would have been transformed naturally into each other as a consequence of this duality symmetry. For example, we could have had defined easily the magnetic current of magnetic monopoles. However, since the duality symmetry does not apply in the presence of matter, the magnetic degrees of freedom, which derive their meaning only from the electronic ones (electrons' spin here), are transformed into the JW fermions for the Heisenberg antiferromagnets. This transformation tells us that the magnetic dipoles are fermions that do not carry a spin. In the next section, we will use this symmetry to predict and argue that it is possible to construct magnetic generators and magnetic circuits made of Heisenberg antiferromagnets. This constitutes the central finding of the present work.

3. The Magnetic Current in the Bond–Mean Field Theory

In the presence of a magnetic field gradient along the chain, the Heisenberg model assumes the form:

$$H_{1D} = J\sum_i \mathbf{S}_i \cdot \mathbf{S}_{i+1} - g\mu_B \sum_i B_i^{\text{ex}} S_i^z, \tag{2}$$

where J is the exchange coupling constant and B_i^{ex} is a position dependent magnetic field that can be taken to vary linearly with position along the chain; i.e., $B_i^{\text{ex}} = B_0^{\text{ex}} x$ with B_0^{ex} a field per unit length. In terms of the JW fermions, Equation (1), this Hamiltonian maps onto:

$$H_{1D} = \frac{J}{2} \sum_i c_i^\dagger c_{i+1} + \text{H.C.} + J \sum_i \left(c_i^\dagger c_i - \frac{1}{2}\right)\left(c_{i+1}^\dagger c_{i+1} - \frac{1}{2}\right) - \sum_i h_i c_i^\dagger c_i + \sum_i \frac{h_i}{2}, \tag{3}$$

where $h_i = g\mu_B B_i^{\text{ex}}$, with g being the Landé factor and μ_B the Bohr magneton. As is well known, a constant magnetic field $h_i \equiv h$ plays the role of the chemical potential for the JW fermions. Such a constant magnetic field is also known to polarize the spins along its direction, with the magnetization $M_z = \chi h$ for $h \ll J$, where χ is the uniform spin susceptibility. A gradient in the magnetic field along the chain is equivalent to tilting the chemical potential. Such tilting causes the JW fermions to flow, thus creating a current of these fermionic particles. For the spin degrees of freedom, the flow of the JW fermions occurs with hopping amplitude $J/2$ and results in the flow of spin flip fluctuations, since the presence of a JW fermion at a given site is a spin up and its absence is identified with a spin down; Equation (1).

The need for a gradient in the magnetic field to drag the magnetic excitations resulting from the magnetic dipoles is similar to the fact that a gradient in an electric field is needed in order to drag electric dipoles; a uniform electric field alone does not act on the dipoles, except by a force couple. We think that this similarity is a consequence of the duality symmetry between electricity and magnetism. This, in turn, supports our claim that the JW transformation bears a signature of this symmetry.

We assume that the gradient in the magnetic field is much smaller than the spin exchange coupling. We thus calculate the magnetic conductivity and magnetic thermal conductivity using the (Green–)Kubo formula in the linear response approximation. The current of the JW fermions is readily defined as the magnetic current, and the current–current correlation function is evaluated in the limit of a uniform magnetic field to get this conductivity.

In Ref. [11], the effect of a uniform field on the Heisenberg chain was investigated in the framework of the BMFT [7]. In brief, the Hamiltonian in the presence of the Zeeman coupling term $g\mu_B B \sum_i S_i^z$ with a uniform magnetic field B along the z-axis, takes on the form:

$$H_{1D} = NJQ^2 + Nh/2 - NM_z(M_z+1)J + \sum_k \Psi_k^\dagger \mathcal{H}_{1D}(h) \Psi_k, \tag{4}$$

where N is the total number of sites, and $M_z = \langle S_i^z \rangle$ is the magnetization per site. The two-component spinor Ψ_k is given by:

$$\Psi_k = \begin{pmatrix} c_k^A \\ c_k^B \end{pmatrix}, \tag{5}$$

and the Hamiltonian density matrix by:

$$\mathcal{H}_{1D}(h) = (2M_z J - h)\sigma_0 - (J_1 \sin k)\sigma_2, \tag{6}$$

where σ_0 is the 2×2 identity matrix and σ_2 the second Pauli matrix. Here, $J_1 = J(1+2Q)$ with $Q = |\langle c_i c_{i+1}^\dagger \rangle|$ having been defined as the spin bond parameter, and $h = g\mu_B B$. The chain is subdivided into two sublattices A and B as a consequence of the strong antiferromagnetic correlations that decay only algebraically with distance because the ground–state correlation length is infinite. Locally, the spins maintain a staggered orientation that justifies the use of the bipartite character. This gives rise to two types of JW fermions at the mean-field level, and the creation and annihilation operators are labeled by the two sublattice indexes. At the mean-field level, the chemical potential renormalizes to $h' = h - 2M_z J = h(1 - 2J\chi)$ if $h \ll J$, or to $h' = h - J$ if $h > h_c$ in the fully saturated state. χ is the uniform spin susceptibility, and $h_c = 2J$ is the magnetic field above which the magnetization saturates [14,15].

Diagonalizing Equation (6) yields the following energy eigenvalues:

$$E_p(k) = -h' + pJ_1|\sin k|; \quad p = \pm.$$

The magnetization per site, M_z, and the bond parameter, Q, are given by [11]:

$$\begin{array}{rcl} Q & = & -\frac{1}{2}\int \frac{dk}{2\pi}|\sin k| \sum_{p=\pm} p n_F[E_p(k)], \\ M_z & = & \frac{1}{2}\int \frac{dk}{2\pi} \sum_{p=\pm} n_F[E_p(k)] - \frac{1}{2}, \end{array} \tag{7}$$

where $n_F(x) = 1/(1+e^{\beta x})$ is the Fermi factor. Here, $\beta = 1/k_B T$ is inverse temperature.

In order to calculate the magnetic particle and thermal conductivities, we will next calculate the current density, Green's function and spectral function for the JW fermions within the BMFT. We deal first with the magnetic conductivity.

4. Current Density, Green and Spectral Functions

4.1. Current Density Operator

Because every spin is carried by a localized electron on the chain, the spins are not mobile, which means that they cannot create a spin current in the same way as in metals where electrons are mobile. The spin fluctuations can however propagate along the chain, thus creating the magnetic current that we seek to calculate. The spin fluctuations' propagation is caused by the kinetic energy of the JW fermions in Hamiltonian (3), and this magnetic current's density operator is therefore given within the tight-binding approach by:

$$\mathbf{j} = -i\frac{J}{2}\sum_i \left\{ (c^{B\dagger}_{2i+1}c^A_{2i} - c^{A\dagger}_{2i}c^B_{2i+1}) + (c^{A\dagger}_{2i}c^B_{2i-1} - c^{B\dagger}_{2i-1}c^A_{2i}) \right\}\hat{\mathbf{x}}, \tag{8}$$

where $\hat{\mathbf{x}}$ is a unit vector along the chain direction. Using the phase configuration ...$\pi - 0 - \pi$... on the intersite bonds along the chain, which is utilized to write the mean-field Hamiltonian in Equation (4) [11], and the Fourier transform $c^\alpha_i = \frac{1}{\sqrt{N}}\sum_k e^{ikr_i}c^\alpha_k$ where $\alpha = A, B$, the operator $\mathbf{j}$ takes on the form:

$$\mathbf{j} = \sum_k i\mathbf{v}(k)(c^{A\dagger}_k c^B_k - c^{B\dagger}_k c^A_k) = \sum_k \mathbf{v}(k)\Psi^\dagger_k \sigma_2 \Psi_k, \tag{9}$$

with $\mathbf{v}(k) = J\cos k\hat{\mathbf{x}}$ being the spin velocity along the chain. Note the cosine function in this spin velocity instead of sine because of the above phase configuration. This spin velocity being in cosine is in agreement with the exact result for the energy spectrum $\frac{\pi}{2}J\sin k$ [14].

4.2. Green and Spectral Functions

The single-particle Green's function is defined by $\mathcal{G} = (i\omega_n - \mathcal{H}_{1D})^{-1}$. Within the BMFT, the latter takes on the 2×2 matrix form:

$$\mathcal{G}(k,\omega_n) = \frac{1}{2}\sum_{p=\pm}\frac{\sigma_0 - p\sigma_2 \sin k/|\sin k|}{i\omega_n - E_p(k)}. \tag{10}$$

The retarded Green's function is $\mathcal{G}_{ret}(k,\omega) = \mathcal{G}(k, i\omega_n \to \omega + i\eta)$, with η a very small positive constant.

The spectral function $\mathcal{A}(k,\omega) = -2\mathrm{Im}\mathcal{G}_{ret}(k,\omega)$ assumes the following expression:

$$\mathcal{A}(k,\omega) = \sum_{p=\pm}\left[\eta\sigma_0 + p\left(\omega - E_p(k)\right)\sigma_2' \frac{\sin k}{|\sin k|}\right] a_p(k,\omega), \tag{11}$$

with $a_p(k,\omega) = [(\omega - E_p(k))^2 + \eta^2]^{-1}$, and the matrix $\sigma_2' = -i\sigma_2 = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$.

5. The j-j Correlation Function and Magnetic Conductivity

5.1. Kubo Formula

The Kubo formula for the current-current correlation function, $\Pi(q,\tau) = -\frac{1}{V}\langle \mathbf{j}^\dagger(q,\tau)\mathbf{j}(q,0)\rangle$, where V is the volume of the sample, yields in the long-wavelength ($q = 0$) limit:

$$\begin{aligned} \Pi(i\omega_m) &= \frac{1}{V}\int_0^\beta d\tau e^{i\omega_m\tau} \sum_{\mathbf{k},\mathbf{k}'} \mathbf{v}(k')\mathbf{v}(k)\langle T_\tau \Psi_{k'}^\dagger(\tau)\sigma_2\Psi_{k'}(\tau)\Psi_k^\dagger(0)\sigma_2\Psi_k(0)\rangle \\ &= \int_0^\beta d\tau e^{i\omega_m\tau}\int_{RBZ}\frac{dk}{2\pi}v^2(k)\mathrm{Tr}[\mathcal{G}(k,\tau)\sigma_2\mathcal{G}(k,-\tau)\sigma_2] \\ &= \int_{RBZ}\frac{dk}{2\pi}\int\frac{d\epsilon}{2\pi}\frac{d\epsilon'}{2\pi}\frac{n_F(\epsilon)-n_F(\epsilon')}{\epsilon-\epsilon'+i\omega_m}v^2(k)\mathrm{Tr}[\mathcal{A}(k,\epsilon')\sigma_2\mathcal{A}(k,\epsilon)\sigma_2], \end{aligned} \tag{12}$$

with $\psi_k^1 = c_k^A$ and $\psi_k^2 = c_k^B$. In going from the first line to the second in Equation (12), the summation over the 3D wavevector reduces to only the 1D component, with the summation over the transverse components, k_y and k_z, yielding an overall factor $1/bc$ where b and c are the lattice parameters in the y and z directions, respectively. Note that the sum over the x-component of $\mathbf{k}$, labeled k, divided by the length of the sample L is replaced by $\int \frac{dk}{2\pi}$ in the limit $L/a \to \infty$ where a is the lattice parameter in the x direction. For convenience, we set $a = b = c = 1$ and will reinstate these parameters in the final results of conductivities. The integral over k in Equation (12) is carried over the reduced Brillouin zone (RBZ) only in order to avoid double counting as a result of the two sublattices.

5.2. The Real Part of the Magnetic Conductivity

We next use the analytical limit $i\omega_n \to \omega + i\eta$ to obtain the retarded conductivity, and write $\frac{1}{x+i\eta} = P(x) - i\pi\delta(x)$, where $P(x)$ is the principal part of x and $\delta(x)$ the Dirac delta distribution, in order to cast the conductivity in the following form:

$$\begin{aligned} \sigma(\omega) &= \int_{RBZ}\frac{dk}{2\pi}\int\frac{d\epsilon}{4\pi}\frac{n_F(\epsilon)-n_F(\epsilon+\omega)}{\omega}v^2(k)\mathrm{Tr}[\mathcal{A}(k,\epsilon+\omega)\sigma_2\mathcal{A}(k,\epsilon)\sigma_2] \\ &= \int_{RBZ}\frac{dk}{2\pi}\int\frac{d\epsilon}{2\pi}\frac{n_F(\epsilon)-n_F(\epsilon+\omega)}{\omega}v^2(k)\sum_{p=\pm}\sum_{p'=\pm} \\ &\quad [\eta^2 - pp'\left(\epsilon+\omega-E_{p'}(k)\right)\left(\epsilon - E_p(k)\right)]a_{p'}(k,\epsilon+\omega)a_p(k,\epsilon). \end{aligned} \tag{13}$$

The contribution to $\sigma(\omega)$ in η^2 gives the usual Drude term. The term in pp' is however negligibly small because the main contribution to the integral comes from $\epsilon = E_p(k)$. Indeed, using the representation:

$$\delta(x) = \frac{1}{\pi}\frac{\eta}{x^2+\eta^2}; \quad \eta \to 0^+$$

for the delta distribution, we write $\eta a_p(k,\epsilon) \approx \pi\delta\left(\epsilon - E_p(k)\right)$. Then, the conductivity reduces to the semi-classical expression [16]:

$$\begin{aligned} \sigma(\omega) &= \sum_{p',p=\pm}\int_{RBZ}\frac{dk}{2\pi}\int\frac{d\epsilon}{2}\frac{n_F(\epsilon)-n_F(\epsilon+\omega)}{\omega}v^2(k) \\ &\quad [\eta - pp'\left(\epsilon+\omega-E_{p'}(k)\right)\left(\epsilon-E_p(k)\right)/\eta]a_{p'}(k,\epsilon+\omega)\delta\left(\epsilon-E_p(k)\right) \\ &= \sum_{p',p=\pm}\int_{RBZ}\frac{dk}{4\pi}v^2(k)\frac{\eta}{[\omega+E_p(k)-E_{p'}(k)]^2+\eta^2}\left(-\frac{\partial n_F}{\partial\epsilon}\right)_{\epsilon=E_p(k)}. \end{aligned} \tag{14}$$

The direct conductivity (DC) is obtained by letting $\omega \to 0$: This gives, after taking into account the doubling of the unit cell due to the bipartite character of the Brillouin zone (BZ),

$$\sigma_{DC} = \frac{1}{2} \sum_{p=\pm} \int_{-\pi}^{\pi} \frac{dk}{4\pi} v^2(k) \tau\big(E_p(k)\big) \left(-\frac{\partial n_F}{\partial \epsilon} \right)_{\epsilon = E_p(k)}, \tag{15}$$

when we assume that the main contribution comes from the region of the BZ with $E_p - E_{p'} = 0$; i.e., near the Fermi energy of the JW fermions since the two bands E_+ and E_- touch at the Fermi energy when the external magnetic field is zero. Here, $\eta^{-1} \equiv \tau\big(E_p(k)\big)$ is identified with an energy-dependent relaxation time. Thus, the magnetic conductivity (14) and DC conductivity (15) have the same form as conductivities of real electrons, with the electronic group velocity and energy spectra replaced by those corresponding to the JW fermions. In principle, one could have predicted this result by using the (broken) duality symmetry of electricity and magnetism and stating that the JW transformation is a consequence of this symmetry. For the present case of magnetic currents and conductivities, the energy spectra result from the dispersion of the spin fluctuations. For this reason, it is legitimate to label the current of these magnetic dipoles as magnetic current because the spins do not move contrary to the (spintronics) experiments where two opposite currents of spin-up and spin-down electrons flow.

In the case of a constant relaxation time $\tau_{\text{mag}} = 1/\eta$, the integral in Equation (15) is simple to evaluate, and one finds:

$$\sigma_{DC} \approx \frac{\tau J_1 a}{\pi \hbar^2 bc} = \frac{l_{\text{mag}}}{\pi \hbar bc}, \tag{16}$$

where $l_{\text{mag}} = J_1 a \tau_{\text{mag}} / \hbar$ is the magnetic mean-free path.

The results of the present work tell us that all the methods and techniques developed for the electronic transport can be implemented in the case of the Heisenberg antiferromagnets once the JW transformation is used to transform the spin degrees of freedom to spinless fermions. In general, for any Heisenberg quantum antiferromagnets in higher dimensions, the 2D and 3D JW transformations [7,8] can be used, but one has to deal with the occurrence of long-range antiferromagnetic order below finite critical temperatures for the 3D systems.

6. The Magnetic Thermal Conductivity

Using the definition of the the energy current operator $\mathbf{j}^E$ by Zotos, Naef, and Prelovsek [17] for the Hamiltonian (2), one gets:

$$\mathbf{j}^E = \frac{J^2}{4} \sum_i \big(i c_{i+1}^\dagger c_{i-1} + \text{H.c.} \big) + \frac{J^2}{2} \sum_i \big(-i c_{i+1}^\dagger c_i + \text{H.c.} \big) \big(n_{i-1} + n_{i+2} - 1 \big). \tag{17}$$

In the absence of a magnetic field, $\langle n_i \rangle = 1/2$ because the magnetization $M_z = \langle n_i \rangle - 1/2 = 0$. If we replace n_i by $\langle n_i \rangle$ in Equation (17), the energy current density simplifies to:

$$\mathbf{j}^E \approx \frac{J^2}{4} \sum_i \big(i c_{i+1}^\dagger c_{i-1} + \text{H.c.} \big) + J^2 M_z \sum_i \big(-i c_{i+1}^\dagger c_i + \text{H.c.} \big) \tag{18}$$

when an external magnetic field is applied along the z-axis. The magnetic thermal current $\mathbf{j}^Q$ is obtained by subtracting $h\mathbf{j}$, where h is the chemical potential of the JW fermions, and $\mathbf{j} = \frac{J}{2} \sum_i \big(-i c_{i+1}^\dagger c_i + \text{H.c.} \big)$ is the particle current density. This gives:

$$\mathbf{j}^Q \approx \mathbf{j}^E_{\text{MF}} - h' \mathbf{j}, \tag{19}$$

where:

$$\mathbf{j}^{E}_{\rm MF} = \frac{J^2}{4}\sum_i \left(ic^{\dagger}_{i+1}c_{i-1} + {\rm H.c.}\right)$$

plays the role of the energy current at the mean-field level. Interestingly, the expression (19) for the magnetic thermal current is the same as that obtained using the mean-field Hamiltonian with the renormalized chemical potential $h' = h - 2M_z J$. Taking into account the bipartite character of the lattice and transforming into Fourier space yield:

$$\mathbf{j}^{Q} \approx \sum_k \Psi^{\dagger}_k \mathcal{Q}\Psi_k, \tag{20}$$

where the 2×2 matrix $\mathcal{Q} = \sum_{i=0}^{1} M_i\sigma_i$ with $M_0 = \frac{J^2}{2}\sin(2k)$ and $M_1 = Jh'\cos k$. In the limit of a weak magnetic field; i.e., $h \ll J$, which is realized for most real 1D Heisenberg antiferromagnets, $h' \ll J$ because $JM_z = J\chi h \ll J$; $\chi \sim 1/J$. In this case, $\mathcal{Q} \approx \frac{J^2}{2}\sin(2k)\sigma_0$. The magnetic thermal conductivity within the linear response is given by the Green–Kubo formalism:

$$\kappa_{\rm mag} = \frac{1}{k_B T^2}\left[L^{22} - \frac{(L^{12})^2}{L^{11}}\right], \tag{21}$$

with:

$$\begin{aligned} L^{22} &= \tfrac{1}{\omega\beta}{\rm Im}\int_0^{\beta}\langle T_\tau \mathbf{j}^{Q^{\dagger}}(\tau)\mathbf{j}^{Q}(0)\rangle d\tau, \\ L^{11} &= \tfrac{1}{\omega\beta}{\rm Im}\int_0^{\beta}\langle T_\tau \mathbf{j}^{\dagger}(\tau)\mathbf{j}(0)\rangle d\tau, \\ L^{12} &= \tfrac{1}{\omega\beta}{\rm Im}\int_0^{\beta}\langle T_\tau \mathbf{j}^{Q^{\dagger}}(\tau)\mathbf{j}(0)\rangle d\tau. \end{aligned} \tag{22}$$

We find that $L^{12} = h'L^{11}$; $\langle T_\tau \mathbf{j}^{Q^{\dagger}}(\tau)\mathbf{j}(0)\rangle = h'\langle T_\tau \mathbf{j}^{\dagger}(\tau)\mathbf{j}(0)\rangle$ because the cross term $\langle T_\tau \mathbf{j}^{E^{\dagger}}(\tau)\mathbf{j}(0)\rangle = 0$. We note that the main contribution to the thermal current comes from the effective hopping of the JW fermions between sites belonging in the same sublattice, which is of order J^2. In zero field where $h' = 0$ because $M_z = 0$, only L^{22} survives, giving the following contributions:

$$\begin{aligned} T\kappa_{\rm mag}(\omega) &= \int_{RBZ}\tfrac{dk}{2\pi}\int\tfrac{d\epsilon}{4\pi}\tfrac{n_F(\epsilon)-n_F(\epsilon+\omega)}{\omega}M_0^2{\rm Tr}[\mathcal{A}(k,\epsilon+\omega)\sigma_0\mathcal{A}(k,\epsilon)\sigma_0] \\ &= \int_{RBZ}\tfrac{dk}{2\pi}\int\tfrac{d\epsilon}{4\pi}\tfrac{n_F(\epsilon)-n_F(\epsilon+\omega)}{\omega}M_0^2\sum_{p=\pm}\sum_{p'=\pm} \\ &\quad \left[\eta^2 - pp'\left(\epsilon+\omega-E_{p'}(k)\right)\left(\epsilon-E_p(k)\right)\right]a_{p'}(k,\epsilon+\omega)a_p(k,\epsilon). \end{aligned} \tag{23}$$

As we did for the magnetic conductivity, we will derive a semi-classical expression for $\kappa_{\rm mag}$. Hlubek et al. [18,19] reported that $\kappa_{\rm mag}$ is only limited by extrinsic scattering processes in the low-T regime. We therefore use a constant imaginary part for self-energy; i.e., ${\rm Im}\Sigma = -\eta$, and write for the term $\eta a_p(k,\epsilon) \approx \pi\delta\left(\epsilon - E_p(k)\right)$ in the spectral function as was done for the magnetic conductivity. The result is:

$$\begin{aligned} T\kappa_{\rm mag} &= \sum_{p',p=\pm}\int_{RBZ}\tfrac{dk}{4\pi}M_0^2(k)\tfrac{\eta}{[\omega+E_p(k)-E_{p'}(k)]^2+\eta^2}\left(-\frac{\partial n_F}{\partial\epsilon}\right)_{\epsilon=E_p(k)} \\ &\approx \tfrac{1}{2}\sum_{p=\pm}\int_{-\pi}^{\pi}\tfrac{dk}{4\pi}M_0^2(k)\tau\left(E_p(k)\right)\left(-\frac{\partial n_F}{\partial\epsilon}\right)_{\epsilon=E_p(k)}, \end{aligned} \tag{24}$$

if the main contribution comes from $E_p = E_{p'}$, which means that the summation over index $p' = p$ contributes only one term. In the low-T limit with $T \ll J$, Equation (24) can be evaluated yielding $\kappa_{\rm mag}$ linear in temperature. We assume the scattering processes, represented here by the scattering rate $\tau(E_p)$, to be constant. In the absence of an external magnetic field, that is when $h' = 0$, one finds:

$$\begin{aligned} \kappa_{\rm mag} &= \tfrac{\pi}{3}\tfrac{k_B^2}{\hbar}\tfrac{J_1 a}{\hbar}\tau\tfrac{1}{bc}T \\ &= \tfrac{\pi}{3}\tfrac{k_B^2}{\hbar}l_{\rm mag}\tfrac{1}{bc}T. \end{aligned} \tag{25}$$

This result is the same as the one found using the kinetic estimate in Ref. [20], namely $\kappa_{\text{mag}} = \int \frac{dk}{2\pi} c_k v_k l_k$, where $c_k = d\epsilon_k n_k / dT$ is the specific heat (ϵ_k and n_k are the energy and the statistical occupation function of the state k), v_k the velocity and l_k the mean free path of a particle with wavevector k. In Equation (25), $l_{\text{mag}} = \frac{J_1 a}{\hbar}\tau$ is the magnetic mean-free path, which is assumed to be the same as the mean-free path for the magnetic conductivity.

A magnetic Wiedemann–Franz law can be defined as $\frac{\kappa_{\text{mag}}}{\sigma_{DC}} = LT$ with $L = \frac{\pi^2 k_B^2}{3}$. Here, L differs from that in the Wiedemann–Franz law for true electrons by the absence of the factor e^2 in the denominator; e being the electron charge. When the same self-energy is used for thermal and particle transport in the Heisenberg antiferromagnets, the Wiedemann–Franz law is satisfied, implying that both transport phenomena are due to the spin fluctuations represented here by the motion of the JW fermions.

7. Discussion and Predictions

As far as potential practical applications are concerned, we predict that a magneto-motive force (mmf) could be realized using a *magnetic battery* made of a sample of a Heisenberg antiferromagnet in the presence of a magnetic field with a gradient. Then, a magnetic current could be generated in a loop connected to this magnetic battery, and also made of the same Heisenberg antiferromagnet in the presence of a uniform magnetic field. The magnetic fields involved need not be large, and can be chosen to be much smaller than the saturation field $h_c = 2J$. The magnetic current obviously carries energy, and can be used in spintronics applications. Note that the magnetic circuits need not be coupled through interfaces to the electric circuits providing the magnetic fields. If the predictions of this work are confirmed experimentally, then we will have achieved some sort of practical realization and extension in matter of the symmetry of Maxwell equations in vacuum under the duality transformation $\mathbf{E} \to \mathbf{B}$ and $\mathbf{B} \to -\mathbf{E}$, where $\mathbf{E}$ and $\mathbf{B}$ are the electric and magnetic fields, respectively. The present proposal for generating magnetic currents could be more practical to realize than the other proposed or used methods that consist of selecting polarized spin-up or spin-down electrons. Because the Heisenberg antiferromagnets are insulators, the magnetic current is not accompanied by charge current at all. As mentioned earlier in this work, the spins in the Heisenberg chain are polarized along any nonzero applied uniform magnetic field. Using an interface with a metal, the magnetic current in a Heisenberg antiferromagnet should in principle give rise to an electromotive force by taking advantage of the inverse spin Hall effect (ISHE) [21]. In addition, by switching the magnetic current on and off in a Heisenberg antiferromagnet, the variation in the magnetic field in these antiferromagnets may in principle be used to induce an electromotive force in an ordinary circuit in a contact-less manner contrary to ISHE.

8. Conclusions

In this work, we used the Jordan–Wigner transformation to argue in favor of the applicability of low-dimensional Heisenberg antiferromagnets in the area of spintronics. We propose that this transformation not only preserves the spin commutation relations, but reflects also the duality symmetry that exists between magnetism and electricity in these materials whose charge (electrons) degrees are localized and their excitations gapped by a large energy. The dominating lowest-energy excitations are due to the electronic spins (magnetic dipoles), which are transformed into spinless fermions by the JW transformation. This transformation tells us that the spin magnetic moments turn into particles of spin zero. The spins in these antiferromagnets form spin liquid states, which are gapless for the Heisenberg chain or ladders with an odd number of legs, and gapped for ladders with an even number of legs. There is an interesting similarity between the gapless spin liquid states and the Fermi liquid states formed by electrons in conventional metals.

Given that the JW fermions behave like electrons as far as Fermi statistics is concerned, they are convenient for defining and calculating the magnetic current and magnetic thermal current for the spin-1/2 Heisenberg antiferromagnets. The magnetic conductivity and magnetic thermal conductivity, calculated in the present work for the Heisenberg chain within the bond–mean-field theory, are found to agree with existing results calculated using other methods. Finally, the central prediction made here is that of generating a magneto-motive force using a Heisenberg chain-like material in the presence of a magnetic field with a gradient. We believe that we succeeded to establish a theoretical framework for what the experiments in spintronics attempt to do, namely treating the magnetic degrees of freedom on the same footing as the electronic ones.

Future work will deal with the Heisenberg ladders given that several materials of this sort exist in reality, and may be of great importance for spintronics.

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