

Article

Grinding Behaviors of Components in Heterogeneous Breakage of Coals of Different Ash Contents in a Ball-and-Race Mill

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Abstract: Coals used for power plants normally have different ash contents, and the breakage of coals by the ball-and-race mill or roller mill is an energy-intensive process. Grinding phenomena in mill of power plants is complex, and it is also not the same with ideal grinding tests in labs. The interaction among various coals would result in changes of grinding behaviors and energy consumption characterization if compared with those of single breakage. In this study, anthracite and bituminous coal of different ash contents were selected to be heterogeneously ground. Quantitation of components in products was realized using the relation between sulfur content of the mixture and mass yield of one component in the mixture. Product fineness t_{10} of the component was determined, and split energy was calculated on the premise of specific energy balance and energy-size reduction model by a genetic algorithm. Experimental results indicate that breakage rate and product fineness t_{10} of the mixture decrease with the increase of hard anthracite content in the mixture. Unlike the single breakage, t_{10} of anthracite in heterogeneous grinding is improved dramatically, and bituminous coal shows the opposite trend. The interaction between components results in the decrease of the specific energy of the mixture if compared with the mass average one of components in single breakage. Breakage resistance of hard anthracite decreases due to the addition of soft bituminous coal, and grinding energy efficiency of anthracite is also improved compared with that of single grinding.

Keywords: grinding behaviors; energy consumption characterization; sulfur content; heterogeneous breakage; split energy

1. Introduction

Particle size reduction is widely involved in various industries, especially in mineral processing. In a common comminution process for mineral liberation, size reduction of the raw ore sample is generally realized by the use of a crusher, ball mill and Isa mill in sequence, making it an energy-intensive process. Due to both the decline in ore grades and the increased complexity of the ore characteristic, it is expected that the energy demanded for achieving an ideal mineral liberation will be greatly

increased [1–3]. Statistical data indicate that the grinding process consumes about 70% of the total energy in mineral preparation plants [4]. Hence, any improvement in the grinding process would lead to a significant reduction of energy required for mineral beneficiation. In China, nearly 60% of the raw coal is applied for electric power generation, and it should be ground to pulverized fuel (PF) for improving combustion efficiency [5]. Generally, ball-and-race mills or roller mills are used to produce the PF, of which >85% of fines are finer than 90 μm PF [6]. It is estimated that the spent energy in this process accounts for 0.5–2% of total electrical power of the coal power plant [7]. Though this proportion is not big, the total energy loss from coal power plants in China is tremendous given the total electric power generation capacity of China. Unfortunately, this issue has not been paid enough attention by electricity producers in China.

Normally, the heat efficiency of boilers used in power plants is closely related to the properties of coals that are burnt inside, therefore, the boiler in a power plant is designed according to the quality of coal. Note that in some parts of the world, supplied raw coals are from various mines, and they show obvious differences in quality. Under this situation, coal blending prior to combustion in a boiler becomes very necessary to ensure an optimum heat efficiency. This, in turn, brings up challenges on how to maintain a high efficiency for the grinding process in the power plants as coals of different sizes, densities, coalification degrees, and ash contents are ground in ball-and-race mills, resulting in particles fed onto the grinding table being ground heterogeneously. Interaction among different components would have an effect on the grinding behavior and energy consumption characterization of mixtures and components. Many investigations have been conducted to study the above-discussed issue faced by coal power plants; however, previous work was carried out on samples in narrow size or density and thereby the findings based on them are not applicable to real industrial process [8,9]. On the other hand, both the structure and grinding mechanism of a vertical roller mill employed in coal power plants are different from those of a conventional lab-scale mill. Regarding the first abovementioned issue facing lab-scale grinding research, Hardgrove mill or lab-scale roller mill were applied to simulate the grinding process of particles in industrial vertical spindle mill [10,11]. Related experiments were first conducted by Austin in 1981, in which a modified Hardgrove mill with a torque meter was used [12]. Based on the extensive grinding results, a model including particle breakage, internal and external classification of ground products was successfully applied to the lab-scale and industrial E mills, respectively [13,14]. Shi used a similar machine (JK Fine-particle Breakage Characteriser (JKFBC)) and applied the classical breakage model (developed from Drop-Weight Tests) to describe the energy-size reduction process [15]. Later on, particle properties were modelled into the classical breakage equation based on grinding tests of coal in JKFBC, which further extended the application scope of this model [16,17]. It appears that grinding in a lab-scale Hardgrove mill can simulate the grinding process of coal in a vertical spindle mill. It is noted that the materials used in the above studies were samples in narrow size or density. For the second issue, mixture breakage was initially conducted in the ball mill [18], and samples were pure minerals for the easy separation of progenies by float-sink test or chemical reaction [19,20]. All of these are conducive to analyze the breakage behavior of the component in the mixture; however, the breakage phenomenon was too ideal to draw some substantial conclusions. The key issue for the heterogeneous grinding of coals is the quantitation of components in the mixture. A great amount of research has been done in regard to overcoming this problem. Cho studied the grinding kinetics of the components in a binary mixture of 1.6 $\text{g}\cdot\text{cm}^{-3}$ sink anthracite and 1.4 $\text{g}\cdot\text{cm}^{-3}$ float bituminous coal in a ball-and-race mill, and two coals in ground products were separated by the float-sink tests [21]. Austin conducted the mixture breakage of anthracite with quartz, cement clinker and another two coals in a small laboratory ball mill under standard conditions, and found the acceleration of grinding rate [22]. Xie also ground anthracite with pure minerals in a ball-and-race mill and compared the changes of grinding behavior of components [23]. Float-sink test is a useful tool for the separation of coal from a mixture that has been subjected to grinding. While for different coals, size-reduction also leads to the liberation of associated minerals, and density distribution of products becomes wide, which may result in the density coincidence of mixture products. Unlike the pure minerals, coal is a

complex material and contains both organic and inorganic substances. As such, which part of coal can be used for quantitation should be discussed. Since the species of minerals in coal are numerous, different associated conditions and selective liberation can result in the unpredictable distribution of minerals in products. Though almost all the inorganic elements in nature can be found in coal, they are usually concentrated in parts of products due to the selective liberation of minerals. Moreover, experimental errors caused by tedious sample preparation processes have negative effects on the accurate quantitation of some rare elements in coal. Hence, specific inorganic elements may also not be possible for quantitation. Based on the above discussion, properties of organic substance or organic elements are potential to distinguish different coals. Xie and his colleagues applied the characteristic ratio of XRD pattern to quantify components in progenies, and therefore, confirmed the grinding behavior of components. It is worth noting that this method can only be used for mixtures of coals with various coalification degrees and also requires the ash content of sample to be sufficiently low in order to avoid the negative effect on the analysis of 002 peak of XRD pattern [24]. For coals in the same coalification degree or higher ash content, another organic element should be selected.

In addition to the breakage behavior of the component in the mixture, energy consumption characterizations of the mixture and component are also important output for heterogeneous grinding. Energy consumed by the component can be calculated by energy split factor. Kapur and his colleagues provided energy split factor in terms of breakage rate functions and production rate of fines. This method was based on the assumption that breakage behavior of the component was environment-independent due to the similar grinding path on a triaxial composition diagram [25]. Xie and his colleagues calculated the energy split factor according to the mass and energy balance for the two-component breakage of coal with one mineral [23]. Combined with the product fineness of the component, energy-size reduction relation was established for the comparison of energy efficiency (product t_{10} for the same energy) between the mixture and single breakage [26].

Coals used in a previous study about mixture breakage by authors were of low ash content (2.62% and 3.17%), which was lower than that of the coals used in power plants. Hence, another two coals (2.96% and 35.27%) were chosen. A series of mixture grinding tests were conducted in a ball-and-race mill, and breakage rate, product fineness, and specific energy were determined. As the sulfur content of the mixture was linearly related to the mass yield of one component of the mixture, product fineness of each component after mixture breakage was quantified based on the above relation. On the premise of the classical breakage model and specific energy balance of the mixture, specific energy of the component was computed by genetic algorithm. Split energies and breakage parameters in the energy-size reduction model of components for various mixed conditions and grinding time were determined to indicate the interaction effect on the grinding energy efficiency of the component.

2. Materials, Equipment and Method

2.1. Materials, Equipment and Grinding Tests

Two coals including the anthracite and bituminous coal were used in this study. Anthracite was sampled from the clean coal stream of the Taixi coal preparation plant, while the bituminous coal was sampled from the middling coal stream of the Linhuan coal preparation plant. These samples were subjected to crushing and sieving tests, and particles of $-2.8 + 2$ mm were selected for grinding tests. Ash contents of anthracite and bituminous coal were 2.96% and 35.27%, respectively.

Energy-size reduction tests were conducted in a ball-and-race mill, namely Hardgrove machine, with the addition of a power meter, to investigate changes of grinding behavior and energy consumption characteristics of components during the heterogeneous breakage process. The structure parameters are as follows: diameter of grinding table 76.2 mm, diameter of grinding ball 25.4 mm, the number of grinding balls 8, table revolution rate 20 rpm, grinding force 284 N, and rated power 90 W. The power meter was connected in the electric circuit of Hardgrove mill. Resolution of the power meter was 0.01 W, and instantaneous power was recorded at the frequency of 1 s. Forty gram samples were

prepared for each grinding test, and eight balls were put on the particle bed evenly. The non-load power was first recorded, and power consumption for the breakage of single coal and mixture was measured later. Power for grinding samples was determined by subtracting the non-load power from the gross one.

Grinding tests of single anthracite and bituminous coal were first conducted, and grinding time was designed as 10 s, 20 s, 30 s, 40 s, 50 s, 70 s, 90 s, 120 s, 150 s, 180 s, and 240 s. For each experiment, size distribution of ground products was confirmed by sieving tests, and yield of unbroken particles in the top size, and product fineness t_{10} (yield of progenies which were smaller than 1/10th of the mean size of feed) were determined. Mixture breakage tests were carried out on mixtures of anthracite (A) and bituminous coal (B) at mass ratios of 3:1, 1:1 and 1:3. Grinding time and treatment on ground products of heterogeneous breakage tests were identical to those of homologous breakage tests.

2.2. Quantification of Components in Heterogeneous Grinding Products

For the quantification of components in heterogeneous grinding products, the relation between some characteristic indexes with the mass yield of a certain component should be determined. Firstly, mixtures of anthracite and bituminous coal were prepared with mass ratios of 1:0, 4:1, 3:1, 2:1, 1:1, 1:2, 1:3, 1:4, and 0:1. Then, these materials were ground to $-74 \mu\text{m}$ fines by a vibrating mill respectively, and element composition of ground products was analyzed by the X-Ray Fluorite Spectroscopy (XRF). In the XRF test, about 20 elements were detected, most of which were inorganic elements. Note that the distribution of inorganic elements in coal samples was not even, and minerals associated with coal were of different sizes. Hence, the selective liberation would happen along with grinding, which would result in the unpredictable distribution of elements in various narrowly-sized progenies. So, the X-Ray Diffraction of two coals were then conducted to assist the selection of a proper characteristic element. XRD patterns of two coals are shown in Figure 1. As shown, mineral information of anthracite in the XRD pattern was weak because the ash content was only 2.96%, and the main associated mineral was kaolinite. The main minerals associated in the bituminous coal were kaolinite, montmorillonite and illite. Evidently, element Si is the common inorganic element of these two coals, but it cannot be regarded as the indicator to distinguish the grinding behaviors of components in the mixture due to the selective liberation. Meanwhile, Figure 1 reveals that there is no pyrite detected by XRD. Therefore, it is easy to conclude that sulfur quantified by XRF exists in the organic macromolecular structure of coal in an organic form. Since the breakage process of organic components was non-selective, the uniform distribution of sulfur can be anticipated in each narrowly-sized particles. Hence, it was reasonable to choose the organic element of sulfur for quantification.

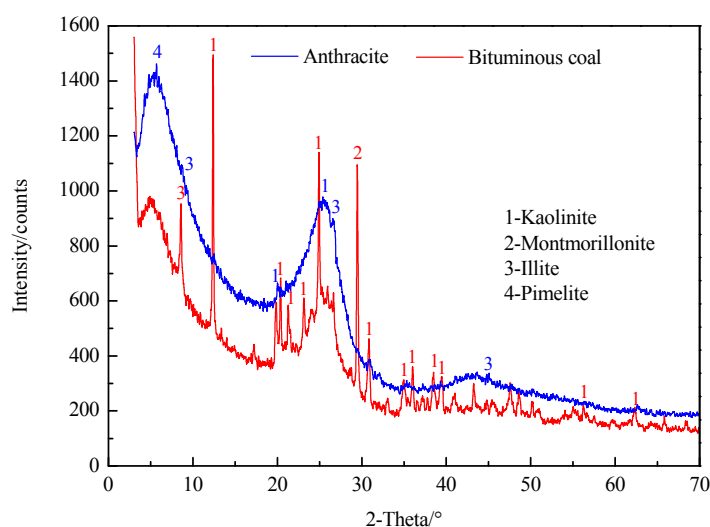


Figure 1. XRD pattern of anthracite and bituminous coal.

Figure 2 shows the relation between sulfur contents of mixtures and mass yields of bituminous coal. A linear equation was developed, with the R^2 of 0.98. Note that the characteristic size of t_{10} for $-2.8 + 2$ mm coals is 0.237 mm ($((2.8 \times 2)^{0.5})/10$). As it is difficult to accurately obtain these fines by sieving, progenies of -0.25 mm were used. For heterogeneous grinding tests of mixtures in various mass ratios of components and grinding time, -0.25 mm products were reground by a vibrating mill to $-74 \mu\text{m}$ fines for the XRF measurement. With the help of the above empirical linear equation, product fineness t_{10} of each component in various heterogeneous grinding conditions was determined.

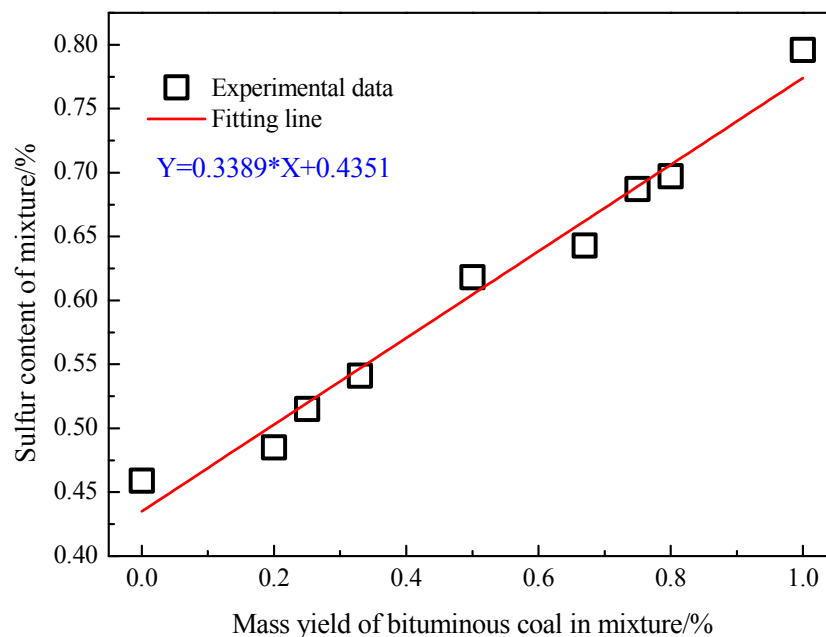


Figure 2. Relation between sulfur content of mixture and mass yield of bituminous coal.

3. Results and Discussion

3.1. Grinding Behavior of Components and Mixtures

Breakage of coarse particles to fines is a progressive process, which can be evaluated by two indicators, namely breakage rate of the top sized particles (BRTSP) and product fineness t_{10} . BRTSPs of mixtures at various mixed ratios (1:0, 3:1, 1:1, 3:1, and 0:1) are compared. Although the longest grinding time of this research is 240 s, it is shown from the size analyses of progenies that particles in the top size are already ground to fines before 90 s. Hence, yields of the top size particles at grinding time periods of 0 s, 10 s, 20 s, 30 s, 40 s, 50 s, and 70 s are plotted in Figure 3. As shown, for these semi-logarithmic curves, BRTSP shows the linear relation with grinding time, namely the first-order law. For the single breakage, yield of unbroken bituminous coal in the top size is a little lower if compared with that of anthracite for each grinding time. That is to say, bituminous coal has a fast breakage rate, and it is relatively easy to break. This is due to the fact that the ash content of the bituminous coal is much higher than that of the anthracite, so that liberated minerals accelerated the size-reduction of bituminous coal. Figure 3 also shows that with the increase in weight percentage of bituminous coal in the mixture, the yield of unbroken particles in the top size decreases, while the breakage rate of the mixture increases.

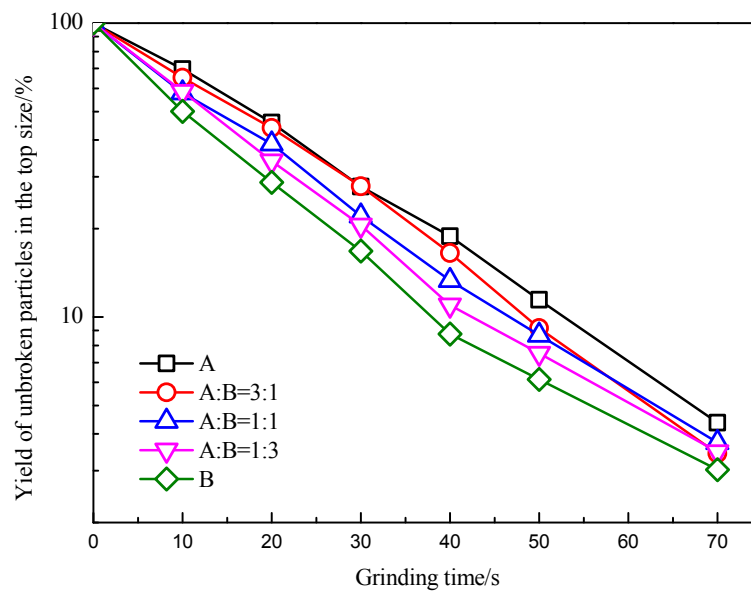


Figure 3. Yield of unbroken particles in the top size for various grinding times.

The results of product fineness t_{10} of mixtures with mixed ratios and grinding time are presented in Figure 4. Similar to the change trending of breakage rate, bituminous coal with a fast BRTSP shows a higher product fineness, and the difference in t_{10} between bituminous coal and anthracite is over 15%. Anthracite has a higher coalification degree, and it is difficult to be broken to fines, especially for those associated with small amounts of minerals. Predictably, t_{10} of mixtures increases with the small yield of anthracite in the mixture. In addition to the direct comparison of breakage rate and product fineness, t_{10} of components in heterogeneous grinding are also determined and compared. As mentioned above, -0.25 mm progenies yielded at various mixture breakage conditions are ground to fines for XRF measurement. Sulfur contents of these samples are shown in Table 1. Obviously, sulfur content increases with grinding time for these three mixtures, which indicates more anthracite fines in products. Based on data in Table 1 and the empirical equation in Figure 2, contents of two components in t_{10} of mixtures are determined. Aiming at the convenient comparison of t_{10} of components in heterogeneous and homogeneous grinding, values of product fineness t_{10} are calculated in terms of the percentage benchmark of the component itself according to data in Figure 4 and the mixed ratios. These data are shown in Figures 5 and 6 for anthracite to bituminous coal, respectively. Compared with the single breakage of anthracite, product fineness t_{10} is improved significantly, and a large value of t_{10} indicates the amount of anthracite in the mixture is small. If the content of anthracite decreases to 25%, t_{10} is more than twice that of homogeneous grinding. On the contrary, t_{10} of bituminous coal in mixture breakage is much lower than that in single breakage. For the hard anthracite, soft bituminous coal in the mixture not only improves grinding phenomena, but also benefits the transition of grinding energy to anthracite. That is why the product fineness of anthracite increases if compared with that of single breakage. In order to explain the interaction between two coals in mixture breakage, energy consumed characterization will be introduced in the next section.

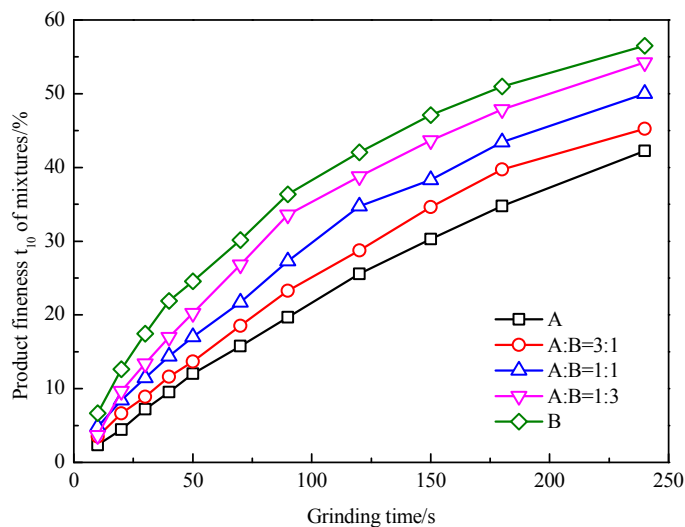


Figure 4. Product fineness t_{10} of mixtures for various grinding times.

Table 1. Sulfur content of -0.25 mm products yielded at various grinding times.

Grinding Time/s	Content of Sulfur/%		
	A:B = 3:1	A:B = 1:1	A:B = 1:3
10	0.507	0.648	0.687
20	0.502	0.618	0.649
30	0.503	0.597	0.650
40	0.502	0.573	0.623
50	0.503	0.565	0.632
70	0.504	0.561	0.639
90	0.501	0.564	0.637
120	0.502	0.561	0.638
150	0.502	0.563	0.638
180	0.501	0.563	0.635
240	0.501	0.559	0.635

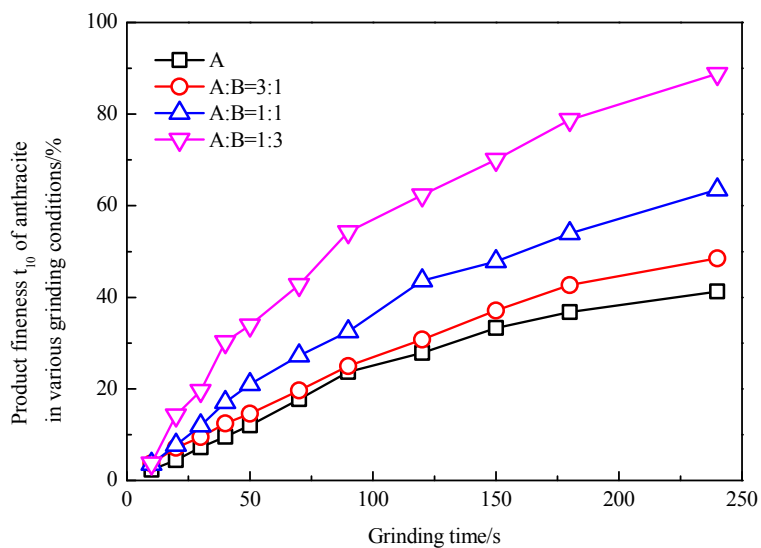


Figure 5. Product fineness t_{10} of anthracite for various grinding conditions and times.

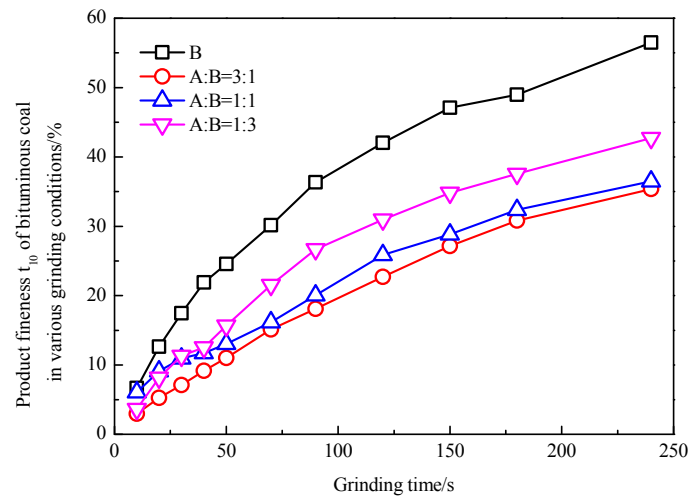


Figure 6. Product fineness t_{10} of bituminous coal for various grinding conditions and times.

3.2. Energy Consumed Characterizations of Mixtures and Components

The grinding process of mixtures should be evaluated not only by breakage rate and product fineness, but also by consumed specific energy. Here, recorded power (W) per second for the heterogeneous grinding of mixtures was converted to specific energy ($\text{kW}\cdot\text{h}\cdot\text{t}^{-1}$). Specific energy and product fineness t_{10} of each mixture breakage for various times are plotted in Figure 7. For the single breakage, bituminous coal shows a higher t_{10} with the same specific energy in comparison with that of anthracite. That is to say, the soft bituminous coal has a higher grinding energy efficiency. Predictably, fineness t_{10} increases with more bituminous coal in the mixture for the same energy input, just as shown in Figure 7. Previous breakage researches of narrowly-sized particles in the ball-and-race mill have illustrated the successful application of classical energy-size reduction model on experimental data [27], and indexes of particle properties are added to that model to improve the utilization [13]. This model is shown as follows:

$$t_{10} = A \times (1 - e^{-b \times E_{cs}}) \tag{1}$$

where t_{10} is the product fineness (%), E_{cs} is the specific energy ($\text{kW}\cdot\text{h}\cdot\text{t}^{-1}$), and A and b are breakage parameters.

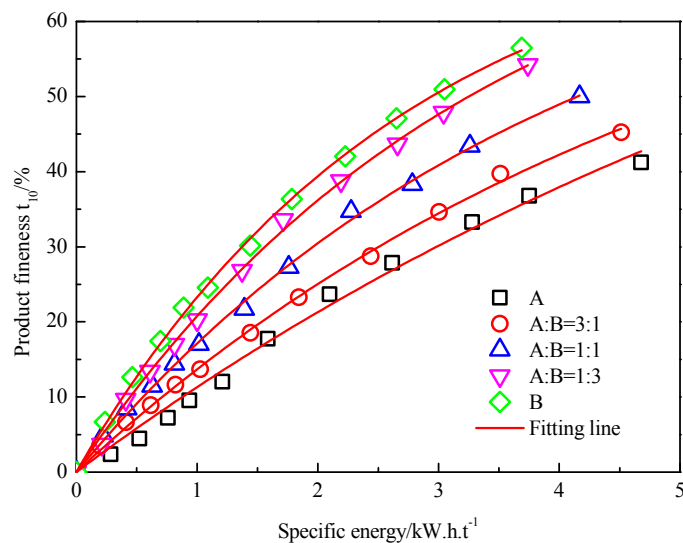


Figure 7. Specific energy vs product fineness t_{10} of mixtures.

In this paper, the classical model is also used to describe the heterogeneous grinding of two coals. Good fitted results, with R^2 above 0.98, are obtained for grinding tests of mixtures in three mixed ratios. Breakage parameters A and b are also determined for each mixture and shown in Table 2. The higher $A*b$ value demonstrates the less resistance to being broken. Hence, bituminous coal, which has the highest $A*b$ value, is easily broken. That is consistent with conclusions of breakage rate and product fineness. With the decrease of bituminous coal in the mixture, the indicator $A*b$ decreases, and more energy would be consumed for yielding fines.

Table 2. Breakage parameters of mixtures.

Breakage Parameters	Mixture Conditions				
	A	A:B = 3:1	A:B = 1:1	A:B = 1:3	B
A	97.63	80.24	77.44	80.77	75.68
b	0.12	0.19	0.25	0.30	0.37
$A*b$	12.01	15.00	19.36	23.99	27.82

Product fineness t_{10} of component has been determined based on the characteristic index of sulfur content as mentioned above. However, the specific energy for yielding fines for each component at various grinding conditions and time is not clear. Recorded power and calculated specific energy are for mixture. Interactions between components can not only change the specific energy consumed by the component, but also have an effect on the specific energy of the mixture if compared with the mass average one. Table 3 lists both measured and calculated mass average specific energies of mixtures at various times. If component interaction does not happen in the mixture breakage, the specific energy should be close to the mass average one. However, data in Table 3 indicate that measured energy is a little lower than the mass average one. So, the mixture breakage is a potential method for energy saving. This difference illustrates that the component interaction may increase the grinding energy efficiency of one component and decrease that of another one. However, this speculation should be further verified by the specific energy split by each component.

Table 3. Comparison of measured and mass average specific energy.

Specific Energy/kW.h.t ⁻¹ Grinding Time/s	Measured Data					Mass Average Data		
	A	A:B = 3:1	A:B = 1:1	A:B = 1:3	B	A:B = 3:1	A:B = 1:1	A:B = 1:3
10	0.28	0.20	0.22	0.21	0.24	0.27	0.26	0.25
20	0.52	0.41	0.42	0.41	0.46	0.51	0.49	0.48
30	0.76	0.62	0.63	0.61	0.70	0.74	0.73	0.71
40	0.94	0.82	0.81	0.81	0.89	0.92	0.91	0.90
50	1.21	1.03	1.01	1.00	1.09	1.18	1.15	1.12
70	1.59	1.44	1.39	1.37	1.44	1.55	1.51	1.48
90	2.10	1.84	1.76	1.71	1.78	2.02	1.94	1.86
120	2.61	2.44	2.28	2.19	2.23	2.52	2.42	2.32
150	3.28	3.00	2.78	2.66	2.65	3.12	2.96	2.81
180	3.75	3.51	3.26	3.04	3.05	3.58	3.40	3.22
240	4.68	4.51	4.17	3.74	3.69	4.43	4.18	3.94

Authors have provided a method to determine the energy split factor of the component for the mixture breakage of coal with pure mineral in the Hardgrove mill [20,21]. In that study, interaction between components was reflected on the change of breakage parameters of mixtures with different mixed ratios. Though the energy balance was realized, it was not correct to calculate the specific energy of the component by the classical breakage model of the mixture. As a result, the energy-size reduction model with parameters for the mixture could also describe the breakage process of two components. In other words, coal, pure mineral and mixture showed the same resistance to being broken, which was not accurate enough. In this case, a new method was put forward to determine

the specific energy of the component. First, the energy (kW.h) balance equation is divided by the mass of the mixture and converted to the specific energy (kW.h.t⁻¹) balance equation [26]. Second, it is assumed that the relation of the specific energy and product fineness of the component in the mixture breakage still follows the classical breakage model. This assumption is the connection of the known product t_{10} to the unknown specific energy of the component. Third, specific energies of components for various grinding conditions and time are calculated according to the specific energy balance of the mixture. Calculation of the third process is conducted by genetic algorithm (GA). Here, the GA toolbox in Matlab is used, and compiled programs are shown as the Appendix A in this paper. The initial population number is set as 50. Boundary conditions of parameters A and b in Equation (1) for components are $[x_{\max}, 100]$ and $[0,1]$, respectively. In addition, the target error is set as 0.1%. If the population mean error is smaller than the target one, the optimum parameters A and b for each component are obtained. Then, specific energy of each grinding condition is calculated by Equation (1), with the results being listed in Table 4. The difference between the calculated and measured specific energy of the mixture is marginal except for the small specific energy of the short grinding time. These data would be used for the energy-size reduction model of the component. Note that the values of t_{10} for components in mixture breakage are in the percentage benchmark of the component itself, namely data in Figures 5 and 6. In addition, Figures 8 and 9 are the relation between t_{10} and specific energy of anthracite and bituminous coal, respectively. Energy efficiency of grinding anthracite increases with more bituminous coal being added in the mixture, and that of bituminous coal shows the opposite trend. These conclusions are similar with the changing law of product fineness t_{10} . Breakage parameters of components are also determined by Equation (1), as shown in Table 5. Breakage resistance of anthracite decreases, which indicates the soft bituminous coal improves the grinding phenomenon of anthracite in comparison with that in single grinding. While for bituminous coal, it acts as grinding media and therefore inhibits the breakage process.

Energy (kW.h) consumed by each component is calculated based on the specific energy (kW.h.t⁻¹) and mass of the component in the mixture. Figure 10 shows the content of energy consumed by anthracite as a function of grinding time. Compared with the mass content of anthracite in the mixture, a higher ratio of energy is split by it for mixtures of 1:1 and 1:3, which benefits the generation of fines. When the mass ratio in the mixture is 3:1, anthracite obtains less energy, however, the ratio is still above 70%. Particles used in this research are in the size of $-2.8 +2$ mm, which is relatively big. Hence, the population of bituminous coal particle in tests conducted at a large mass ratio of anthracite to bituminous coal is small. For the mixture of 3:1, not all the anthracite particles can be surrounded by the soft bituminous coal. This situation may affect the energy split of the component in the mixture.

Table 4. Specific energies of components at various grinding conditions and times.

Specific Energy/kW.h.t ⁻¹	Mixture Conditions											
	A:B = 3:1				A:B = 1:1				A:B = 1:3			
	Grinding Time/s	A	B	Calculated Mixture	Measured Mixture	A	B	Calculated Mixture	Measured Mixture	A	B	Calculated Mixture
10	0.22	0.31	0.24	0.20	0.20	0.29	0.25	0.22	0.10	0.20	0.17	0.21
20	0.43	0.55	0.46	0.41	0.44	0.44	0.44	0.42	0.40	0.46	0.45	0.41
30	0.58	0.75	0.62	0.62	0.69	0.63	0.66	0.63	0.57	0.65	0.63	0.61
40	0.77	0.99	0.82	0.82	0.94	0.71	0.82	0.81	0.94	0.73	0.78	0.81
50	0.92	1.20	0.99	1.03	1.16	0.86	1.01	1.01	1.08	0.93	0.97	1.00
70	1.29	1.71	1.39	1.44	1.56	1.14	1.35	1.39	1.46	1.34	1.37	1.37
90	1.71	2.09	1.81	1.84	1.96	1.41	1.68	1.76	2.05	1.74	1.81	1.71
120	2.22	2.74	2.35	2.44	2.85	2.00	2.42	2.28	2.55	2.09	2.20	2.19
150	2.86	3.42	3.00	3.00	3.16	2.37	2.76	2.78	3.15	2.44	2.62	2.66
180	3.50	4.03	3.63	3.51	3.95	2.73	3.34	3.26	4.05	2.70	3.04	3.04
240	4.30	4.87	4.44	4.51	5.25	3.11	4.18	4.17	5.73	3.25	3.87	3.74

Table 5. Breakage parameters of components.

Breakage Parameters	Anthracite				Bituminous Coal			
	Single	A:B = 3:1	A:B = 1:1	A:B = 1:3	Single	A:B = 3:1	A:B = 1:1	A:B = 1:3
A	97.6257	78.721	94.2295	99.958	75.684	73.043	56.1382	81.387
b	0.12305	0.223	0.21653	0.383	0.3676	0.136	0.3024	0.229
A*b	12.01	17.55	20.40	38.28	27.82	9.93	16.98	18.64

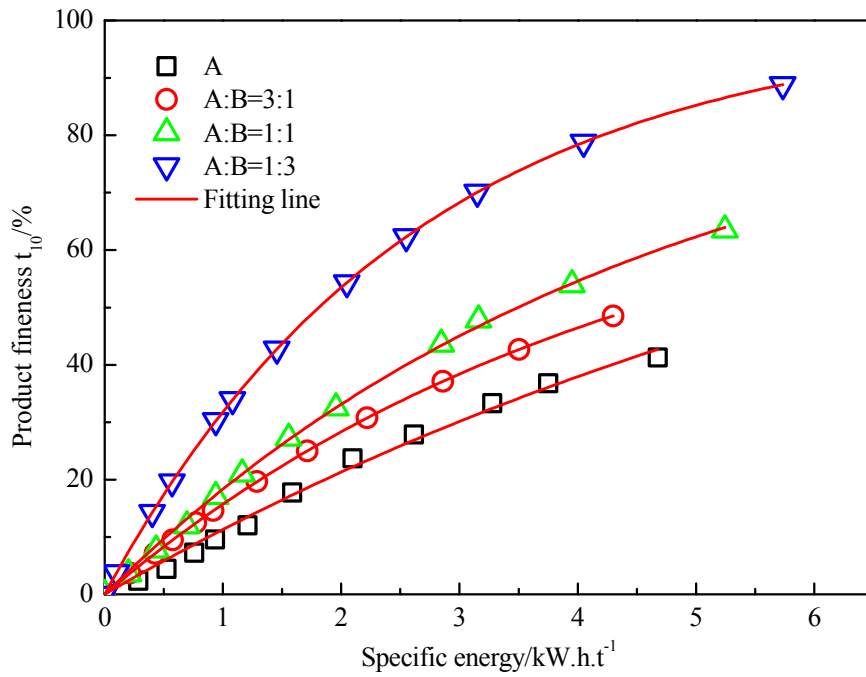


Figure 8. Specific energy vs product fineness t_{10} of anthracite in mixture breakage.

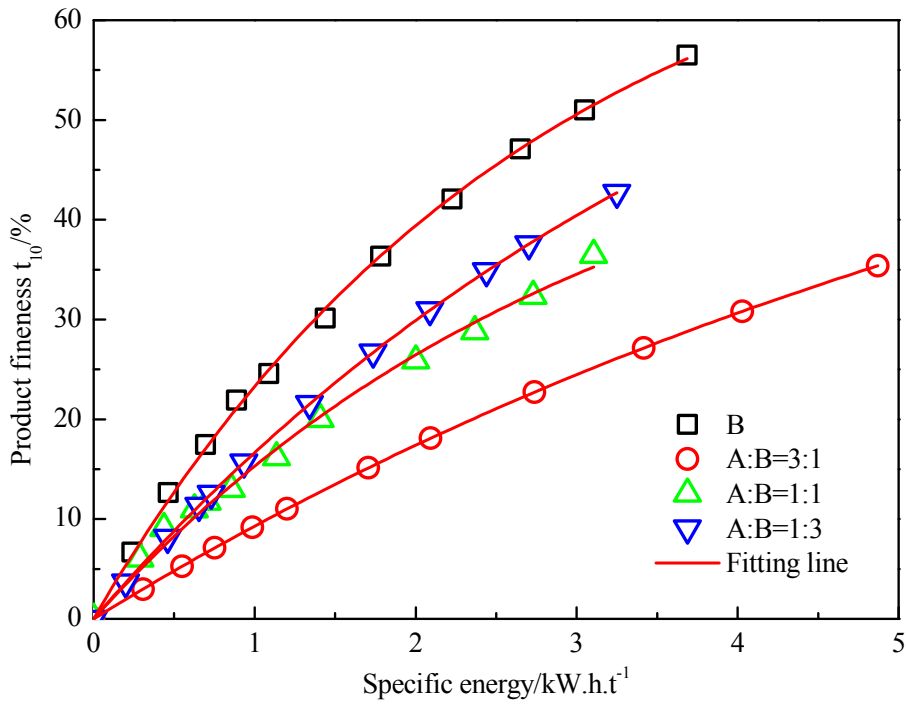


Figure 9. Specific energy vs product fineness t_{10} of bituminous coal in mixture breakage.

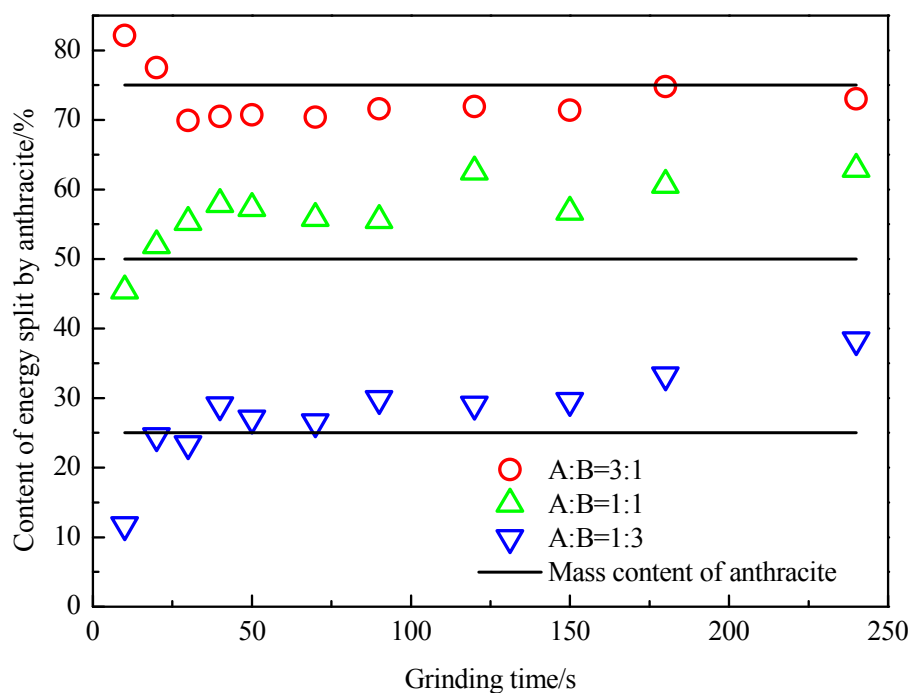


Figure 10. Content of energy split by anthracite in various grinding conditions and times.

4. Conclusions

This paper reports grinding behaviors of components in heterogeneous breakage of two coals of different ash contents in a ball-and-race mill. Quantitation of two coals is conducted by the relation of sulfur content of mixture and mass yield of bituminous coal. Values of product fineness t_{10} of components at various grinding conditions and time periods are determined, and specific energy and energy split by component are calculated by a genetic algorithm on the premise of energy balance of mixture, classical energy-size reduction model and t_{10} of components. The main conclusions of this paper are as follows:

- (1) XRD results show that sulfur in two coals is in organic form, and its content is quantified by XRF. Sulfur contents of mixtures show a linear relation with the mass yield of bituminous coal, which contributes to determining the product fineness t_{10} of two coals at various grinding conditions and time periods.
- (2) The breakage rate of the mixture obeys the first-order law. In the mixture breakage, the breakage rate increases with increasing the mass ratio of bituminous coal of the mixture. Compared with the single breakage of anthracite, t_{10} is improved significantly after mixing with the soft bituminous coal. If the content of anthracite decreases to 25%, t_{10} is more than twice that of homogeneous grinding, but t_{10} of bituminous coal is reduced.
- (3) The classical energy-size reduction model can be applied for the mixture breakage of coals in the Hardgrove mill. Breakage indicator A^*b of the mixture increases with adding more soft bituminous coal. Specific energy of the mixture is a little lower than the mass average one of components due to the component interaction in mixture breakage. The relation between t_{10} and the specific energy of the component indicates that energy efficiency of anthracite grinding increases during the heterogeneous grinding. Added bituminous coal surrounds anthracite particles and improves the grinding phenomenon if compared with the single breakage. Content of energy split by anthracite is bigger than the mass yield of it in the mixture, which indicates the easy transfer of energy to anthracite.

Author Contributions: Q.L., X.S. and W.X. designed experiments of this study, J.D., Z.Z., X.W., Y.Z. and J.W. conducted the heterogeneous and homogeneous grinding tests, and also analyzed size distribution of ground products, Q.L. and X.Z. conducted XRF measurement and analyzed data, W.X. and B.L. contributed to the writing the paper. All authors have read and agreed to the published version of the manuscript.

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Conflicts of Interest: The authors declare no conflict of interest.

Appendix A

```
function z = hanshu(x)
syms A1 b1 A2 b2 y1 y2 x1 x2 r c;
syms x1_1 x1_2 x1_3 x1_4 x1_5 x1_6 x1_7 x1_8 x1_9 x1_10 x1_11;
syms x2_1 x2_2 x2_3 x2_4 x2_5 x2_6 x2_7 x2_8 x2_9 x2_10 x2_11;
A1 = x(1)
b1 = x(2)
A2 = x(3)
b2 = x(4)
y1 = [3.76 14.27 19.61 30.25 33.92 42.75 54.32 62.31 70.06 78.77 88.84]; 't10 of anthracite
y2 = [3.63 8.14 11.33 12.55 15.68 21.52 26.69 30.95 34.84 37.56 42.72]; 't10 of bituminous coal
x1 = -log(1 - y1./A1)/b1; 'specific energy of anthracite
x2 = -log(1 - y2./A2)/b2; 'specific energy of bituminous coal
r = 0.25. × x1 + 0.75. × x2;
c = [0.21 0.41 0.61 0.81 1.00 1.37 1.71 2.19 2.66 3.04 3.74];
z = abs(abs(c(1,1) - r(1,1))./max(c(1,1),r(1,1)) + abs(c(1,2) - r(1,2))./max(c(1,2),r(1,2)) +
abs(c(1,3) - r(1,3))./max(c(1,3),r(1,3)) + abs(c(1,4) - r(1,4))./max(c(1,4),r(1,4)) + abs(c(1,5) -
r(1,5))./max(c(1,5),r(1,5)) + abs(c(1,6) - r(1,6))./max(c(1,6),r(1,6)) + abs(c(1,7) - r(1,7))./max(c(1,7),r(1,7))
+ abs(c(1,8) - r(1,8))./max(c(1,8),r(1,8)) + abs(c(1,9) - r(1,9))./max(c(1,9),r(1,9)) + abs(c(1,10) -
r(1,10))./max(c(1,10),r(1,10)))
```

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